R20

UNIT — III

ANTENNA ARRAYS

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UNIT — III

Antenna Arrays.

Introduction:

→ The field radiated by a small linear antenna is not distributed uniformly in the direction perpendicular to the axis of the antenna. In case of a short dipole, the maximum radiation takes place in the direction right angles to the axis of the dipole. But it decreases to minimum when the polar angle decreases.

→ Hence to increase the field strength in the desired direction by using group of antennas excited simultaneously. Such a group of antennas is called array of antennas or simply antenna array.

→ Antenna array is, the radiating system in which several antennas are spaced properly so as to get greater field strength at a far distance from the radiating system by combining radiations at point from all the antennas.

→ The individual element is generally called element of an array.

→ The antenna array is said to be linear if the elements of the antenna array are equally spaced along a straight line. The linear array is said to be uniform linear array if all the elements are fed with a current of equal magnitude with progressive uniform phase shift along the line.

Various forms of Antenna Arrays: (2 element)

1. Broad side Array
2. End fire Array
3. Collinear Array.
4. Parasitic Array.

1. Broad Side Array:

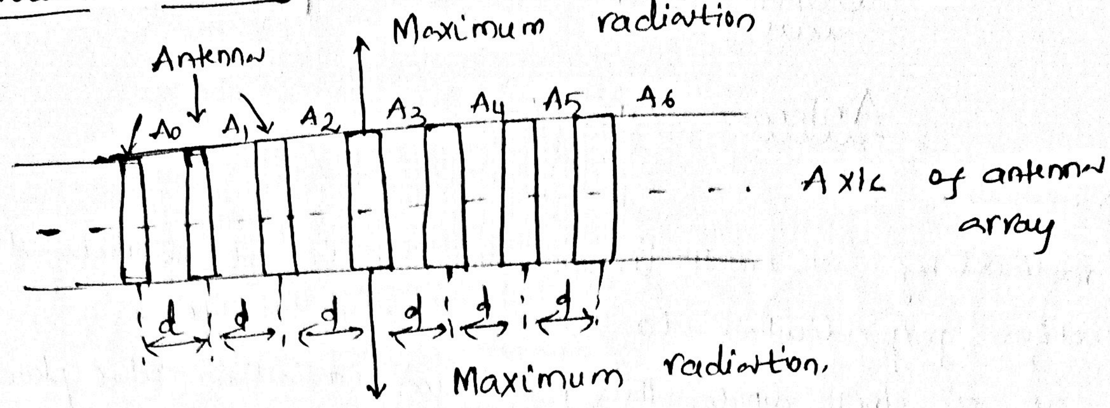


Fig ; Broadside array of radiation.

→ The broad side array is the array of antennas in which all the elements are placed parallel to each other and the direction of maximum radiation is always perpendicular to the plane consisting elements., shown in above fig.

→ The straight line perpendicular to the axis of individual antenna is known as axis of antenna array.

each element is ⊥r to the axis of antenna array.

→ The spacing between two elements is denoted by 'd'.

→ All the elements fed with equal magnitude and phase of currents.

→ So broad side array is the arrangement of antennas in which max radiation is in the direction perpendicular to the axis of array and plane containing the elements of array.

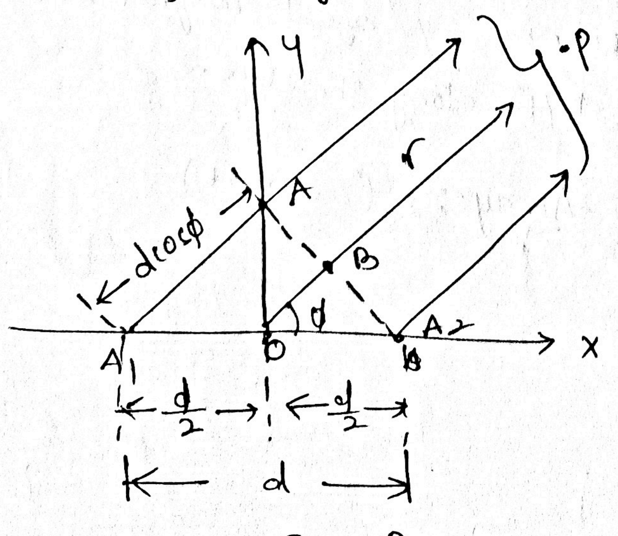


Fig: Broadside array with two isotropic point source with equal amplitude and phase.

→ consider two isotropic point sources spaced (3) equally w.r.t the origin of the co-ordinate system as shown in above fig.

→ let p is the far away from the region, distance is r.

Now we compare the wave radiation between A_1 and A_2

→ This is due to path difference.

$\therefore$ Path difference $= d\cos\phi$ → (1)

The eq (1) in terms of wavelength is

Path difference $= \frac{d}{\lambda}\cos\phi$ → (2)

→ The phase angle is 2π times the path difference.

Hence phase angle $= \psi = 2\pi$ (path difference)

$= 2\pi\left(\frac{d}{\lambda}\cos\phi\right)$ rad

$= \left(\frac{2\pi}{\lambda}\right) d\cos\phi$. → (3)

But phase shift $\beta = \frac{2\pi}{\lambda}$ so, the eq becomes

$\psi = \beta d\cos\phi$.

2). End fire Array:

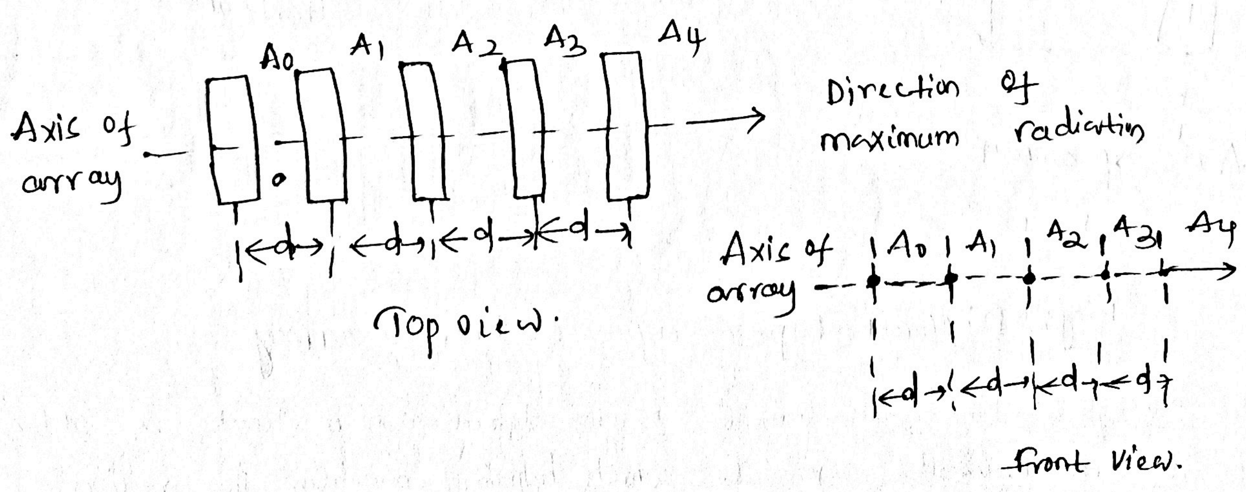


fig: End fire array

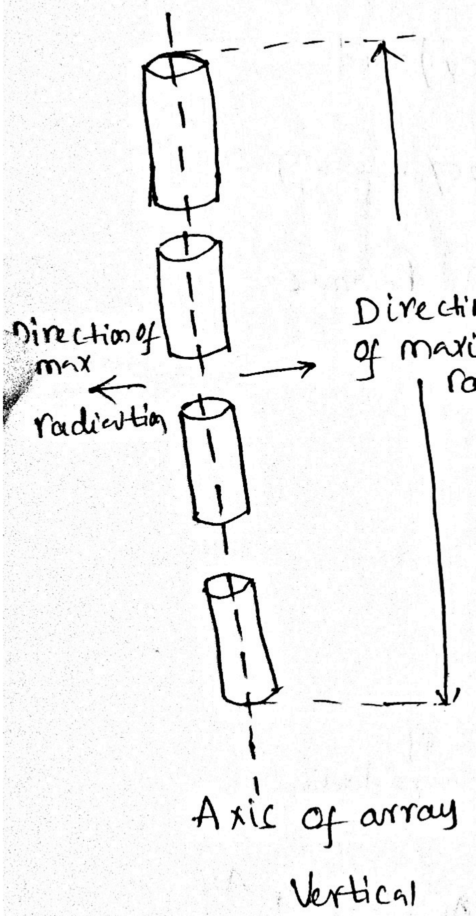
→ This array is very much similar to the broad side array from the point of view of arrangement.

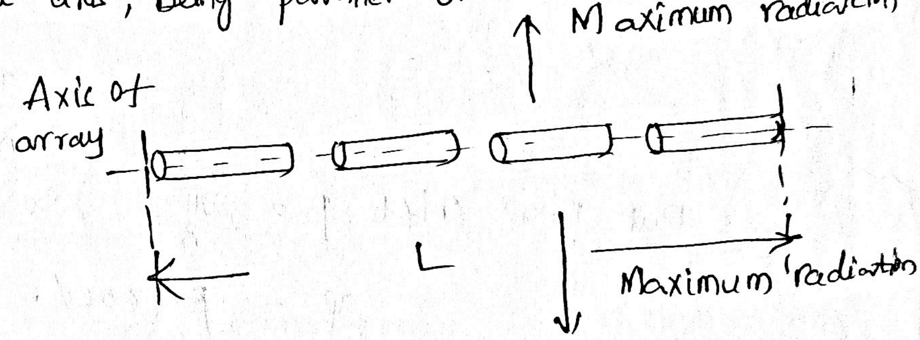
But main difference is in the direction of maximum radiation.

→ In broad side array, the direction of maximum radiation is ⊥r to the axis of array while in the end fire array, the direction of the maximum radiation is along the axis of array.

→ In end fire array all the antennas are fed individually with currents of equal magnitudes but their phases vary progressively along the line to get entire arrangement Onidirectional finally.

Collinear array: It consist of two or more half-wave dipoles which are placed end to end. These antennas are placed on same axis, being paraller or collinear.





* maximum radiation takes place perpendicular to the line of array.

Horizontal.

fig: Different types of collinear array.

Parasitic array: Parasitic element is an element which depends on other's feed. It does not have its own feed. which help in increasing the radion indirectly.

$$T_{4+1}(x) = T_5(x) = 2x\,T_4(x) - T_{4-1}(x)$$

$$= 2x\,T_4(x) - T_3(x).$$

$$= 2x\left[8x^4 - 8x^2 + 1\right] - \left[4x^3 - 3x\right]$$

$$= 16x^5 - 16x^3 + 2x - 4x^3 + 3x.$$

$$= 16x^5 - 20x^3 + 5x.$$

Similarly T_6 also,

$$T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1.$$

→ The nulls of the Tchebyscheff pattern are given by the roots of the mth degree Tchebyscheff polynomial as follows.

$$T_m(x) = \cos(m\cos^{-1}x) = \cos m\delta$$

where $\cos\delta = x$.

→ Then the nulls are given by roots

$$\cos m\delta = 0$$

$$m\delta = \cos^{-1}(0)$$

$$m\delta = (2k-1)\frac{\pi}{2}$$

$$\delta = \frac{(2k-1)\pi}{2m}, \quad \text{where } k = 1, 2, 3, \ldots m.$$

Two point Sources with Currents Equal in Magnitude and Phase:

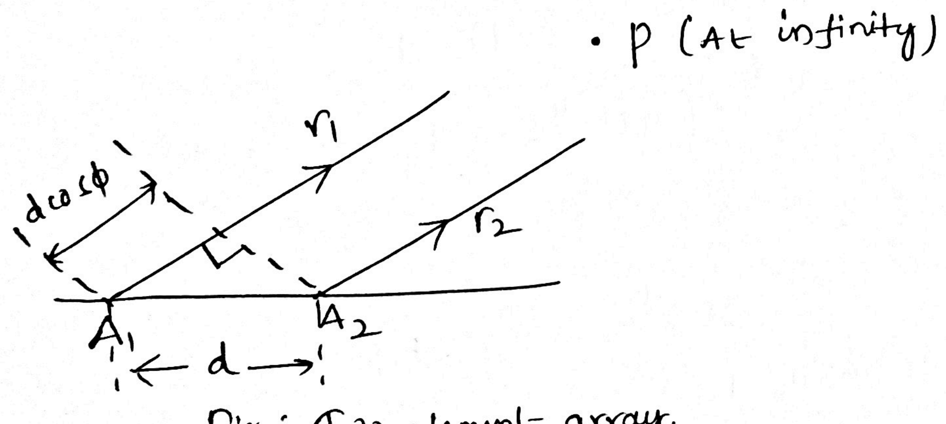


Fig ; Two element array.

→ Consider two point sources A_1 and A_2 separated by distance d as shown in the fig. Consider that both the point sources are supplied with currents equal in magnitude and phase.

Let $m=2$, then from eq 1-a

$T_2(x) = \cos(2 \cdot \cos^{-1} x)$

$T_2(x) = \cos(2\delta) \quad \because \delta = \cos^{-1} x.$

But by trigonometric property, $\cos 2\delta = 2\cos^2\delta - 1$

$\therefore T_2(x) = 2\cos^2\delta - 1$

Substituting $\delta = \cos^{-1} x.$

$\therefore T_2(x) = 2x^2 - 1$

Let $m=3$ from eq 1-a

$T_3(x) = \cos(3\cos^{-1} x)$

$= \cos(3\delta) \quad \cos 3\delta = 4\cos^3\delta - 3\cos\delta$

$= 4x^3 - 3x.$

Let $m=4$ from eq 1-a,

$T_4(x) = \cos(4\cos^{-1} x)$

$= \cos 4\delta \quad \cos 4\delta = 2\cos^2 2\delta - 1$

$= 2\cos^2 2\delta - 1$

But $\cos 2\delta = \cos^2\delta - 1$, hence the polynomial becomes,

$T_4(x) = 2(2\cos^2\delta - 1)^2 - 1$

$= 2(4\cos^4\delta - 4\cos^2\delta + 1) - 1$

$= 8\cos^4\delta - 8\cos^2\delta + 1$

$= 8x^4 - 8x^2 + 1.$

Now the polynomials with higher values of m can be obtained by using reccuressive formula given by

$T_{m+1}(x) = 2x T_m(x) - T_{m-1}(x).$

→ consider point P far away from the array. (6) Let the distance between point P and point sources A_1 and A_2 be r_1 and r_2 respectively. These radial distances are extremely large as compared with the distance of separation between two point sources.

$$r_1 = r_2 = r.$$

→ A_2 will reach earlier at point P than that from point source A_1 because of the path differences.

$$\text{Path difference} = d\cos\phi. \quad \rightarrow (1)$$

The path difference can be expressed interms of wave length as

$$\text{Path difference} = \frac{d\cos\phi}{\lambda} \quad \rightarrow (2)$$

Hence the phase angle ψ is given by

$$\text{Phase angle } \psi = 2\pi \text{ (path difference)}$$

$$\therefore \psi = 2\pi \left(\frac{d\cos\phi}{\lambda}\right)$$

$$= \frac{2\pi}{\lambda} d\cos\phi \text{ rad} \quad \rightarrow (3)$$

But phase shift $\beta = \frac{2\pi}{\lambda}$, thus eq(3) becomes,

$$\psi = \beta d\cos\phi \text{ rad} \quad \rightarrow (4)$$

Let
$$E_1 = E_0 \cdot e^{-j\frac{\psi}{2}} \quad \rightarrow (5)$$

$$E_2 = E_0 \cdot e^{j\frac{\psi}{2}} \quad \rightarrow (6)$$

$$E_T = E_1 + E_2 = E_0 \cdot e^{-j\frac{\psi}{2}} + E_0 \cdot e^{j\frac{\psi}{2}}$$

$$\therefore E_T = E_0\left(e^{-j\frac{\psi}{2}} + e^{j\frac{\psi}{2}}\right)$$

Rearranging the terms on R.H.S, we get,

$$\therefore E_T = 2E_0 \left[\frac{e^{j\frac{\psi}{2}} + e^{-j\frac{\psi}{2}}}{2}\right] \quad \rightarrow (7)$$

By trigonometric identity, $\frac{e^{j\theta} + e^{-j\theta}}{2} = \cos\theta$ (7)

Hence eq (7) can be written as

$$E_T = 2E_0 \cos(\psi/2) \rightarrow (8)$$

Substituting the value of ψ from eq (6) in eq (8) we get

$$E_T = 2E_0 \cos\left(\frac{\beta d \cos\phi}{2}\right) \rightarrow (9)$$

amplitude is $2E_0$, phase shift is $\frac{\beta d \cos\phi}{2}$

Array factor is $A.F = \frac{|E_T|}{|E_{max}|}$

But maximum field is $E_{max} = 2E_0$

$$\therefore A.F = \frac{|E_T|}{|2E_0|} = \cos\left(\pi . \frac{d}{\lambda} \cos\phi\right)$$

Maxima direction

The condition for maximum is given by

$$\cos\left(\frac{\beta d \cos\phi}{2}\right) = \pm 1$$

Let spacing between the two point sources be $\lambda/2$ then

$$\cos\left[\frac{\beta(\lambda/2)\cos\phi}{2}\right] = \pm 1$$

i.e $$\cos\left[\frac{\frac{2\pi}{\lambda} . \frac{\lambda}{2}\cos\phi}{2}\right] = \pm 1$$

$$\cos\left(\frac{\pi}{2}\cos\phi\right) = \pm 1$$

$$\frac{\pi}{2}\cos\phi_{max} = \cos^{-1}(\pm 1) = \pm n\pi, \text{ where } n = 0, 1, 2, \ldots$$

If $n = 0$, then

$$\frac{\pi}{2}\cos\phi_{max} = 0$$

$$\cos\phi_{max} = 0$$

$$\phi_{max} = 90° \text{ or } 270°.$$

Minima directions:

The condition for minima is given by 8

$$\cos\left(\frac{\beta d \cos\phi}{2}\right) = 0$$

assuming $d = \lambda/2$ and $\beta = \frac{2\pi}{\lambda}$ we can write

$$\cos\left(\frac{\pi}{2}\cos\phi_{min}\right) = 0$$

$$\therefore \frac{\pi}{2}\cos\phi_{min} = \cos^{-1} 0 = \pm(2n+1)\pi/2, \text{ where } n = 0, 1, 2, \ldots$$

If $n = 0$ then

$$\frac{\pi}{2}\cos\phi_{min} = \pm\frac{\pi}{2}$$

$$\cos\phi_{min} = \pm 1$$

$$\phi_{min} = 0^\circ \text{ or } 180^\circ$$

Half power point directions:

When the power is half, the voltage or current is $\frac{1}{\sqrt{2}}$ times the maximum value. Hence the condition for half power point is given by,

$$\cos\left(\frac{\beta d \cos\phi}{2}\right) = \pm\frac{1}{\sqrt{2}}$$

Let $d = \frac{\lambda}{2}$ and $\beta = \frac{2\pi}{\lambda}$

$$\cos\left(\frac{\pi}{2}\cos\phi\right) = \pm\frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2}\cos\phi = \cos^{-1}\left(\pm\frac{1}{\sqrt{2}}\right) = \pm(2n+1)\pi/4, \text{ where } n = 0, 1, 2, \ldots$$

If $n = 0$, then

$$\frac{\pi}{2}\cos\phi_{HPPD} = \pm\frac{\pi}{4}$$

$$\cos\phi_{HPPD} = \pm\frac{1}{2}$$

$$\phi_{HPPD} = \cos^{-1}\left(\pm\frac{1}{2}\right)$$

$$\therefore \phi_{HPPD} = 60^\circ \text{ or } 120^\circ$$

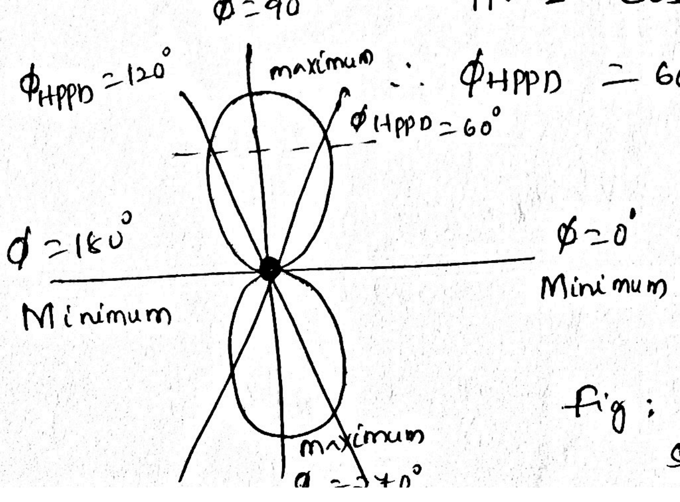


Fig: Field pattern for two point source with spacing $d = \lambda/2$ with equal currents in magnitude and phase.

Let m = 2 . Then from eq .

(9) Two point sources with currents equal in magnitudes but opposite in phase:

→ consider two point sources separated by distance d and supplied with currents equal in magnitude but opposite in phase.

The total electric field $E_T = (-E_1) + (E_2) \rightarrow ①$

$$E_1 = E_0 e^{-j\psi/2} \rightarrow ②$$

$$E_2 = E_0 e^{j\psi/2} \rightarrow ③$$

Substituting the values of E_1 and E_2 in eq ①

$$E_T = -E_0 \cdot e^{-j\psi/2} + e^{j\psi/2}$$

$$= E_0 \left(-e^{-j\psi/2} + e^{j\psi/2}\right)$$

Rearranging the terms in above equation, we get,

$$\therefore E_T = (j2) E_0 \left(\frac{e^{j\psi/2} - e^{-j\psi/2}}{j2} \right)$$

By trigonometric identity, $\frac{e^{j\theta} - e^{-j\theta}}{2} = \sin\frac{\theta}{2} \rightarrow ④$

Hence, eq ④ can be written as,

$$E_T = j2E_0 \sin\left(\frac{\psi}{2}\right) \rightarrow ⑤$$

Phase angle $\psi = \beta d \cos\phi \rightarrow ⑥$

Substituting value of phase angle in eq ⑤, we get

$$E_T = j(2E_0) \sin\left(\frac{\beta d \cos\phi}{2}\right) \rightarrow ⑦$$

Maxima direction:

The condition for maxima is given by,

$$\sin\left(\frac{\beta d \cos\phi}{2}\right) = \pm 1 \rightarrow ⑧$$

Substitute the values of $d = \frac{\lambda}{2}$, $\beta = \frac{2\pi}{\lambda}$ in eq ⑧

$$\sin\left(\frac{\pi}{2}\cos\phi\right) = \pm 1$$

$\frac{\pi}{2}\cos\phi = \pm(2n+1)\frac{\pi}{2}$, where $n = 0, 1, 2, \ldots$

If $n=0$, then

$$\frac{\pi}{2}\cos\phi_{max} = \pm\frac{\pi}{2}$$

$$\cos\phi_{max} = \pm 1$$

$$\phi_{max} = 0^\circ \text{ and } 180^\circ \quad \rightarrow (9)$$

Minima direction:

The conditions for minima is given by

$$\sin\left(\frac{\beta d\cos\phi}{2}\right) = 0 \quad \rightarrow (10)$$

$d = \frac{\lambda}{2}$, $\beta = \frac{2\pi}{\lambda}$ in eq (10)

$\sin\left(\frac{\pi}{2}\cos\phi\right) = 0$ where $n = 0, 1, 2, \ldots$

$\frac{\pi}{2}\cos\phi = \pm n\pi,$

If $n=0$ then $\frac{\pi}{2}\cos\phi_{min} = 0$

$$\cos\phi_{min} = 0$$

$$\phi_{min} = \pm 90^\circ \text{ or } -90^\circ.$$

Half Power Point Direction (HPPD)

When the power is half of max value, the voltage or current equals to $\frac{1}{\sqrt{2}}$ times the respective maximum value. Hence

$$\sin\left(\frac{\beta d\cos\phi}{2}\right) = \pm\frac{1}{\sqrt{2}}$$

Let $d = \lambda/2$ and $\beta = \frac{2\pi}{\lambda}$

$$\sin\left(\frac{\pi}{2}\cos\phi\right) = \pm\frac{1}{\sqrt{2}}$$

$\frac{\pi}{2}\cos\phi = \pm(2n+1)\,\pi/4$ where $n = 0, 1, 2.$

If $n=0$, we can write,

$$\frac{\pi}{2}\cos\phi_{HPPD} = \pm\frac{\pi}{4}$$

$\cos\phi_{HPPD} = \pm\frac{1}{2}$ $\therefore \phi_{HPPD} = 60^\circ \text{ or } 120^\circ.$

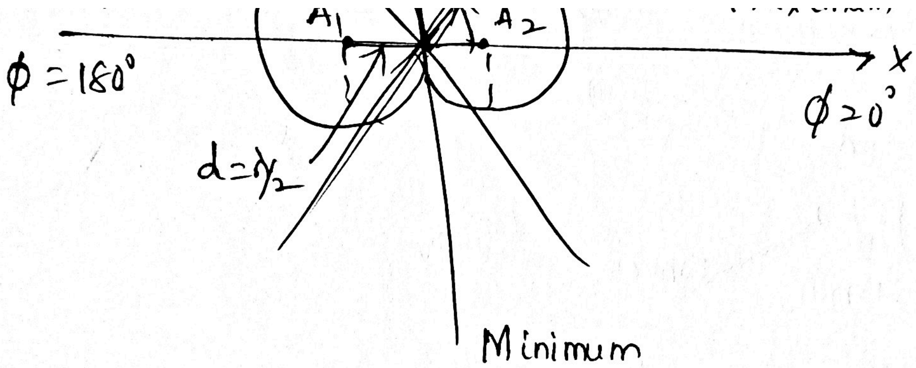


fig: field pattern for two point sources with spacing $d = \lambda/2$ fed with currents equal in magnitude but out of phase by 180°

Two point sources with currents unequal in magnitudes and with any phase:

Where $E_2 = kE_1$

θ = phase angle

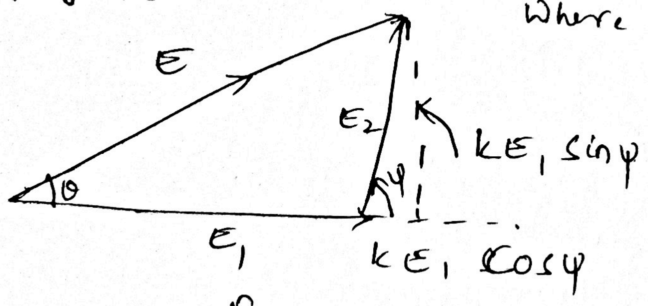
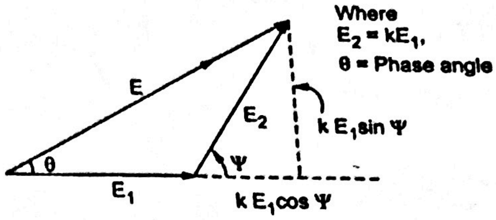


fig: Vector diagram of fields E_1 and E_2.

Consider the two sources, with amplitudes E_1 and E_2.

→ Let us assume that E_1 is greater than E_2 in magnitude.

$$\psi = \frac{2\pi}{\lambda} \cos\phi + \alpha. \quad \to (1)$$



Vector Diagram of fields E_1 and E_2

where α is the phase angle with which current I_2 leads current I_1. Now if a then the condition is similar to the two point sources with currents equal in magnitude and phase. Similarly if α = 180°, then the condition is similar to the two point source with currents equal in magnitude but opposite in phase. Assume value of phase difference a as $0 < \alpha < 180°$. Then the resultant field at point P is given by,

$$E_T = E_1\, e^{-j0} + E_2\, e^{j\psi}$$

$$E_T = E_1 + E_2\, e^{j\psi}$$

$$E_T = E_1\,(E_1 + \frac{E_2}{E_1}\, e^{j\psi})$$

Let $\frac{E_2}{E_1}$= k note that $E_2 > E_1$, the value of k is less than unity. Moreover the value of k is given by $0 \leq k \leq 1$

Then $E_T = E_1[1+k(\cos\psi + j\sin\psi)]$

The magnitude of the resultant field at point P is given by

$$|E_T| = E_1\sqrt{(1 + k\cos\psi)\text{^}2\,) + (k\sin\psi)^2}$$

The phase difference between two fields at the far point P is given by

$$\theta = \tan^{-1}\frac{k\sin\psi}{1 + k\cos\psi}$$

n Element Uniform Linear Arrays

n Element Uniform Linear arrays:

At higher frequencies, for point to point communication it is necessary to have a pattern with single beam radiation. Such highly directive single beam pattern can be obtained by increasing the point sources in the array from 2 to n say.

→ An array of n elements is said to be linear array if all the individual elements are spaced equally along a line. An array is said to be uniform array if the elements in the array are fed with currents with equal magnitude and with uniform progressive phase shift along the line.

→ Consider a general n element linear and uniform array with all the individual elements spaced equally at distance d from each other and all elements are fed with currents equal in magnitude and uniform progressive phase shift along line as shown in the fig.

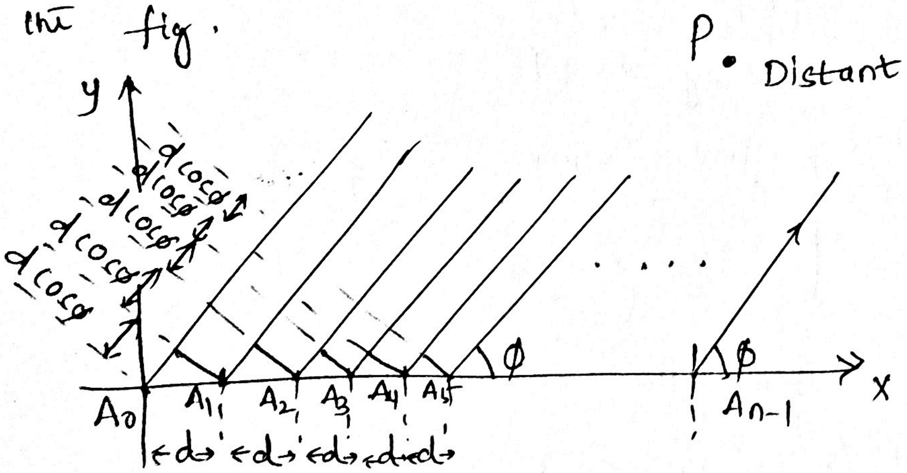


Fig: Uniform, linear array of n elements.

→ The total resultant field at the distant point P is obtained by adding the fields due to n individual sources vertically.

Hence we can write,

$$E_T = E_0 \cdot e^{j0} + E_0 e^{j\psi} + E_0 e^{2j\psi} + \ldots + E_0 e^{j(n-1)\psi}$$

$$\therefore E_T = E_0 \left[1 + e^{j\psi} + e^{j2\psi} + \ldots e^{j(n-1)\psi} \right] \rightarrow ①$$

(13)

Note that $\psi = (\beta \cos\phi + \alpha)$ indicates the ~~total~~ phase difference of the fields from adjacent sources calculated at point P. Similarly α is the progressive phase shift between two adjacent point sources. The value of α may ~~be~~ lie between $0°$ and $180°$. If $\alpha = 0$ we get n element uniform linear broad side array. If $\alpha = 180°$, we get n element uniform linear end fire array.

Multiply eq ① by $e^{j\psi}$, we get,

$$E_T e^{j\psi} = E_0\left[e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \dots + e^{jn\psi}\right] \rightarrow ②$$

→ Subtracting eq ② from ① we get,

$$E_T - E_T e^{j\psi} = E_0\left\{\left[1 + e^{j\psi} + e^{j2\psi} + \dots + e^{j(n-1)\psi}\right] - \left[e^{j\psi} + e^{j2\psi} + \dots + e^{jn\psi}\right]\right\}$$

$$E_T(1 - e^{j\psi}) = E_0(1 - e^{jn\psi})$$

$$\therefore E_T = E_0\left[\frac{1 - e^{jn\psi}}{1 - e^{j\psi}}\right]$$

Simplifying mathematically we get

$$E_T = E_0\left[\frac{e^{jn\frac{\psi}{2}}\left(e^{-jn\frac{\psi}{2}} - e^{jn\frac{\psi}{2}}\right)}{e^{j\frac{\psi}{2}}\left(e^{-j\frac{\psi}{2}} - e^{j\frac{\psi}{2}}\right)}\right]$$

According to trigonometric identity,

$$e^{-j\theta} - e^{j\theta} = -2j\sin\theta$$

The resultant field is given by

$$E_T = E_0\left[\frac{\left(-j2\sin\frac{n\psi}{2}\right)e^{jn\frac{\psi}{2}}}{\left(-j2\sin\frac{\psi}{2}\right)e^{j\frac{\psi}{2}}}\right] = E_0\left[\frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}\right]e^{j\left(\frac{n-1}{2}\right)\psi}$$

$$\therefore E_T = E_0\left[\frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}\right]$$

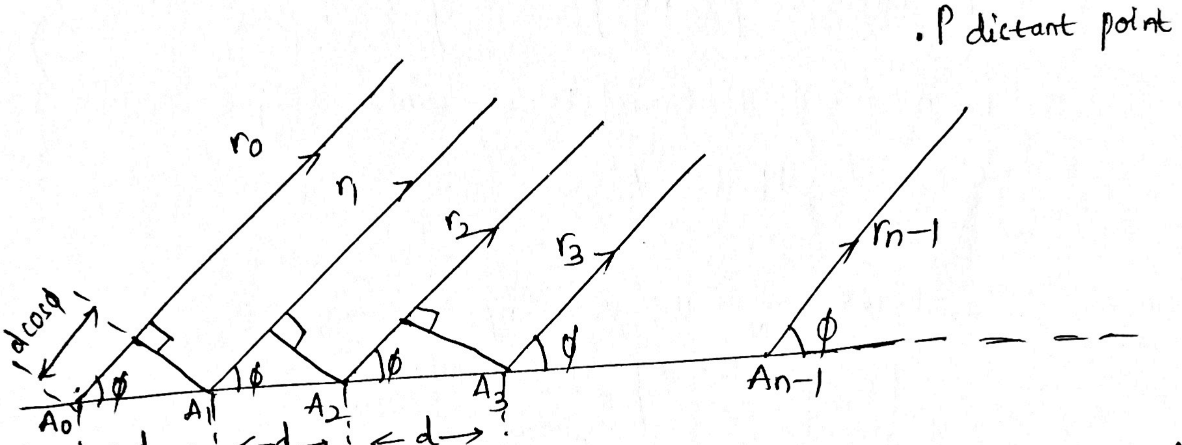
Phase angle

$$\theta = \left(\frac{n-1}{2}\right)\cdot\psi = \left(\frac{n-1}{2}\right)(\beta d\cos\phi + \alpha).$$

Array of n elements with equal spacing and currents Equal in Magnitude and Phase — Broad Side Array

→ Consider a 'n' number of identical radiators carry currents which are equal in magnitude and in phase. The identical radiators are equispaced. Hence the maximum radiation occurs in the directions normal to the line of array. Hence such an array is known as Uniform broadside array.

Consider a broad side array with 'n' identical radiators as shown in the fig.



The electric field produced at point P due to an element A0 is given by,
$$E_0 = \frac{I\,dL\sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_0}\right]e^{-j\beta r_0} \rightarrow (1)$$

→ The distance is 'd' between any two array elements is very small as compared to the radial distance of point P from $A_0, A_1, \ldots A_{n-1}$.
We can assume $r_0, r_1, r_2, \ldots r_{n-1}$ are approximately same.

The electric field due to A_1 is given by
$$E_1 = \frac{I\,dL\sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_1}\right]e^{-j\beta r_1}$$

but $r_1 = r_0 - d\cos\phi$

$$\therefore E_1 = \frac{I\,dL\sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_0}\right]e^{-j\beta(r_0-d\cos\phi)}$$

$$= \frac{I\,dL\sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_0}\right]e^{-j\beta r_0}\,e^{j\beta d\cos\phi}$$

$$\therefore E_1 = E_0 \cdot e^{j\beta d\cos\phi} \rightarrow (2)$$

Similarly

$$E_2 = \frac{I\,dL\sin\theta}{4\pi\omega\epsilon_0}\left[j\frac{\beta^2}{r_2}\right]e^{-j\beta r_2}$$

$$= \frac{I\,dL\sin\theta}{4\pi\omega\epsilon_0}\left[j\frac{\beta^2}{r_1}\right]e^{-j\beta(r_1 - d\cos\phi)}$$

$$= \left\{\frac{I\,dL\sin\theta}{4\pi\omega\epsilon_0}\left[j\frac{\beta^2}{r_1}\right]e^{-j\beta r_1}\right\}e^{j\beta d\cos\phi} \qquad r_2 = r_1 - d\cos\phi$$

The term inside the bracket represents E_1.

$$\therefore E_2 = E_1\, e^{j\beta d\cos\phi}.$$

From eq (2), substituting the value of E_1, we get,

$$E_2 = \left[E_0\, e^{j\beta d\cos\phi}\right]e^{j\beta d\cos\phi}$$

$$\therefore E_2 = E_0 \cdot e^{j2\beta d\cos\phi} \rightarrow (3)$$

Similarly, the electrifield due to an element, A_{n-1} is given by,

$$E_{n-1} = E_0 \cdot e^{j(n-1)\beta d\cos\phi} \rightarrow (4)$$

The total electricfield at point P is given by,

$$E_T = E_0 + E_1 + E_2 + \cdots E_{n-1}$$

$$E_T = E_0 + E_0\, e^{j\beta d\cos\phi} + E_0\, e^{j2\beta d\cos\phi} + \cdots + E_0\, e^{j(n-1)\beta d\cos\phi}$$

Let $\beta d\cos\phi = \psi$ then

$$E_T = E_0 + E_0\, e^{j\psi} + E_0\, e^{j2\psi} + \cdots + E_0\, e^{j(n-1)\psi}$$

$$\therefore E_T = E_0\left[1 + e^{j\psi} + e^{j2\psi} + \cdots + e^{j(n-1)\psi}\right] \rightarrow (5)$$

consider a series is given by

$$S = 1 + r + r^2 + r^3 + \cdots r^{n-1} \qquad -(i)$$

where $r = e^{j\psi}$.

Multiplying both sides of the (i) by r,

$$Sr = r + r^2 + r^3 + r^4 + \cdots r^n \quad \text{— (ii)}$$

Subtracting eq (ii) from (i) we get

$$S - Sr = 1 - r^n$$

$$\therefore S(1-r) = 1 - r^n$$

$$\therefore S = \frac{1-r^n}{1-r} \quad \text{— (iii)}$$

Using eq (iii), eq 5) can be modified as,

$$E_T = E_0 \left[\frac{1 - e^{jn\varphi}}{1 - e^{j\varphi}} \right]$$

$$\therefore \frac{E_T}{E_0} = \frac{e^{\frac{jn\varphi}{2}} \left[e^{-\frac{jn\varphi}{2}} - e^{\frac{jn\varphi}{2}} \right]}{e^{\frac{j\varphi}{2}} \left[e^{-\frac{j\varphi}{2}} - e^{\frac{j\varphi}{2}} \right]} \quad (6)$$

From the trigonometric identities

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

and $e^{-j\theta} - e^{j\theta} = -j2\sin\theta$

Using above trigonometric identities, eq (6) can be written as,

$$\frac{E_T}{E_0} = \frac{e^{\frac{jn\varphi}{2}} \left[-j2\sin\left(\frac{n\varphi}{2}\right) \right]}{e^{\frac{j\varphi}{2}} \left[-j2\sin\left(\frac{\varphi}{2}\right) \right]}$$

$$\therefore \frac{E_T}{E_0} = e^{\frac{j(n-1)\varphi}{2}} \left[\frac{\sin\left(\frac{n\varphi}{2}\right)}{\sin\left(\frac{\varphi}{2}\right)} \right] \rightarrow (7)$$

$$\left| \frac{E_T}{E_0} \right| = \frac{\sin\frac{n\varphi}{2}}{\sin\frac{\varphi}{2}}.$$

(a)

(16) (17)

Broad side Array: Array of n element with equal spacing and Current Equal in Magnitude and phase

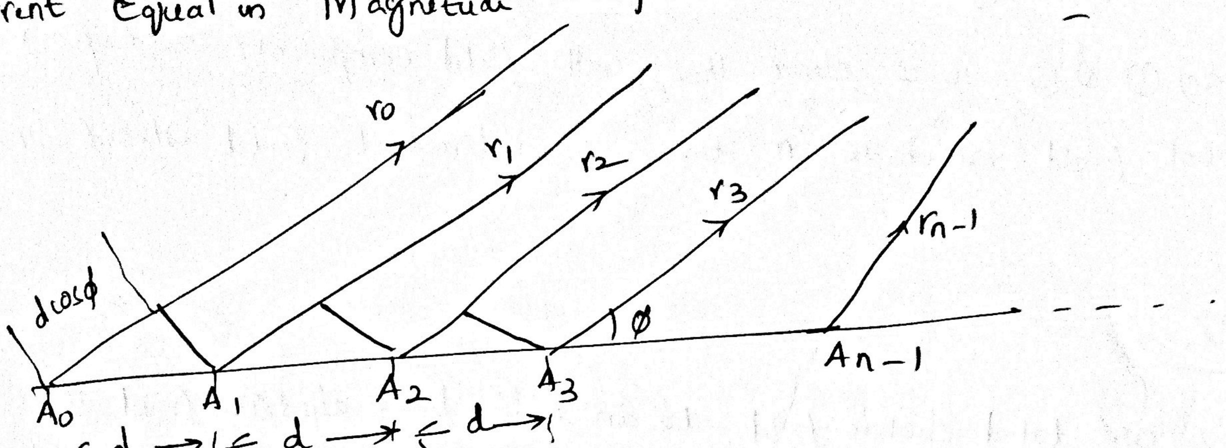


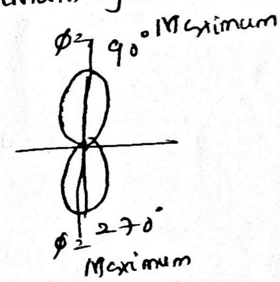
fig: Broad side Array

Array factor $\left|\frac{E_T}{E_0}\right| = \left[\frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}}\right]$ $\left(\because e^{j\left(\frac{n-1}{2}\right)\psi} \cdot \frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}} = \frac{E_T}{E_0}\right)$

Properties of Broad side Array:

1. **Major Lobe:**

In case of broad side array, the field is maximum in the direction normal to the axis of the array. Thus the condition for maximum field at point P is given by,



$\psi = 0$, i.e $\beta d\cos\phi = 0$

i.e $\cos\phi = 0$

i.e $\phi = \cos^{-1}(0) = 90°$ or $270°$ — (1)

Thus $\phi = 90°$ and $\phi = 270°$ are called direction of principle maxima

2. **Magnitude of Major lobe:**

Maximum radiation occurs when $\psi = 0$ Hence we can write,

$$|\text{Major lobe}| = \left|\frac{E_T}{E_0}\right| = \lim_{\psi\to 0}\left\{\frac{\frac{d}{d\psi}\left(\sin\frac{n\psi}{2}\right)}{\frac{d}{d\psi}\left(\sin\frac{\psi}{2}\right)}\right\}$$

$$= \lim_{\psi\to 0}\left\{\frac{\cos\frac{n\psi}{2}\cdot\frac{n\psi}{2}}{\cos\frac{\psi}{2}\cdot\frac{\psi}{2}}\right\}$$

| Major label| = n

where n is the no. of elements in the array —②

from eq ① & ② it is clear that, all field components add up together to give total field which is 'n' times the individual field when $\phi = 90^\circ$ or 270°.

3. Nulls:

The ratio of total electric field to an individual electric field is given by

$$\left|\frac{E_T}{E_0}\right| = \frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}} = 0$$

Thus condition of minima is given by,

$$\sin\frac{n\psi}{2} = 0 \quad \text{but} \quad \sin\frac{\psi}{2} \neq 0$$

Hence $\sin\frac{n\psi}{2} = 0$

$$\frac{n\psi}{2} = \sin^{-1}(0) = \pm m\pi, \text{ Where } m = 1, 2, 3 \ldots$$

Now $\psi = \beta d\cos\phi = \frac{2\pi d}{\lambda}\cos\phi$

$$= \frac{n}{\cancel{2}}\left(\frac{\cancel{2}\cancel{\pi}d}{\lambda}\right)\cos\phi_{min} = \pm m\cancel{\pi}$$

$$= \frac{nd}{\lambda}\cos\phi_{min} = \pm m$$

$$\phi_{min} = \cos^{-1}\left(\pm\frac{m\lambda}{nd}\right) \rightarrow ③$$

Where n = no. of elements in array

d = Spacing between elements in meter

λ = Wave length in meter

m = constant = 1, 2, 3

The equation 3 gives the direction of nulls.

when $\psi = 0$.

(b)

(20)

4. Subsidary maxima (or side lobes)

The direction of the subsidary maxima or side lobes can be obtained if in equation $\left|\frac{E_T}{E_0}\right|$, $\sin\left(\frac{n\psi}{2}\right) = \pm 1$

$$\frac{n\psi}{2} = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \dots$$

Hence $\left(\frac{n\psi}{2}\right) = \pm 1$ is not considered. If $\frac{n\psi}{2} = \frac{\pi}{2}$ then $\sin\frac{n\psi}{2} = 1$

Which is the direction of principle maxima.

Hence we can skip $\frac{n\psi}{2} = \pm\pi/2$ value

Thus $\psi = \pm\frac{3\pi}{n}, \pm\frac{5\pi}{n}, \pm\frac{7\pi}{n}, \dots$

$$\psi = \beta d\cos\phi = \left(\frac{2\pi}{\lambda}\right) d\cos\phi$$

Hence $\frac{2\pi}{\lambda} d\cos\phi = \pm\frac{3\pi}{n}, \pm\frac{5\pi}{n}, \pm\frac{7\pi}{n}$

$$\therefore \cos\phi = \frac{\lambda}{2\pi d}\left[\pm\frac{(2m+1)}{n}\pi\right] \quad \text{where } m = 1, 2, 3, \dots$$

$$\therefore \phi = \cos^{-1}\left[\pm\frac{\lambda(2m+1)}{2nd}\right] \quad (4)$$

The eq (4) represents the directions where certain radiation which is not maximum. Hence it represents directions of subsidary maxima or side lobes.

5. Bandwidth of major lobes

The bandwidth is defined as the angle between first nulls. It's angle twice the angle between first null and major lobe maximum direction

$$BWFN = 2\times\gamma, \quad \text{wher } \gamma = 90 - \phi$$

$$\phi_{min} = \cos^{-1}\left(\pm\frac{m\lambda}{nd}\right) \quad \text{where } m = 1, 2, 3, \dots$$

$$90 - \phi_{min} = \gamma \quad \text{i.e. } 90 - \gamma = \phi_{min}$$

$$90 - \gamma = \cos^{-1}\left(\pm\frac{m\lambda}{nd}\right)$$

Taking cosine of angle on both the sides, we get

$$\cos(90-\gamma) = \cos\left[\cos^{-1}\left(\pm\frac{m\lambda}{nd}\right)\right]$$

$$\sin\gamma = \frac{\pm m\lambda}{nd}$$

If γ is very small, $\sin\gamma = \gamma$

so, $\gamma = \pm\frac{m\lambda}{nd}$

for first null i.e $m=1$

$$\gamma = \pm\frac{\lambda}{nd}$$

$$\therefore \text{BWFN} = 2\gamma = \frac{2\lambda}{nd}$$

But $nd = (n-1)d$ if n is very large. The $n.d$ indicates length of array in meter. This is denoted by L.

$$\text{BWFN} = \frac{2\lambda}{L}\text{ rad} = \frac{2}{\left(\frac{L}{\lambda}\right)}\text{ rad.}$$

$$\text{BWFN} = \frac{114.6\lambda}{L} \text{ or } \frac{114.6}{\left(\frac{L}{\lambda}\right)} \text{ in degrees.}$$

$$\text{HPBW} = \frac{\text{BWFN}}{2} = \frac{\frac{2\lambda}{L}}{2} = \frac{1\lambda}{L} = \frac{1}{\left(\frac{L}{\lambda}\right)}\text{ rad. or } \frac{57.3}{\frac{L}{\lambda}}\text{ degrees}$$

6. <u>Directivity:</u>

The directivity in case of broad side array is defined as,

$$G_{D\,max} = \frac{\text{Maximum radiation intensity}}{\text{Average radiation intensity}} = \frac{U_{max}}{U_{avg}} = \frac{U_{max}}{U_0}$$

$$U_0 = \frac{P_{rad}}{4\pi} = \frac{1}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} |E(\theta,\phi)|^2 \sin\theta\, d\theta\, d\phi$$

$$\left|\frac{E_T}{E_0}\right| = n \quad , \quad |E_T| = n|E_0|.$$

for normalized condition let us assume $E_0 = 1$ then $|E_T| = n$

Thus field from array is maximum in any direction θ, when $\psi = 0$.
Hence normalized field pattern is given by,

$$E_{Normalized} = \left| \frac{E_T}{E_{T\,max}} \right| = \frac{|E_0|}{n|E_0|} = \frac{1}{n}$$

Hence the field is given by,

$$E_{normalized} = \frac{\sin \frac{n\psi}{2}}{n\left(\sin \frac{\psi}{2}\right)} \rightarrow \text{Array factor}$$

Where $\psi = \beta d \cos\phi$

$$E = \frac{1}{n}\left[\frac{\sin \frac{n\beta d \cos\phi}{2}}{\sin \frac{\beta d \cos\phi}{2}}\right]$$

assuming d is very small as compared to length of array, so,

$$\sin\frac{\beta d \cos\phi}{2} = \frac{\beta d \cos\phi}{2}$$

Substituting value of E in U_0, we get

$$U_0 = \frac{1}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\left[\frac{1}{n}\cdot\frac{\sin \frac{n\beta d \cos\phi}{2}}{\sin \frac{\beta d \cos\phi}{2}}\right]^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{1}{4\pi}\int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi}\left[\frac{\sin \frac{n}{2}\beta d\cos\phi}{\frac{n}{2}\beta d \cos\phi}\right]^2 \sin\theta \, d\theta$$

$$= \frac{1}{4\pi}[2\pi] \cdot \int_{\theta=0}^{\pi}\left[\frac{\sin z}{z}\right]^2 \sin\theta \, d\theta$$

$$z = \frac{n}{2}\beta d \cos\phi$$

$$dz = -\frac{n}{2}\beta d \sin\theta \cdot d\theta$$

$$\sin\theta \cdot d\theta = \frac{-dz}{\frac{n}{2}\beta d}$$

$\theta = \pi$, $z = -\frac{n}{2}\beta d$ and

$\theta = 0$, $z = +\frac{n}{2}\beta d$

Re writing above equation we get,

$$U_0 = \frac{1}{2}\int_{+n/2\,\beta d}^{-n/2\,\beta d}\left[\frac{\sin z}{z}\right]^2 \cdot \frac{dz}{-\frac{n}{2}\beta d}$$

$$= -\frac{1}{n\beta d}\int_{\frac{n}{2}\beta d}^{-n/2\,\beta d}\left[\frac{\sin z}{z}\right]^2 dz$$

for Large array, n is large

$$U_0 = \frac{-1}{n\beta d}\int_{+\infty}^{-\infty}\left[\frac{\sin z}{z}\right]^2 dz \quad \text{Interchanging the limits of integration}$$

$$U_0 = +\frac{1}{n\beta d}\int_{-\infty}^{+\infty}\left[\frac{\sin z}{z}\right]^2 dz$$

$$\int_{-\infty}^{\infty}\left[\frac{\sin z}{z}\right]^2 dz = \pi$$

$$= \frac{1}{n\beta d}\times\pi \qquad = \frac{\pi}{n\beta d}$$

$$G_D\ max = \frac{U_{max}}{U_0}$$

$U_{max} = 1$ at $\phi = 90^\circ$ substituting the value of U_0

$$= \frac{1}{\frac{\pi}{n\beta d}} = \frac{n\beta d}{\pi}$$

$$\beta = \frac{2\pi}{\lambda} \qquad \therefore\ G_D\ max = \frac{n\left(\frac{2\pi}{\lambda}\right)d}{\pi}$$

$$= \frac{2nd}{\lambda} = 2n\left(\frac{d}{\lambda}\right)$$

Total length $L = (n-1)d \simeq nd$ if n is very large

$$G_D\ max = 2\left(\frac{L}{\lambda}\right)$$

2-B

Array of n Elements with Equal spacing and currents Equal in Magnitude but with Progressive phase shift — End Fire Array.

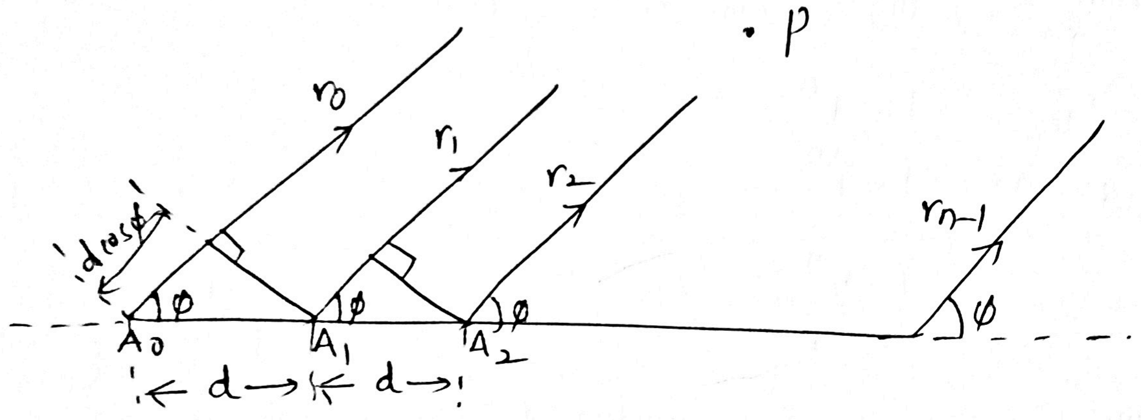


fig: End fire array

→ The current supplied to A_1 is given by,

$$I_1 = I_0 \cdot e^{-j\beta d}$$

Similarly the current supplied to A_2 is given by,

$$I_2 = I_1 \cdot e^{-j\beta d} = \left[I_0 \cdot e^{-j\beta d}\right] e^{-j\beta d} = I_0 \cdot e^{-j2\beta d}$$

Thus the current supplied to the last element is given by,

$$I_{n-1} = I_0 \, e^{-j(n-1)\beta d}$$

The electric field produced at point P, due to A_0 is given by

$$E_0 = \frac{I\,dL\sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_0}\right] e^{-j\beta r_0} \rightarrow ①$$

The electric field produced at point P, due to A_1 is given by,

$$E_1 = \frac{I\,dL\sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_1}\right] e^{-j\beta r_1} \cdot e^{-j\beta d}$$

but $r_1 = r_0 - d\cos\phi$

$$\therefore E_1 = \frac{I\,dL\sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_0}\right] e^{-j\beta(r_0 - d\cos\phi)} \cdot e^{-j\beta d}$$

$$= E_0 \cdot e^{j\beta d(\cos\phi - 1)} \rightarrow ②$$

Properties of End fire Array:

(25)

1. Major Lobe:

for the end fire array where currents supplied to the antennas are equal in amplitude but the phase changes progressively through array, the phase angle is given by,

$$\psi = \beta d(\cos\phi - 1) \quad -①$$

In case of end fire array, the condition of principle maxima is given by, $\psi = 0$ i.e $\beta d(\cos\phi - 1) = 0$

i.e $\cos\phi = 1$

i.e $\phi = 0^\circ$

Thus $\phi = 0^\circ$ indicates the direction of principle maxima. Also it indicates that the maximum radiation is along the axis of array or line of array.

2. Magnitude of major lobe:

The maximum radiation occurs when $\psi = 0$. Thus we can write

$$|\text{Major lobe}| = \lim_{\psi \to 0} \left\{ \frac{\frac{d}{d\psi}\left(\sin\frac{n\psi}{2}\right)}{\frac{d}{d\psi}\left(\sin\frac{\psi}{2}\right)} \right\}$$

$$= \lim_{\psi \to 0} \left\{ \frac{\left(\cos n\frac{\psi}{2}\right)\left(\frac{n}{2}\right)}{\left(\cos\frac{\psi}{2}\right)\left(\frac{1}{2}\right)} \right\}$$

$$\therefore |\text{Major lobe}| = n.$$

Where, n is the number of elements in the array.

3. Nulls:

The ratio of the total field to the individual field is given by

$$\left|\frac{E_T}{E_0}\right| = \frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}$$

To find direction of minima, equating ratio of magnitudes to zero,

$$\therefore \left|\frac{E_T}{E_0}\right| = \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} = 0$$

Thus condition of minima is given by,

$$\sin \frac{n\psi}{2} = 0, \text{ but } \sin \frac{\psi}{2} \neq 0$$

Hence we can write,

$$\sin \frac{n\psi}{2} = 0$$

$$\frac{n\psi}{2} = \pm m\pi, \text{ where } m = 1, 2, 3, \ldots$$

Substituting value of ψ from eq ①, we get,

$$\therefore \frac{n\beta d(\cos\phi - 1)}{2} = \pm m\pi$$

Put $\beta = \frac{2\pi}{\lambda}$, we get

$$\therefore \frac{nd}{\lambda}(\cos\phi - 1) = \pm m.$$

Note that value of $(\cos\phi - 1)$ is always less than 1. Hence it is always negative. Hence only considering -ve values R.H.S

$$\frac{nd}{\lambda}(\cos\phi - 1) = -m$$

$$\cos\phi - 1 = \frac{-m\lambda}{nd}$$

$$\cos\phi = 1 - \frac{m\lambda}{nd}$$

$$\phi_{min} = \cos^{-1}\left[1 - \frac{m\lambda}{nd}\right]$$

where m = Constant = 1, 2, 3, ...

n = No. of elements in array

d = Spacing between element in meter

λ = Wave length in meter

$$\cos\phi_{min} - 1 = \pm \frac{m\lambda}{nd}$$

Expressing term on L.H.S in terms of half angles

$$2\sin^2 \frac{\phi_{min}}{2} = \pm \frac{m\lambda}{nd}$$

$$\sin^2 \frac{\phi_{min}}{2} = \pm \frac{m\lambda}{2nd}$$

$$\sin \frac{\phi_{min}}{2} = \pm \sqrt{\frac{m\lambda}{2nd}}$$

$$\frac{\phi_{min}}{2} = \sin^{-1}\left[\pm\sqrt{\frac{m\lambda}{2nd}}\right]$$

$$\phi_{min} = 2\sin^{-1}\left[\pm\sqrt{\frac{m\lambda}{2nd}}\right]$$

$[\because \cos\theta - 1 = 2\sin^2 \theta/2]$

4. Subsidary maxima (or side lobes):

The direction of the subsidary maxima or side lobes can be obtained if in equation $\sin(\frac{n\psi}{2}) = \pm 1$

$$\frac{n\psi}{2} = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \dots$$

Hence $\frac{n\psi}{2} = \pm\frac{\pi}{2}$ is skipped because with the value of $\frac{n\psi}{2}$ we get $\sin\frac{n\psi}{2} = 1$, which the direction of principle maxima

Thus we can write

$$\frac{n\psi}{2} = \pm(2m+1)\pi/2, \text{ where } m = 1, 2, 3, \dots$$

Putting the value of ψ

$$\frac{n\beta d(\cos\phi - 1)}{\cancel{2}} = \pm(2m+1)\pi/\cancel{2}$$

$$n\beta d(\cos\phi - 1) = \pm(2m+1)\pi$$

Put $\beta = \frac{2\pi}{\lambda}$

$$n\left(\frac{2\pi}{\lambda}\right) d(\cos\phi - 1) = \pm(2m+1)\pi$$

$$\cos\phi - 1 = \pm(2m+1)\frac{\lambda}{2nd}$$

$$\cos\phi = 1 - (2m+1)\frac{\lambda}{2nd}$$

$$\phi = \cos^{-1}\left[1 - \frac{(2m+1)\lambda}{2nd}\right]$$

5. Bandwidth or major lobe's

The beam width of end fire array is greater than that of broad side array.

Beam width = 2X Angle between first nulls and maximum of the major lobe i.e θ_{min}

$$\phi_{min} = 2\sin^{-1}\left[\pm\sqrt{\frac{m\lambda}{2nd}}\right]$$

$$\sin\frac{\phi_{min}}{2} = \pm\sqrt{\frac{m\lambda}{2nd}}$$

If ϕ_{min} is very low, then we can write $\sin\frac{\phi_{min}}{2} \simeq \frac{\phi_{min}}{2}$

Using this property, $\frac{\sin\phi_{min}}{2} = \pm\sqrt{\frac{m\lambda}{2nd}}$

$$\phi_{min} = \pm\sqrt{\frac{4m\lambda}{2nd}} = \pm\sqrt{\frac{2m\lambda}{nd}}$$

$$nd = L \rightarrow \pm\sqrt{\frac{2m\lambda}{L}} = \pm\sqrt{\frac{2m}{L/\lambda}}$$

$$BWFN = 2\phi_{min} = \pm 2\sqrt{\frac{2m}{L/\lambda}}$$

Expressing BWFN in degrees,

$$BWFN = \pm 2\sqrt{\frac{2m}{L/\lambda}} \times 57.3 = \pm 114.6\sqrt{\frac{2m}{L/\lambda}} \text{ degree.}$$

For $m = 1$

$$BWFN = \pm 2\sqrt{\frac{2}{L/\lambda}} \text{ rad} = 114.6\sqrt{2/L/\lambda} \text{ degree}$$

6. Directivity;

Similar to the broadside array, the directivity for the end fire array is given by,

$$G_{Dmax} = \frac{U_{max}}{U_0}$$

for EFA $U_{max} = 1$, and $U_0 = \frac{\pi}{2n\beta d}$

$$G_{Dmax} = \frac{1}{\frac{\pi}{2n\beta d}} = \frac{2n\beta d}{\pi}$$

$$= 2n\left(\frac{2\pi}{\lambda}\right) \cdot \frac{d}{\pi}$$

$$= \frac{4nd}{\lambda} \qquad nd = L$$

$$= 4\left(\frac{L}{\lambda}\right).$$

Array of n Elements with Equal Spacing and Currents with Equal Amplitude and Progressive phase-shift - End Fire Array with Increased Directivity:

→ In End fire Array the maximum radiation can be obtained along the axis of the uniform end fire array, if the progressive phase shift α between the elements is given by

$$\alpha = \pm\beta d = -\beta d \text{ for maximum in } \phi = 0^\circ \text{ direction}$$
$$= +\beta d \text{ for maximum in } \phi = 180^\circ \text{ direction} \rightarrow (1)$$

→ It is found that the field produced in the direction $\phi = 0^\circ$ is maximum, but the directivity is not maximum. In many applications it is necessary to have the maximum possible directivity of the linear array.

→ In 1938, Hansen and Woodyard proposed certain conditions for the end-fire case which are helpful in enhancing the directivity without altering other characteristics of the end fire array. These conditions are popularly known as "Hansen-Woodyard" conditions for End fire Radiation.

According to Hansen-Woodyard condition, the phase shift between closely spaced radiations of a very long array should be

$$\alpha = -\left(\beta d + \frac{2.94}{n}\right) \simeq -\left(\beta d + \frac{\pi}{n}\right) \text{ for maximum in } \phi = 0^\circ$$

and

$$\alpha = +\left(\beta d + \frac{2.94}{n}\right) \simeq +\left(\beta d + \frac{\pi}{n}\right) \text{ for maximum in } \phi = 180^\circ \rightarrow (2)$$

Note that with the above condition also, maximum possible directivity can not be achieved. The maximum may not even occur at $\phi = 0^\circ$ and $\phi = 180^\circ$.

The enhanced directivity due to Hansen - Woodyard conditions can be realized by using eq ② along with assumptions for $|\psi|$ values given as below. 30

i) for maximum radiation along $\phi = 0°$

$$\left.\begin{aligned} |\psi| &= |\beta d \cos\phi + \alpha|_{\phi=0°} = \frac{\pi}{n} \\ \text{and } |\psi| &= |\beta d \cos\phi + \alpha|_{\phi=180°} = \pi \end{aligned}\right\} \quad ③$$

ii) for maximum radiation along $\phi = 180°$

$$\left.\begin{aligned} |\psi| &= |\beta d \cos\phi + \alpha|_{\phi=180°} = \frac{\pi}{n} \\ \text{and } |\psi| &= |\beta d \cos\phi + \alpha|_{\phi=0°} = \pi \end{aligned}\right\} \quad ④$$

→ Even though equations 3 and 4 represent conditions obtained from equation ② only, the precaution must be taken to fulfil the condition $|\psi| = \pi$, for each array.

The equation ② satisfy the conditions $\phi = 0°$ and $\phi = 180°$.

The spacing between two elements as,

$$d = \left(\frac{n-1}{n}\right)\frac{\lambda}{4} \rightarrow ⑤$$

If the no. of elements considerably large, then we can write

$$\therefore d = \frac{\lambda}{4} \rightarrow ⑥$$

→ The Hansen-Woodyard conditions illustrate enhanced directivity if the spacing between the two adjacent elements is approximately $\lambda/4$

The array factor of the n-element array is given by,

$$(AF)_n = \frac{1}{n}\left[\frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}}\right] \rightarrow ⑦$$

But $\psi = \beta d \cos\phi + \alpha$. Putting in eq (7) we get

$$(AF)_n = \frac{1}{n}\left[\frac{\sin\frac{n}{2}(\beta d \cos\phi + \alpha)}{\sin\frac{1}{2}(\beta d\cos\phi + \alpha)}\right] \rightarrow (8)$$

For smaller values of ψ we can approximate $\sin\frac{\psi}{2} \simeq \frac{\psi}{2}$

$$(AF)_n = \frac{1}{n}\left[\frac{\sin\frac{n}{2}(\beta d\cos\phi + \alpha)}{\frac{1}{2}(\beta d \cos\phi + \alpha)}\right]$$

$$= \frac{\sin\frac{n}{2}(\beta d\cos\phi + \alpha)}{\frac{n}{2}(\beta d\cos\phi + \alpha)} \rightarrow (9)$$

Let the progressive phase shift be $\alpha = -pd$, where p is constant.

Then eq (9) becomes

$$(AP)_n = \frac{\sin\frac{n}{2}(\beta d\cos\phi - pd)}{\frac{n}{2}(\beta d\cos\phi - pd)}$$

$$= \frac{\sin\frac{nd}{2}(\beta\cos\phi - p)}{\frac{nd}{2}(\beta\cos\phi - p)} \rightarrow (10)$$

Let $\frac{nd}{2} = q$, then

$$(AP)_n = \frac{\sin q(\beta\cos\phi - p)}{q(\beta\cos\phi - p)} \rightarrow (11)$$

Let $z = q(\beta\cos\phi - p)$.

$\rightarrow (12)$

Hence $(AF)_n = \frac{\sin z}{z}$

The radiation intensity is given by,

$$U(\phi) = |(AF)_n|^2 = \left[\frac{\sin z}{z}\right]^2 \rightarrow (13)$$

(32)

At $\phi = 0$, the radiation intensity is given by,

$$U(\phi = 0) = \left[\frac{\sin Z}{Z}\right]^2 = \left[\frac{\sin q(\beta - p)}{q(\beta - p)}\right]^2 \rightarrow (14)$$

Dividing eq (13) by (14) we get

$$U(\phi)_n = \left[\frac{q(\beta - p)}{\sin q(\beta - p)} \cdot \frac{\sin q(\beta\cos\phi - p)}{q(\beta\cos\phi - p)}\right]^2$$

Let $Z = q(\beta - p)$. then equation becomes

$$U(\phi)_n = \left[\frac{Z}{\sin Z} \cdot \frac{\sin Z}{Z}\right]^2 \rightarrow (15)$$

The directivity of the array factor is given by,

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{U_{max}}{\left(\frac{P_{rad}}{4\pi}\right)} = \frac{U_{max}}{U_0} \rightarrow (16)$$

The average radiation intensity is given by,

$$U_0 = \frac{P_{rad}}{4\pi} = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} U(\theta)\sin\theta \cdot d\theta \cdot d\phi$$

$$\therefore U_0 = \frac{1}{2}\left[\frac{Z}{\sin Z}\right]^2 \int_0^{\pi} \frac{\sin Z}{Z} \sin\theta \cdot d\theta.$$

$$U_0 = \frac{1}{2}\left[\frac{q(\beta - p)}{\sin q(\beta - p)}\right]^2 \int_0^{\pi} \frac{\sin q(\beta\cos\phi - p)}{q(\beta\cos\phi - p)} \cdot \sin\theta \cdot d\theta \rightarrow (17)$$

$$= \frac{1}{2\beta q}\left[\frac{v}{\sin v}\right]^2 \left[\frac{\pi}{2} + \frac{\cos(2v) - 1}{2v} + \sin(2v)\right]$$

$$U_0 = \frac{1}{2\beta q}\left[g(v)\right] \quad \text{where } v = q(\beta - p) \rightarrow (18)$$

When the $g(v)$ is plotted against v, its max value appears when $v = v(\beta - p) = \frac{nd}{2}(\beta - p) = -1.47$.

$$\alpha = -pd = -\left(\beta d + \frac{2.94}{n}\right). \rightarrow (19)$$

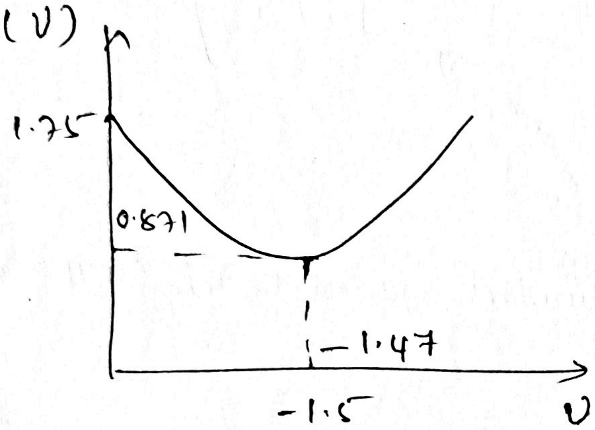
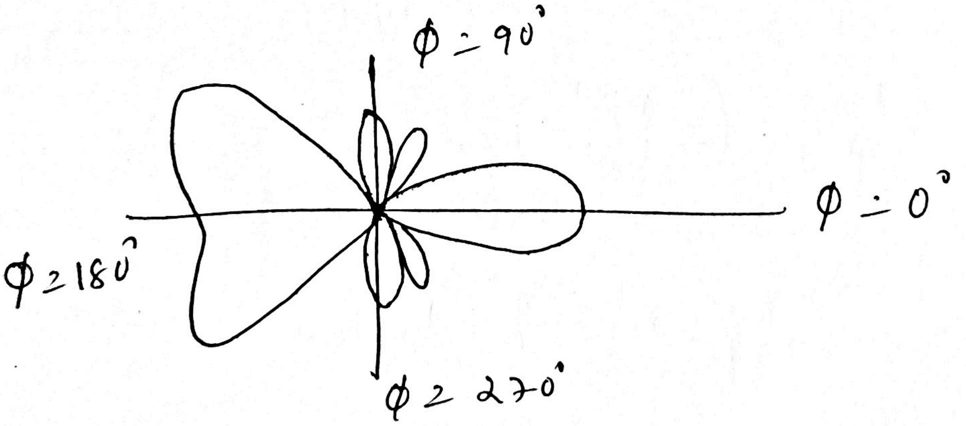


fig: variation of $g(v)$ as function of v.

→ The eq (19) gives the condition for the end fire array with enhanced directivity.

The field pattern for 4 element end fire array with equal amplitude and $\lambda/2$ spacing for increased directivity



field pattern of 4 element end fire array.

Directivity of End fire Array with Increased Directivity.

For an end fire array the increased directivity and maximum radiation in $\phi = 0^\circ$ direction,

$$\text{So,}\quad U_0 = \frac{1}{n\beta d}\left(\frac{\pi}{2}\right)^2\left[\frac{\pi}{2} + \frac{2}{\pi} - 1.8515\right] \quad \rightarrow (1)$$

$$\therefore U_0 = \frac{0.878}{n\beta d}$$

Multiplying numerator and denominator quantities by 2π in equation (1) we get,

$$U_0 = \frac{0.878 \times 2\pi}{n\beta d \times 2\pi} = \frac{1.756\pi}{2\pi n\beta d} = \frac{1.756}{\pi}\left(\frac{\pi}{2n\beta d}\right)$$

$$= 0.599\left(\frac{\pi}{2n\beta d}\right) \quad \rightarrow (2)$$

$$D = \frac{U_{max}}{U_0} = \frac{1}{0.559\left(\frac{\pi}{2n\beta d}\right)}$$

$$\therefore D = \frac{1}{0.559}\left(\frac{2n\beta d}{\pi}\right)$$

$$\beta = \frac{2\pi}{\lambda}$$

$$= 1.789\left(\frac{2n\left(\frac{2\pi}{\lambda}\right)d}{\pi}\right)$$

$$= 1.789\left(\frac{4nd}{\lambda}\right)$$

$$= 1.789\left[4\left(\frac{L}{\lambda}\right)\right] \quad \text{Where } L = (n-1)d \simeq nd \rightarrow (3).$$

	Broad side array	End fire array	Hansen-Woodyard
Directivity	$D = 2n\left(\frac{d}{\lambda}\right) = 2\left(\frac{L}{\lambda}\right)$ $L >> d$	$D = 4n\left(\frac{d}{\lambda}\right) = 4\left(\frac{L}{\lambda}\right)$	$D = 1.789\left[4n\left(\frac{d}{\lambda}\right)\right]$ $= 1.789\left[4\left(\frac{L}{\lambda}\right)\right]$ $L >> d$.

Principle of Pattern Multiplication:

→ The method of obtaining the pattern of arrays is pattern multiplication method. This is shown in below fig.

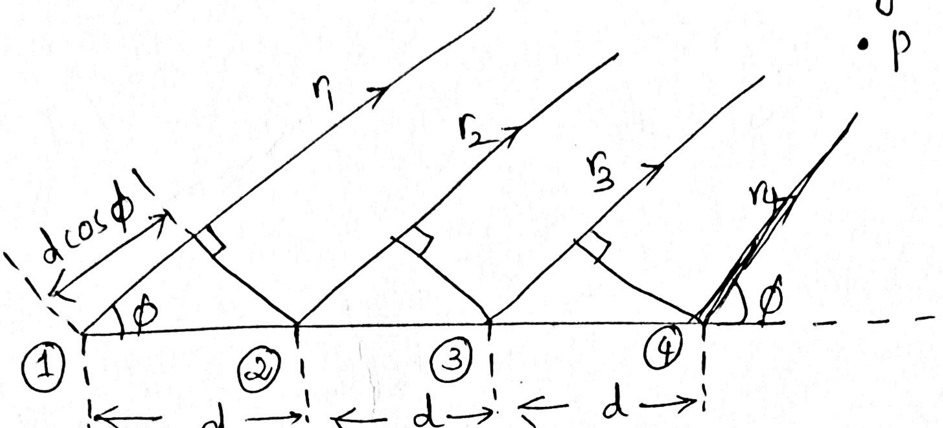


fig: four element linear array.

→ This method is very useful in the design of arrays because it makes possible to draw the patterns of complicated arrays rapidly.

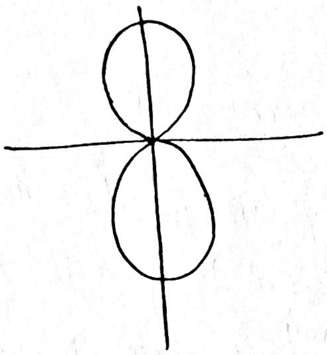
In above fig, we have to used 4 element array of equispaced identical antennas. The spacing between two units be $d = \lambda/2$.

→ We assumed that all the elements are supplied with equal magnitude currents which are in phase.

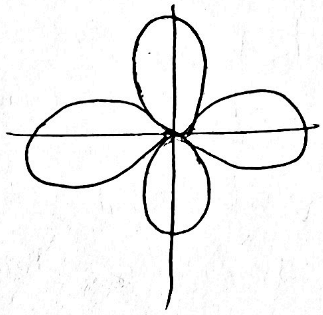
At the point 'P' at which the resultant field has to be obtained is far away, we can assume the radiation from the antenna in the form of parallel lines.

The radiation pattern of antenna (1) and (2) treated to be operating as a single unit is shown in fig (a). Similarly the radiation pattern of the antennas (3) and (4), spaced $\lambda/2$ distance apart and fed with equal current inphase, treated to be operated as single unit is again shown in fig (b). Now instead of considering two separate elements (1) and (2), we can replace it by a single antenna located at a point midway between them $\left(\frac{d}{2}\right)$ as shown

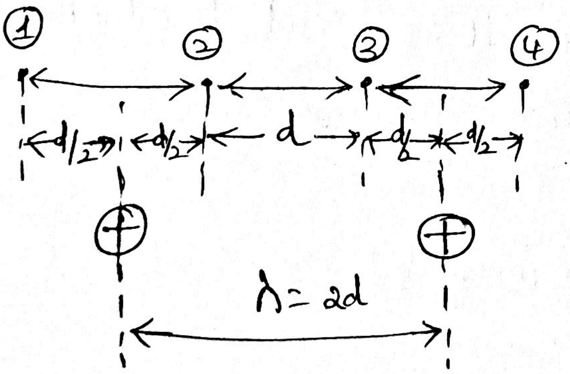
Similarly replacing antennas (3) and (4) by single antenna having same pattern as shown in the fig (c). Now both the antennas have bidirection pattern i.e figure eight pattern spaced distance λ apart from each other, fed with equal currents in phase is shown in fig (b). Now the resultant radiation pattern of four element array can be obtained as the multiplication of pattern as shown in the fig. Note that this multiplication is polar graphical multiplication for different values of ϕ.



(a). Radiation pattern of two antennas spaced at distance $\lambda/2$ and fed with equal currents in phase.

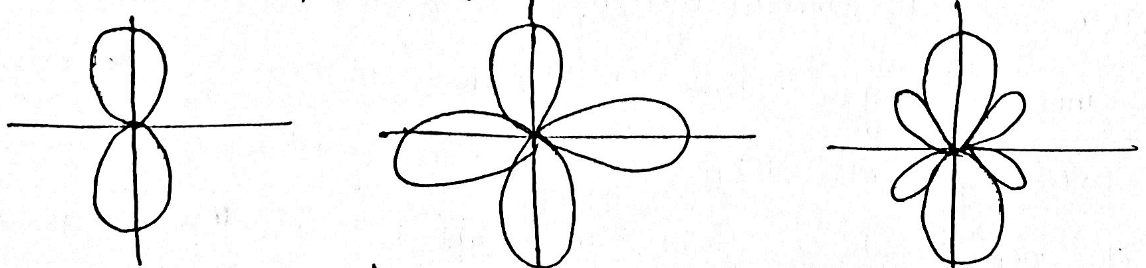


(b). Radiation pattern of two antennas spaced at distance λ and fed with equal currents in phase.



Array of 4 identical elements.
Replacement of array by two single antennas placed at distance λ apart.

(c). Antenna ① and ② and ③ and ④ replaced by single antenna separately.



(d). Multiplication of pattern.

fig: Illustration of pattern multiplication method.