NEC NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

COURSE FILE

Academic year	**: 2022-23**
Department	**: ME**
Course Name	**: B.Tech**
Student's Batch	**: 2022-23**
Regulation	**: R20**
Year and Semester	**: II B.Tech I Semester**
Name of the Subject	: Mechanics of Solids
Subject Code	: R20ME2105
Faculty In charge	: T. Ashok Kumar

T. Ashok

Signature of Faculty

Head of the Department

Professor & Head
Dept. of Mechanical Engineering
NARASARAOPETA ENGINEERING COLLEGE
NARASARAOPET - 522 601, Guntur (Dt), A.P.

NEC NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

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NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

INSTITUTE VISION AND MISSION

NEC NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

INSTITUTE VISION AND MISSION

VISION:

To emerge as a **Centre of excellence** in technical education with a blend of effective **student centric teaching learning** practices as well as **research** for the transformation of **lives and community**.

MISSION:

1. Provide the best class infrastructure to explore the field of engineering and research.
2. Build a passionate and a determined team of faculty with student centric teaching, imbibing experiential and innovative skills.
3. Imbibe lifelong learning skills, entrepreneurial skills and ethical values in students for addressing societal problems.

PRINCIPAL

NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

DEPARTMENT VISION AND MISSION

NARASARAOPETA
NEC ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

DEPARTMENT VISION AND MISSION

VISION:

To strive for making competent **Mechanical Engineering Professionals** to cater the real time needs of Industry and **Research** Organizations of high repute with **Entrepreneurial Skills and Ethical Values.**

MISSION:

M1. To train the students with State of Art Infrastructure to make them industry ready professionals and to promote them for higher studies and research.

M2. To employ committed faculty for developing competent mechanical engineering graduates to deal with complex problems.

M3. To support the students in developing professionalism and make them socially committed mechanical engineers with morals and ethical values.

HOD-ME

NEC NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

PROGRAM EDUCATIONAL OBJECTIVES (PEOs)

AND

PROGRAM SPECIFIC OUTCOMES (PSOs)

NARASARAOPETA
NEC ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

PROGRAM EDUCATIONAL OBJECTIVES (PEOs)

PEO 1: Excel in profession with sound knowledge in mathematics and applied sciences

PEO 2: Demonstrate leadership qualities and team spirit in achieving goals

PEO 3: Pursue higher studies to ace in research and develop as entrepreneurs.

PROGRAM SPECIFIC OUTCOMES (PSOs)

PSO1. The students will be able to apply knowledge of modern tools in manufacturing enabling to conquer the challenges of Modern Industry.

PSO2. The students will be able to design various thermal engineering systems by applying the principles of thermal sciences.

PSO3. The students will be able to design different mechanisms and machine components of transmission of power and automation in modern industry.

HOD-ME

NARASARAOPETA
ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

PROGRAM OUTCOMES
(POs)

NARASARAOPETA
NEC ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

PROGRAM OUTCOMES (POs):

Engineering Graduates will be able to:

1. Engineering knowledge: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.

2. Problem analysis: Identify, formulate, review research literature, and analyse complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.

3. Design/development of solutions: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

4. Conduct investigations of complex problems: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

5. Modern tool usage: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modelling to complex engineering activities with an understanding of the limitations.

6. The engineer and society: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

7. Environment and sustainability: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

8. Ethics: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

9. Individual and team work: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

10. Communication: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

11. Project management and finance: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

12. Life-long learning: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

HOD-ME

NEC NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

BLOOM'S TAXONOMY LEVELS

REVISED Bloom's Taxonomy Action Verbs

Definitions	I. Remembering	II. Understanding	III. Applying	IV. Analyzing	V. Evaluating	VI. Creating
Bloom's Definition	Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers.	Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas.	Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations.	Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria.	Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions.
Verbs	• Choose • Define • Find • How • Label • List • Match • Name • Omit • Recall • Relate • Select • Show • Spell • Tell • What • When • Where • Which • Who • Why	• Classify • Compare • Contrast • Demonstrate • Explain • Extend • Illustrate • Infer • Interpret • Outline • Relate • Rephrase • Show • Summarize • Translate	• Apply • Build • Choose • Construct • Develop • Experiment with • Identify • Interview • Make use of • Model • Organize • Plan • Select • Solve • Utilize	• Analyze • Assume • Categorize • Classify • Compare • Conclusion • Contrast • Discover • Dissect • Distinguish • Divide • Examine • Function • Inference • Inspect • List • Motive • Relationships • Simplify • Survey • Take part in • Test for • Theme	• Agree • Appraise • Assess • Award • Choose • Compare • Conclude • Criteria • Criticize • Decide • Deduct • Defend • Determine • Disprove • Estimate • Evaluate • Explain • Importance • Influence • Interpret • Judge • Justify • Mark • Measure • Opinion • Perceive • Prioritize • Prove • Rate • Recommend • Rule on • Select • Support • Value	• Adapt • Build • Change • Choose • Combine • Compile • Compose • Construct • Create • Delete • Design • Develop • Discuss • Elaborate • Estimate • Formulate • Happen • Imagine • Improve • Invent • Make up • Maximize • Minimize • Modify • Original • Originate • Plan • Predict • Propose • Solution • Solve • Suppose • Test • Theory

derson, L. W., & Krathwohl, D. R. (2001). A taxonomy for learning, teaching, and assessing, Abridged Edition. Boston, MA: Allyn and Bacon.

NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

COURSE OUTCOMES
(COs)

NEC NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING
R20 REGULATION - COURSE OUTCOMES

II B. TECH I SEMESTER

Course Name: MECHANICS OF SOLIDS		Course Code: C215
CO	**After successful completion of this course, the students will be able to:**	
C215.1	**Illustrate** the concepts of stress and strain and thermal stress in members, strain energy due gradually, suddenly applied loads.	
C215.2	**Analyze** shear force diagrams and bending moment diagrams to the different loads for the different support arrangements.	
C215.3	**Determine** shear stresses induced in the beams which are made with different cross sections like rectangular, circular, I, T sections.	
C215.4	**Solve** the equations of slope and deflection for different support arrangements by double integration method, Macaulay's method.	
C215.5	**Determine** stresses induced in cylinders subjected to internal, external pressures.	

NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

COURSE INFORMATION SHEET

NEC	Narasaraopeta Engineering College (Autonomous) Yallmanda(Post), Narasaraopet- 522601 **Department of Mechanical Engineering**

COURSE INFORMATION SHEET

PROGRAMME: B.Tech Mechanical Engineering	
COURSE: **MECHANICS OF SOLIDS**	Semester : **I** CREDITS: **3**
COURSE CODE: **R20ME2105** REGULATION: **Autonomous**	COURSE TYPE (CORE /ELECTIVE / BREADTH/ S&H): **CORE**
COURSE AREA/DOMAIN: **MECHANICS OF MATERIALS**	PERIODS: 6 Per Week.

COURSE PRE-REQUISITES:

C.CODE	COURSE NAME	DESCRIPTION	SEM
R16ME1204	**Engineering Mechanics**	To impart the basic concepts and fundamentals of Engineering Mechanics and the principles of various force systems under static and dynamic conditions. To develop the problem solving skills of engineering mechanics essential for mechanical engineering.	**I/II**

COURSE OUTCOMES:

SNO	Course Outcome Statement
CO1	**CO 1: Illustrate** the concepts of stress and strain and thermal stress in members, strain energy due gradually, suddenly applied loads.
CO2	**CO 2: Analyze** shear force diagrams and bending moment diagrams to the different loads for the different support arrangements.
CO3	**CO 3: Determine** shear stresses induced in the beams which are made with different cross sections like rectangular, circular, I, T sections.
CO4	**CO 4: Solve** the equations of slope and deflection for different support arrangements by double integration method, Macaulay's method.
CO5	**CO 5: Determine** stresses induced in cylinders subjected to internal, external pressures.

SYLLABUS:

UNIT	DETAILS
I	**SIMPLE STRESSES & STRAINS:** Concept of stress and strain- Types of stresses & strains- tensile, compressive, shear –Hooke's law – stress – strain diagram for mild steel – Factor of safety – Lateral strain, Poisson's ratio & volumetric strain – Bars of varying cross section – composite bars. Elastic moduli and the relationship between them. Temperature stresses. **STRAIN ENERGY & IMPACT LOADING:** Strain energy - Resilience – Stress due to various types of axial loads- Gradually applied suddenly applied and impact loadings.
II	**SHEAR FORCE AND BENDING MOMENT:** Definition of beam – Types of beams - concepts of SF & BM with point load, Uniformly Distributed Load, uniformly varying loads and combination of these loads, Point of contra flexure, Relation between S.F., B.M and rate of loading at a section of a beam.
III	**FLEXURAL STRESSES:** Theories of simple bending – Assumptions - derivation of bending equation, - Neutral axis, Moment of resistance, determination of bending stresses, section modulus of rectangular and circular sections (solid and hallow), I & T sections. **SHEAR STRESSES:** Shear stress distribution across various beams sections- rectangular, circular, I and T sections.
IV	**DEFLECTION OF BEAMS:** Member bending into a circular arc –slope, deflection and radius of curvature. Determination of slope and deflection for cantilever and simply supported beams subjected to point loads and U.D.L by Double integration method, Macaulay's method, Moment area method.
V	**THIN CYLINDERS:** Thin cylinders - longitudinal and circumferential stresses, Derivation of formulae and calculations of hoop stress, longitudinal stress in a cylinder subjected to internal pressure. **THICK CYLINDERS:** Derivation of formulae for radial and hoop stresses, Lame's equation, and cylinders subjected to inside & outside pressure, compound cylinders. **TORSION OF SHAFTS:** Theory of pure torsion, Torsional moment of resistance, derivation of Torsion equation, assumptions in the theory of pure torsion, polar modulus, power transmitted by a circular shaft, shafts in series, shafts in parallel.

TEXT BOOKS	
T	**BOOK TITLE/AUTHORS/PUBLISHER**
T1	Strength of materials by Bhavikatti
T2	Strength of materials by R. K. Bansal
REFERENCE BOOKS	
R	**BOOK TITLE/AUTHORS/PUBLISHER**
R1	Mechanics of Materials (In Si Units) by Beer and Johnson
R2	Strength of Materials (Mechanics of Materials) by James M.Gere and Barry J.Goodno
R3	Strength of Materials (Mechanics of Solids) by R.K. Rajput

TOPICS BEYOND SYLLABUS/ADVANCED TOPICS:

SNO	DESCRIPTION	Associated PO & PSO
1	Statically Indeterminate Structures	PO1, PO2,PO3, PO5 & PSO1
2	Finite Element Analysis	PO1, PO2,PO3, PO5 & PSO1

WEB SOURCE REFERENCES:

1	https://nptel.ac.in/courses/105102090/
2	https://nptel.ac.in/courses/105104160/
3	https://nptel.ac.in/courses/105106172/
4	https://nptel.ac.in/courses/105106116/
5	https://nptel.ac.in/courses/112107146/
6	https://nptel.ac.in/courses/105105108/
7	https://nptel.ac.in/courses/112107147/
8	https://nptel.ac.in/courses/112106141/
9	https://nptel.ac.in/courses/112101095/

DELIVERY/INSTRUCTIONAL METHODOLOGIES:

✔ Chalk & Talk	✔ PPT	✔ Active Learning
✔ Web Resources	✔ Students Seminars	✔ Case Study
☐ Blended Learning	✔ Quiz	☐ Tutorials
☐ Project based learning	☐ NPTEL/MOOCS	☐ Simulation
☐ Flipped Learning	☐ Industrial Visit	☐ Model Demonstration
☐ Brain storming	☐ Role Play	☑ Virtual Labs

MAPPING CO'S WITH PO'S

CO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3
CO1	3	2	1	2	-	-	-	-	-	-	-	-	-	-	3
CO2	3	3	2	3	-	-	-	-	-	-	-	-	-	-	3
CO3	3	3	2	3	-	-	-	-	-	-	-	-	-	-	3
CO4	3	3	2	3	-	-	-	-	-	-	-	-	-	-	3
CO5	3	2	2	2	-	-	-	-	-	-	-	-	-	-	3
Average	3.00	2.67	1.67	2.67	-	-	-	-	-	-	-	-	-	-	3.00

MAPPING COURSE WITH POs & PSOs

Course	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3
R16ME2102	3.00	2.67	1.67	2.67	-	-	-	-	-	-	-	-	-	-	3.00

Course Outcome Assessment Methods			Weightages		Final Course Outcome (100%)
Direct Assessment	Cumulative Internal Examinations (CIE)	Descriptive Test	30%	90%	
		Objective Test			
		Assignment Test			
	Semester End Examinations (SEE)		70%		
Indirect Assessment	Course End Survey			10%	

Rubrics for overall attainment of course outcomes:
If 50% of the students crossed 50% of the marks: Attainment Level 1
If 60% of the students crossed 50% of the marks: Attainment Level 2
If 70% of the students crossed 50% of the marks: Attainment Level 3

Course Instructor **Course Coordinator** **Module Coordinator** **Head of the Department**

ANNEXURE I:

(A) PROGRAM OUTCOMES (POs) Engineering Graduates will be able to:

1. Engineering knowledge: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.

2. Problem analysis: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.

3. Design/development of solutions: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

4. Conduct investigations of complex problems: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

5. Modern tool usage: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.

6. The engineer and society: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

7. Environment and sustainability: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

8. Ethics: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

9. Individual and team work: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

10. Communication: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

11. Project management and finance: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

12. Life-long learning: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

(B) PROGRAM SPECIFIC OUTCOMES (PSOs) :

PSO1.The students will be able to understand the modern tools of machining which gives them good expertise on advanced manufacturing methods.

PSO2.The students will be able to design different heat transfer devices with emphasis on combustion and power production.

PSO3.The students are able to design different mechanisms and machine components suitable to automation industry.

Cognitive levels as per Revised Blooms Taxonomy:

Cognitive Domain	LEVEL	Key words
Remember	K1	Defines, describes, identifies, knows, labels, lists, matches, names, outlines, recalls, recognizes, reproduces, selects, states.
Understand	K2	Comprehends, converts, defends, distinguishes, estimates, explains, extends, generalizes, gives an example, infers, interprets, paraphrases, predicts, rewrites, summarizes, translates.
Apply	K3	Applies, changes, computes, constructs, demonstrates, discovers, manipulates, modifies, operates, predicts, prepares, produces, relates, selcts, shows, solves, uses.
Analyse	K4	Analyzes, breaks down, compares, contrasts, diagrams, deconstructs, differentiates, discriminates, distinguishes, identifies, illustrates, infers, outlines, relates, selects, separates.
Evaluate	K5	Appraises, compares, concludes, contrasts, criticizes, critiques, defends, describes, discriminates, evaluates, explains, interprets, justifies, relates, summarizes, supports
Create	K6	Categorizes, combines, compiles, composes, creates, devises, designs, explains, generates, modifies, organizes, plans, rearranges, reconstructs, relates, reorganizes, revises, rewrites, summarizes, tells, write

Unit wise Sample assessment questions

COURSE OUTCOMES: Students are able to

CO 1: **Illustrate** the concepts of stress and strain and thermal stress in members, strain energy due gradually, suddenly applied loads.

CO 2: **Analyze** shear force diagrams and bending moment diagrams to the different loads for the different support arrangements.

CO 3: **Determine** shear stresses induced in the beams which are made with different cross sections like rectangular, circular, I, T sections.

CO 4: **Solve** the equations of slope and deflection for different support arrangements by double integration method, Macaulay's method.

CO 5: **Determine** stresses induced in cylinders subjected to internal, external pressures.

S NO	QUESTION	KNOWLEDGE LEVEL	CO
	UNIT I		
1	Explain about Stress- Strain diagram of (Mild Steel) ductile materials under tensile test?	K1	CO1
2	Derive the relation among the three elastic constants (Modulus of Elasticity and Shear Modulus, and 'Bulk Modulus)?	K2	CO1
3	Explain about different types of Stresses and Strains?	K1	CO1
4	Define Strain Energy and derive the expressions for stress due to gradually applied loads, suddenly applied loads and Impact loads?	K2	CO1
5	A reinforced concrete column 300mmX300mm in section. The column in reinforced with 8 steel bars of 20mm diameter. The column carries a load of 360KN. Find the stresses in the concrete and steel bars? Take E (Steel) = $2.1 X 10^5$ N/mm^2, E (Concrete) = $1.4 X 10^4$ N/mm^2?	K3	CO1
	UNIT 2		
1	Explain about following the terms: i) Shear Force ii) Shear Force Diagram iii) Bending Moment iv) bending Moment Diagram v) Pont of Contra Flexure.	K1	CO2
2	A cantilever of length L carrying a point load **W** at free end and a U.D.L. ω per unit run over the whole length. Draw S.F. and B.M. diagrams?	K2	CO2
3	A simply supported beam of length L carrying a U.D.L. of w per unit run over the whole span. Draw S.F. and B.M. diagrams?	K2	CO2
4	A beam AB, 10m long has supports at its ends A&B. It carries a point load of 5KN at 3m from A and a point load of 5KN at 7m from A and a uniformly	K3	CO2

	distributed load of 1KN/m between the two point loads. Draw SF and BM Diagrams for the beam?		
UNIT 3			
1	Derive the Bending Moment equation and mention the assumptions made in the theory of simple bending?	**K2**	**CO3**
2	A cast iron bracket subjected to bending has the cross-section of I-form with unequal flanges. The total depth of the section is 280mm and the metal is 40mm thick throughout. The top flange is 200mm wide and the bottom flange is 120mm wide. Find the position of the neutral axis and moment of inertia of the section about the neutral axis and the maximum bending moment that should be imposed on this section if the tensile stress in the top flange is not to exceed 20N/mm2. What is then the value of the maximum compressive stress in the bottom flange?	**K2&K3**	**CO3**
3	Prove that the shear stress at any point in the cross section of a beam which is subjected to a shear force F, is given by $\tau = F \times \frac{A\bar{y}}{I \times b}$	**K2**	**CO3**
4	Derive an expression for the shear stress at any point in a circular section of a beam, which is subjected to a shear force F. And prove that the maximum shear stress in a circular section of a beam is 4/3 times the average shear stress.	**K2**	**CO3**
UNIT 4			
1	Derive an expression for the slope and deflection of a beam subjected to uniform bending moment (Bending into Circular ARC)?	**K2**	**CO4**
2	Find the deflection of a simply supported beam of length L carrying a uniform distributed load of w per unit length? (Double Integration Method)	**K3**	**CO4**
3	Find the deflection of a simply supported beam of length L carrying a point load W at a distance 'a' from left support and at a distance 'b' from right supports by using MACAULAY'S METHOD?	**K3**	**CO4**
4	Find the slope and deflection of a simply supported beam AB of length L and carrying a U.D.L. of w per unit length over the entire span by any method.	**K2**	**CO4**
UNIT 5			
1	Derive the Expression for Hoop stress and longitudinal stress in a thin cylinder shells subjected to internal fluid pressure?	**K2**	**CO5**
2	Derive the Expression for effect of internal pressure on the dimensions of a thin cylindrical shell?	**K2**	**CO5**
3	A cylindrical vessel is 1.5m diameter and 4m long is closed at ends by rigid plates. It is subjected to an internal pressure of 3N/mm^2. If the maximum principal	**K3**	**CO5**

	stress is not to exceed 150 N/ mm^2, find the thickness of the shell. Assume E = 2 X 10^5 N /mm^2 and poisson's ratio = 0.25. Find the changes in diameter, length and volume of the shell?		
4	Derive the expressions for the stresses in a thick cylindrical shell subjected to an internal fluid pressure? (OR) Derive the Lame's equations?	K2	CO5
5	A compound cylinder is made by shrinking a cylinder of external diameter 300mm and internal diameter of 250mm over another cylinder of external diameter 250mm and internal diameter 200mm. the radial pressure at the junction after shrinking is 8 N/ mm2. Find the final stresses set up across the section, when the compound cylinder is subjected to an internal fluid pressure of 84.5 N/ mm^2?	K3	CO5

Model Question Paper-I

Code: R20ME2105

R20

Narasaraopeta Engineering College

(Autonomous)

Yallmanda(Post), Narasaraopet- 522601

II B. Tech I Semester Regular Examinations

MECHANICS OF SOLIDS

MECHANICAL ENGINEERING

[OUTCOME BASED EDUCATION PATTERN]

Time: 3 Hrs **Max. Marks: 70**

Note: 1. Answering the question in Part-A is compulsory

Execution Plan

Sl. No	Activities	Time (Minutes)
1	To study the Question Paper and choose to attempt	5
2	Part-A 33Minutes x 5Questions	165
3	Quick revision & Winding up	10
	Total	180

PART–A (12 Marks)

Answer ALL Questions.

PART–A (70Marks)

Answer any FIVE Questions

S. No		Question	Cognitive Level	CO	Marks
1	a	A tensile test was conducted on a mild steel bar. The following data was obtained from the test: Diameter of the steel bar = 20mm, Gauge length of the bar = 150mm, Load at the elastic limit = 200KN, Extension at a load of 100KN = 0.2mm, Maximum load= 300KN, Total extension = 50mm, Diameter of the rod at failure = 12.5mm. **Determine**: i) The Young's Modulus, ii) Stress at the elastic limit, iii) Ultimate Stress, iv) Percentage elongation, v) Percentage decrease in area. OR	K3	1	14
	b	A steel rod 100mm in diameter and 2.5m long is subjected to a suddenly applied pull of 400KN. **Determine** the strain energy if E = 200 KN/mm^2 ii) Why Nominal Breaking stress is less than the ultimate strength as obtained from uniaxial tensile test of a ductile material?	K3	1	7+7
2	a	A beam 8m long is symmetrically supported over a 4m span. The overhanging ends at the left and right carry point loads of 60KN and 40KN respectively, while the length between supports carries a uniformly distributed load of 20KN. **Draw** shear force and bending moment diagrams. OR	K2	2	14

	b	i)Define Shear force diagram and Bending Moment Diagram? **ii) Derive** the Bending Moment equation and mention the assumptions made in the theory of simple bending?	**K2**	**3**	7+7
3	**a**	A cast iron bracket subjected to bending has the cross-section of I-form with unequal flanges. The total depth of the section is 280mm and the metal is 40mm thick throughout. The top flange is 200mm wide and the bottom flange is 120mm wide. Find the position of the neutral axis and moment of inertia of the section about the neutral axis and the maximum bending moment that should be imposed on this section if the tensile stress in the top flange is not to exceed 20N/mm2. What is then the value of the maximum compressive stress in the bottom flange? **OR**	**K3**	**3**	**14**
	b	Derive an expression for the shear stress at any point in a circular section of a beam, which is subjected to a shear force F. And prove that the maximum shear stress in a circular section of a beam is 4/3 times the average shear stress.	**K3**	**3**	**14**
4	**a**	What is Macaulay's Method? Where it is used? Find the expression for deflection at any section of a simply supported beam with an eccentric point load using Macaulay's method. **OR**	**K2**	**4**	**14**
	b	**Find** the deflection of a simply supported beam of length L carrying a uniform distributed load of w per unit length? (Double Integration Method)	**K3**	**4**	**14**
5	**a**	Make a neat sketch of a circular shaft subjected to a twisting moment? **Derive** the torsion formula? What assumptions are taken while deriving formula for a circular shaft? **OR**	**K3**	**4**	**14**
	b	A compound cylinder is made by shrinking a cylinder of external diameter 300mm and internal diameter of 250mm over another cylinder of external diameter 250mm and internal diameter 200mm. the radial pressure at the junction after shrinking is 8 N/ mm2. Find the final stresses set up across the section, when the compound cylinder is subjected to an internal fluid pressure of 84.5 N/ mm^2?	**K3**	**5**	**14**

NEC NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

ACADEMIC CALENDAR

NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

ACADEMIC CALENDAR
(B.Tech. 2021 Admitted Batch, Academic Year 2022-23)

2021 Batch 2nd Year 1st Semester			
Description	**From Date**	**To Date**	**Duration**
Commencement of Class Work	5-09-2022		
1st Spell of Instructions	5-09-2022	22-10-2022	7 Weeks
Assignment Test-I	26-9-2022	31-09-2022	
I Mid examinations	24-10-2022	29-10-2022	1 Week
2nd Spell of Instructions	31-10-2022	17-12-2022	7 Weeks
Assignment Test-II	21-11-2022	26-11-2022	
II Mid examinations	19-12-2022	24-12-2022	1 Week
Preparation & Practicals	26-12-2022	31-12-2022	1Week
Semester End Examinations	02-01-2023	14-01-2023	2 Weeks
2021 Batch 2nd Year 2nd Semester			
Commencement of Class Work	16-01-2023		
1st Spell of Instructions	16-01-2023	04-03-2023	7 Weeks
Assignment Test-I	06-02-2023	11-02-2023	
I Mid examinations	06-03-2023	11-03-2023	1 Week
2nd Spell of Instructions	13-03-2023	29-04-2023	7 Weeks
Assignment Test-II	03-04-2023	08-04-2023	
II Mid examinations	01-05-2023	06-05-2023	1 Week
Preparation & Practicals	08-05-2023	13-05-2023	1 Week
Semester End Examinations	15-05-2023	27-05-2023	2 Weeks
Commencement of 3rd Year 1st Sem Class Work	05-06-2023		

PRINCIPAL

NARASARAOPETA ENGINEERING COLLEGE
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DEPARTMENT OF MECHANICAL ENGINEERING

TIME TABLE

NARASARAOPETA ENGINEERING COLLEGE: NARASARAOPET (AUTONOMOUS)
DEPARTMENT OF MECHANICAL ENGINEERING
II B.TECH I SEM TIME TABLE
Section-A

ROOM NO: 1312

Wef: 24/07/2023

	1	2	BREAK	3	4		5	6	7
TIMINGS	9.10-10.00	10.00-10.50	10.50-11.00	11.00-11.50	11.50-12.40	12.40-1.30	1.30-2.20	2.20-3.10	3.10-4.00
MON	NMT		-	MOS		L	FM&HM	M&I	
TUE	TD	SM/ M&I LAB				U	MOS		FM&HM
WED	TD			FM&HM		N	NM&T	ES	M&I
THU	MOS			TD		C	MOS&M/ FM&HM LAB		
FRI	ES	MOS&M/ FM&HM LAB				H	M&I		TD
SAT	FM&HM			NM&T			SM/ M&I LAB		

CODE	SUBJECT	FACULTY
NM&T	Numerical Methods and Transformations	Mr.MD.Shareef
FM&HM	Fluid Mechanics &Hydraulic Machinery	Mr.M.Venkaiah
M&I	Metrology& Instrumentation	Mr.K.Kiran Chand
TD	Thermodynamics	Dr.M.Naveen Kumar
MOS	Mechanics of Solids	Mr.T.Ashok Kumar
FM&HM LAB	Fluid Mechanics &Hydraulic Machinery Lab	Dr.M.Sreenivasa Kumar/Mr.M.Venkaiah
MOS&M LAB	MOS & Metallurgy Lab	Mr.G.Bhargav/B.Ajay Kumar
M&I LAB	Metrology& Instrumentation Lab	Dr.M.Rama Kotaiah/T.N.V.Mahesh
SM	Solid Modelling	Mr.K.John Babu/R.Chinna Rao
ES	Environmental Studies	Dr.K.Srinivasulu

2nd YEAR INCHARGE& CLASS TEACHER: Dr. M.VENKANNA BABU

MENTORS: A.Pavan Kumar / K.Kiran Chand

Signature of HOD

Signature of Principal

NARASARAOPETA
NEC ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

SYLLABUS COPY

II B.TECH I SEMESTER	L	T	P	INTERNAL MARKS	EXTERNAL MARKS	TOTAL MARKS	CREDITS
	2	1	0	30	70	100	3
Code: R20ME2105	**MECHANICS OF SOLIDS**						

COURSE OBJECTIVES:

- To understand stresses and deformation in a member due to an axial loading. Also to estimate the thermal stresses, strains and strain energy in members subjected to axial loading.
- Understand the concept of shear force and bending moment with respect to beams and to draw the shear force and bending moment diagrams.
- Understand bending and shear stresses in beams of various cross sections under different loading conditions.
- Understand and analyze beam deflections using various methods like double integration approach, Macaulay's method.
- Study the pressure vessels, their classification and to estimate various stresses such as radial, circumferential, longitudinal and shrinkage induced in them, concepts of torsion.

COURSE OUTCOMES:

After successful completion of this course, the students will be able to:

CO 1: Illustrate the concepts of stress and strain and thermal stress in members, strain energy due gradually, suddenly applied loads.

CO 2: Analyze shear force diagrams and bending moment diagrams to the different loads for the different support arrangements.

CO 3: Determine shear stresses induced in the beams which are made with different cross sections like rectangular, circular, I, T sections.

CO 4: Solve the equations of slope and deflection for different support arrangements by double integration method, Macaulay's method.

CO 5: Determine stresses induced in cylinders subjected to internal, external pressures.

UNIT– I

SIMPLE STRESSES & STRAINS: Concept of stress and strain- Types of stresses & strains- tensile, compressive, shear –Hooke's law – stress – strain diagram for mild steel – Factor of safety – Lateral strain, Poisson's ratio & volumetric strain – Bars of varying cross section – composite bars. Elastic moduli and the relationship between them. Temperature stresses.

STRAIN ENERGY & IMPACT LOADING: Strain energy - Resilience – Stress due to various types of axial loads- Gradually applied suddenly applied and impact loadings.

UNIT–II

SHEAR FORCE AND BENDING MOMENT: Definition of beam – Types of beams - concepts of SF & BM with point load, Uniformly Distributed Load, uniformly varying loads and combination of these loads, Point of contra flexure, Relation between S.F., B.M and rate of loading at a section of a beam.

UNIT–III

FLEXURAL STRESSES: Theories of simple bending – Assumptions - derivation of bending equation, - Neutral axis, Moment of resistance, determination of bending stresses, section modulus of rectangular and circular sections (solid and hallow), I & T sections.

SHEAR STRESSES: Shear stress distribution across various beams sections- rectangular, circular, I and T sections.

UNIT-IV

DEFLECTION OF BEAMS: Member bending into a circular arc –slope, deflection and radius of curvature. Determination of slope and deflection for cantilever and simply supported beams subjected to point loads and U.D.L by Double integration method, Macaulay's method, Moment area method.

UNIT-V

THIN CYLINDERS: Thin cylinders - longitudinal and circumferential stresses, Derivation of formulae and calculations of hoop stress, longitudinal stress in a cylinder subjected to internal pressure

THICK CYLINDERS: Derivation of formulae for radial and hoop stresses, Lame's equation, cylinders subjected to inside & outside pressure, compound cylinders.

TORSION OF SHAFTS: Theory of pure torsion, Torsional moment of resistance, derivation of Torsion equation, assumptions in the theory of pure torsion, polar modulus, power transmitted by a circular shaft, shafts in series, shafts in parallel.

TEXT BOOKS:

1. Mechanics of Materials by B.C. Punmia, Ashok Kumar Jain, Arun Kumar
2. Strength of materials by S. Ramamrutham, Dhanpat Rai Publications.
3. Strength of materials by R. K. Bansal, Lakshmi publications

REFERENCE BOOKS:

1. Introduction to solid mechanics by Irving H. Shames, James M. Pitarresi, Pearson Publications.
2. Mechanics of Materials (In Si Units) by Beer and Johnson, Tata McGraw-Hil.
3. Strength of Materials (Mechanics of Materials) by James M.Gere and Barry J.Goodno, PWS-KENT Publishing Company, 1990
4. Strength of Materials (Mechanics of Solids) by R.K. Rajput, S.Chand Publications.

WEB REFERENCES:

1. URL: https://nptel.ac.in/courses/112107146/23
2. https://nptel.ac.in/courses/105105108/19
3. https://nptel.ac.in/courses/112105125/pdf/module-9%20lesson-2.pdf
4. https://nptel.ac.in/courses/112105164/36

E-BOOKS: 1. https://easyengineering.net/a-textbook-of-strength-of-materials/

NARASARAOPETA
ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

LESSON PLAN

Narasaraopeta Engineering College
(Autonomous)
Yallmanda(Post), Narasaraopet- 522601

DEPARTMENT OF MECHANICAL ENGINEERING
LESSON PLAN

Course Code	Course Title (Regulation)	Sem	Branch	Contact Periods/Week	Sections
R20ME2105	MECHANICS OF SOLIDS	I	Mechanical Engineering	6	A

SNO	Course Outcome Statement
CO1	**Illustrate** the concepts of stress and strain and thermal stress in members, strain energy due gradually, suddenly applied loads. **(K1&K2)**
CO2	**Analyze** shear force diagrams and bending moment diagrams to the different loads for the different support arrangements. **(K3&K4)**
CO3	**Determine** shear stresses induced in the beams which are made with different cross sections like rectangular, circular, I, T sections. **(K3&K4)**
CO4	**Solve** the equations of slope and deflection for different support arrangements by double integration method, Macaulay's method. **(K5)**
CO5	**Determine** stresses induced in cylinders subjected to internal, external pressures. **(K3&K4)**

Unit No	Outcome		Topics/Activity	Ref Text book	Total Periods	Delivery Method
1	CO1. **Illustrate** the concepts of stress and strain and thermal stress in members, strain energy due gradually, suddenly applied loads. **(K1&K2)**		**Unit-1. SIMPLE STRESSES & STRAINS**			
		1.1	Explanation about Elasticity and plasticity, types of stresses & strains-tensile ,compressive, shear –Hooke's law	T2,R1	1	Chalk & Talk, PPT
		1.2	Stress – strain diagram for mild steel – Working stress – Factor of safety – Lateral strain, Poisson's ratio & volumetric strain	T2,R1	1	Chalk & Talk, PPT
		1.3	Problems on Bars of varying section – composite bars.	T2,R1	2	Chalk & Talk, PPT
		1.4	Relation between elastic constants-E,G&K.	T2,R1	2	Chalk & Talk, PPT
		1.5	Determination of Temperature stresses.	T2,R1	2	Chalk & Talk, PPT
		1.6	Explanation about Principal planes and principal stresses and Mohr's circle.	T2,R1	2	Chalk & Talk, PPT
		1.7	Determination of Strain Energy- sudden, gradual and Impact loadings.	T2,R1	2	Chalk & Talk, PPT
2	CO2. **Analyze** shear force diagrams and bending moment diagrams to the different loads for the different support arrangements. **(K3&K4)**		**Unit-2. SHEAR FORCE AND BENDING MOMENT**			
		2.1	Types of beams-cantilever beam, simply supported beam, overhanging beam	T2,R1	2	Chalk & Talk, PPT
		2.2	S.F and B.M diagrams for cantilever beams subjected to point loads, Uniform Distributed Load (U.D.L), uniformly varying loads and combination of these loads	T2,R1	2	Chalk & Talk, PPT
		2.3	S.F and B.M diagrams for simply supported beams subjected to point loads, Uniform Distributed Load (U.D.L), uniformly varying loads and combination of these loads	T2,R1	2	Chalk & Talk, PPT

		2.4	S.F and B.M diagrams for overhanging beams subjected to point loads, Uniform Distributed Load (U.D.L), uniformly varying loads and combination of these loads	T2,R1	2	Chalk & Talk, PPT
		2.5	Problems on S.F and B.M diagrams for cantilever, simply supported and overhanging beams subjected to point loads, Uniform Distributed Load (U.D.L), uniformly varying loads and combination of these loads	T2,R1	2	Chalk & Talk, PPT
		2.6	Relation between S.F., B.M and rate of loading at a section of a beam.	T2,R1	2	Chalk & Talk, PPT
	CO 3. **Determine** shear stresses induced in the beams which are made with different cross sections like rectangular, circular, I, T sections. **(K3&K4)**		**Unit-3. FLEXURAL STRESSES & SHEAR STRESSES**			
		3.1	Theories of bending, Assumptions and derivation of bending equation	T2,R1	3	Chalk & Talk, PPT,
		3.2	Determination bending stresses–section modulus of rectangular and circular sections.	T2,R1	3	Chalk & Talk, PPT
		3.3	Determination bending stresses of I, T, section.	T2,R1	3	Chalk & Talk, PPT
		3.4	Shear stress distribution across various beams sections like rectangular, circular, I, T sections.	T2,R1	3	Chalk & Talk, PPT
			MID I EXAMINATION DURING SEVENTH WEEK			
4	CO 4. **Solve** the equations of slope and deflection for different support arrangements by double integration method, Macaulay's method. **(K5)**		**Unit-4. DEFLECTION OF BEAMS**			
		4.1	Bending into a circular arc, slope, deflection and radius of curvature.	T2, R1	3	Chalk & Talk, PPT Chalk & Talk, PPT
		4.2	Double integration and Macaulay's methods	T2, R1	3	Chalk & Talk, PPT
		4.3	Determination of slope and deflection for cantilever and simply supported beams subjected to point loads, U.D.L	T2, R1	3	Chalk & Talk, PPT
		4.4	Mohr's theorems and Moment area method.	T2, R1	3	
5	CO 5. **Determine** stresses induced in cylinders subjected to internal, external pressures. **(K3&K4)**		**Unit 5. THIN CYLINDERS & THICK CYLINDERS**			
		5.1	Derivation of formula for longitudinal and circumferential stresses	T2, R1	2	Chalk & Talk, PPT
		5.2	Determination of volumetric strains, changes in diameter, and volume of thin cylinders.	T2, R1	2	Chalk & Talk, PPT
		5.3	Determination of lame's equation for cylinders subjected to inside & outside pressures.	T2, R1	2	Chalk & Talk, PPT
		5.4	Problems on Compound cylinders.		2	Chalk & Talk, PPT
			TORSION & COLUMNS			
		5.5	Derivation of Torsion of Circular shafts.	T2, R1	2	Chalk & Talk, PPT
		5.6	Problems on Shafts in series, Shafts in parallel.	T2, R1	2	Chalk & Talk, PPT
				Total	60	

MID II EXAMINATION DURING FOURTEENTH WEEK
END EXAMINATIONS

TEXT BOOKS

T	BOOK TITLE/AUTHORS/PUBLISHER
T1	Strength of materials by Bhavikatti
T2	Strength of materials by R. K. Bansal

REFERENCE BOOKS

R	BOOK TITLE/AUTHORS/PUBLISHER
R1	Mechanics of Materials (In Si Units) by Beer and Johnson
R2	Strength of Materials (Mechanics of Materials) by James M.Gere and Barry J.Goodno
R3	Strength of Materials (Mechanics of Solids) by R.K. Rajput

Faculty **HOD** **Principal**

NEC NARASARAOPETA ENGINEERING COLLEGE
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DEPARTMENT OF MECHANICAL ENGINEERING

CO-POs & CO-PSOs MAPPING (COURSE ARTICULATION MATRIX)

NARASARAOPETA
NEC ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING
COURSE ARTICULATION MATRIX
R20-REGULATION

Explanation of Course Articulation Matrix Table to be ascertained:

- Course Articulation Matrix correlates the individual COs of a course with POs and PSOs.
- The Course Outcomes are mapped with POs and PSOs in the scale of 1 to 3.
- The strength of correlation is indicated as **3** for **Substantial (High)** correlation, **2** for **Moderate (Medium)** correlation, and 1 for **Slight (Low)** correlation.

II B.Tech I SEMESTER

Course Code: C215	Course Name: MECHANICS OF SOLIDS														
COs	POs&PSOs														
	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3
C215.1	3	3	2	-	-	-	-	-	-	-	-	-	-	-	3
C215.2	3	3	2	2	-	-	-	-	-	-	-	-	-	-	3
C215.3	3	3	2	-	-	-	-	-	-	-	-	-	-	-	3
C215.4	3	3	2	2	-	-	-	-	-	-	-	-	-	-	3
C215.5	3	3	2	-	-	-	-	-	-	-	-	-	-	-	3
C215	3.00	3.00	2.00	2.00	-	-	-	-	-	-	-	-	-	-	3.00

NARASARAOPETA ENGINEERING COLLEGE
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DEPARTMENT OF MECHANICAL ENGINEERING

WEB REFERENCES

WEB SOURCE REFERENCES:

1	https://nptel.ac.in/courses/105102090/
2	https://nptel.ac.in/courses/105104160/
3	https://nptel.ac.in/courses/105106172/
4	https://nptel.ac.in/courses/105106116/
5	https://nptel.ac.in/courses/112107146/
6	https://nptel.ac.in/courses/105105108/
7	https://nptel.ac.in/courses/112107147/
8	https://nptel.ac.in/courses/112106141/
9	https://nptel.ac.in/courses/112101095/

NEC NARASARAOPETA ENGINEERING COLLEGE
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STUDENT'S ROLL LIST

NEC **NARASARAOPETA ENGINEERING COLLEGE**

(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

21 BATCH II-II STUDENT LIST

S.No	HT No.	Student Name
1	21471A0301	ANGIREKULA VEERANJANEYULU
2	21471A0302	BATTULA YUVA RAJU
3	21471A0303	BOMMIREDDY VENU
4	21471A0304	JEEDIMALLA SRI LAKSHMI NILENDRA
5	21471A0305	KONATHAM VENKATA NARAYANA
6	21471A0306	KUNCHALA ANKA RAO
7	21471A0307	PATHAN RIYAZ
8	21471A0309	SASAPU SAI SANTOSH
9	21471A0310	SOUBHAGYAPU SAI RAM
10	21471A0311	MUNAGA RAMANJANEYULU
11	21471A0312	YELCHURI HEMALATHA MEGHANA
12	21471A0314	ALLAM TIRUMALA RAJU
13	22475A0301	LUKALAPU RAMBABU
14	22475A0302	THUNUGUNTLA NAGA THARUN
15	22475A0303	MAILAVARAPU PAVAN KALYAN
16	22475A0304	KOLLI GOWRI SANKARA RAO
17	22475A0305	VANTAKU GANAPATHI LAKSHMI NAIDU
18	22475A0306	SYED HUSSAIN
19	22475A0307	BEHERA SANJAY KUMAR
20	22475A0308	YASAM MANIKANTA
21	22475A0309	MALLADI GOPI PURNA
22	22475A0310	UNGATI LOKESH

23	22475A0311	RAMAVATH VASU DEVA NAIK
24	22475A0312	PASALA SYAM KUMAR
25	22475A0313	THURIMELLA VAMSI GANESH
26	22475A0314	KUKKAMALLA KARTHIK
27	22475A0315	VUTLA KISHORE
28	22475A0316	DHARMANA APPALA NAIDU
29	22475A0317	NIKKU SURESH
30	22475A0318	GORANTLA SIVA KOTESWARA RAO
31	22475A0319	POGUNOLLA KARUN KUMAR
32	22475A0321	BANDLAMUDI NAGA RAJU
33	22475A0322	BOJJA SYAM BABU
34	22475A0323	ATHULURI PURNA VENKATA RAMARAO
35	22475A0324	BATTULA LAKSHMI NARAYANA
36	22475A0325	GUDIKANDULA ANJANEYULU
37	22475A0326	AYINAMPUDI KISHORE BABU
38	22475A0327	KETHABOYINA MAHESH
39	22475A0328	SHAIK BABULAL
40	22475A0329	BANDARU VENU GOPAL
41	22475A0330	YADAVALLI LOKESH
42	22475A0331	CHOUDAM VENKATESH
43	22475A0332	CHATTI MURALI KRISHNA
44	22475A0333	DARAM PRUDHVI KRISHNA
45	22475A0334	SHAIK NAGUR BASHA
46	22475A0335	GOLLAPUDI SARATH KUMAR
47	22475A0336	ADAKA VINOD
48	22475A0337	JANNI ARUN
49	22475A0338	NOWPADA MEGHANADH
50	22475A0339	BALAGA YUGANDHAR
51	22475A0340	NEYYELA KUMAR BEHERA

52	22475A0341	KUNITI PAVAN KUMAR
53	22475A0342	BHUKYA DIWAKAR NAIK
54	22475A0343	VOONA NARENDRA
55	22475A0344	CHANDARLAPATI GANESH
56	22475A0345	BALAGA MOHAN
57	22475A0346	BOMMALI MAHESH
58	22475A0347	DUDDETI NAGA SAI
59	22475A0348	BASWA DILLESWARA RAO
60	22475A0349	KORRAPATI MOHAN KRISHNA
61	22475A0350	NAKKANABOINA NAGA SRIDHAR
62	22475A0351	GONDU GANESH PAVAN
63	22475A0352	LINGA SRINIVAS

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NEC NARASARAOPETA ENGINEERING COLLEGE
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DEPARTMENT OF MECHANICAL ENGINEERING

HAND WRITTEN/PRINTED LECTURE NOTES

UNIT – I
SIMPLE STRESSES AND STRAINS

INTRODUCTION AND REVIEW

Preamble

Engineering science is usually subdivided into number of topics such as

1. Solid Mechanics

2. Fluid Mechanics

3. Heat Transfer

4. Properties of materials and soon Although there are close links between them in terms of the physical principles involved and methods of analysis employed.

The solid mechanics as a subject may be defined as a branch of applied mechanics that deals with behaviours of solid bodies subjected to various types of loadings. This is usually subdivided into further two streams i.e Mechanics of rigid bodies or simply Mechanics and Mechanics of deformable solids.

The mechanics of deformable solids which is branch of applied mechanics is known by several names i.e. strength of materials, mechanics of materials etc.

Mechanics of rigid bodies:

The mechanics of rigid bodies is primarily concerned with the static and dynamic behaviour under external forces of engineering components and systems which are treated as infinitely strong and undeformable Primarily we deal here with the forces and motions associated with particles and rigid bodies.

Mechanics of deformable solids :

Mechanics of solids:

The mechanics of deformable solids is more concerned with the internal forces and associated changes in the geometry of the components involved. Of particular importance are the properties of the materials used, the strength of which will determine whether the components fail by breaking in service, and the stiffness of which will determine whether the amount of deformation they suffer is acceptable. Therefore, the subject of mechanics of materials or strength of materials is central to the whole activity of engineering design. Usually the objectives in analysis here will be the determination of the stresses, strains, and deflections produced by loads. Theoretical analyses and experimental results have an equal roles in this field.

Analysis of stress and strain :

Concept of stress : Let us introduce the concept of stress as we know that the main problem of engineering mechanics of material is the investigation of the internal resistance of the body, i.e. the nature of forces set up within a body to balance the effect of the externally applied forces.

The externally applied forces are termed as loads. These externally applied forces may be due to any one of the reason.

(i) due to service conditions

(ii) due to environment in which the component works

(iii) through contact with other members

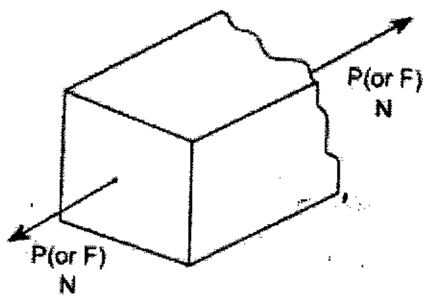
(iv) due to fluid pressures

(v) due to gravity or inertia forces.

As we know that in mechanics of deformable solids, externally applied forces acts on a body and body suffers a deformation. From equilibrium point of view, this action should be opposed or reacted by internal forces which are set up within the particles of material due to cohesion.

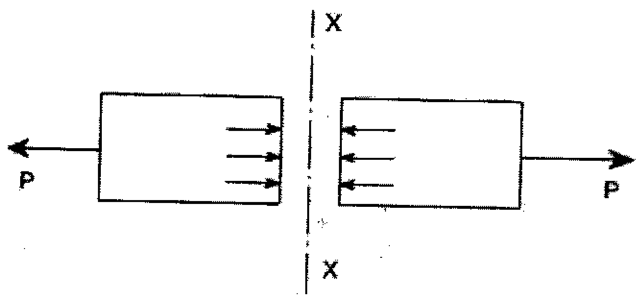
These internal forces give rise to a concept of stress. Therefore, let us define a stress Therefore, let us define a term stress

Stress:



Let us consider a rectangular bar of some cross – sectional area and subjected to some load or force (in Newtons)

Let us imagine that the same rectangular bar is assumed to be cut into two halves at section XX. The each portion of this rectangular bar is in equilibrium under the action of load P and the internal forces acting at the section XX has been shown



Now stress is defined as the force intensity or force per unit area. Here we use a symbol to represent the stress.

$$\sigma = \frac{P}{A}$$

Where A is the area of the X – section

Here we are using an assumption that the total force or total load carried by the rectangular bar is uniformly distributed over its cross – section.

But the stress distributions may be for from uniform, with local regions of high stress known as stress concentrations.

If the force carried by a component is not uniformly distributed over its cross – sectional area, A, we must consider a small area, ' A' which carries a small load P, of the total force 'P', Then definition of stress is

$$\sigma = \frac{\delta F}{\delta A}$$

As a particular stress generally holds true only at a point, therefore it is defined mathematically as

$$\sigma = \lim_{\delta A \to 0} \frac{\delta F}{\delta A}$$

Units :

The basic units of stress in S.I units i.e. (International system) are N / m^2 (or Pa)

MPa = 10^6 Pa

GPa = 10^9 Pa

KPa = 10^3 Pa

Some times N / mm^2 units are also used, because this is an equivalent to MPa. While US customary unit is pound per square inch psi.

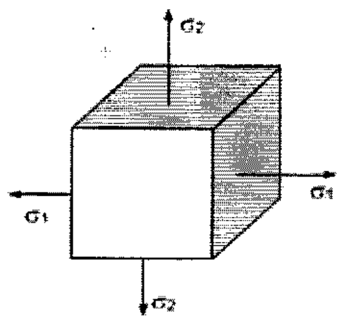
<u>TYPES OF STRESSES :</u>

only two basic stresses exists : (1) normal stress and (2) shear shear stress. Other stresses either are similar to these basic stresses or are a combination of these e.g. bending stress is a combination tensile, compressive and shear stresses. Torsional stress, as encountered in twisting of a shaft is a shearing stress.

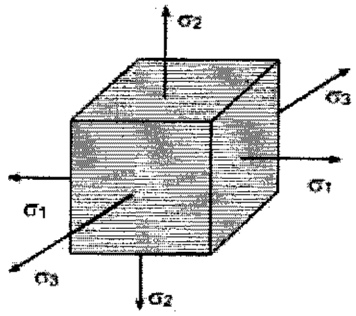
Let us define the normal stresses and shear stresses in the following sections.

Normal stresses : We have defined stress as force per unit area. If the stresses are normal to the areas concerned, then these are termed as normal stresses. The normal stresses are generally denoted by a Greek letter ( ) l

This is also known as uniaxial state of stress, because the stresses acts only in one direction however, such a state rarely exists, therefore we have biaxial and triaxial state of stresses where either the two mutually perpendicular normal stresses acts or three mutually perpendicular normal stresses acts as shown in the figures below :



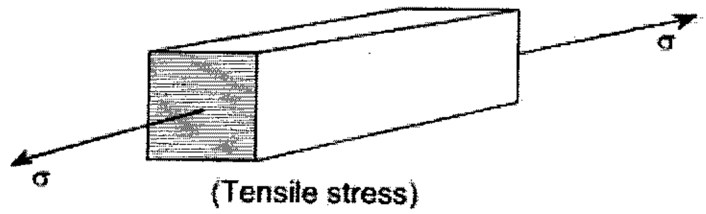
(Biaxial state of stress)



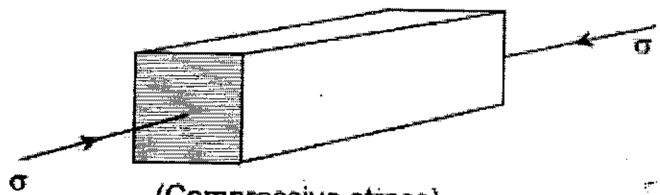
(Triaxial state of stress)

Tensile or compressive stresses :

The normal stresses can be either tensile or compressive whether the stresses acts out of the area or into the area

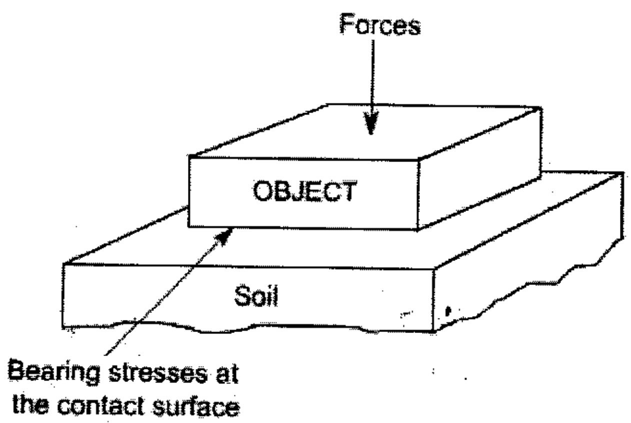


(Tensile stress)



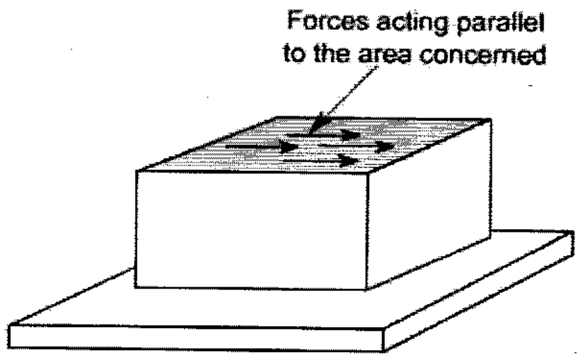
(Compressive stress)

Bearing Stress : When one object presses against another, it is referred to a bearing stress (They are in fact the compressive stresses).



Shear stresses :

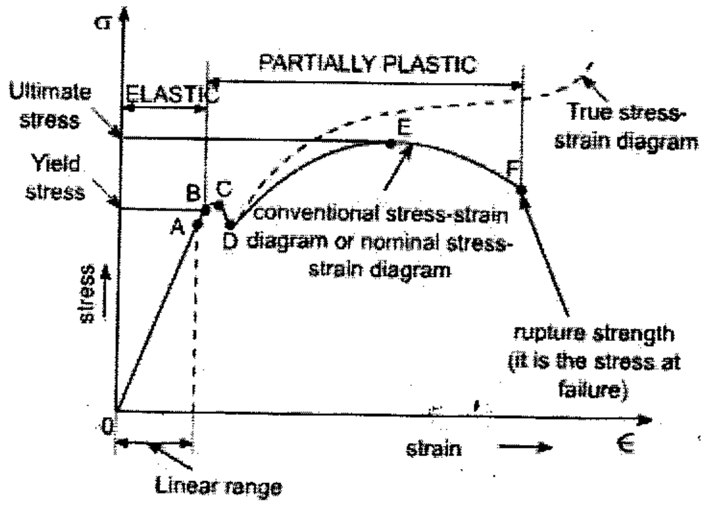
Let us consider now the situation, where the cross – sectional area of a block of material is subject to a distribution of forces which are parallel, rather than normal, to the area concerned. Such forces are associated with a shearing of the material, and are referred to as shear forces. The resulting force interistes are known as shear stresses.



The resulting force intensities are known as shear stresses, the mean shear stress being equal to

$$\tau = \frac{P}{A}$$

Where P is the total force and A the area over which it acts.



<u>**Nominal stress – Strain OR Conventional Stress – Strain diagrams:**</u>

Stresses are usually computed on the basis of the original area of the specimen; such stresses are often referred to as conventional or nominal stresses.

<u>**True stress – Strain Diagram:**</u>

Since when a material is subjected to a uniaxial load, some contraction or expansion always takes place. Thus, dividing the applied force by the corresponding actual area of the specimen at the same instant gives the so called true stress.

SALIENT POINTS OF THE GRAPH:

(A) So it is evident from the graph that the strain is proportional to strain or elongation is proportional to the load giving a st. line relationship. This law of proportionality is valid upto a point A.

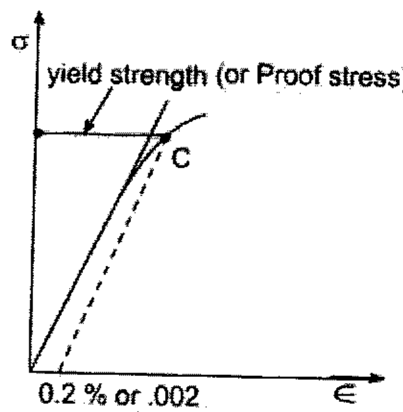
Or we can say that point A is some ultimate point when the linear nature of the graph ceases or there is a deviation from the linear nature. This point is known as **the limit of proportionality or the proportionality limit.**

(B) For a short period beyond the point A, the material may still be elastic in the sense that the deformations are completely recovered when the load is removed. The limiting point B is termed as **Elastic Limit.**

(C) and (D) - Beyond the elastic limit plastic deformation occurs and strains are not totally recoverable. There will be thus permanent deformation or permanent set when load is removed. These two points are termed as upper and lower yield points respectively. The stress at the yield point is called the yield strength.

A study a stress – strain diagrams shows that the yield point is so near the proportional limit that for most purpose the two may be taken as one. However, it is much easier to locate the former. For material which do not possess a well define yield points, In order to find the yield point or yield strength, an offset method is applied.

In this method a line is drawn parallel to the straight line portion of initial stress diagram by off setting this by an amount equal to 0.2% of the strain as shown as below and this happens especially for the low carbon steel.



(E) A further increase in the load will cause marked deformation in the whole volume of the metal. The maximum load which the specimen can with stand without failure is called the load at the ultimate strength.

The highest point 'E' of the diagram corresponds to the ultimate strength of a material.

σ_u = Stress which the specimen can with stand without failure & is known as Ultimate Strength or Tensile Strength.

σ_u is equal to load at E divided by the original cross-sectional area of the bar.

(F) Beyond point E, the bar begins to forms neck. The load falling from the maximum until fracture occurs at F.

[Beyond point E, the cross-sectional area of the specimen begins to reduce rapidly over a relatively small length of bar and the bar is said to form a neck. This necking takes place whilst the load reduces, and fracture of the bar finally occurs at point F]

Note: Owing to large reduction in area produced by the necking process the actual stress at fracture is often greater than the above value. Since the designers are interested in maximum loads which can be carried by the complete cross section, hence the stress at fracture is seldom of any practical value.

Percentage Elongation:

The ductility of a material in tension can be characterized by its elongation and by the reduction in area at the cross section where fracture occurs.

It is the ratio of the extension in length of the specimen after fracture to its initial gauge length, expressed in percent.

$$\delta = \frac{(l_1 - l_g)}{l_1} \times 100$$

l_1 = gauge length of specimen after fracture(or the distance between the gage marks at fracture)

l_g= gauge length before fracture(i.e. initial gauge length)

For 50 mm gage length, steel may here a % elongation of the order of 10% to 40%.

Ductile and Brittle Materials:

Based on this behaviour, the materials may be classified as ductile or brittle materials

Ductile Materials:

It we just examine the earlier tension curve one can notice that the extension of the materials over the plastic range is considerably in excess of that associated with elastic loading. The Capacity of materials to allow these large deformations or large extensions without failure is termed as ductility. The materials with high ductility are termed as ductile materials.

Brittle Materials:

A brittle material is one which exhibits a relatively small extensions or deformations to fracture, so that the partially plastic region of the tensile test graph is much reduced.

This type of graph is shown by the cast iron or steels with high carbon contents or concrete.

ELASTIC CONSTANTS

In considering the elastic behavior of an isotropic materials under, normal, shear and hydrostatic loading, we introduce a total of four elastic constants namely E, G, K, and μ .

It turns out that not all of these are independent to the others. In fact, given any two of them, the other two can be foundout . Let us define these elastic constants

(i) E = Young's Modulus of Rigidity

= Stress / strain

(ii) G = Shear Modulus or Modulus of rigidity

= Shear stress / Shear strain

(iii) μ = Possion's ratio

= lateral strain / longitudinal strain

(iv) K = Bulk Modulus of elasticity

= Volumetric stress / Volumetric strain

Where

Volumetric strain = sum of linear stress in x, y and z

direction. Volumetric stress = stress which cause the change

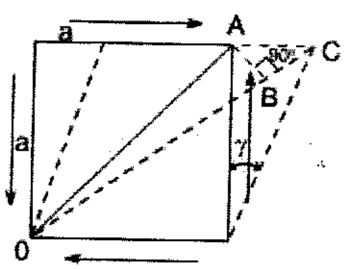
in volume. Let us find the relations between them

RELATION AMONG ELASTIC CONSTANTS

Relation between E, G and μ :

Let us establish a relation among the elastic constants E,G and . Consider a cube of material of side 'a' subjected to the action of the shear and complementary shear stresses as shown in the figure and producing the strained shape as shown in the figure below.

Assuming that the strains are small and the angle A C B may be taken as 45^0.



Therefore strain on the diagonal OA

= Change in length / original length

Since angle between OA and OB is very small hence OA OB therefore BC, is the change in the length of the diagonal OA

Thus, strain on diagonal OA = $\frac{BC}{OA}$

$$= \frac{AC\cos 45^0}{OA}$$

$$OA = \frac{a}{\sin 45^0} = a.\sqrt{2}$$

hence strain = $\frac{AC}{a\sqrt{2}} . \frac{1}{\sqrt{2}}$

$$= \frac{AC}{2a}$$

but AC = $a\gamma$

where γ = shear strain

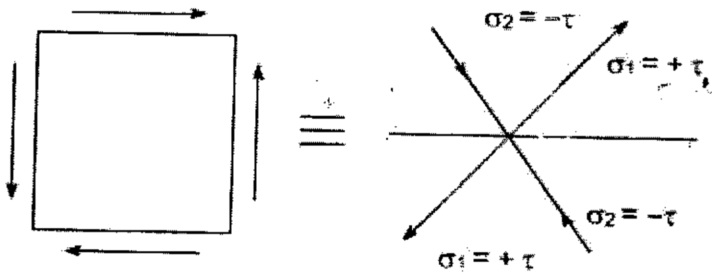
Thus, the strain on diagonal = $\frac{a\gamma}{2a} = \frac{\gamma}{2}$

From the definition

$$G = \frac{\tau}{\gamma} \text{ or } \gamma = \frac{\tau}{G}$$

thus, the strain on diagonal = $\frac{\gamma}{2} = \frac{\tau}{2G}$

Now this shear stress system is equivalent or can be replaced by a system of direct stresses at 45^0 as shown below. One set will be compressive, the other tensile, and both will be equal in value to the applied shear strain.



Thus, for the direct state of stress system which applies along the diagonals:

$$\text{strain on diagonal} = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

$$= \frac{\tau}{E} - \mu\frac{(-\tau)}{E}$$

$$= \frac{\tau}{E}(1+\mu)$$

equating the two strains one may get

$$\frac{\tau}{2G} = \frac{\tau}{E}(1+\mu)$$

or $$\boxed{E = 2G(1+\mu)}$$

We have introduced a total of four elastic constants, i.e E, G, K and μ. It turns out that not all of these are independent of the others. Infact given any two of then, the other two can be found.

Again $E = 3K(1 - 2\gamma)$

$$\Rightarrow \frac{E}{3(1-2\gamma)} = K$$

if $\gamma = 0.5$ $K = \infty$

$$\epsilon_v = \frac{(1-2\gamma)}{E}(\epsilon_x + \epsilon_y + \epsilon_z) = 3\frac{\sigma}{E}(1-2\gamma)$$

(for $\epsilon_x = \epsilon_y = \epsilon_z$ hydrostatic state of stress)

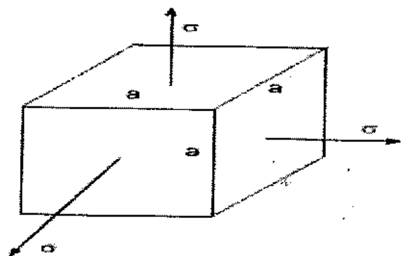
$\epsilon_v = 0$ if $\gamma = 0.5$

irrespective of the stresses i.e, the material is incompressible.

When $\mu = 0.5$ Value of k is infinite, rather than a zero value of E and volumetric strain is zero, or in other words, the material is incompressible.

Relation between E, K and :

Consider a cube subjected to three equal stresses as shown in the figure below



The total strain in one direction or along one edge due to the application of hydrostatic stress or volumetric stress is given as

$$= \frac{\sigma}{E} - \gamma\frac{\sigma}{E} - \gamma\frac{\sigma}{E}$$

$$= \frac{\sigma}{E}(1 - 2\gamma)$$

volumetre strain = 3.linear strain

volumetre strain = $\epsilon_x + \epsilon_y + \epsilon_z$

or thus, $\epsilon_x = \epsilon_y = \epsilon_z$

volumetric strain $= 3\frac{\sigma}{E}(1-2\gamma)$

By definition

$$\text{Bulk Modulus of Elasticity } (K) = \frac{\text{Volumetric stress}(\sigma)}{\text{Volumetric strain}}$$

or

$$\text{Volumetric strain} = \frac{\sigma}{k}$$

Equating the two strains we get

$$\frac{\sigma}{k} = 3.\frac{\sigma}{E}(1-2\gamma)$$

$$\boxed{E = 3K(1-2\gamma)}$$

Relation between E, G and K :

$$\boxed{E = \frac{9GK}{(3K+G)}}$$

Relation between E, K and □ :

From the already derived relations, E can be eliminated

$$E = 2G(1+\gamma)$$
$$E = 3K(1-2\gamma)$$

Thus, we get

$$3k(1-2\gamma) = 2G(1+\gamma)$$

therefore

$$\gamma = \frac{(3K-2G)}{2(G+3K)}$$

or

$$\boxed{\gamma = 0.5(3K-2G)(G+3K)}$$

Engineering Brief about the elastic constants :

We have introduced a total of four elastic constants i.e E, G, K and / It may be seen that not all of these are independent of the others. Infact given any two of them, the other two can be determined. Futher, it may be noted that

$$E = 3K(1-2\gamma)$$

or

$$K = \frac{E}{(1-2\gamma)}$$

if $\gamma = 0.5$; $K = \infty$

Also $\epsilon_v = \frac{(1-2\gamma)}{E}(\sigma_x + \sigma_y + \sigma_z)$

$= \frac{(1-2\gamma)}{E}.3\sigma$ (for hydrostatic state of stress i.e $\sigma_x = \sigma_y = \sigma_z = \sigma$)

hence if $\mu = 0.5$, the value of K becomes infinite, rather than a zero value of E and the volumetric strain is zero or in otherwords, the material becomes incompressible

Futher, it may be noted that under condition of simple tension and simple shear, all real materials tend to experience displacements in the directions of the applied forces and Under hydrostatic loading they tend to increase in volume. In otherwords the value of the elastic constants E, G and K cannot be negative

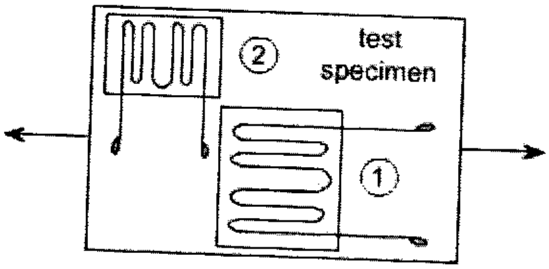
Therefore, the relations

E = 2 G (1 +μ)

E = 3 K (1   )

Yields $\boxed{-1 \le v \le 0.5}$

Determination of Poisson's ratio: Poisson's ratio can be determined easily by simultaneous use of two strain gauges on a test specimen subjected to uniaxial tensile or compressive load. One gage is mounted parallel to the longitudnal axis of the specimen and other is mounted perpendicular to the longitudnal axis as shown below:



Compression Test: Machines used for compression testing are basically similar to those used for tensile testing often the same machine can be used to perform both tests.

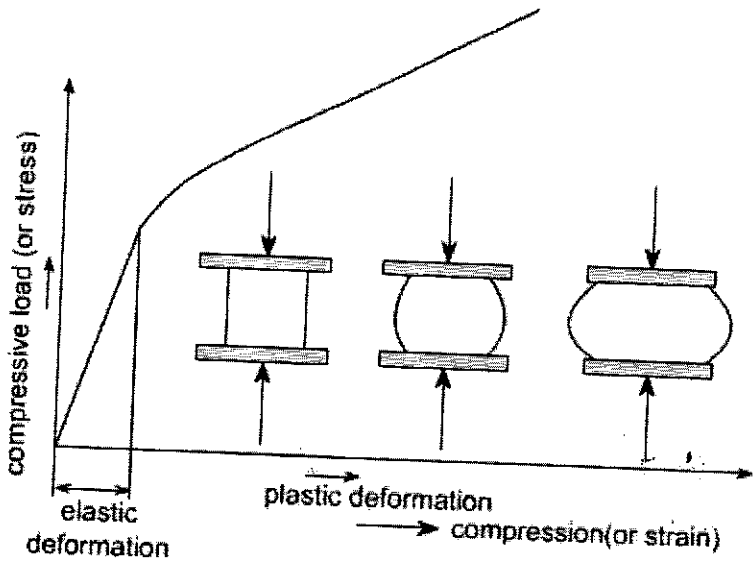
Shape of the specimen: The shape of the machine to be used for the different materials are as follows:

(i) **For metals and certain plastics:** The specimen may be in the from of a cylinder

(ii) **For building materials:** Such as concrete or stone the shape of the specimen may be in the from of a cube.

Shape of stress stain diagram

(a) Ductile materials: For ductile material such as mild steel, the load Vs compression diagram would be as follows



(1) The ductile materials such as steel, Aluminum, and copper have stress – strain diagrams similar to ones which we have for tensile test, there would be an elastic range which is then followed by a plastic region.

(2) The ductile materials (steel, Aluminum, copper) proportional limits in compression test are very much close to those in tension.

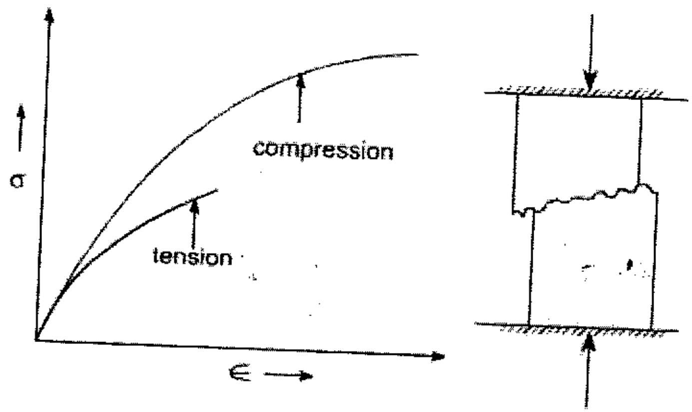
(3) In tension test, a specimen is being stretched, necking may occur, and ultimately fracture fakes place. On the other hand when a small specimen of the ductile material is compressed, it begins to bulge on sides and becomes barrel shaped as shown in the figure above. With increasing load, the specimen is flattened out, thus offering increased resistance to further shortening (which means that the stress – strains curve goes upward) this effect is indicated in the diagram.

Brittle materials (in compression test)

Brittle materials in compression typically have an initial linear region followed by a region in which the shortening increases at a higher rate than does the load. Thus, the compression stress – strain diagram has a shape that is similar to the shape of the tensile diagram.

However, brittle materials usually reach much higher ultimate stresses in compression than in tension.

For cast iron, the shape may be like this



Brittle materials in compression behave elastically up to certain load, and then fail suddenly by splitting or by craking in the way as shown in figure. The brittle fracture is performed by separation and is not accompanied by noticeable plastic deformation.

Practice Problems:

PROB 1: A standard mild steel tensile test specimen has a diameter of 16 mm and a gauge length of 80 mm such a specimen was tested to destruction, and the following results obtained.

Load at yield point = 87 kN

Extension at yield point = 173×16^{-6} m

Ultimate load = 124 kN

Total extension at fracture = 24 mm

Diameter of specimen at fracture = 9.8 mm

Cross - sectional area at fracture = 75.4 mm^2

Cross - sectional Area 'A' = 200 mm^2

Compute the followings:

(i) Modulus of elasticity of steel

(ii) The ultimate tensile stream

(iii) The yield stress

(iv) The percentage elongation

(v) The Percentage reduction in Area.

PROB 2:

A light alloy specimen has a diameter of 16mm and a gauge Length of 80 mm. When tested in tension, the load extension graph proved linear up to a load of 6kN, at which point the extension was 0.034 mm. Determine the limits of proportionality stress and the modulus of elasticity of material.

Note: For a 16mm diameter specimen, the Cross – sectional area A = 200 mm^2

This is according to tables Determine the limit of proportion try stream & the modulus of elasticity for the material.

Ans: 30 MN /m^2, 70.5 GN /m^2

solution:

Limit of proportionaly stress $= \dfrac{6 \text{ kN}}{200\times10^{-6}}$

$= 30 \text{ MN/m}^2$

Young Modulus $E = \dfrac{\text{Stress}}{\text{Strain}}$

$\text{strain} = \dfrac{.034}{80}$

$E = 30\times10^6 \Big/ \dfrac{.034}{80}$

$= 70.5 \text{ GN/m}^2$

Strain Energy

Strain Energy of the member is defined as the internal work done in defóming the body by the action of externally applied forces. This energy in elastic bodies is known as **elastic strain energy** :

Strain Energy in uniaxial Loading

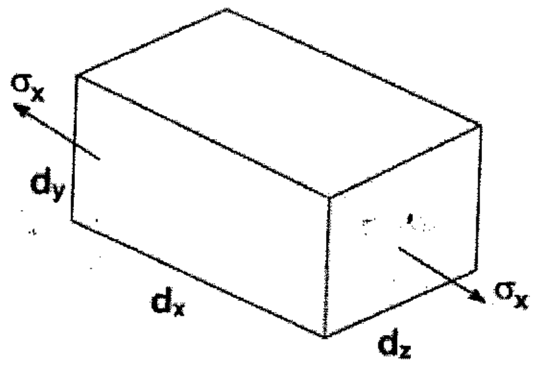


Fig .1

Let as consider an infinitesimal element of dimensions as shown in Fig .1. Let the element be subjected to normal stress σ_x.

The forces acting on the face of this element is σ_x. dy. dz

where

dydz = Area of the element due to the application of forces, the element deforms to an amount = σ_x dx

$$= \frac{\text{Change in length}}{\text{Orginal in length}}$$

Assuming the element material to be as linearly elastic the stress is directly proportional to strain as shown in Fig . 2.

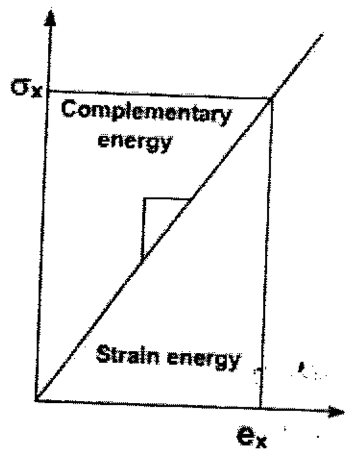


Fig .2

From Fig .2 the force that acts on the element increases linearly from zero until it attains its full value.

For a perfectly elastic body the above work done is the internal strain energy "du".

$$\boxed{du = \frac{1}{2}\sigma_x dydz\, \epsilon_x\, dx} \quad .....(2)$$

$$= \frac{1}{2}\sigma_x\, \epsilon_x\, dxdydz$$

$$\boxed{du = \frac{1}{2}\sigma_x\, \epsilon_x\, dv} \quad ....(3)$$

where dv = dxdydz

= Volume of the element

By rearranging the above equation we can write

$$U_o = \boxed{\frac{du}{dv} = \frac{1}{2}\sigma_x\, \epsilon_x} \quad .....(4)$$

The equation (4) represents the strain energy in elastic body per unit volume of the material its strain energy – density 'u_o'.

From Hook's Law for elastic bodies, it may be recalled that

$$\boxed{\sigma = E\epsilon}$$

$$\boxed{U_o = \frac{du}{dv} = \frac{\sigma_x^2}{2E} = \frac{E\epsilon_x^2}{2}} \quad ......(5)$$

$$\boxed{U = \int_{Vol} \frac{\sigma_x^2}{2E} dv} \quad ....(6)$$

In the case of a rod of uniform cross – section subjected at its ends an equal and opposite forces of magnitude P as shown in the Fig .3.

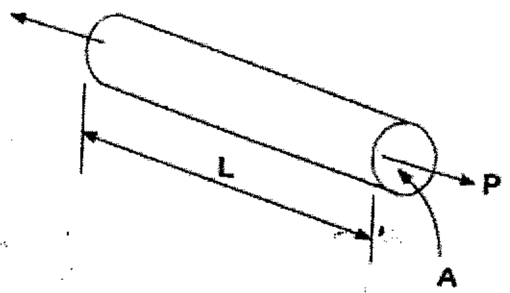


Fig .3

$$U = \int_{Vol}^{L} \frac{\sigma_x^2}{2E} dv \qquad \sigma_x = \frac{P}{A}$$

$$U = \int_0^L \frac{P^2}{2EA^2} A dx \qquad dv = Adx = \text{Element volume}$$

A = Area of the bar.
L = Length of the bar

$$\boxed{U = \frac{P^2 L}{2AE}} \qquad .....(7)$$

Modulus of resilience :

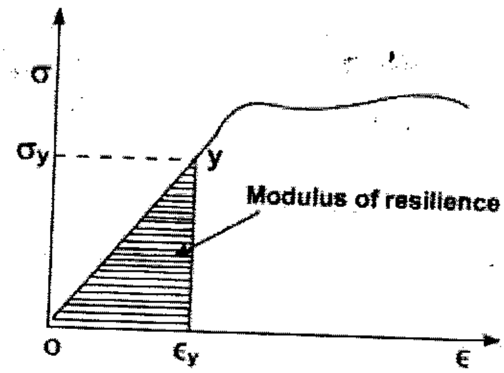


Fig .4

Suppose 'σ_x' in strain energy equation is put equal to σ_y i.e. the stress at proportional limit or yield point. The resulting strain energy gives an index of the materials ability to store or absorb energy without permanent deformation

$$\text{So} \quad \boxed{U_y = \frac{\sigma_y^2}{2E}} \qquad .....(8)$$

The quantity resulting from the above equation is called the Modulus of resilience

The modulus of resilience is equal to the area under the straight line portion 'OY' of the stress – strain diagram as shown in Fig .4 and represents the energy per unit volume that the material can absorb without yielding. Hence this is used to differentiate materials for applications where energy must be absorbed by members.

Modulus of Toughness :

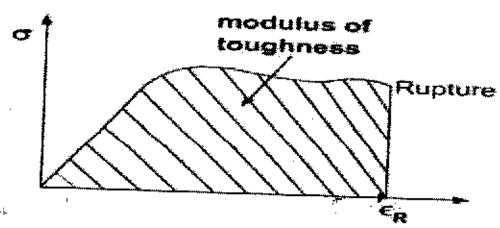


Fig .5

Suppose '∣' [strain] in strain energy expression is replaced by R∣strain at rupture, the resulting strain energy density is called modulus of toughness

$$U = \int_0^{\epsilon} E\epsilon_x \, dx = \frac{E\epsilon_R^2}{2} dv$$

$$\boxed{U = \frac{E\epsilon_R^2}{2}} \qquad \ldots\ldots(9)$$

From the stress – strain diagram, the area under the complete curve gives the measure of modules of toughness. It is the materials.

Ability to absorb energy upto fracture. It is clear that the toughness of a material is related to its ductility as well as to its ultimate strength and that the capacity of a structure to withstand an impact Load depends upon the toughness of the material used.

UNIT-II

SHEAR FORCE AND BENDING MOMENT

Concept of Shear Force and Bending moment in beams:

When the beam is loaded in some arbitrarily manner, the internal forces and moments are developed and the terms shear force and bending moments come into pictures which are helpful to analyze the beams further. Let us define these terms

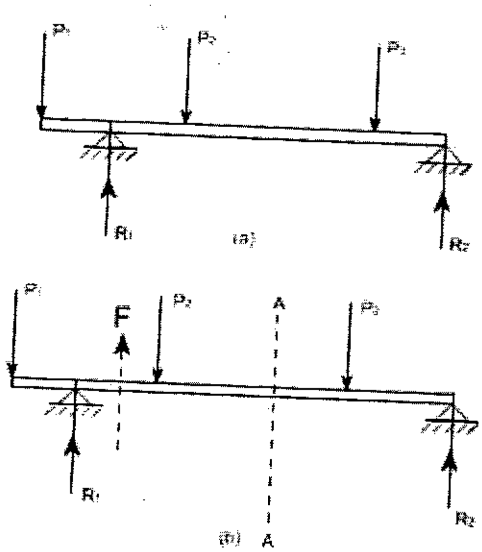


Fig 1

Now let us consider the beam as shown in fig 1(a) which is supporting the loads P_1, P_2, P_3 and is simply supported at two points creating the reactions R_1 and R_2 respectively. Now let us assume that the beam is to divided into or imagined to be cut into two portions at a section AA. Now let us assume that the resultant of loads and reactions to the left of AA is 'F' vertically upwards, and since the entire beam is to remain in equilibrium, thus the resultant of forces to the right of AA must also be F, acting downwards. This forces 'F' is as a shear force. The shearing force at any x-section of a beam represents the tendency for the portion of the beam to one side of the section to slide or shear laterally relative to the other portion.

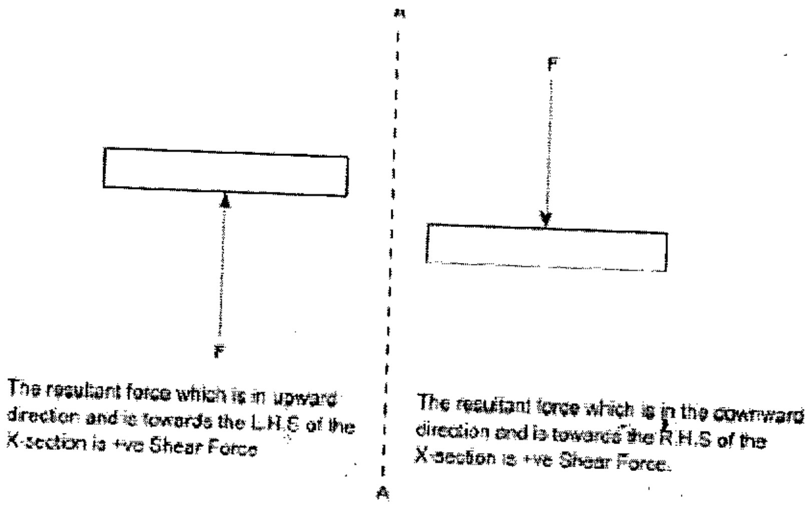
Therefore, now we are in a position to define the shear force 'F' to as follows:

At any x-section of a beam, the shear force 'F' is the algebraic sum of all the lateral components of the forces acting on either side of the x-section.

Sign Convention for Shear Force:

The usual sign conventions to be followed for the shear forces have been illustrated in figures 2 and 3.

Fig 2: Positive Shear Force



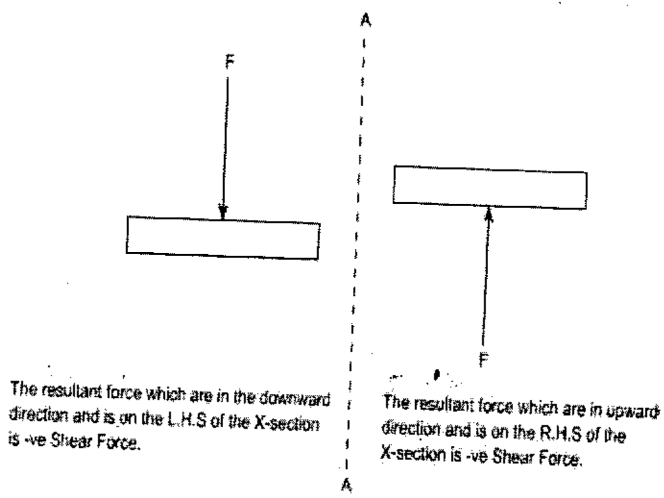


Fig 3: Negative Shear Force

Bending Moment:

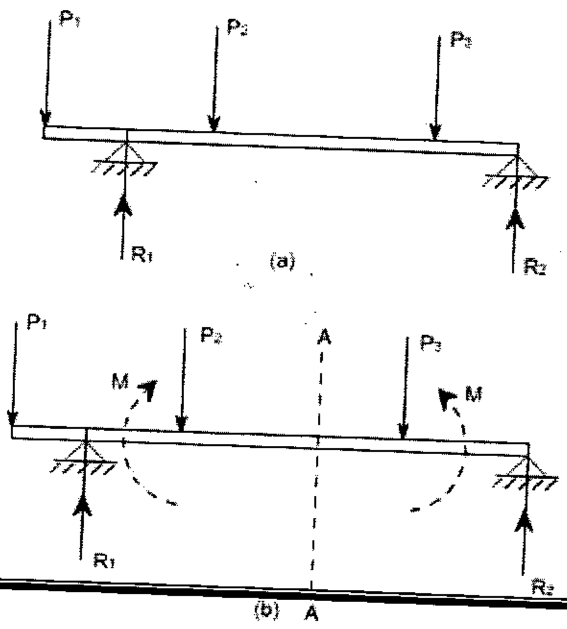


Fig 4

Let us again consider the beam which is simply supported at the two prints, carrying loads P_1, P_2 and P_3 and having the reactions R_1 and R_2 at the supports Fig 4. Now, let us imagine that the beam is cut into two potions at the x-section AA. In a similar manner, as done for the case of shear force, if we say that the resultant moment about the section AA of all the loads and reactions to the left of the x-section at AA is M in C.W direction, then moment of forces to the right of x-section AA must be 'M' in C.C.W. Then 'M' is called as the Bending moment and is abbreviated as B.M. Now one can define the bending moment to be simply as the algebraic sum of the moments about an x-section of all the forces acting on either side of the section

Sign Conventions for the Bending Moment:

For the bending moment, following sign conventions may be adopted as indicated in Fig 5 and Fig 6.

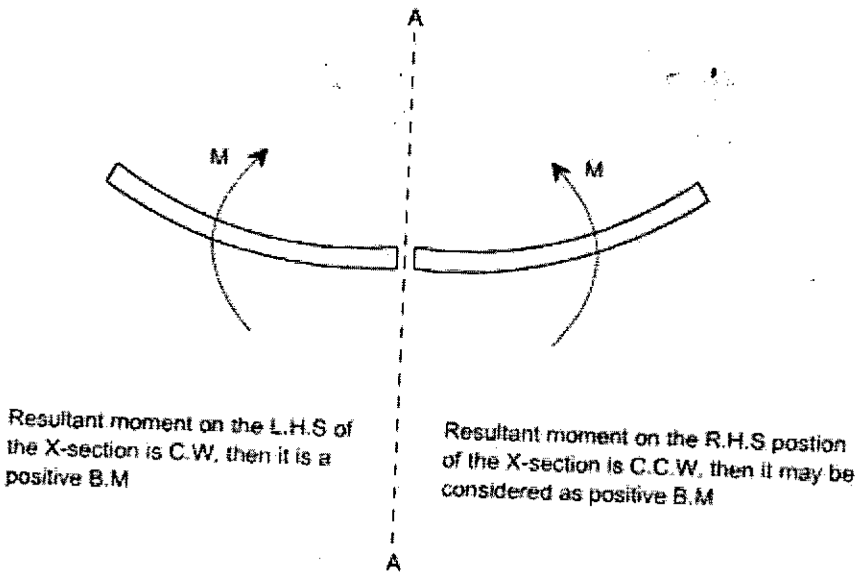


Fig 5: Positive Bending Moment

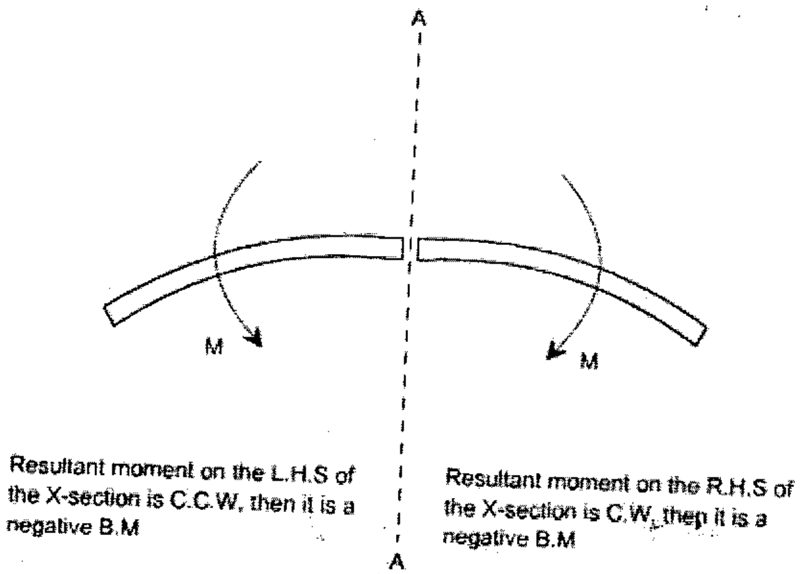


Fig 6: Negative Bending Moment

Some times, the terms 'Sagging' and Hogging are generally used for the positive and negative bending moments respectively.

Bending Moment and Shear Force Diagrams:

The diagrams which illustrate the variations in B.M and S.F values along the length of the beam for any fixed loading conditions would be helpful to analyze the beam further.

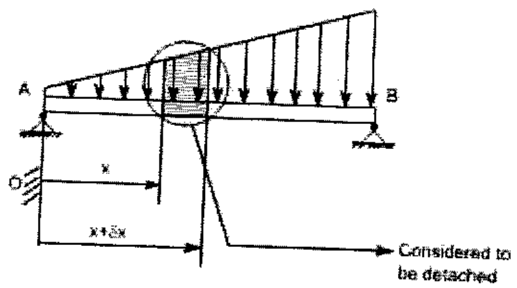
Thus, a shear force diagram is a graphical plot, which depicts how the internal shear force 'F' varies along the length of beam. If x dentotes the length of the beam, then F is function x i.e. F(x).

Similarly a bending moment diagram is a graphical plot which depicts how the internal bending moment 'M' varies along the length of the beam. Again M is a function x i.e. M(x).

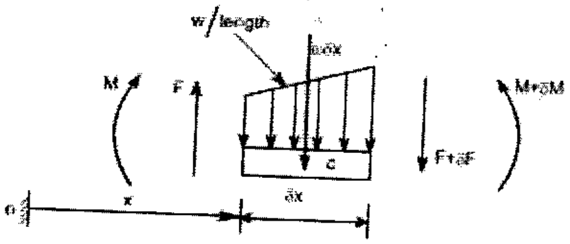
Basic Relationship Between The Rate of Loading, Shear Force and Bending Moment:

The construction of the shear force diagram and bending moment diagrams is greatly simplified if the relationship among load, shear force and bending moment is established.

Let us consider a simply supported beam AB carrying a uniformly distributed load w/length. Let us imagine to cut a short slice of length dx cut out from this loaded beam at distance 'x' from the origin '0'.



Let us detach this portion of the beam and draw its free body diagram.



The forces acting on the free body diagram of the detached portion of this loaded beam are the following

- The shearing force F and F+ F at the section x and x + x respectively.

- The bending moment at the sections x and x + x be M and M + dM respectively.

- Force due to external loading, if 'w' is the mean rate of loading per unit length then the total loading on this slice of length x is w. x, which is approximately acting through the centre 'c'. If the loading is assumed to be uniformly distributed then it would pass exactly through the centre 'c'.

This small element must be in equilibrium under the action of these forces and couples.

Now let us take the moments at the point 'c'. Such that

$$M + F.\frac{\delta x}{2} + (F + \delta F).\frac{\delta x}{2} = M + \delta M$$

$$\Rightarrow F.\frac{\delta x}{2} + (F + \delta F).\frac{\delta x}{2} = \delta M$$

$$\Rightarrow F.\frac{\delta x}{2} + F.\frac{\delta x}{2} + \delta F.\frac{\delta x}{2} = \delta M \text{ [Neglecting the product of } \delta F \text{ and } \delta x \text{ being small quantities]}$$

$$\Rightarrow F.\delta x = \delta M$$

$$\Rightarrow F = \frac{\delta M}{\delta x}$$

Under the limits $\delta x \to 0$

$$\boxed{F = \frac{dM}{dx}} \qquad \text{..........} \quad (1)$$

Resolving the forces vertically we get

$$w.\delta x + (F + \delta F) = F$$

$$\Rightarrow w = -\frac{\delta F}{\delta x}$$

Under the limits $\delta x \to 0$

$$\Rightarrow w = -\frac{dF}{dx} \text{ or } -\frac{d}{dx}\left(\frac{dM}{dx}\right)$$

$$\boxed{w = -\frac{dF}{dx} = -\frac{d^2M}{dx^2}} \qquad \text{..........}(2)$$

Conclusions: From the above relations,the following important conclusions may be drawn

- From Equation (1), the area of the shear force diagram between any two points, from the basic calculus is the bending moment diagram

$$M = \int F.dx$$

- The slope of bending moment diagram is the shear force,thus

$$F = \frac{dM}{dx}$$

Thus, if F=0; the slope of the bending moment diagram is zero and the bending moment is therefore constant.'

- The maximum or minimum Bending moment occurs where $\frac{dM}{dx}=0$.

The slope of the shear force diagram is equal to the magnitude of the intensity of the distributed loading at any position along the beam. The –ve sign is as a consequence of our particular choice of sign conventions

Procedure for drawing shear force and bending moment diagram:

Preamble:

The advantage of plotting a variation of shear force F and bending moment M in a beam as a function of 'x' measured from one end of the beam is that it becomes easier to determine the maximum absolute value of shear force and bending moment.

Further, the determination of value of M as a function of 'x' becomes of paramount importance so as to determine the value of deflection of beam subjected to a given loading.

Construction of shear force and bending moment diagrams:

A shear force diagram can be constructed from the loading diagram of the beam. In order to draw this, first the reactions must be determined always. Then the vertical components of forces and reactions are successively summed from the left end of the beam to preserve the mathematical sign conventions adopted. The shear at a section is simply equal to the sum of all the vertical forces to the left of the section.

When the successive summation process is used, the shear force diagram should end up with the previously calculated shear (reaction at right end of the beam. No shear force acts through the beam just beyond the last vertical force or reaction. If the shear force diagram closes in this fashion, then it gives an important check on mathematical calculations.

The bending moment diagram is obtained by proceeding continuously along the length of beam from the left hand end and summing up the areas of shear force diagrams giving due regard to sign. The process of obtaining the moment diagram from the shear force diagram by summation is exactly the same as that for drawing shear force diagram from load diagram.

It may also be observed that a constant shear force produces a uniform change in the bending moment, resulting in straight line in the moment diagram. If no shear force exists along a certain portion of a beam, then it indicates that there is no change in moment takes place. It may also further observe that dm/dx= F therefore, from the fundamental theorem of calculus the maximum or minimum moment occurs where the shear is zero. In order to check the validity of the bending moment diagram, the terminal conditions for the moment must be satisfied. If the end is free or pinned, the computed sum must be equal to zero. If the end is built in, the moment computed by the summation must be equal to the one calculated initially for the reaction. These conditions must always be satisfied.

Illustrative problems:

In the following sections some illustrative problems have been discussed so as to illustrate the procedure for drawing the shear force and bending moment diagrams

1. A cantilever of length carries a concentrated load 'W' at its free end.

Draw shear force and bending moment.

Solution:

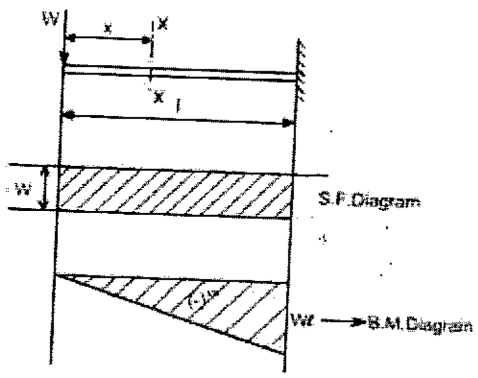
At a section a distance x from free end consider the forces to the left, then F = -W (for all values of x) -ve sign means the shear force to the left of the x-section are in downward direction and therefore negative

Taking moments about the section gives (obviously to the left of the section)

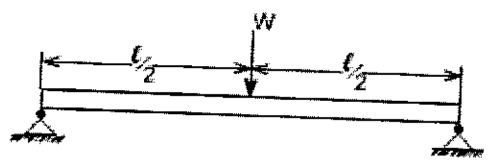
M = -Wx (-ve sign means that the moment on the left hand side of the portion is in the anticlockwise direction and is therefore taken as –ve according to the sign convention)

so that the maximum bending moment occurs at the fixed end i.e. M = -W l

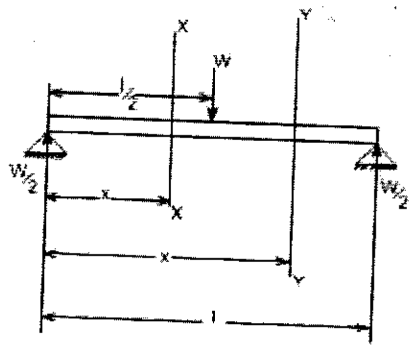
From equilibrium consideration, the fixing moment applied at the fixed end is Wl and the reaction is W. the shear force and bending moment are shown as,



2. Simply supported beam subjected to a central load (i.e. load acting at the mid-way)



By symmetry the reactions at the two supports would be W/2 and W/2. now consider any section X-X from the left end then, the beam is under the action of following forces.



So the shear force at any X-section would be = W/2 [Which is constant upto x < l/2]

If we consider another section Y-Y which is beyond l/2 then

$S.F_{Y\text{-}Y} = \frac{W}{2} - W = \frac{-W}{2}$ for all values greater = l/2

Hence S.F diagram can be plotted as,
.For B.M diagram:

If we just take the moments to the left of the cross-section,

$B.M_{X\text{-}X} = \frac{W}{2}x$ for x lies between 0 and l/2

$B.M_{at\ x=\frac{l}{2}} = \frac{W}{2}\frac{l}{2}$ i.e B.M at x = 0

$= \frac{Wl}{4}$

$B.M_{Y\text{-}Y} = \frac{W}{2}x - W\left(x - \frac{l}{2}\right)$

Again

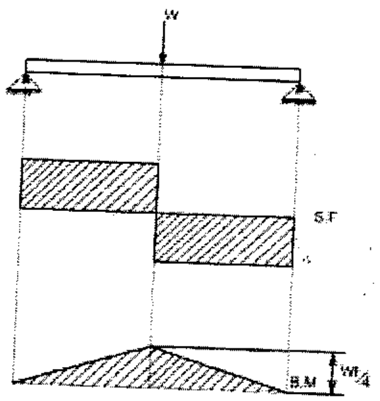
$= \frac{W}{2}x - Wx + \frac{Wl}{2}$

$= -\frac{W}{2}x + \frac{Wl}{2}$

$BM_{at\ x=l} = -\frac{Wl}{2} + \frac{Wl}{2}$

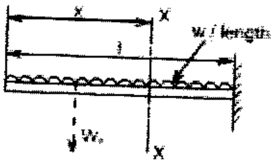
$= 0$

Which when plotted will give a straight relation i.e.



It may be observed that at the point of application of load there is an abrupt change in the shear force, at this point the B.M is maximum.

3. A cantilever beam subjected to U.d.L, draw S.F and B.M diagram.



Here the cantilever beam is subjected to a uniformly distributed load whose intensity is given w / length.

Consider any cross-section XX which is at a distance of x from the free end. If we just take the resultant of all the forces on the left of the X-section, then

$S.F_{xx} = -Wx$ for all values of 'x'. ---------- **(1)**

$S.F_{xx} = 0$

$S.F_{xx \text{ at } x=l} = -Wl$

So if we just plot the equation No. (1), then it will give a straight line relation. Bending Moment at X-X is obtained by treating the load to the left of X-X as a concentrated load of the same value acting through the centre of gravity.

Therefore, the bending moment at any cross-section X-X is

$$B.M_{X\text{-}X} = -Wx\,\frac{x}{2}$$

$$= -W\frac{x^2}{2}$$

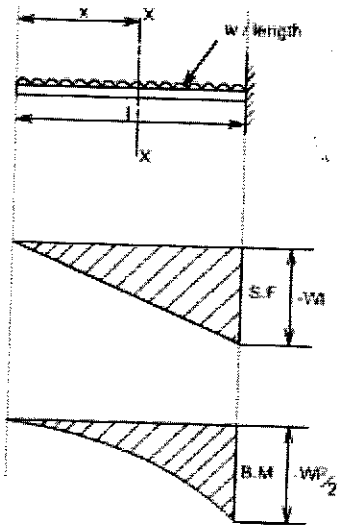
The above equation is a quadratic in x, when B.M is plotted against x this will produces a parabolic variation.

The extreme values of this would be at x = 0 and x = l

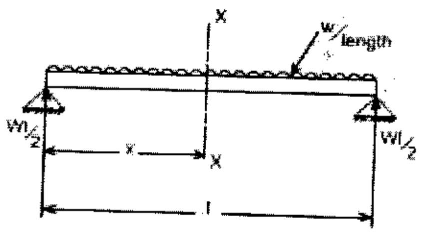
$$B.M_{\text{at } x=l} = -\frac{Wl^2}{2}$$

$$= \frac{Wl}{2} - Wx$$

Hence S.F and B.M diagram can be plotted as follows:



4. Simply supported beam subjected to a uniformly distributed load [U.D.L].



The total load carried by the span would be

= intensity of loading x length

= w x l

By symmetry the reactions at the end supports are each wl/2

If x is the distance of the section considered from the left hand end of the beam.

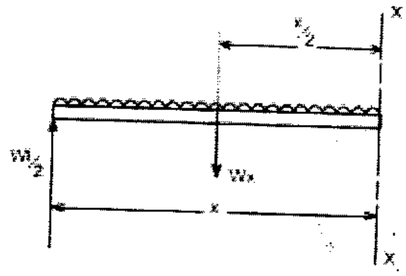
S.F at any X-section X-X is

$$= \frac{Wl}{2} - Wx$$
$$= W\left(\frac{l}{2} - x\right)$$

Giving a straight relation, having a slope equal to the rate of loading or intensity of the loading.

$$S.F_{at\ x=0} = \frac{wl}{2} - wx$$
so at
$$S.F_{at\ x=\frac{l}{2}} = 0$$ hence the S.F is zero at the centre
$$S.F_{at\ x=l} = -\frac{wl}{2}$$

The bending moment at the section x is found by treating the distributed load as acting at its centre of gravity, which at a distance of x/2 from the section



$$B.M_{x\text{-}x} = \frac{Wl}{2}x - Wx.\frac{x}{2}$$

so the

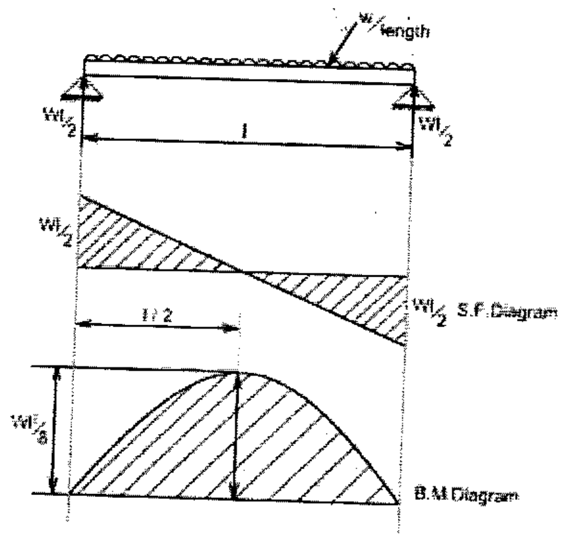
$$= W.\frac{x}{2}(l - 2) \quad \ldots\ldots(2)$$

$B.M_{at\ x = 0} = 0$

$B.M_{at\ x = l} = 0$

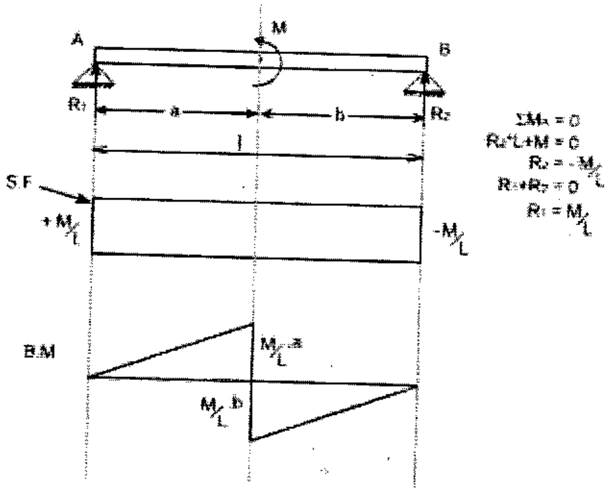
$B.M\Big|_{at\ x = l} = -\frac{Wl^2}{8}$

So the equation (2) when plotted against x gives rise to a parabolic curve and the shear force and bending moment can be drawn in the following way will appear as follows:



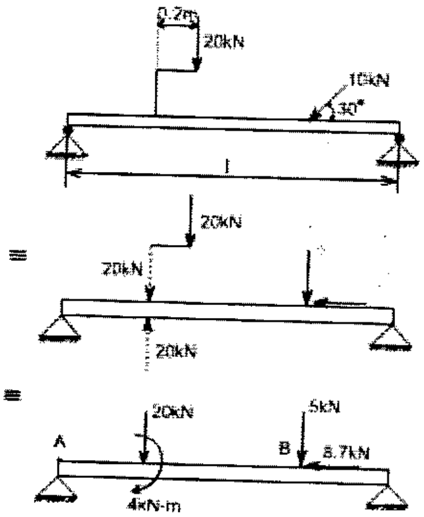
5. Couple.

When the beam is subjected to couple, the shear force and Bending moment diagrams may be drawn exactly in the same fashion as discussed earlier.



6. Eccentric loads.

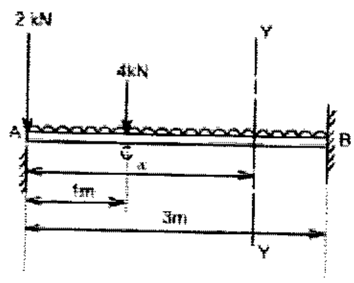
When the beam is subjected to an eccentric loads, the eccentric load are to be changed into a couple/ force as the case may be, In the illustrative example given below, the 20 kN load acting at a distance of 0.2m may be converted to an equivalent of 20 kN force and a couple of 2 kN.m. similarly a 10 kN force which is acting at an angle of 30^0 may be resolved into horizontal and vertical components.The rest of the procedure for drawing the shear force and Bending moment remains the same.



6. Loading changes or there is an abrupt change of loading:

When there is an aabrupt change of loading or loads changes, the problem may be tackled in a systematic way.consider a cantilever beam of 3 meters length. It carries a uniformly distributed load of 2 kN/m and a concentrated loads of 2kN at the free end and 4kN at 2 meters from fixed end.The shearing force and bending moment diagrams are required to be drawn and state the maximum values of the shearing force and bending moment.

Solution



Consider any cross section x-x, at a distance x from the free end

Shear Force at x-x = -2 -2x   0 < x < 1

S.F at x = 0 i.e. at A = -2 kN

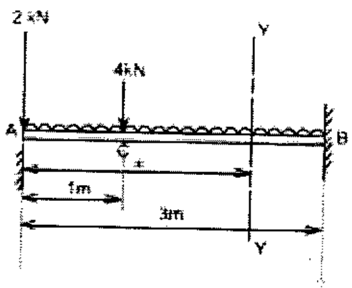
S.F at x = 1 = -2-2 = - 4kN

S.F at C (x = 1) = -2 -2x - 4   Concentrated load

= - 2 - 4 -2x1 kN

= - 8 kN

Again consider any cross-section YY, located at a distance x from the free end



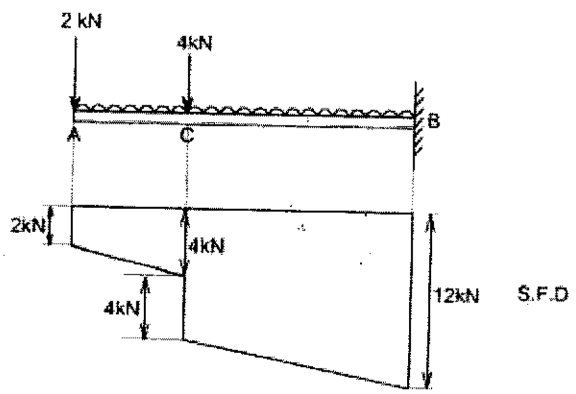
S.F at Y-Y = -2 - 2x - 4   1< x < 3

This equation again gives S.F at point C equal to -8kN

S.F at x = 3 m = -2 -4 -2x3

= -12 kN

Hence the shear force diagram can be drawn as below:



For bending moment diagrams – Again write down the equations for the respective cross sections, as consider above

Bending Moment at xx = -2x - 2x.x/2 valid upto AC

B.M at x = 0 = 0

B.M at x =1m = -3 kN.m

For the portion CB, the bending moment equation can be written for the x-section at Y-Y .

B.M at YY = -2x - 2x.x/2 - 4(x -1)

This equation again gives,

B.M at point C = - 2.1 - 1 - 0 i.e. at x = 1

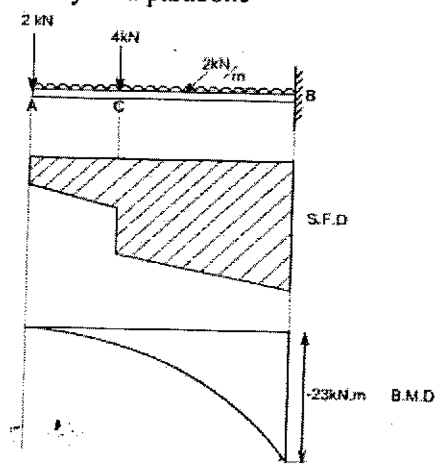
= -3 kN.m

at point B i.e. at x = 3 m

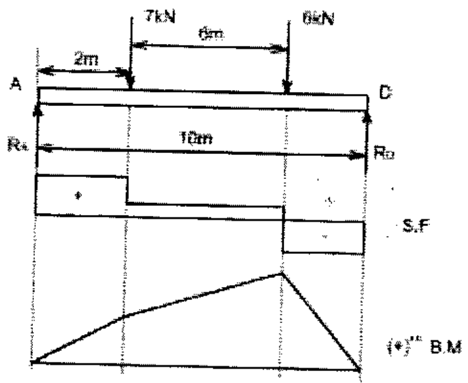
= - 6 - 9 - 8

= - 23 kN-m

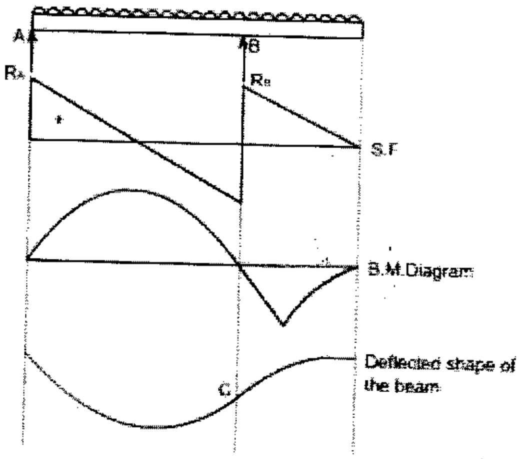
The variation of the bending moment diagrams would obviously be a parabolic curve Hence the bending moment diagram would be



Point of Contraflexure:

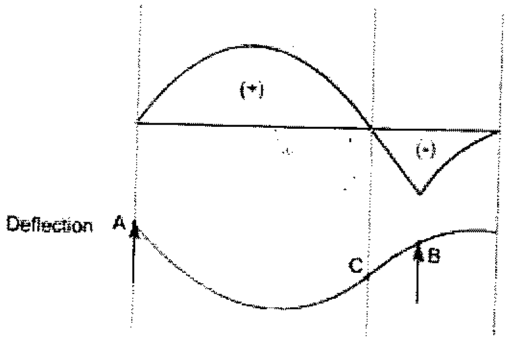


Consider the loaded beam a shown below along with the shear force and Bending moment diagrams for It may be observed that this case, the bending moment diagram is completely positive so that the curvature of the beam varies along its length, but it is always concave upwards or sagging.However if we consider a again a loaded beam as shown below along with the S.F and B.M diagrams, then



It may be noticed that for the beam loaded as in this case,

The bending moment diagram is partly positive and partly negative.If we plot the deflected shape of the beam just below the bending moment



This diagram shows that L.H.S of the beam 'sags' while the R.H.S of the beam 'hogs'

The point C on the beam where the curvature changes from sagging to hogging is a point of contraflexure.

UNIT - III

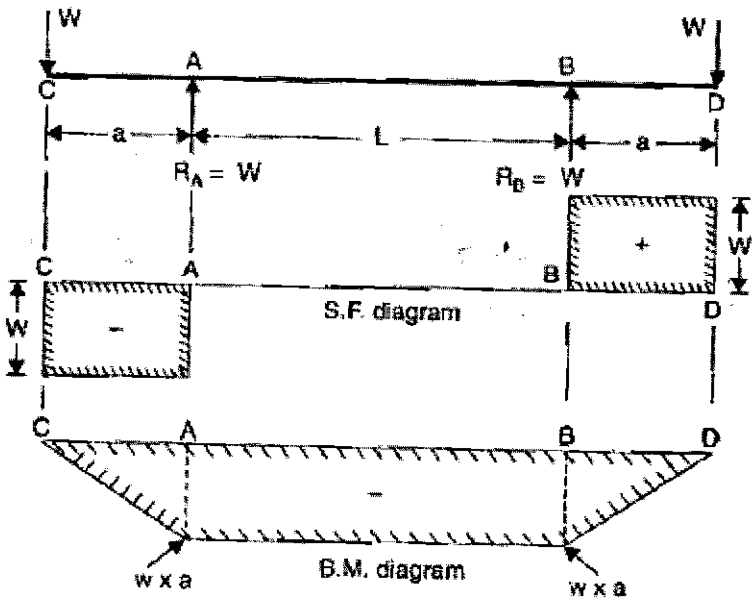
FLEXURAL STRESSES OR BENDING STRESSES IN BEAMS

BENDING STRESSES:

The stresses induced by bending moment are known as bending stresses.

PURE BENDING OR SIMPLE BENDING:

If a length of a beam is subjected to a constant bending moment & shear force is zero, then the stresses set up in that length of the beam are known as bending stresses and that length of the beam is said to be in pure bending.



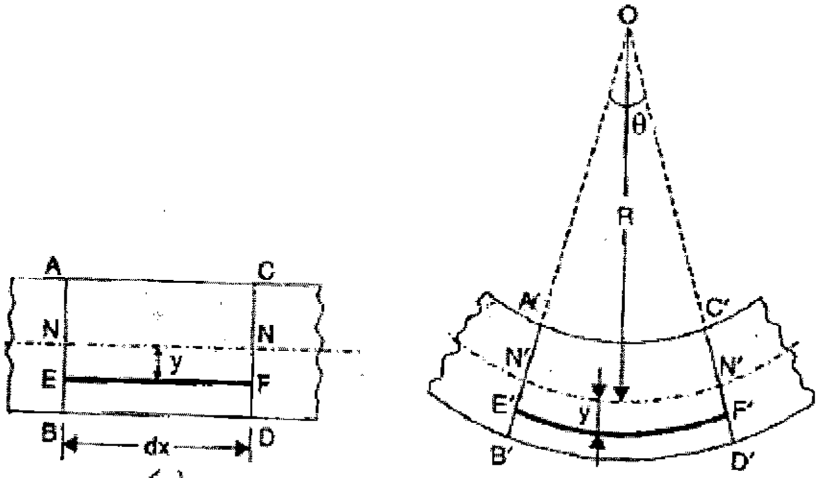
There is no shear force between A and B but bending moment between A and B is constant. This condition of beam between A & B is known as pure bending.

ASSUMPTIONS MADE IN THE THEORY OF SIMPLE BENDING:

1. The material of the beam is homogeneous and isotropic.
2. The value of Young's modulus of elasticity is same in tension and compression.
3. The transverse sections which were plane before bending remain plane after bending also.
4. The beam is initially straight and all longitudinal filaments bend into circular arcs with a common centre of curvature.
5. The radius of curvature is large compared with the dimensions of the cross-section.

THEORY OF SIMPLE BENDING OR DERIVATION OF BENDING EQUATION:

A small length δx of a beam subjected to a simple bending as shown in the figure (a) and due to action of bending, the par of length δx will be deformed as shown in the figure (b).



(a) (b)

Neutral layer or surface (N-N):

A layer which is neither shortened nor elongated is known as neutral layer.

Neutral axis (N-A):

The line of intersection of neutral layer on a cross-section of beam is known as neutral axis.

- Due to the decrease in length of the layers above N-N, these layers will be subjected to compressive stresses.
- Due to the increase in length of the layers above N-N, these layers will be subjected to tensile stresses.
- The amount by which a layer increases or decreases in length, depends upon the position of the layer w.r.t. N-N. This theory of bending is known as theory of simple bending.

Let

R = Radius of neutral layer N'-N'.

θ = Angle subjected at O by A'B' and C'D' produced.

y = Distance from the neutral layer.

Original length of the layer = EF = δx = NN = N'N'

From the above figure (b), N'N' = **R θ**

Increase in length of the EF = E'F' –EF = **(R + y) θ – R θ = y θ**

- Strain in the layer EF = e_{EF} = Increase in length / original length = **y θ / R θ = y / R**
- **e_{EF} α y**

$$\therefore \quad \sigma = E \times \frac{y}{R} = \frac{E}{R} \times y$$

$$\therefore \quad \frac{\sigma}{y} = \frac{E}{R}$$

- **σ α y**

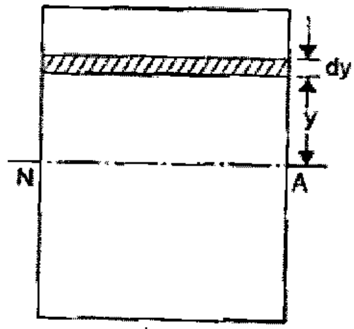
The below figure shows the cross section of the beam.

force on the layer

$= \text{Stress on layer} \times \text{Area of layer}$

$= \sigma \times dA$

$= \dfrac{E}{R} \times y \times dA$



Total force on the beam section

$= \int \dfrac{E}{R} \times y \times dA$

$= \dfrac{E}{R} \int y \times dA$

But for pure Bending, Total force = 0

$\therefore \quad \dfrac{E}{R} \int y \times dA = 0$

$\int y \times dA = 0$

- y dA represents the moment of area dA about neutral axis. That is the centroidal axis of a section gives the position of the neutral axis.

Force on layer $= \dfrac{E}{R} \times y \times dA$

Moment of this force about N.A.

$= \text{Force on layer} \times y$

$= \dfrac{E}{R} \times y \times dA \times y$

$= \dfrac{E}{R} \times y^2 \times dA$

Total moment of the forces on the section of the beam (or moment of resistance)

$\therefore \quad M = \int \dfrac{E}{R} \times y^2 \times dA = \dfrac{E}{R} \int y^2 \times dA$

$\therefore \quad M = \dfrac{E}{R} \times I \quad \text{or} \quad \dfrac{M}{I} = \dfrac{E}{R}$

$\therefore \quad \dfrac{M}{I} = \dfrac{\sigma}{y} = \dfrac{E}{R}$

The above equation is known as **Bending Equation.** And it is applicable at **bending moment is maximum.**

SECTION MODULUS (Z):

It is the ratio of moment of inertia of a section about the neutral axis to the distance of the outermost layer from the neutral axis.

$$Z = \frac{I}{y_{max}}$$

I = M.O.I. about neutral axis

y_{max} = Distance of the outermost layer from the neutral axis

$$\therefore \quad M = \sigma_{max} \cdot Z$$

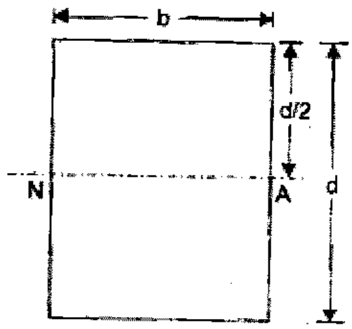
- Hence moment of resistance offered by the section is maximum when Z is maximum. Hence Z represents the strength of the section.

1. **Rectangular section:**

$$I = \frac{bd^3}{12}$$

$$y_{max} = \frac{d}{2}$$

$$Z = \frac{bd^2}{6}$$

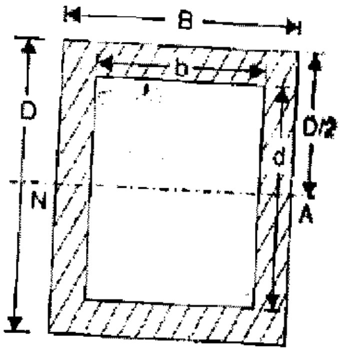


2. **Hollow Rectangular Section:**

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$y_{max} = \left(\frac{D}{2}\right)$$

$$Z = \frac{1}{6D}\left[BD^3 - bd^3\right]$$

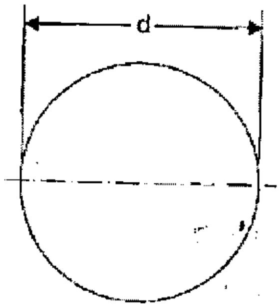


3. **Circular Section:**

$$I = \frac{\pi}{64} d^4$$

$$y_{max} = \frac{d}{2}$$

$$Z = \frac{\pi}{32} d^3$$

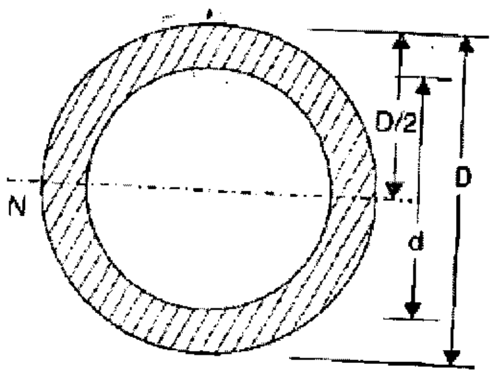


4. Hollow Circular Section:

$$I = \frac{\pi}{64}\left[D^4 - d^4\right]$$

$$y_{max} = \frac{D}{2}$$

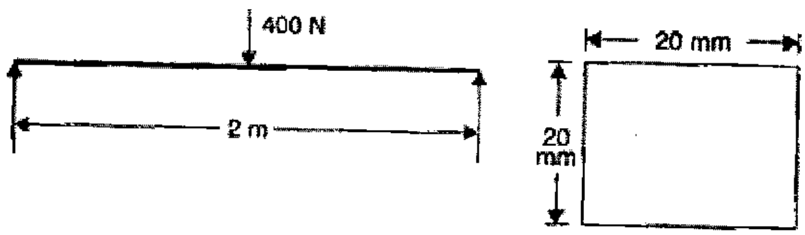
$$Z = \frac{\pi}{32D}\left[D^4 - d^4\right]$$



1. **A square beam 20mm X 20mm in section and 2m long is supported at the ends. The beam fails when a point load of 400N is applied at the centre of the beam. What uniformly distributed load per meter length will break a cantilever of the same material 4omm wide, 60mm deep and 3m long?**

ANS:

For simply supported beam:



Let σ_{max} = Max. stress induced

$$Z = \frac{bd^2}{6} = \frac{20 \times 20^2}{6} = \frac{4000}{3}\ \text{mm}^3 \qquad \& \qquad M = \frac{w \times L}{4} = \frac{400 \times 2}{4} = 200\ \text{Nm}$$

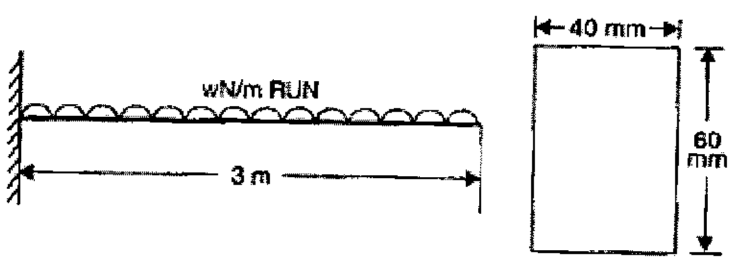
$$= 200 \times 1000 = 200000\ \text{Nmm}$$

$$M = \sigma_{max} . Z$$

$$200000 = \sigma_{max} \times \frac{4000}{3}$$

$$\sigma_{max} = \frac{200000 \times 3}{4000} = 150\ \text{N/mm}^2$$

For cantilever beam:



Let w = Uniformly distributed load per m run.

$$Z = \frac{40 \times 60^2}{6} = 24000 \text{ mm}^3$$

Maximum B.M. for a cantilever

$$= \frac{wL^2}{2} = \frac{w \times 3^2}{2} = 4.5w \text{ Nm}$$

$$\therefore \quad M = 4.5 \times 1000w \text{ Nmm}$$

$$M = \sigma_{max} \cdot Z$$

$$4.5 \times 1000w = 150 \times 24000$$

$$\therefore \quad w = \frac{150 \times 24000}{4.5 \times 1000} = 800 \text{ N/m.}$$

2. A timber beam of rectangular section of length 8m is simply supported. The beam carries a U.D.L. of 12KN/m run over the entire length and a point load of 10KN at 3m from the left support. If the depth is two times the width and the stress in the timber is not to exceed 8N/mm², find the suitable dimensions of the section?

ANS:

Given Data:

Length, $L = 8$ m

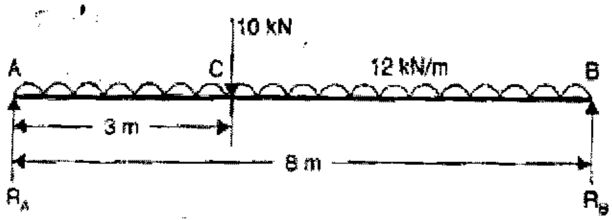
U.D.L., $w = 12$ kN/m $= 12000$ N/m

Point load, $W = 10$ kN $= 10000$ N

Depth of beam = 2 × Width of beam

$\therefore$ $d = 2b$

Stress, $\sigma_{max} = 8$ N/mm²



Taking moments about A, we get

$$R_B \times 8 = 12000 \times 8 \times 4 + 10000 \times 3$$

$$\therefore \quad R_B = \frac{12000 \times 32 + 30000}{8} = 51750 \text{ N}$$

$$\therefore \quad R_A = \text{Total load} - R_B$$

$$= (12000 \times 8 + 10000) - 51750 = 54250 \text{ N}$$

Now S.F. at A = $+R_A = +54250$ N

S.F. just L.H.S. at C = $54250 - 12000 \times 3 = +18250$ N

S.F. just R.H.S. of C = $18250 - 10000 = 8250$ N

S.F. at B = $-R_B = -51750$ N

The S.F. is changing sign between section CB and hence at some section in C and B the S.F. will be zero.

Let S.F. is zero at x metre from B.

Equating the S.F. at this section to zero, we have

$$12000 \times x - R_B = 0$$

or

$$12000 \times x - 51750 = 0$$

$$\therefore \quad x = \frac{51750}{12000} = 4.3125 \text{ m}$$

B.M. $= M = R_B \times 4.3125 - 12000 \times 4.3125 \times \dfrac{4.3125}{2}$

$= 51750 \times 4.3125 - 111585.9375$

$= 111585.9375 \text{ Nm} = 111585.9375 \times 1000 \text{ Nmm}$ & $Z = \dfrac{bd^2}{6} = \dfrac{b \times (2b)^2}{6} = \dfrac{2b^3}{3}$

$M = \sigma_{max} . Z$

$111585.9375 \times 1000 = 8 \times \dfrac{2b^3}{3}$ $b = (20.9223 \times 10^6)^{1/3} = \mathbf{275.5\ mm.}$

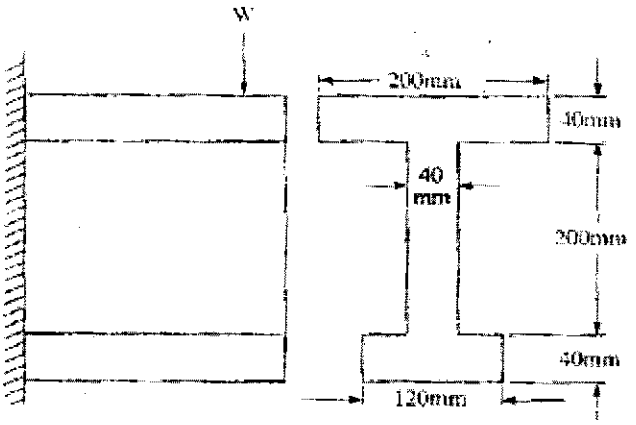
& $d = 2 \times 275.5 = \mathbf{551\ mm.}$ Ans.

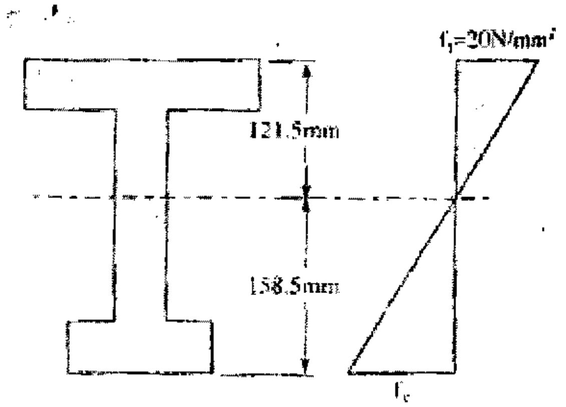
3. A cast iron bracket subjected to bending has the cross-section of I-form with unequal flanges. The total depth of the section is 280mm and the metal is 40mm thick throughout. The top flange is 200mm wide and the bottom flange is 120mm wide. Find the position of the neutral axis and moment of inertia of the section about the neutral axis and the maximum bending moment that should be imposed on this section if the tensile stress in the top flange is not to exceed 20N/mm². What is then the value of the maximum compressive stress in the bottom flange?

ANS:

Let the maximum bending moment be M *Nmm.*

Let the max. compressive stress be f_c $N/mm.^2$





Component	Area a mm²	Centroidal distance y from top edge y(mm)	ay (mm)³	ay² (mm)⁴	I_{xx} (mm)⁴
Top Flange	8000	20	16×10^4	320×10^4	$\dfrac{200 \times 40^3}{12} = 106.67 \times 10^4$
Web	8000	140	112×10^4	15680×10^4	$\dfrac{40 \times 200^3}{12} = 2666.67 \times 10^4$
Bottom Flange	4800	260	124.8×10^4	32448×10^4	$\dfrac{120 \times 40^3}{12} = 64 \times 10^4$
Total	20800		252.8×10^4	48448×10^4	2837.34×10^4

∴ Distance of the neutral axis from the upper edge

$$= \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{252.8\times10^4}{20800} = 121.5\ mm.$$

Moment of inertia about the upper edge

$$= I_{uu} = \Sigma I_{self} + \Sigma ay^2$$

$$2837.34 \times 10^4 + 48448 \times 10^4 = 511285.34 \times 10^4\ mm^4$$

But

$$I_{uu} = I_{xx} + (\Sigma a)\ \bar{y}^2$$

$$\therefore \quad 51285.34 \times 10^4 = I_{xx} + 20800 \times 121.5^2$$

$$I_{xx} = (51285.34 - 30705.48)\ 10^4 = 20579.86 \times 10^4\ mm^4$$

$$\therefore \quad \frac{M}{I} = \frac{f_t}{y_t} \Rightarrow M = \frac{f_t}{y_t} \times I$$

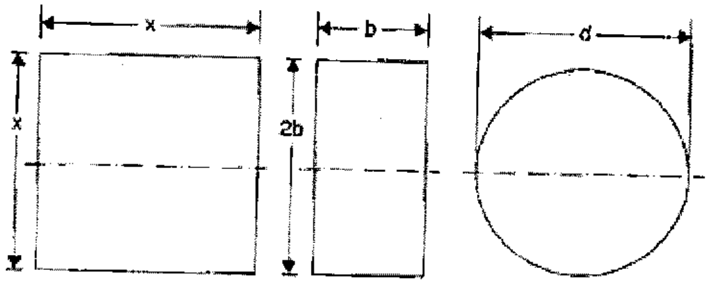
$$M = \frac{20}{121.5} \times 20579.86 \times 10^4\ Nmm = 33870\ Nm$$

$$\therefore \quad f_c = \frac{158.5}{121.5} \times 20\ N/mm^2. = 26.09\ N/mm^2.$$

4. Three beams have the same length, same allowable bending stress and the same bending moment. The cross-section of the beams are a square, rectangle with depth twice the width and a circle. Find the ratios of the weights of the circular and the rectangular beams with respect to square beams?

ANS:

The below figure shows a square, a rectangle and a circular section.



Let x = Side of a square beam

b = Width of rectangular beam

∴ $2b$ = Depth of the rectangular beam

d = Diameter of a circular section.

The moment of resistance of a beam is given by,

$$M = \sigma \times Z$$

where Z = Section modulus.

Section modulus of a square beam $= \frac{I}{y} = \frac{\frac{bd^3}{12}}{\left(\frac{d}{2}\right)} = \frac{x.x^3}{12} \times \frac{2}{x} = \frac{x^3}{6}$

Section modulus of a rectangular beam $= \frac{\frac{bd^3}{12}}{\frac{d}{2}} = \frac{\frac{b\times(2b)^3}{12}}{\left(\frac{2b}{2}\right)} = \frac{b\times 8b^3}{12}\times\frac{2}{2b} = \frac{2}{3}b^3$

Section modulus of a circular beam $= \frac{\frac{\pi d^4}{64}}{\frac{d}{2}} = \frac{\pi d^4}{64}\times\frac{2}{d} = \frac{\pi d^3}{32}$

Equating the section modulus of a square beam with that of a rectangular beam, we get

$$\frac{x^3}{6} = \frac{2}{3}b^3$$

$$\therefore \quad b^3 = \frac{3x^3}{6\times 2} = \frac{x^3}{4} = 0.25x^3$$

$$b = (0.25)^{1/3}\,x = 0.63x$$

Equating the section modulus of a square beam with that of a circular beam, we get

$$\frac{x^3}{6} = \frac{\pi d^3}{32}$$

$$\therefore \quad d^3 = \frac{32x^3}{6\pi} \quad \text{or} \quad d = \left(\frac{32}{6\pi}\right)^{1/3}.\,x = 1.1927x$$

The weights of the beams are proportional to their cross-sectional areas. Hence

$$\frac{\text{Weight of rectangular beam}}{\text{Weight of square beam}} = \frac{\text{Area of rectangular beam}}{\text{Area of square beam}}$$

$$= \frac{b\times 2b}{x\times x} = \frac{0.63x\times 2\times 0.63x}{x\times x}$$

$$= \mathbf{0.7938.} \quad \text{Ans.}$$

And

$$\frac{\text{Weight of circular beam}}{\text{Weight of square beam}} = \frac{\text{Area of circular beam}}{\text{Area of square beam}}$$

$$= \frac{\frac{\pi d^2}{4}}{x^2} = \frac{\pi d^2}{4x^2} = \frac{x\times(1.1927x)^2}{4x^2}$$

$$= \mathbf{1.1172.} \quad \text{Ans.}$$

SHEAR STRESSES IN BEAMS

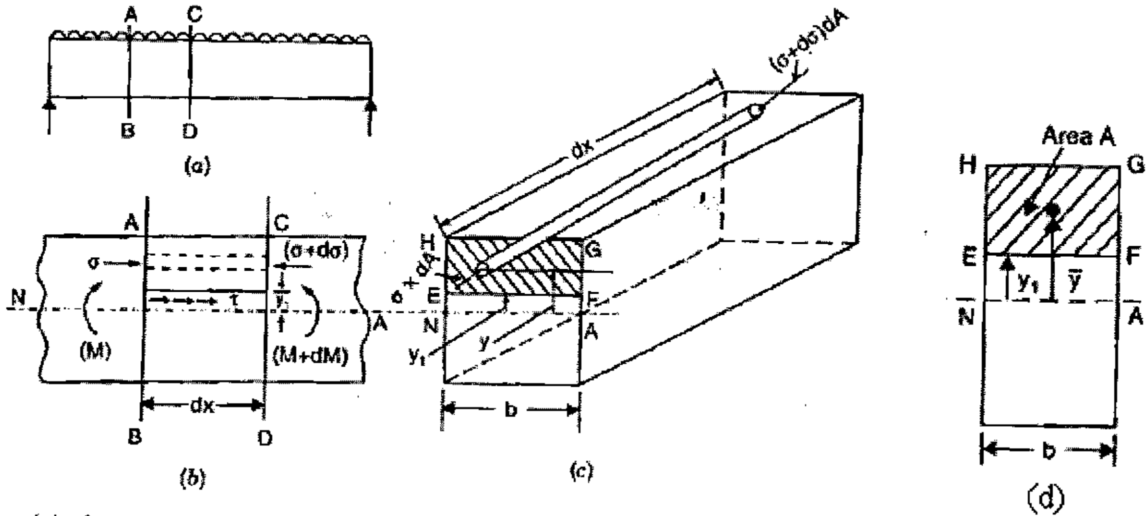
SHEAR STRESSES:

The stresses induced by shear force are known as bending stresses.

1. **Prove that the shear stress at any point in the cross section of a beam which is subjected to a shear force F, is given by**

$$\tau = F \times \frac{A\bar{y}}{I \times b}$$

ANS:



The figure (a) shows a simply supported beam carrying a U.D.L.

The figure (b) shows two sections AB & CD at a distance dx apart.

The figure (c) & (d) shows the cross section of the beam.

Let at section AB, F = Shear force & M = Bending moment

Let at section CD, F + dF = Shear force & M + dM = Bending moment

Let dA = Area of elemental cylinder & dx = Length of the elemental cylinder

y = Distance of elemental cylinder from neutral axis

σ = Intensity of bending stress on the elemental cylinder on the section AB

σ + dσ = Intensity of bending stress on the end of the elemental cylinder on the section CD

Bending stress on the end of elemental cylinder on the section AB, will be

$$\sigma = \frac{M}{I} \times y$$

Bending stress on the end of elemental cylinder on the section CD, will be

$$\sigma + d\sigma = \frac{(M + dM)}{I} \times y$$

Force on the end of the elemental cylinder on the section AB

$= \text{Stress} \times \text{Area of elemental cylinder} = \sigma \times dA = \frac{M}{I} \times y \times dA$

Similarly, force on the elemental cylinder on the section CD

$$= (\sigma + d\sigma) dA = \frac{(M + dM)}{I} \times y \times dA$$

$\therefore$ Net unbalanced force on the elemental cylinder $= \frac{dM}{I} \times y \times dA$

$\therefore$ Total unbalanced force $= \int \frac{dM}{I} \times y \times dA = \frac{dM}{I} \int y \times dA = \frac{dM}{I} \times A \times \bar{y}$

Where A = Area of the section above the level EF

$\bar{y}$ = Distance of the C.G. of the area A from the neutral axis.

$\therefore$ Shear resistance (or shear force) at the level EF = Total unbalanced force $= \frac{dM}{I} \times A \times \bar{y}$

Let τ = Intensity of horizontal shear at the level EF

b = Width of beam at the level EF

$\therefore$ Shear force due to τ = Shear stress $\times$ Shear area $= \tau \times b \times dx$

Equating the two values of shear force given by equations

$$\therefore \quad \tau \times b \times dx = \frac{dM}{I} \times A \times \bar{y}$$

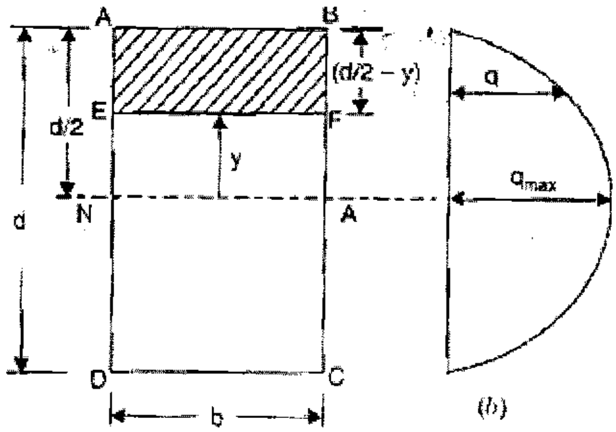
$$\therefore \quad \tau = \frac{dM}{dx} \cdot \frac{A\bar{y}}{I \times b} = F \times \frac{A\bar{y}}{I \times b}$$

2. Prove that the shear stress distribution in a RECTANGULAR section of a beam which is subjected to a shear force F is given by

$$\tau = \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right)$$

And show that for a rectangular section of the maximum shear stress is 1.5 times the average stress?

ANS: Let the shear force acting at the section = F



(a) (b)

Consider a level EF at a distance y from the neutral axis.

b = Actual width of the section at the level EF

A = Area of the section above y $= \left(\frac{d}{2} - y\right) \times b$

$\bar{y}$ = Distance of the C.G. of area A from neutral axis $= y + \frac{1}{2}\left(\frac{d}{2} - y\right) = \frac{1}{2}\left(y + \frac{d}{2}\right)$

The shear stress at the level EF = $\tau = F \cdot \frac{A\bar{y}}{b \times I} = \frac{F\left(\frac{d}{2} - y\right) \times b \times \frac{1}{2}\left(y + \frac{d}{2}\right)}{b \times I} = \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right)$

Therefore the variation of τ with respect to y is parabola.

At the Top edge, y = d / 2 and hence

$$\tau = \frac{F}{2I}\left[\frac{d^2}{4} - \left(\frac{d}{2}\right)^2\right] = \frac{F}{2I} \times 0 = 0$$

At the neutral axis, y = 0 and hence

$$\tau = \frac{F}{2I}\left(\frac{d^2}{4} - 0\right) = \frac{Fd^2}{8I} = \frac{Fd}{8 \times \frac{bd^3}{12}} = 1.5\,\frac{F}{bd} \qquad ...(i$$

Now average shear stress, $\tau_{avg} = \frac{\text{Shear force}}{\text{Area of section}} = \frac{F}{b \times d}$

Substituting the above value in equation (*i*), we get

$$\tau = 1.5 \times \tau_{avg} = \tau_{max}$$

3. A rectangular beam 100mm wide and 250mm deep is subjected to a maximum shear force of 50KN. Determine: (i) Average shear stress, (ii) maximum shear stress, and (iii) shear stress at a distance of 25mm above the neutral axis.

ANS: Given Data:

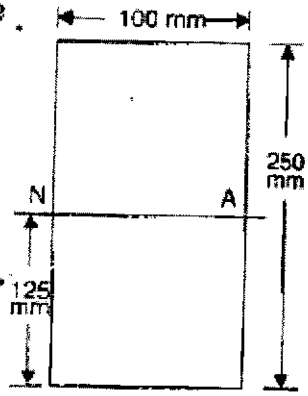
Width,	$b = 100$ mm
Depth,	$d = 250$ mm
Maximum shear force,	$F = 50$ kN $= 50,000$ N.

(*i*) Average shear stress $= \tau_{avg} = \frac{F}{\text{Area}} = \frac{50,000}{100 \times 250} = 2$ **N/mm²**.

(*ii*) Maximum shear stress $= \tau_{max} = 1.5 \times \tau_{avg} = 3$ **N/mm²**.

(*iii*) The shear stress at a distance y from N.A. is

$$= \tau = \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right) = \frac{50000}{2I}\left(\frac{250^2}{4} - 25^2\right) = 2.88 \text{ N/mm}^2.$$



4. Derive an expression for the shear stress at any point in a circular section of a beam, which is subjected to a shear force F. And prove that the maximum shear stress in a circular section of a beam is 4/3 times the average shear stress.

ANS:

The shear stress at the level EF = $\tau = F \cdot \dfrac{A\bar{y}}{b \times I}$

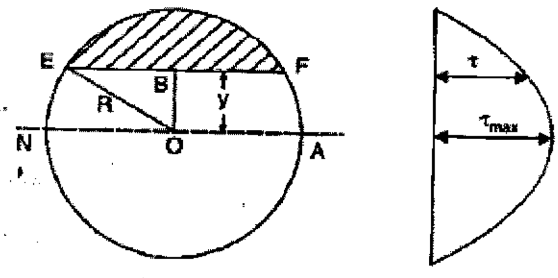
Consider a strip of thickness dy at a distance y from N.A.

Let dA is the area of strip.

Then $dA = b \times dy = EF \times dy$

$= 2 \times EB \times dy$

$= 2 \times \sqrt{R^2 - y^2} \times dy$



(a) (b)

Moment of this area dA about N.A. $= y \times dA = y \times 2\sqrt{R^2 - y^2} \times dy = 2y\sqrt{R^2 - y^2}\, dy.$

Moment of the whole shaded area about the N.A. $= A\bar{y} = \int_y^R 2y\sqrt{R^2 - y^2}\, dy$

$$= -\int_y^R (-2y)\sqrt{R^2 - y^2}\, dy.$$

$$= \frac{2}{3}(R^2 - y^2)^{3/2}$$

$$b = EF = 2 \times EB = 2 \times \sqrt{R^2 - y^2}$$

$$\therefore \quad \tau = F \cdot \frac{A\bar{y}}{b \times I} = \frac{F \times \frac{2}{3}(R^2 - y^2)^{3/2}}{I \times b} = \frac{\frac{2}{3}F(R^2 - y^2)^{3/2}}{I \times 2\sqrt{R^2 - y^2}} = \frac{F}{EI}(R^2 - y^2)$$

$\therefore$ At $y = 0$ *i.e.*, at the neutral axis, the shear stress is maximum and is given by

$$\tau_{max} = \frac{F}{3I}R^2 = \frac{F \times R^2}{3 \times \frac{\pi}{4}R^4} = \frac{4}{3} \times \frac{F}{\pi R^2}$$

But average shear stress,

$$\tau_{avg} = \frac{\text{Shear force}}{\text{Area of circular section}} = \frac{F}{\pi R^2}$$

$$\therefore \quad \tau_{max} = \frac{4}{3} \times \tau_{avg}$$

5. How will you prove that the shear stress changes abruptly at the junction of the flange and the web of an I-section?

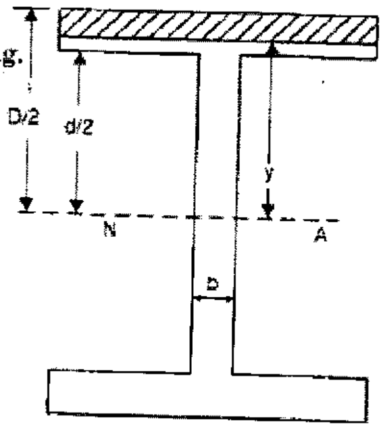
ANS: Let B = overall width of the section,

D = Overall depth of the section,

b = Thickness of the web, and d = Depth of web.

(i) Shear stress distribution in the flange

Consider a section at a distance y from N.A. in the flange as shown in Fig.



Width of the section = B

Shaded area of flange, $A = B\left(\frac{D}{2} - y\right)$

Distance of the C.G. of the shaded area from neutral axis is given as

$$\bar{y} = y + \frac{1}{2}\left(\frac{D}{2} - y\right) = \frac{1}{2}\left(\frac{D}{2} + y\right)$$

Hence shear stress in the flange becomes,

$$\tau = \frac{F \times A\bar{y}}{I \times B} = \frac{F \times B\left(\frac{D}{2} - y\right) \times \frac{1}{2}\left(\frac{D}{2} + y\right)}{I \times B} = \frac{F}{2I}\left(\frac{D^2}{2} - y^2\right)$$

(a) For the upper edge of the flange, $y = \frac{D}{2}$

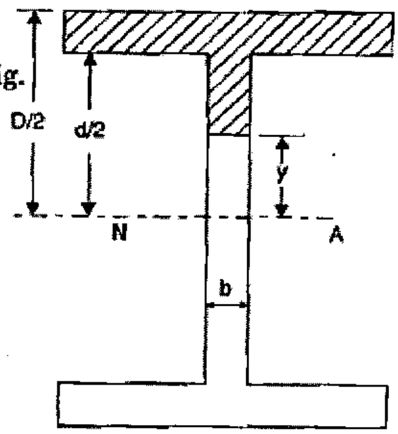
Hence shear stress, $\tau = \frac{F}{2I}\left[\frac{D^2}{4} - \left(\frac{D}{2}\right)^2\right] = 0.$

(b) For the lower edge of the flange, $y = \frac{d}{2}$

Hence $\tau = \frac{F}{2I}\left[\frac{D^2}{4} - \left(\frac{d}{2}\right)^2\right] = \frac{F}{2I}\left(\frac{D^2}{4} - \frac{d^2}{4}\right) = \frac{F}{8I}(D^2 - d^2)$

(ii) Shear stress distribution in the web

Consider a section at a distance y in the web from the N.A. as shown in Fig.



Width of the section = b.

$\therefore A\bar{y}$ = Moment of the flange area about N.A.

+ moment of the shaded area of web about N.A.

$$= B\left(\frac{D}{2} - \frac{d}{2}\right) \times \frac{1}{2}\left(\frac{D}{2} + \frac{d}{2}\right) + b\left(\frac{d}{2} - y\right) \times \frac{1}{2}\left(\frac{d}{2} + y\right)$$

$$= \frac{B}{8}(D^2 - d^2) + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)$$

Hence the shear stress in the web becomes as

$$\tau = \frac{F \times A\bar{y}}{I \times b} = \frac{F}{I \times b} \times \left[\frac{B}{8}(D^2 - d^2) + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)\right]$$

At the neutral axis, $y = 0$ and hence shear stress is maximum.

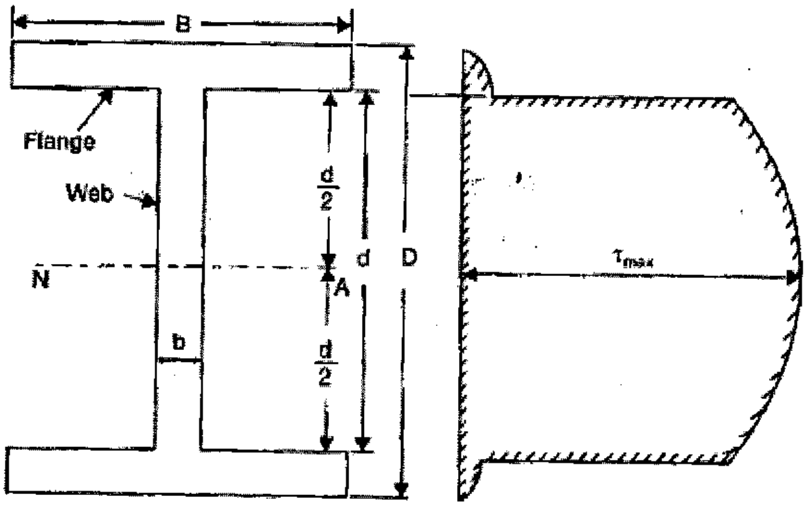
$$\therefore \qquad \tau_{max} = \frac{F}{I \times b}\left[\frac{B}{8}(D^2 - d^2) + \frac{b}{2} \times \frac{d^2}{4}\right] = \frac{F}{I \times b}\left[\frac{B(D^2 - d^2)}{8} + \frac{bd^2}{8}\right]$$

At the junction of top of the web and bottom of flange, $y = \frac{d}{2}$

Hence shear stress is given by,

$$\tau = \frac{F}{I \times b}\left[\frac{B}{8}(D^2 - d^2) + \frac{b}{2}\left(\frac{d^2}{4} - \left(\frac{d}{2}\right)^2\right)\right] = \frac{F \times B \times (D^2 - d^2)}{8I \times b}$$

The shear stress distribution for I- section is shown in the below figure.



(a) (b)

6. The shear force acting on a beam at an I-section with unequal flanges is 50KN. The section is shown in the below figure. The moment of inertia of the section about N.A. is 2.849X10^4. Calculate the shear stress at the N.A. and also draw the shear stress distribution over the depth of the section.

ANS:

Given Data:

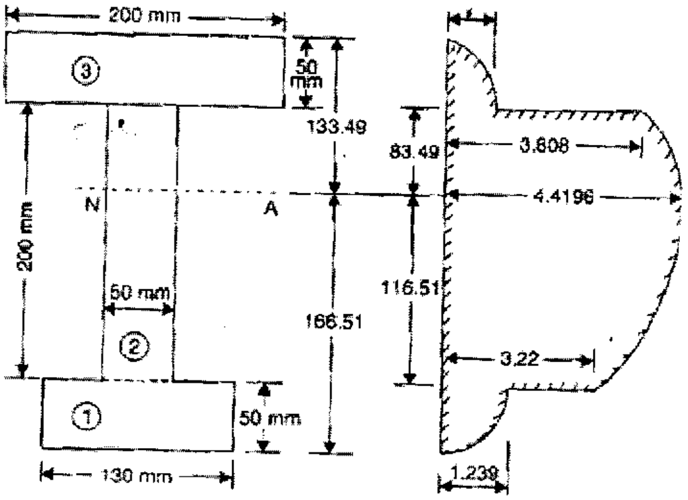
Shear force, $F = 50 \text{ KN} = 50{,}000 \text{ N}$

Moment of inertia about N.A.,

$I = 2.849 \times 10^8 \text{ mm}^4$.

Distance of the center of gravity from the bottom surface = $y^* = \dfrac{A_1y_1 + A_2y_2 + A_3y_3}{(A_1 + A_2 + A_3)}$

where A_1 = Area of bottom flange
$= 130 \times 50 = 6500\ mm^2$
A_2 = Area of web $= 200 \times 50 = 10000\ mm^2$
A_3 = Area of top flange $= 200 \times 50 = 10000\ mm^2$
y_1 = Distance of C.G. of A_1 from bottom face
$= \frac{50}{2} = 25$ mm
y_2 = Distance of C.G. of A_2 from bottom face
$= 50 + \frac{200}{2} = 150$ mm
y_3 = Distance of C.G. of A_3 from bottom face
$= 50 + 200 + \frac{50}{2} = 275$ mm



$$\therefore \quad y^* = \frac{6500 \times 25 \times 10000 \times 150 + 10000 \times 275}{6500 + 10000 + 10000} = 166.51 \text{ mm}$$

Hence N.A. is at a distance of 166.51mm from the bottom face & 133.49mm from upper top fiber.

Shear Stress Distribution:

$$\tau = \frac{F \times A\bar{y}}{I \times b}$$

(i) Shear stress at the extreme edges of the flanges is zero.

(ii) The shear stress in the upper flange just at junction of upper flange and web

$A\bar{y}$ = Moment of the area of the upper flange about N.A.
= Area of upper flange × Distance of the C.G. of upper flange from N.A.
$= (200 \times 50) \times (133.49 - 25) = 1084900$
b = Width of upper flange = 200 mm

$$\tau = \frac{50000 \times 1084900}{2.849 \times 108 \times 200} = 0.9520 \text{ N/mm}^2.$$

(iii) The shear stress in the web just at the junction of the web and upper flange will suddenly increase from 0.952 to 0.952 X 200 / 50 = 3.808 N/ mm².

(iv) The shear stress will be maximum at the N.A.

$A\bar{y}$ = Moment of total area (about N.A.) about N.A.
= Moment of area of upper flange about N.A. + Moment of area of web about N.A.

$$= 200 \times 50 \times (133.49 - 25) + (133.49 - 50) \times 50 \times \frac{(133.49 - 50)}{2} = 1259164.5$$

$b = 50$ mm

$$\tau_{max} = \frac{50000 \times 1259164.5}{2.849 \times 10^8 \times 50} = 4.4196 \text{ N/mm}^2.$$

(v) The shear stress in the lower flange just at the junction of the lower flange and the web.

$A\bar{y}$ = Moment of the area of the lower flange about N.A.

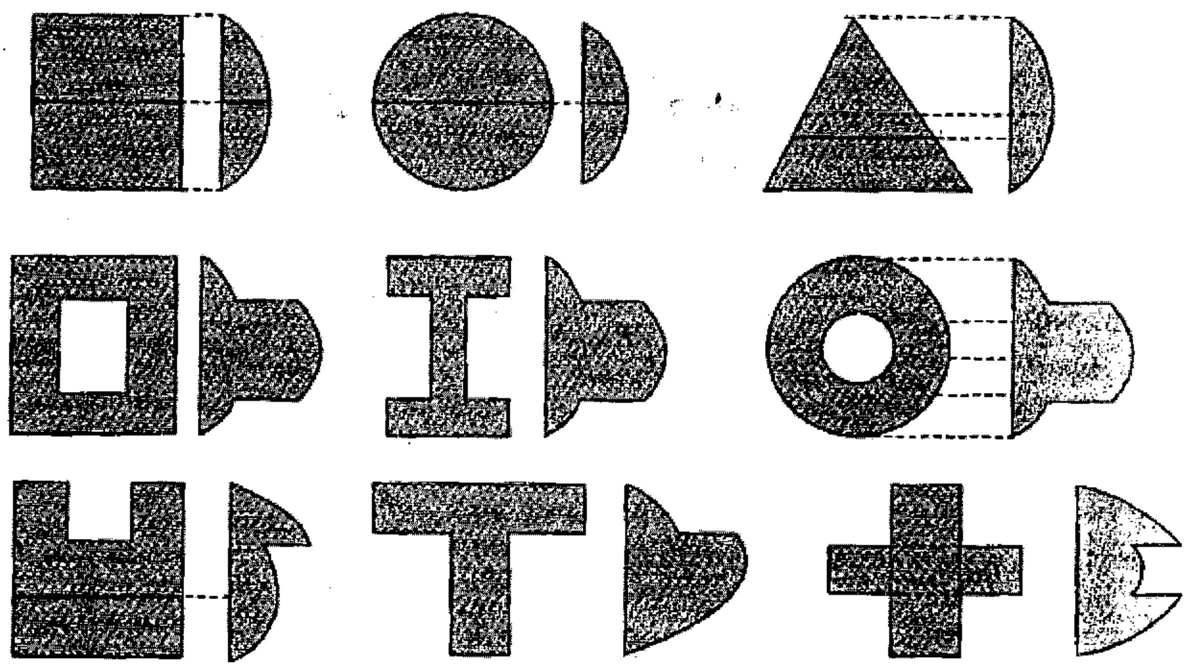
$= 130 \times 50 \times (166.51 - 25) = 918125$

b = Width of lower flange = 130 mm

$$\tau = \frac{50000 \times 918125}{2.849 \times 10^8 \times 130} = 1.239 \text{ N/mm}^2.$$

(vi) The shear stress in the web just at the junction of the web and lower flange will suddenly increase from 1.239 to 1.239 X 130 / 50 = 3.22 N/mm².

SHEAR STRESS DISTRIBUTION FOR BEAMS OF VARIOUS SECTIONS

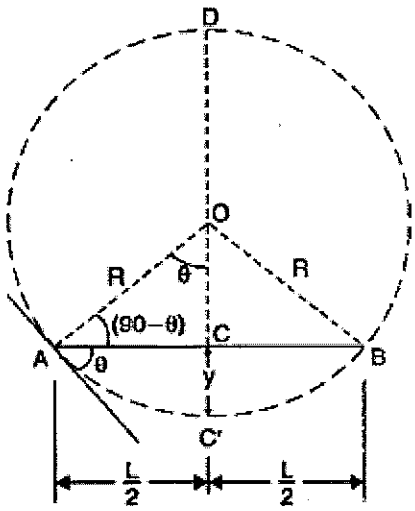


UNIT-4

DEFLECTION OF BEAMS

1. Derive an expression for the slope and deflection of a beam subjected to uniform bending moment?

Ans:



Let R = Radius of curvature of the deflected beam,
y = Deflection of the beam at the centre (*i.e.*, distance CC'),
I = Moment of inertia of the beam section,
E = Young's modulus for the beam material, and
θ = Slope of the beam at the end A (*i.e.*, the angle made by the tangent at A with the beam AB). For a practical beam the deflection y is a small quantity.

Hence,

$$\frac{dy}{dx} = \tan\theta = \theta.$$

$$AC = BC = \frac{L}{2}$$

&

$$AC \times CB = DC \times CC'$$

$$\frac{L}{2} \times \frac{L}{2} = (2R - y) \times y$$

$$\frac{L^2}{4} = 2Ry - y^2$$

Hence neglecting y^2, we get,

$$\frac{L^2}{4} = 2Ry$$

$$\therefore \quad y = \frac{L^2}{8R}$$

But from bending equation, we have

$$\frac{M}{I} = \frac{E}{R}$$

$$R = \frac{E \times I}{M}$$

&

$$y = \frac{L^2}{8 \times \frac{EI}{M}}$$

$$y = \frac{ML^2}{8EI}$$

From triangle AOB, we know that

$$\sin\theta = \frac{AC}{AO} = \frac{\left(\frac{L}{2}\right)}{R} = \frac{L}{2R} \longrightarrow \theta = \frac{L}{2R} = \frac{L}{2 \times \frac{EI}{M}} = \frac{M \times L}{2EI}$$

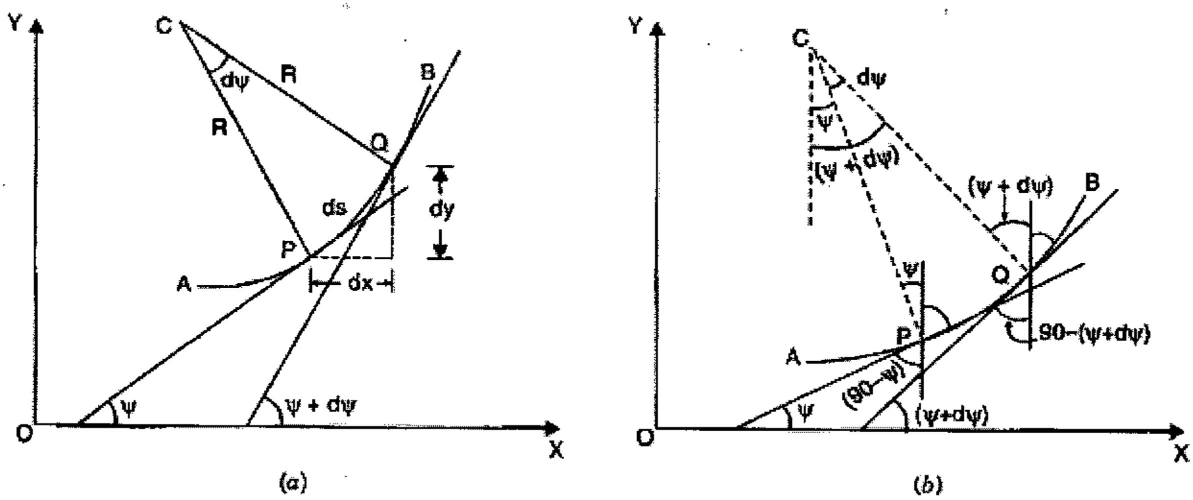
The above equation gives the slope of the deflected beam at A or at B.

2. Prove that the relation that Bending moment =

$$M = EI\,\frac{d^2y}{dx^2}$$ I = Moment of inertia of the beam section, E = Young's modulus for the beam material,

Ans:

Let the curve AB represents the deflection of a beam as shown in the below figure. Consider a small portion PQ of this beam. Let the tangents at P and Q make angle ψ + dψ with x-axis . Normal at P and Q will meet at C such that PC = QC = R



The point C is known as centre of curvature of the curve PQ.

Let the length of PQ is equal to ds.

From Fig. 12.2 (b), we see that

Angle $PCQ = d\psi$

$\therefore \quad PQ = ds = R.d\psi$

$$R = \frac{ds}{d\psi} \qquad \text{...}(i)$$

But if x and y be the co-ordinates of P, then

$$\tan\psi = \frac{dy}{dx} \qquad \text{...}(ii)$$

$$\sin\psi = \frac{dy}{ds}$$

$$\cos\psi = \frac{dx}{ds}$$

Now equation (i) can be written as

$$R = \frac{ds}{d\psi} = \frac{\left(\frac{ds}{dx}\right)}{\left(\frac{d\psi}{dx}\right)} = \frac{\left(\frac{1}{\cos\psi}\right)}{\left(\frac{d\psi}{dx}\right)}$$

$$R = \frac{\sec\psi}{\left(\frac{d\psi}{dx}\right)} \qquad \text{...}(iii)$$

Differentiating equation (ii) w.r.t. x, we get

$$\frac{d\psi}{dx} = \frac{\left(\frac{d^2y}{dx^2}\right)}{\sec^2 \psi}$$

Substituting this value of $\frac{d\psi}{dx}$ in equation (*iii*), we get

$$R = \frac{\sec \psi}{\left(\frac{\frac{d^2y}{dx^2}}{\sec^2 \psi}\right)} = \frac{\sec \psi \,.\sec^2 \psi}{\frac{d^2y}{dx^2}} = \frac{\sec^3 \psi}{\left(\frac{d^2y}{dx^2}\right)}$$

Taking the reciprocal to both sides, we get

$$\frac{1}{R} = \frac{\frac{d^2y}{dx^2}}{\sec^3 \psi} = \frac{\frac{d^2y}{dx^2}}{(\sec^2 \psi)^{3/2}}$$

$$= \frac{\frac{d^2y}{dx^2}}{(1 + \tan^2 \psi)^{3/2}}$$

For a practical beam, the slope $\tan \psi$ at any point is a small quantity. Hence $\tan^2 \psi$ be neglected.

$$\therefore \quad \frac{1}{R} = \frac{d^2y}{dx^2}$$

From the bending equation, we have

$$\frac{M}{I} = \frac{E}{R}$$

or

$$\frac{1}{R} = \frac{M}{EI}$$

Equating equations (*iv*) and (*v*), we get

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$\therefore \quad M = EI\frac{d^2y}{dx^2}$$

Differentiating the above equation w.r.t. x, we get

$$\frac{dM}{dx} = EI\frac{d^3y}{dx^3}$$

But $\frac{dM}{dx} = F$ shear force (See page 288)

$$\therefore \quad F = EI\frac{d^3y}{dx^3}$$

$$\frac{dF}{dx} = EI\frac{d^4y}{dx^4}$$

$\frac{dF}{dx} = w$ the rate of loading

$$w = EI\frac{d^4y}{dx^4}$$

Hence, the relation between curvature, slope, deflection etc. at a section is given by:

Deflection $= y$

Slope $= \frac{dy}{dx}$

Bending moment $= EI\frac{d^2y}{dx^2}$

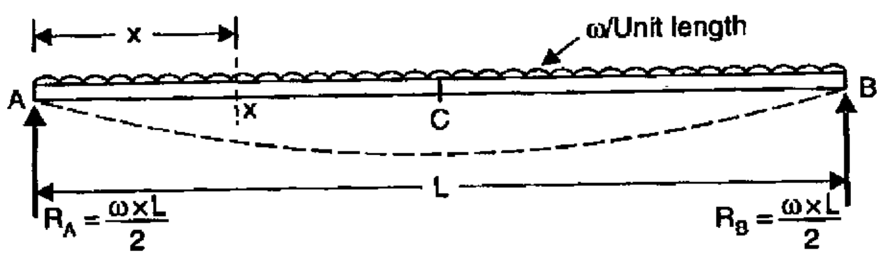
Shearing force $= EI\frac{d^3y}{dx^3}$

The rate of loading $= EI\frac{d^4y}{dx^4}$.

---

3. Find the deflection of a simply supported beam of length L carrying a uniform distributed load of w per unit length?

Ans:



$$R_A = R_B = \frac{w \times L}{2} \quad \& \quad M_x = R_A \times x - w \times x \times \frac{x}{2} = \frac{w.L}{2}.x - \frac{w.x^2}{2}$$

We know that B.M. at any section is

$$M = EI\frac{d^2y}{dx^2}$$

Equating the two values of B.M., we get

$$EI\frac{d^2y}{dx^2} = \frac{w.L}{2}x - \frac{w.x^2}{2}$$

Integrating the above equation, we get

$$EI\frac{dy}{dx} = \frac{w.L}{2}.\frac{x^2}{2} - \frac{w}{2}.\frac{x^3}{3} + C_1 \quad \text{------------ (i)}$$

Integrating the above equation again, we get

$$EI.y = \frac{w.L}{4}.\frac{x^3}{3} - \frac{w}{6}.\frac{x^4}{4} + C_1x + C_2 \quad \text{------------ (ii)}$$

The boundary conditions are:

(i) at x=0, y=0 and (ii) at x=L, y=0

Substituting first boundary condition *i.e.*, $x = 0, y = 0$ in equation (*ii*), we get

$$0 = 0 - 0 + 0 + C_2 \quad \text{or} \quad C_2 = 0$$

Substituting the second boundary condition *i.e.*, at $x = L, y = 0$ in equation (*ii*), we get

$$0 = \frac{w.L}{4}.\frac{L^3}{3} - \frac{w}{6}.\frac{L^4}{4} + C_1.L \qquad (C_2 \text{ is already zero})$$

$$= \frac{w.L^4}{12} - \frac{w.L^4}{24} + C_1 L$$

$$C_1 = -\frac{wL^3}{12} + \frac{wL^3}{24} = -\frac{wL^3}{24}$$

Substituting the value of C_1 in equations (*i*) and (*ii*), we get

$$EI\frac{dy}{dx} = \frac{w.L}{4}.x^2 - \frac{w}{6}x^3 - \frac{wL^3}{24} \qquad ...(iii)$$

and

$$EI.y = \frac{w.L}{12}x^3 - \frac{w}{24}.x^4 + \left(-\frac{wL^3}{24}\right)x + 0$$

or

$$EIy = \frac{w.L}{12}x^3 - \frac{w}{24}.x^4 - \frac{wL^3}{24}x \qquad \text{------------------------ (iv)}$$

Let θ_A = Slope at support A. This is equal to $\left(\frac{dy}{dx}\right)_{at\ A}$

and θ_B = Slop at support $B = \left(\frac{dy}{dx}\right)_{at\ B}$

At A, $x = 0$ and $\frac{dy}{dx} = \theta_A$.

Substituting these values in equation (*iii*), we get

$$EI.\theta_A = \frac{wL}{4}\times 0 - \frac{w}{6}\times 0 - \frac{wL^3}{24}$$

$$= \frac{wL^3}{24} = -\frac{WL^2}{24}$$

$$\therefore \quad \theta_A = -\frac{WL^2}{24EI} \quad \& \quad \text{By symmetry,} \quad \theta_B = -\frac{WL^2}{24EI}$$

$$EI.y_C = \frac{w.L}{12}.\left(\frac{L}{2}\right)^3 - \frac{w}{24}.\left(\frac{L}{2}\right)^4 - \frac{wL^3}{24}.\left(\frac{L}{2}\right)$$

$$= \frac{w.L^4}{96} - \frac{wL^4}{384} - \frac{wL^4}{48} = -\frac{5w.L^4}{384}$$

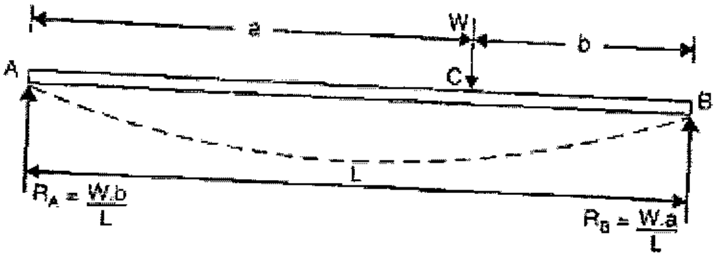
$$y_C = -\frac{5}{384}.\frac{wL^4}{EI} = -\frac{5}{384}.\frac{W.L^3}{EI} \qquad (\because \ w.L = W = \text{Total load})$$

---

4. Find the deflection of a simply supported beam of length L carrying a point load W at a distance 'a' from left support and at a distance 'b' from right supports by using MACAULAY'S METHOD?

Ans:

$$R_A = \frac{W.b}{L} \quad \text{and} \quad R_B = \frac{W.a}{L} \quad \& \quad M_x = R_A \times x = \frac{W.b}{L}\times x$$



The above equation of B.M. holds good for the values of x between 0 and 'a'. The B.M. at any section between C and B at a distance x from A is given by,

$$M_x = R_A . x - W \times (x - a)$$

$$= \frac{W.b}{L} . x - W(x - a)$$

The above equation of B.M. holds good for all values of x between $x = a$ and $x = b$.

The B.M. for all sections of the beam can be expressed in a single equation written as

$$M_x = \frac{W.b}{L} x \quad \vdots \quad - W(x - a) \qquad ...(i)$$

$$M = EI \frac{d^2y}{dx^2} \qquad ...(ii)$$

Hence equating (i) and (ii), we get

$$EI \frac{d^2y}{dx^2} = \frac{W.b}{L} . x \quad \vdots \quad - W(x - a) \qquad ...(iii)$$

Integrating the above equation, we get

$$EI \frac{dy}{dx} = \frac{W.b}{L} \frac{x^2}{2} + C_1 \quad \vdots \quad - \frac{W(x - a)^2}{2} \qquad ...(iv)$$

Integrating equation (iv) once again, we get

$$EIy = \frac{W.b}{2L} . \frac{x^3}{3} + C_1 x + C_2 \quad \vdots \quad - \frac{W}{2} \frac{(x - a)^3}{3} \qquad ...(v)$$

The boundary conditions are:

(i) At $x = 0$, $y = 0$ and **(ii) At $x = L$, $y = 0$**

(i) At A, $x = 0$ and $y = 0$. Substituting these values in equation (v) upto dotted line only,

$$0 = 0 + 0 + C_2$$

$$\therefore \quad C_2 = 0$$

(ii) At B, $x = L$ and $y = 0$. Substituting these values in equation (v), we get

$$0 = \frac{W.b}{2L} . \frac{L^3}{3} + C_1 \times L + 0 - \frac{W}{2} \frac{(L - a)^3}{3}$$

($\because$ $C_2 = 0$. Here complete Eq. (v) is to be taken)

$$= \frac{W . b . L^2}{6} + C_1 \times L - \frac{W}{2} \frac{b^3}{3} \qquad (\because \; L - a = b)$$

$$\therefore \quad C_1 \times L = \frac{W}{6} \cdot b^3 - \frac{W.b.L^2}{6} = -\frac{W.b}{6}(L^2 - b^2)$$

$$\therefore \quad C_1 = -\frac{W.b}{6L}(L^2 - b^2) \qquad \text{...(vi)}$$

Substituting the value of C_1 in equation (*iv*), we get

$$EI\frac{dy}{dx} = \frac{W.b}{L}\frac{x^2}{2} + \left[-\frac{W.b}{6L}\left(L^2 - b^2\right)\right] \quad \vdots \quad -\frac{W(x-a)^2}{2}$$

$$= \frac{W.b.x^2}{2L} - \frac{W.b}{6L}\left(L^2 - b^2\right) \quad \vdots \quad -\frac{W(x-a)^2}{2} \qquad \text{...(vii)}$$

Equation (*vii*) gives the slope at any point in the beam. Slope is maximum at A or B. To find the slope at A, substitute $x = 0$ in the above equation upto dotted line as point A lies in AC.

$$\therefore \quad EI.\theta_A = \frac{W.b}{2L} \times 0 - \frac{Wb}{6L}(L^2 - b^2) \qquad \left(\because \frac{dy}{dx} \text{ at } A = \theta_A\right)$$

$$= -\frac{Wb}{6L}(L^2 - b^2)$$

$$\therefore \quad \theta_A = -\frac{Wb}{6EIL}(L^2 - b^2) \qquad \text{(as given before)}$$

Substituting the values of C_1 and C_2 in equation (*v*), we get

$$EIy = \frac{W.b}{6L}.x^3 + \left[-\frac{Wb}{6L}(L^2 - b^2)\right]x + 0 \quad \vdots \quad -\frac{W}{6}(x-a)^3$$

$$EIy_c = \frac{W.b}{6L}.a^3 - \frac{W.b}{6L}(L^2 - b^2)a = \frac{W.b}{6L}.a\,(a^2 - L^2 + b^2)$$

$$y_c = -\frac{Wa^2.b^2}{3EIL}$$

---

5. Explain about MOMENT AREA METHOD?

<u>Ans:</u>

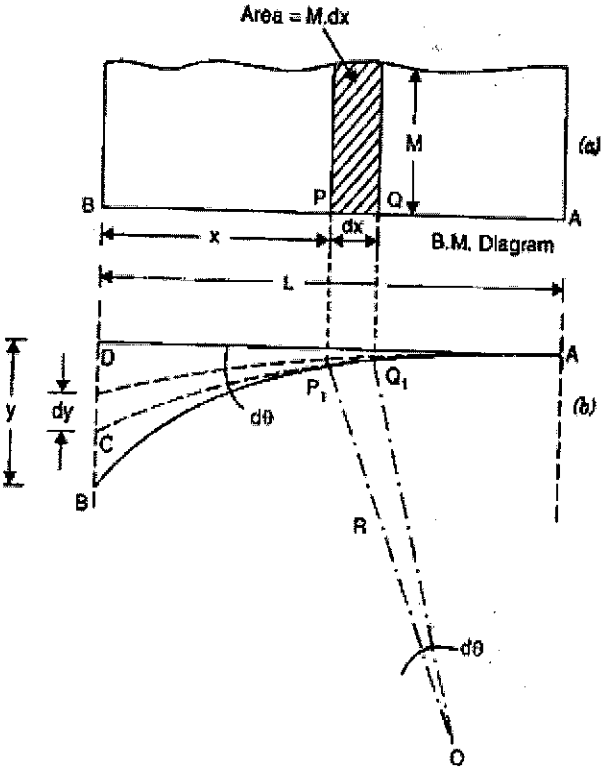
Consider an element PQ of small length dx at a distance x from B. The corresponding points on the deflected beam are P_1Q_1 as shown in Fig. 12.17 (*b*).

Let R = Radius of curvature of deflected part P_1Q_1

$d\theta$ = Angle subtended by the arc P_1Q_1 at the centre O

P_1C = Tangent at point P_1

Q_1D = Tangent at point Q_1.



For the deflected part P_1Q_1 of the beam, we have

$$P_1Q_1 = R.d\theta$$

But $P_1Q_1 \approx dx$

$\therefore$ $dx = R.d\theta$

$\therefore$ $d\theta = \frac{dx}{R}$

But for a loaded beam, we have

$$\frac{M}{I} = \frac{E}{R} \quad \text{or} \quad R = \frac{EI}{M}$$

Substituting the values of R in equation (i), we get

$$d\theta = \frac{dx}{\left(\frac{EI}{M}\right)} = \frac{M\,dx}{EI}$$

Since the slope at point A is assumed zero, hence total slope at B is obtained by integrating the above equation between the limits 0 and L.

$\therefore$ $$\theta = \int_0^L \frac{M.dx}{EI} = \frac{1}{EI}\int_0^L M.dx$$

But $M.dx$ represents the area of B. M. diagram of length dx. Hence $\int_0^L M.dx$ represents the area of B. M. diagram between A and B.

$\therefore$ $$\theta = \frac{1}{EI} \text{ [Area of B. M. diagram between } A \text{ and } B]$$

But θ = slope at $B = \theta_B$

$\therefore$ Slope at B,

$$\theta_B = \frac{\text{Area of } B.\,M. \text{ diagram between } A \text{ and } B}{EI}$$

If the slope at A is not zero then, we have

"Total change of slope between B and A is equal to the erea of B. M. diagram between B and A divided by the flexural rigidity EI"

or $$\theta_B - \theta_A = \frac{\text{Area of B.M. between } A \text{ and } B}{EI}$$

Now the deflection, due to bending of the portion P_1Q_1 is given by

$$dy = x.d\theta$$

Substituting the value of $d\theta$ from equation (ii), we get

$$dy = x\,.\,\frac{M.dx}{EI} \quad ...(iii)$$

Since deflection at A is assumed to be zero, hence the total deflection at B is obtained by integrating the above equation between the limits zero and L.

$$\therefore \quad y = \int_0^L \frac{xM.dx}{EI} = \frac{1}{EI}\int_0^L xM.dx$$

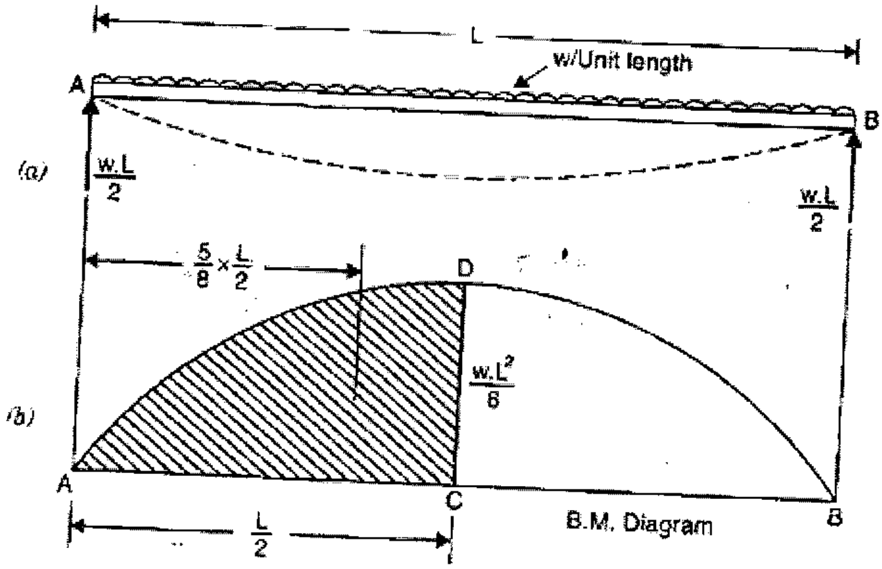
But $x \times M.dx$ represents the moment of area of the B.M. diagram of length dx about point B.

$$\therefore \quad y = \frac{1}{EI} \times A \times \bar{x} = \frac{A\bar{x}}{EI}$$

---

6. Find the slope and deflection of a simply supported beam AB of length L and carrying a U.D.L. of w per unit length over the entire span by using MOHR'S THEOREM?

Ans:



(i) Now using Mohr's theorem for slope, we get

Slope at $A = \dfrac{\text{Area of B.M. diagram between } A \text{ and } C}{EI}$

But area of B.M. diagram between A and C

$= \text{Area of parabola } ACD$

$= \dfrac{2}{3} \times AC \times CD$

$= \dfrac{2}{3} \times \dfrac{L}{2} \times \dfrac{wL^2}{8} = \dfrac{w.L^3}{24}$

$\therefore$ Slope at $A = \dfrac{w.L^3}{24EI}$

(ii) Now using Mohr's theorem for deflection, we get from equation (12.17) as

$$y = \frac{A\bar{x}}{EI}$$

$\bar{x}$ = Distance of C.G. of area A from A

$= \dfrac{5}{8} \times AC = \dfrac{5}{8} \times \dfrac{L}{2} = \dfrac{5L}{16}$

$$y = \frac{\frac{w.L^3}{24} \times \frac{5L}{16}}{EI} = \frac{5}{384}\frac{w.L^4}{EI}$$

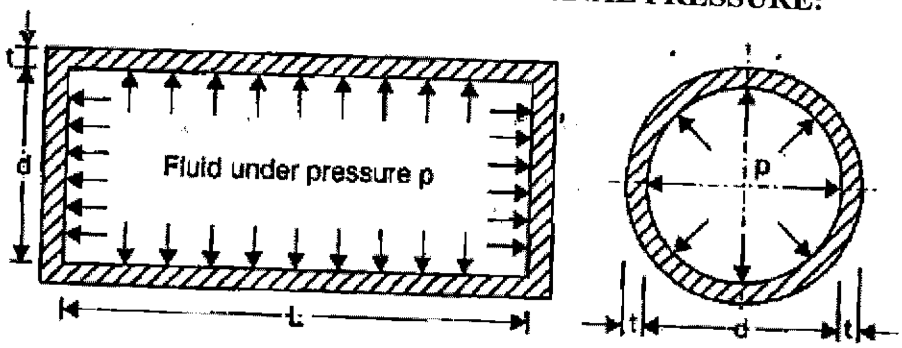
UNIT-5

THIN CYLINDERS

1. Derive the Expression for Hoop stress and longitudinal stress in a thin cylinder shells subjected to internal fluid pressure? (OR) Show that in thin cylinder shells subjected to internal fluid pressure, the Hoop stress is twice the longitudinal stress?

Ans:

THIN CYLINDRICAL VESSEL SUBJECTED TO INTERNAL PRESSURE:



The figure shows a thin cylindrical vessel in which a fluid under pressure is stored.

Let d = internal diameter of the thin cylinder

t = thickness of the wall of the cylinder

p = Internal pressure of the fluid

L = Length of the cylinder

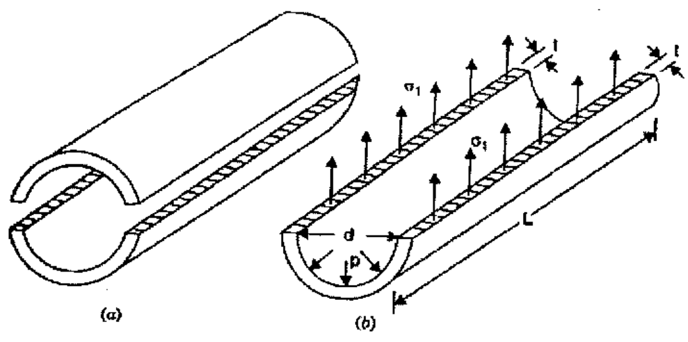
EXPRESSION FOR CIRCUMFERENTIAL STRESS (HOOP STRESS):

Let p = Internal pressure of fluid

d = Internal diameter of the cylinder

t = Thickness of the wall of the cylinder

σ_1 = Circumferential or hoop stress in the material



Force due to fluid pressure $= p \times$ Area on which p is acting

$= p \times (d \times L)$ ...(*i*)

($\because$ p is acting on projected area $d \times L$)

Force due to circumferential stress

$= \sigma_1 \times$ Area on which σ_1 is acting

$= \sigma_1 \times (L \times t + L \times t)$

$= \sigma_1 \times 2Lt = 2\sigma_1 \times L \times t$ ...(ii)

Equating (i) and (ii), we get

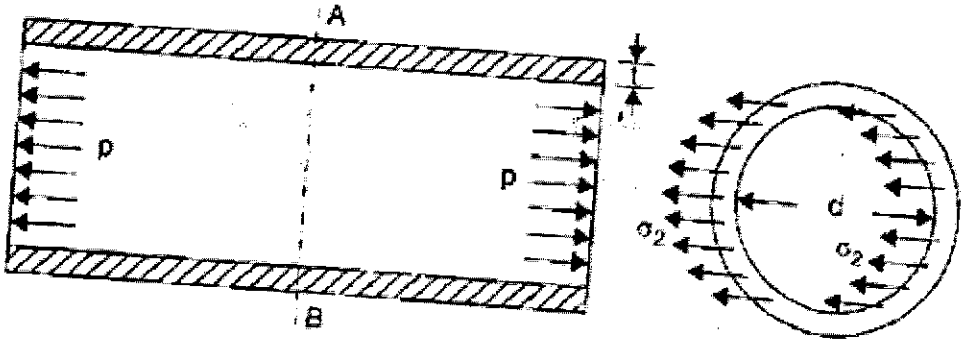
$$p \times d \times L = 2\sigma_1 \times L \times t$$

$$\therefore \quad \sigma_1 = \frac{pd}{2t} \text{ (cancelling } L\text{)}$$

EXPRESSION FOR LONGITUDINAL STRESS:

Let p = Internal pressure of fluid, d = Internal diameter of the cylinder

t = Thickness of the wall of the cylinder, σ_2 = Longitudinal stress in the material



(a)

(b)
Longitudinal stress (σ_2) develop

Resisting force = $\sigma_2 \times$ Area on which σ_2 is acting

$= \sigma_2 \times \pi d \times t$

∴ Hence in the limiting case

Force due to fluid pressure = Resisting force

$$p \times \frac{\pi}{4} d^2 = \sigma_2 \times \pi d \times t$$

$$\therefore \quad \sigma_2 = \frac{p \times \frac{\pi}{4} d^2}{\pi d \times t} = \frac{pd}{4t}$$

$$\sigma_2 = \frac{pd}{2 \times 2t} = \frac{1}{2} \times \sigma_1$$

Longitudinal stress = Half of circumferential stress.

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\frac{pd}{2t} - \frac{pd}{4t}}{2} = \frac{pd}{8t}$$

3. Derive the Expression for effect of internal pressure on the dimensions of a thin cylindrical shell?

Ans:

EXPRESSION FOR EFFECT OF INTERNAL PRESSURE ON THE DIMENSIONS OF A THIN CYLINDRICAL SHELL: $\sigma_1 = \dfrac{pd}{2t}$ & $\sigma_2 = \dfrac{p \times d}{4t}$,

Let e_1 = Circumferential strain,

e_2 = Longitudinal strain.

Then circumferential strain,

$$e_1 = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E}$$

But change in volume (δV) = Final volume – Original volume

Original volume (V) = Area of cylindrical shell × Length

$$= \frac{\pi}{4} d^2 \times L$$

Final volume

$$= \text{(Final area of cross-section)} \times \text{Final length}$$

$$= \frac{\pi}{4}[d + \delta d]^2 \times [L + \delta L]$$

$$= \frac{\pi}{4}[d^2 + (\delta d)^2 + 2d\,\delta d] \times [L + \delta L]$$

$$= \frac{\pi}{4}[d^2 L + (\delta d)^2 L + 2d\,L\delta d + \delta L d^2 + \delta L\,(\delta d)^2 + 2d\,\delta d \delta L]$$

Neglecting the smaller quantities such as $(\delta d)^2 L$, $\delta L(\delta d)^2$ and $2d\,\delta d\delta L$, we get

Final volume

$$= \frac{\pi}{4}[d^2 L + 2d\,L\delta d + \delta L\,d^2]$$

$\therefore$ Change in volume (δV)

$$= \frac{\pi}{4}[d^2 L + 2\,dL\delta d + \delta L d^2] - \frac{\pi}{4} d^2 \times L$$

$$= \frac{\pi}{4}[2d\,L\delta d + \delta L d^2]$$

$\therefore$ Volumetric strains

$$= \frac{\delta V}{V} = \frac{\frac{\pi}{4}[2d\,L\delta d + \delta L d^2]}{\frac{\pi}{4} d^2 \times L}$$

$$= \frac{2\delta d}{d} + \frac{\delta L}{L}$$

$$= 2e_1 + e_2 \qquad \left(\because \frac{\delta d}{d} = e_1, \frac{\delta L}{L} = e_2\right)$$

$$= 2 \times \frac{pd}{2Et}\left[1 - \frac{\mu}{2}\right] + \frac{pd}{2Et}\left(\frac{1}{2} - \mu\right)$$

(Substituting the values of e_1 and e_2)

$$= \frac{pd}{2Et}\left(2 - \frac{2\mu}{2} + \frac{1}{2} - \mu\right)$$

$$= \frac{pd}{2Et}\left(2 + \frac{1}{2} - \mu - \mu\right)$$

$$= \frac{pd}{2Et}\left(\frac{5}{2} - 2\mu\right)$$

Also change in volume (δV) = $V(2e_1 + e_2)$.

4. A cylindrical vessel is 1.5m diameter and 4m long is closed at ends by rigid plates. It is subjected to an internal pressure of 3N/mm2. If the maximum principal stress is not to exceed 150 N/ mm^2, find the thickness of the shell. Assume E = 2 X 10^5 N /mm^2 and poisson's ratio = 0.25. Find the changes in diameter, length and volume of the shell?

Ans:

Dia., $d = 1.5 \text{ m} = 1500 \text{ mm}$

Length, $L = 4 \text{ m} = 4000 \text{ mm}$

Internal pressure, $p = 3 \text{ N/mm}^2$

Max. principal stress $= 150 \text{ N/mm}^2$

Max. principal stress means the circumferential stress

$\therefore$ Circumferential stress, $\sigma_1 = 150 \text{ N/mm}^2$

Value of $E = 2 \times 10^5 \text{ N/mm}^2$,

Poisson's ratio, $\mu = 0.25$

Let t = thickness of the shell,

δd = change in diameter,

δL = change in length, and

δV = change in volume.

$$\sigma_1 = \frac{p \times d}{2t}$$

$$t = \frac{p \times d}{2 \times \sigma_1} = \frac{3 \times 1500}{2 \times 150}$$

$$= \textbf{15 mm.}$$

$$\delta d = \frac{pd^2}{2t \times E}\left(1 - \frac{1}{2} \times \mu\right)$$

$$= \frac{3 \times 1500^2}{2 \times 15 \times 2 \times 10^5}\left(1 - \frac{1}{2} \times 0.25\right) = \textbf{0.984 mm.}$$

&

$$\delta L = \frac{p \times d \times L}{2t \times E}\left(\frac{1}{2} - \mu\right)$$

$$= \frac{3 \times 1500 \times 4000}{2 \times 15 \times 2 \times 10^5}\left(\frac{1}{2} - 0.25\right)$$

$$= \textbf{0.75 mm.}$$

$$\frac{\delta V}{V} = \frac{p \times d}{2E \times t}\left(\frac{5}{2} - 2 \times \mu\right)$$

$$= \frac{3 \times 1500}{2 \times 2 \times 10^5 \times 15}\left(\frac{5}{2} - 2 \times 0.25\right) = \frac{3 \times 1500 \times 2}{4 \times 10^5 \times 15}$$

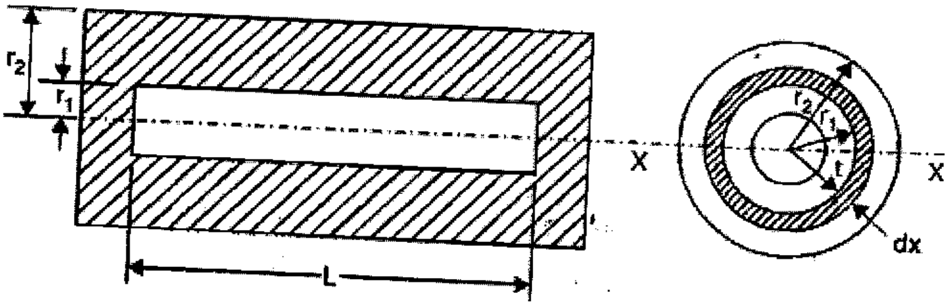
$$\delta V = \frac{3}{2000} \times V = \frac{3}{2000} \times \left(\frac{\pi}{4} \times d^2 \times L\right)$$

$$= \frac{3}{2000} \times \left(\frac{\pi}{4} \times 1500^2 \times 4000\right) = \textbf{10602875 mm}^3.$$

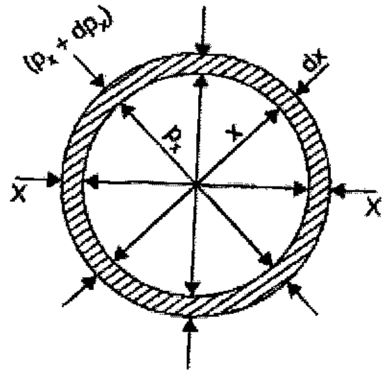
THICK CYLINDERS

1. Derive the expressions for the stresses in a thick cylindrical shell subjected to an internal fluid pressure? (OR) Derive the Lame's equations?

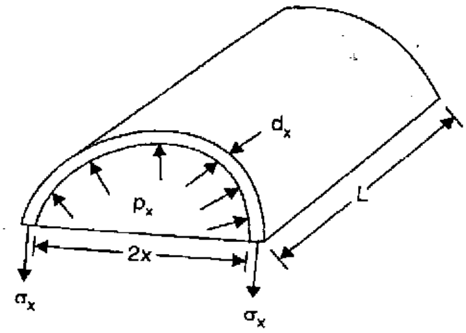
Ans:

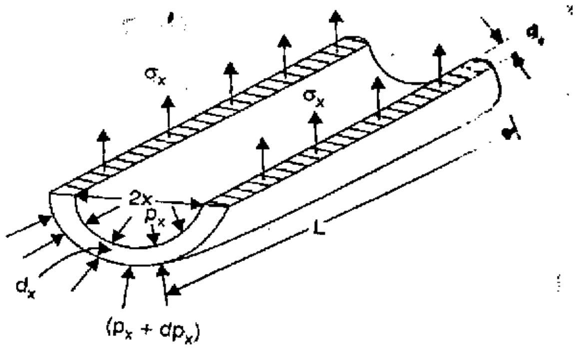


Let r_2 = External radius of the cylinder,
r_1 = Internal radius of the cylinder, and
L = Length of cylinder.



Let p_x = Radial pressure on the inner surface of the ring
$p_x + dp_x$ = Radial pressure on the outer surface of the ring
σ_x = Hoop stress induced in the ring.





Bursting force

$= p_x (2xL) - (p_x + dp_x) \times 2(x + dx) . L$

$= 2L\ [p_x . x - (p_x . x + p_x . dx + x.dp_x + dp_x . dx)]$

$= 2L\ [-p_x . dx - x . dp_x]$

$= -2L\ (p_x dx + x . dp_x)$ (Neglecting $dp_x . dx$

Resisting force = Hoop stress × Area on which it acts = $\sigma_x \times 2dx . L$

$$\sigma_x \times 2dx . L = -2L\ (p_x . dx + x . dp_x)$$

$$\sigma_x = -p_x - x\frac{dp_x}{dx}$$

The longitudinal strain (e_2) at this point is given by,

$$e_2 = \frac{\sigma_2}{E} - \frac{\mu\sigma_x}{E} + \frac{\mu p_x}{E}$$

But longitudinal strain is constant.

$$\therefore \quad \frac{\sigma_2}{E} - \frac{\mu\sigma_x}{E} + \frac{\mu p_x}{E} = \text{constant}$$

But σ_2 is also constant, and for the material of the cylinder E and μ are constant.

$\therefore \quad \sigma_x - p_x = \text{constant}$

$= 2a$ where a is constant

$$\therefore \quad \sigma_x = p_x + 2a$$

Equating the two values of σ_x given by equations (*iii*) and (*iv*), we get

$$p_x + 2a = -p_x - x\frac{dp_x}{dx}$$

$$x \cdot \frac{dp_x}{dx} = -p_x - p_x - 2a = -2p_x - 2a$$

$$\frac{dp_x}{dx} = -\frac{2p_x}{x} - \frac{2a}{x} = \frac{-2(p_x + a)}{x}$$

$$\frac{dp_x}{(p_x + a)} = -\frac{2dx}{x}$$

Integrating the above equation, we get

$$\log_e (p_x + a) = -2\log_e x + \log_e b$$

The above equation can also be written as

$$\log_e (p_x + a) = -\log_e x^2 + \log_e b$$

$$= \log_e \frac{b}{x^2}$$

$$\therefore \quad p_x + a = \frac{b}{x^2}$$

$$p_x = \frac{b}{x^2} - a$$

Substituting the values of p_x in equation (*iv*), we get

$$\sigma_x = \frac{b}{x^2} - a + 2a = \frac{b}{x^2} + a$$

These equations are called Lame's Equations.

Boundary conditions are:

(*i*) at $x = r_1, p_x = p_0$ or the pressure of fluid inside the cylinder, and
(*ii*) at $x = r_2, p_x = 0$ or atmosphere pressure.

After knowing the values of 'a' and 'b', the hoop stress can be calculated at any radius.

2. Find the thickness of a metal necessary for a cylindrical shell of internal diameter 160 mm to withstand an internal pressure of 8 N / mm². The maximum hoop stress in the section is to exceed 35 N /mm² ?

Ans:

Internal dia. $= 160$ mm

$\therefore$ Internal radius, $r_1 = \frac{160}{2} = 80$ mm

Internal pressure $= 8$ N/mm²

This means at $x = 80$ mm, $p_x = 8$ N/mm²

Maximum hoop stress, $\sigma_x = 35$ N/mm²

$$p_x = \frac{b}{x^2} - a \quad \ldots(i)$$

$$\sigma_x = \frac{b}{x^2} + a \quad \ldots(ii)$$

Substituting $x = 80$ mm and $p_x = 8$ N/mm² in equation (*i*), we get

$$8 = \frac{b}{80^2} - a = \frac{b}{6400} - a$$

Substituting $x = 80$ mm and $\sigma_x = 35$ N/mm² in equation (*ii*), we get

$$35 = \frac{b}{80^2} + a = \frac{b}{6400} + a \quad \ldots(iv)$$

Subtracting equation (*iii*) from equation (*iv*), we get

$$27 = 2a \quad \text{or} \quad a = \frac{27}{2} = 13.5$$

Substituting the value of a in equation (*iii*), we get

$$8 = \frac{b}{6400} - 13.5$$

$$b = (8 + 13.5) \times 6400 = 21.5 \times 6400$$

Substituting the values of 'a' and 'b' in equation (*i*),

$$p_x = \frac{21.5 \times 6400}{x^2} - 13.5$$

But at the outer surface, the pressure is zero. Hence at $x = r_2, p_x = 0$. Substituting these in the above equation, we get

$$0 = \frac{21.5 \times 6400}{r_2^2} - 13.5$$

$$r_2^2 = \frac{21.5 \times 6400}{13.5} \quad \text{or} \quad r_2 = \sqrt{\frac{21.5 \times 6400}{13.5}} = 100.96 \text{ mm}$$

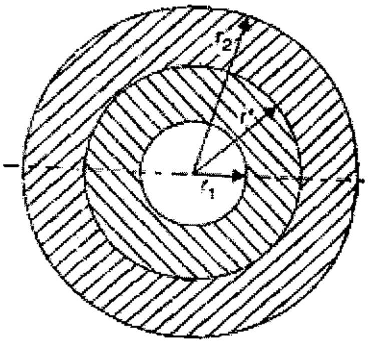
$\therefore$ Thickness of the shell, $t = r_2 - r_1$

$= 100.96 - 80 = 20.96$ mm. **Ans.**

3. Determine the hoop stresses in a thick compound cylinder?

Ans:

Let r_2 = Outer radius of compound cylinder
r_1 = Inner radius of compound cylinder
r^* = Radius at the junction of the two cylinders
p^* = Radial pressure at the junction of the two cylinders.



(i) For outer cylinder

The Lame's equations at a radius x for outer cylinder are given by

$$p_x = \frac{b_1}{x^2} - a_1 \quad ...(i) \qquad \sigma_x = \frac{b_1}{x^2} + a_1 \quad ...(ii)$$

where a_1, b_1 are constants for outer cylinder.

At $x = r_2, p_x = 0$. And at $x = r^*, p_x = p^*$

Substituting these conditions in equation (i), we get

$$0 = \frac{b_1}{r_2^2} - a_1 \quad ...(iii) \qquad p^* = \frac{b_1}{r^{*2}} - a_1 \quad ...(iv)$$

From equations (iii) and (iv), the constants a_1 and b_1 can be determined. These values substituted in equation (ii). And then hoop stresses in the outer cylinder due to shrinking can obtained.

(ii) For inner cylinder

The Lame's equations for inner cylinder at a radius x are given by

$$p_x = \frac{b_2}{x^2} - a_2, \quad \sigma_x = \frac{b_2}{x^2} + a_2$$

where a_2, b_2 are constants for inner cylinder.

At $x = r_1, p_x = 0$ as fluid under pressure is not admitted into the inner cylinder. At $x = r^*, p_x = p^*$.

Substituting these values in the above value of p_x, we get

$$0 = \frac{b_2}{r_1^2} - a_2 \quad ...(v) \quad \text{and} \quad p^* = \frac{b_2}{r^{*2}} - a_2 \quad ...(vi)$$

$$p_x = \frac{B}{x^2} - A \quad ...(vii) \quad \text{and} \quad \sigma_x = \frac{B}{x^2} + A \quad ...(viii)$$

---

4. A compound cylinder is made by shrinking a cylinder of external diameter 300mm and internal diameter of 250mm over another cylinder of external diameter 250mm and internal diameter 200mm. the radial pressure at the junction after shrinking is 8 N/ mm². Find the final stresses set up across the section, when the compound cylinder is subjected to an internal fluid pressure of 84.5 N/ mm²?

Ans:

For outer cylinder :

External diameter = 300 mm

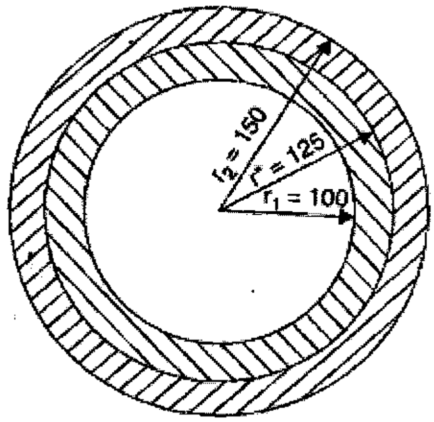
∴ External radius,

$$r_2 = \frac{300}{2} = 150 \text{ mm}$$

Internal diameter = 250 mm

∴ Radius at the junction,

$$r^* = \frac{250}{2} = 125 \text{ mm.}$$



For inner cylinder :

Internal diameter = 200 mm

∴ Internal radius

$$r_1 = \frac{200}{2} = 100 \text{ mm}$$

Radial pressure due to shrinking at the junction,

$$p^* = 8 \text{ N/mm}^2$$

Fluid pressure in the compound cylinder,

$$p = 84.5 \text{ N/mm}^2.$$

(i) *Stresses due to shrinking in the outer and inner cylinders before the fluid pressu admitted.*

(a) Lame's equations for outer cylinders are :

$$p_x = \frac{b_1}{x^2} - a_1 \quad \ldots(i) \qquad \text{and} \qquad \sigma_x = \frac{b_1}{x^2} + a_1$$

At $x = 150$ mm, $p_x = 0$.

Substituting these values in equation (*i*),

$$0 = \frac{b_1}{150^2} - a_1 = \frac{b_1}{22500} - a_1$$

At $x = r^* = 125$ mm, $p_x = p^* = 8$ N/mm^2.

Substituting these values in equation (*i*), we get

$$8 = \frac{b_1}{125^2} - a_1 = \frac{b_1}{15625} - a_1$$

Subtracting equation (*iii*) from equation (*iv*), we get

$$8 = -\frac{b_1}{22500} + \frac{b_1}{15625} = \frac{(-15625 + 22500)\, b_1}{22500 \times 15625}$$

$$\therefore \quad b_1 = \frac{8 \times 22500 \times 15625}{(-15625 + 22500)} = 409090.9$$

Substituting the value of b_1 in equation (*iii*), we get

$$0 = \frac{409090.9}{22500} - a_1 \quad \text{or} \quad a_1 = \frac{409090.9}{22500} = 18.18$$

Substituting the values of a_1 and b_1 in equation (*ii*), we get

$$\sigma_x = \frac{409090.9}{x^2} + 18.18$$

The above equation gives the hoop stress in the outer cylinder due to shrinking. The hoop stress at the outer and inner surface of the outer cylinder is obtained by substituting $x = 150$ mm and $x = 125$ mm respectively in the above equation.

$$\therefore \quad \sigma_{150} = \frac{409090.9}{150^2} + 18.18 = 36.36 \text{ N/mm}^2 \text{ (tensile)}$$

and $$\sigma_{125} = \frac{409090.9}{125^2} + 18.18 = 44.36 \text{ N/mm}^2 \text{ (tensile)}.$$

(*b*) Lame's equations for the inner cylinder are :

$$p_x = \frac{b_2}{x^2} - a_2 \quad \text{...}(v) \text{ and } \quad \sigma_x = \frac{b_2}{x^2} + a_2 \quad \text{...}(vi)$$

At $x = r_1 = 100$ mm, $p_x = 0$ (There is no fluid under pressure.)

Substituting these values in equation (*v*), we get

$$0 = \frac{b_2}{100^2} - a_2 = \frac{b_2}{10000} - a_2 \quad \text{...}(vii)$$

At $x = r^* = 125$ mm, $p_x = p^* = 8$ N/mm^2. Substituting these values in equation (*v*), we get

$$8 = \frac{b_2}{125^2} - a_2 = \frac{b_2}{15625} - a_2 \quad \text{...}(viii)$$

Subtracting equation (*vii*) from equation (*viii*), we get

$$8 = \frac{b_2}{15625} - \frac{b_2}{10000}$$

$$= \frac{b_2(10000 - 15625)}{15625 \times 10000} = \frac{-5625\, b_2}{15625 \times 10000}$$

$$\therefore \quad b_2 = -\frac{8 \times 15625 \times 10000}{5625} = -222222.2$$

Substituting the value of b_2 in equation (*vii*), we get

$$0 = -\frac{222222.2}{10000} - a_2$$

$$a_2 = -22.22$$

Substituting the values of a_2 and b_2 in equation (*vi*), we get

$$\sigma_x = -\frac{222222.2}{x^2} - 22.22$$

Hence the hoop stress for the inner cylinder is obtained by substituting $x = 125$ mm respectively in the above equation.

$$\therefore \quad \sigma_{125} = -\frac{222222.2}{125^2} - 22.22$$

$$= -14.22 - 22.22 = -36.44 \text{ N/mm}^2 \text{ (compressive)}$$

$$\sigma_{100} = -\frac{222222.2}{100^2} - 22.22$$

$$= -22.22 - 22.22 = -44.44 \text{ N/mm}^2 \text{ (compressive)}$$

(ii) Stresses due to fluid pressure alone

When the fluid under pressure is admitted inside the compound cylinder, the two cylinders together will be considered as one single unit. The hoop stresses are calculated by Lame's equations, which are

$$p_x = \frac{B}{x^2} - A \quad ...(ix) \quad \text{and} \quad \sigma_x = \frac{B}{x^2} + A \quad ...(x)$$

where A and B are constants.

At $x = 100$ mm, $p_x = p = 84.5$ N/mm². Substituting the values in equation (*ix*), we get

$$84.5 = \frac{B}{100^2} - A = \frac{B}{10000} - A \quad ...(xi)$$

At $x = 150$ mm, $p_x = 0$. Substituting these values in equation (*ix*), we get

$$0 = \frac{B}{150^2} - A = \frac{B}{22500} - A \quad ...(xii)$$

Subtracting equation (*xii*) from equation (*xi*), we get

$$84.5 = \frac{B}{10000} - \frac{B}{22500}$$

$$= \frac{B(22500 - 10000)}{10000 \times 22500} = \frac{12500 \times B}{10000 \times 22500}$$

$$\therefore \quad B = \frac{84.5 \times 10000 \times 22500}{12500} = 1521000$$

Substituting this value in equation (*xii*), we get

$$0 = \frac{1521000}{22500} - A \quad \text{or} \quad A = \frac{1521000}{22500} = 67.6$$

Substituting the values of A and B in equation (x), we get

$$\sigma_x = \frac{1521000}{x^2} + 67.6$$

Hence the hoop stresses due to internal fluid pressure alone are given by,

$$\sigma_{100} = \frac{1521000}{100^2} + 67.6 = 219.7 \text{ N/mm}^2 \text{ (tensile)}$$

$$\sigma_{125} = \frac{1521000}{125^2} + 67.6 = 97.344 + 67.6$$
$$= 164.94 \text{ N/mm}^2$$

$$\sigma_{150} = \frac{1521000}{150^2} + 67.6 = 67.6 + 67.6 = 135.2 \text{ N/mm}^2.$$

The resultant stresses will be the algebraic sum of the initial stresses due to shrinking and those due to internal fluid pressure.

Inner cylinder

$F_{100} = \sigma_{100}$ due to shrinkage + σ_{100} due to internal fluid pressure
$= -44.44 + 219.7 =$ **175.26 N/mm² (tensile).**

$F_{125} = \sigma_{125}$ due to shrinkage + σ_{125} due to internal fluid pressure
$= -36.44 + 164.94 =$ **128.5 N/mm² (tensile).**

Outer cylinder

$F_{125} = \sigma_{125}$ due to shrinkage + σ_{125} due to internal fluid pressure
$= 44.36 + 164.94 =$ **209.3 N/mm² (tensile).**

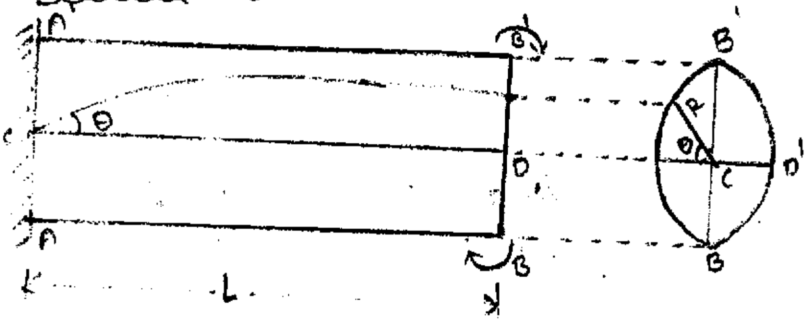
$F_{150} = \sigma_{150}$ due to shrinkage + σ_{150} due to internal fluid pressure
$= 36.36 + 135.2 =$ **171.56 N/mm² (tensile).**

UNIT-V TORSION AND COLUMNS

TORSION OF CIRCULAR SHAFTS

(1) Make a neat sketch of a circular shaft subjected to a twisting moment. Show clearly the variation of shear angle, angles of twist and shear stress in shaft. Derive the torsion formula. What assumptions are taken while deriving formula for a circular shaft?

Ans: DERIVATION OF TORSION EQUATION:



Consider a shaft fixed at one end AA and free end BB is subjected to torque (T) at the end BB. Now distorsion at the outer surface $\tau = \tan\phi = \phi = \frac{DD'}{CD} = \frac{DD'}{L}$ —— (1)

And shear strain ∴ $DD' = R\theta$ —— (2)

from ① & ②

$$\phi = \frac{R\theta}{2} \quad \text{—— ③}$$

Now Shear modulus or modulus of rigidity / rigidity of modulus

$$C = \frac{\tau}{\phi} = \frac{\text{Shear stress}}{\text{Shear strain}}$$

$$= \frac{\tau}{\frac{R\theta}{L}}$$

$$\Rightarrow \frac{\tau}{R} = \frac{C\theta}{L} \quad \text{—— ④}$$

If θ is constant where C and L are constant

then

$$\Rightarrow \tau/R = \text{Constant}$$

Let q at r $\quad \tau/R = q/r$

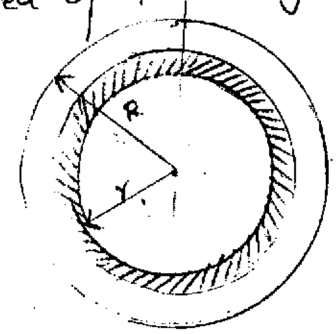
$$q/r = \frac{C\theta}{L} = \frac{\tau}{R} \quad \text{— ⑤}$$

from the Equation no: (5)

The shear stress is maximum at outer surface and is zero at the center of the shaft (axis of the shaft)

Consider an elemental circular ring of thickness (dr) at a distance 'r' from the center

∴ Area of the ring $= dA = (2\pi r)\,dr$



∴ Turning force on the ring

$\Rightarrow dF = q \times dA = \left(\frac{T}{R} \times r\right) \times dr$

∴ Turning moment on the ring

$=$ Torque $= dT = dF \times r$

$\Rightarrow dT = \left(T/R \times r^2\right) dA$

∴ Total turning momentum or Total torque

$$\Rightarrow T = \int_0^R dT = \int_0^R \tau/R \cdot r^2 \cdot dA$$

$$\Rightarrow T = \tau/R \int_0^R r^2 \, dA$$

$$\Rightarrow T = \tau/R \, (J) \quad \text{——— (6)}$$

From eqn (5) & (6)

$$\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{L}$$

where T = Torque / Turning moment / Twisting moment / Torsional Resistance of moment. (N-m)

J = polar moment of interia

$J = \int r^2 dA \ (m^4)$

τ = Shear stress at the Outer Surface (N/m²)

R = Radius of the Shaft

C = Shear modulus / modulus of Rigidity / Rigidity modulus (N/m²)

θ = Angle of twist / twisting angle (Degree)

L = Length of the Shaft.

Torsional Shear StressWhen a machine member is subjected to the actionof two equal and opposite couples acting in parallel planes(or torque or twisting moment), then the machine memberis said to be subjected to torsion. The stress set up by torsionis known as torsional shear stress. It is zero at the centroidalaxis and maximum at the outer surface.Consider a shaft fixed at one end and subjected to atorque (T) at the other end as shown in Fig. 5.1. As a resultof this torque, every cross-section of the shaft is subjectedto torsional shear stress. We have discussed above that thetorsional shear stress is zero at the centroidal axis and maximum at the outer surface. The maximum torsional shear stress at the outer surface of the shaft may be obtained from the followingequation:

$$\frac{\tau}{r} = \frac{T}{J} = \frac{C \cdot \theta}{l} \qquad \ldots(i)$$

where τ = Torsional shear stress induced at the outer surface of the shaft or maximum shear stress,

r = Radius of the shaft,

T = Torque or twisting moment,

J = Second moment of area of the section about its polar axis or polar moment of inertia,

C = Modulus of rigidity for the shaft material,

l = Length of the shaft, and

θ = Angle of twist in radians on a length l.

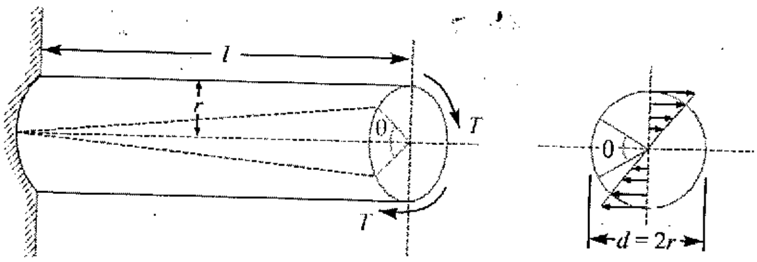


Fig. 5.1. Torsional shear stress.

The equation (i) is known as ***torsion equation.*** It is based on the following assumptions:

1. The material of the shaft is uniform throughout.
2. The twist along the length of the shaft is uniform.
3. The normal cross-sections of the shaft, which were plane and circular before twist, remain plane and circular after twist.
4. All diameters of the normal cross-section which were straight before twist, remain straight with their magnitude unchanged, after twist.
5. The maximum shear stress induced in the shaft due to the twisting moment does not exceed its elastic limit value.

Notes : 1. Since the torsional shear stress on any cross-section normal to the axis is directly proportional to the distance from the centre of the axis, therefore the torsional shear stress at a distance x from the centre of the shaft is given by

$$\frac{\tau_x}{x} = \frac{\tau}{r}$$

2. From equation (i), we know that

$$\frac{T}{J} = \frac{\tau}{r} \quad \text{or} \quad T = \tau \times \frac{J}{r}$$

For a solid shaft of diameter (d), the polar moment of inertia,

$$J = I_{XX} + I_{YY} = \frac{\pi}{64} \times d^4 + \frac{\pi}{64} \times d^4 = \frac{\pi}{32} \times d^4$$

$$\therefore \quad T = \tau \times \frac{\pi}{32} \times d^4 \times \frac{2}{d} = \frac{\pi}{16} \times \tau \times d^3$$

In case of a hollow shaft with external diameter (d_o) and internal diameter (d_i), the polar moment of inertia,

$$J = \frac{\pi}{32}[(d_o)^4 - (d_i)^4] \text{ and } r = \frac{d_o}{2}$$

$$\therefore \quad T = \tau \times \frac{\pi}{32}[(d_o)^4 - (d_i)^4] \times \frac{2}{d_o} = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right]$$

$$= \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4) \qquad \ldots\left(\text{Substituting, } k = \frac{d_i}{d_o}\right)$$

3. The expression ($C \times J$) is called ***torsional rigidity*** of the shaft.

4. The strength of the shaft means the maximum torque transmitted by it. Therefore, in order to design a shaft for strength, the above equations are used. The power transmitted by the shaft (in watts) is given by

$$P = \frac{2\pi N . T}{60} = T.\omega \qquad \ldots\left(\because \omega = \frac{2\pi N}{60}\right)$$

Example 1 *A steel shaft 35 mm in diameter and 1.2 m long held rigidly at one end has a hand wheel 500 mm in diameter keyed to the other end. The modulus of rigidity of steel is 80 GPa.*

1. What load applied to tangent to the rim of the wheel produce a torsional shear of 60 MPa?

2. How many degrees will the wheel turn when this load is applied?

Solution. Given : $d = 35$ mm or $r = 17.5$ mm ; $l = 1.2$ m = 1200 mm ; $D = 500$ mm or $R = 250$ mm ; $C = 80$ GPa = 80 kN/mm² = 80×10^3 N/mm² ; $\tau = 60$ MPa = 60 N/mm²

1. Load applied to the tangent to the rim of the wheel

Let W = Load applied (in newton) to tangent to the rim of the wheel.

We know that torque applied to the hand wheel,

$$T = W.R = W \times 250 = 250\, W \text{ N-mm}$$

and polar moment of inertia of the shaft,

$$J = \frac{\pi}{32} \times d^4 = \frac{\pi}{32}(35)^4 = 147.34 \times 10^3 \text{ mm}^4$$

We know that $\frac{T}{J} = \frac{\tau}{r}$

$$\therefore \quad \frac{250\, W}{147.34 \times 10^3} = \frac{60}{17.5} \quad \text{or} \quad W = \frac{60 \times 147.34 \times 10^3}{17.5 \times 250} = 2020 \text{ N}$$

$$\frac{T}{J} = \frac{C.\theta}{l}$$ **Ans.**

2. Number of degrees which the wheel will turn when load W = 2020 N is applied

Let θ = Required number of degrees.

We know that

$$\therefore \quad \theta = \frac{T.l}{C.J} = \frac{250 \times 2020 \times 1200}{80 \times 10^3 \times 147.34 \times 10^3} = 0.05°$$

Example 2. *A hollow shaft is required to transmit 600 kW at 110 r.p.m., the maximum torque being 20% greater than the mean. The shear stress is not to exceed 63 MPa and twist in a length of 3 metres not to exceed 1.4 degrees. Find the external diameter of the shaft, if the internal diameter to the external diameter is 3/8. Take modulus of rigidity as 84 GPa.*

Solution. Given : $P = 600$ kW = 600×10^3 W; $N = 110$ r.p.m. ; $T_{max} = 1.2\, T_{mean}$; $\tau = 63$ MPa = 63 N/mm² ; $l = 3$ m = 3000 mm ; $\theta = 1.4 \times \pi / 180 = 0.024$ rad ; $k = d_i / d_o = 3/8$; $C = 84$ GPa = 84×10^9 N/m² = 84×10^3 N/mm²

Let T_{mean} = Mean torque transmitted by the shaft,

d_o = External diameter of the shaft, and

d_i = Internal diameter of the shaft.

We know that power transmitted by the shaft (P),

$$600 \times 10^3 = \frac{2\pi N.T_{mean}}{60} = \frac{2\pi \times 110 \times T_{mean}}{60} = 11.52\, T_{mean}$$

$\therefore$ $T_{mean} = 600 \times 10^3/11.52 = 52 \times 10^3$ N-m $= 52 \times 10^6$ N-mm

and maximum torque transmitted by the shaft,

$$T_{max} = 1.2\, T_{mean} = 1.2 \times 52 \times 10^6 = 62.4 \times 10^6 \text{ N-mm}$$

Now let us find the diameter of the shaft considering strength and stiffness.

1. Considering strength of the shaft

We know that maximum torque transmitted by the shaft,

$$T_{max} = \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4)$$

$$62.4 \times 10^6 = \frac{\pi}{16} \times 63 \times (d_o)^3 \left[1 - \left(\frac{3}{8}\right)^4\right] = 12.12\,(d_o)^3$$

$\therefore$ $(d_o)^3 = 62.4 \times 10^6/12.12 = 5.15 \times 10^6$ or $d_o = 172.7$ mm ...(*i*)

2. Considering stiffness of the shaft

We know that polar moment of inertia of a hollow circular section,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{32}(d_o)^4\left[1 - \left(\frac{d_i}{d_o}\right)^4\right]$$

$$= \frac{\pi}{32}(d_o)^4(1 - k^4) = \frac{\pi}{32}(d_o)^4\left[1 - \left(\frac{3}{8}\right)^4\right] = 0.0962\,(d_o)^4$$

We also know that

$$\frac{T}{J} = \frac{C.\theta}{l}$$

$$\frac{62.4 \times 10^6}{0.0962\,(d_o)^4} = \frac{84 \times 10^3 \times 0.024}{3000} \quad \text{or} \quad \frac{648.6 \times 10^6}{(d_o)^4} = 0.672$$

$\therefore$ $(d_o)^4 = 648.6 \times 10^6/0.672 = 964 \times 10^6$ or $d_o = 176.2$ mm ...(*ii*)

Taking larger of the two values, we shall provide

$d_o = 176.2$ say 180 mm **Ans.**

5.3 Shafts in Series and Parallel

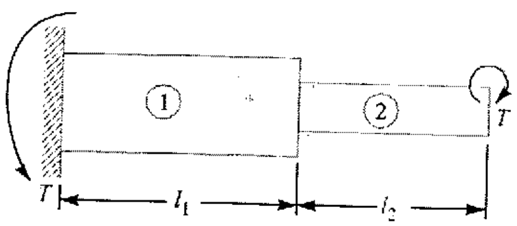
When two shafts of different diameters are connected together to form one shaft, it is then known as ***composite shaft.*** If the driving torque is applied at one end and the resisting torque at the other end, then the shafts are said to be connected in series as shown in Fig. 5.2 (*a*). In such cases, each shaft transmits the same torque and the total angle of twist is equal to the sum of the angle of twists of the two shafts.

Mathematically, total angle of twist,

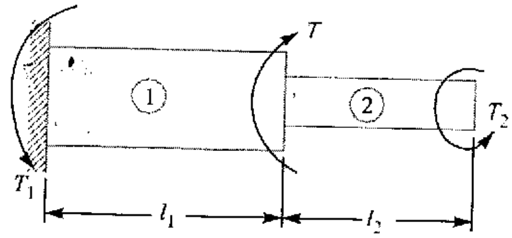
$$\theta = \theta_1 + \theta_2 = \frac{T.l_1}{C_1 J_1} + \frac{T.l_2}{C_2 J_2}$$

If the shafts are made of the same material, then $C_1 = C_2 = C$.

$$\therefore \quad \theta = \frac{T.l_1}{CJ_1} + \frac{T.l_2}{CJ_2} = \frac{T}{C}\left[\frac{l_1}{J_1} + \frac{l_2}{J_2}\right]$$



(*a*) Shafts in series.



(*b*) Shafts in parallel.

When the driving torque (T) is applied at the junction of the two shafts, and the resisting torques T_1 and T_2 at the other ends of the shafts, then the shafts are said to be connected in parallel, as shown in Fig. 5.2 (*b*). In such cases, the angle of twist is same for both the shafts, *i.e.*

$$\theta_1 = \theta_2$$

or
$$\frac{T_1 l_1}{C_1 J_1} = \frac{T_2 l_2}{C_2 J_2} \quad \text{or} \quad \frac{T_1}{T_2} = \frac{l_2}{l_1} \times \frac{C_1}{C_2} \times \frac{J_1}{J_2}$$

and
$$T = T_1 + T_2$$

If the shafts are made of the same material, then $C_1 = C_2$.

$$\therefore \quad \frac{T_1}{T_2} = \frac{l_2}{l_1} \times \frac{J_1}{J_2}$$

Example 3. *A steel shaft ABCD having a total length of 3.5 m consists of three lengths having different sections as follows:*

AB is hollow having outside and inside diameters of 100 mm and 62.5 mm respectively, and BC and CD are solid. BC has a diameter of 100 mm and CD has a diameter of 87.5 mm. If the angle of twist is the same for each section, determine the length of each section. Find the value of the applied torque and the total angle of twist, if the maximum shear stress in the hollow portion is 47.5 MPa and shear modulus, C = 82.5 GPa.

Solution. Given: $L = 3.5$ m ; $d_o = 100$ mm ; $d_i = 62.5$ mm ; $d_2 = 100$ mm ; $d_3 = 87.5$ mm ; $\tau = 47.5$ MPa $= 47.5$ N/mm²; $C = 82.5$ GPa $= 82.5 \times 10^3$ N/mm²

The shaft *ABCD* is shown in Fig. 5.3.

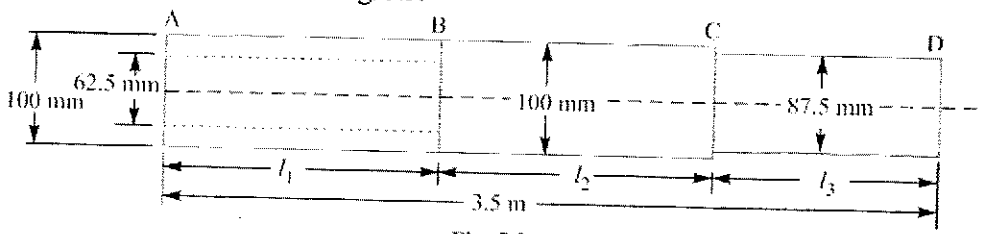


Fig. 5.3

Length of each section

Let l_1, l_2 and l_3 = Length of sections *AB*, *BC* and *CD* respectively.

We know that polar moment of inertia of the hollow shaft *AB*,

$$J_1 = \frac{\pi}{32}[(d_o)^4 - (d_i)^4] = \frac{\pi}{32}[(100)^4 - (62.5)^4] = 8.32 \times 10^6 \text{ mm}^4$$

Polar moment of inertia of the solid shaft *BC*,

$$J_2 = \frac{\pi}{32}(d_2)^4 = \frac{\pi}{32}(100)^4 = 9.82 \times 10^6 \text{ mm}^4$$

and polar moment of inertia of the solid shaft *CD*,

$$J_3 = \frac{\pi}{32}(d_3)^4 = \frac{\pi}{32}(87.5)^4 = 5.75 \times 10^6 \text{ mm}^4$$

We also know that angle of twist,

$$\theta = T \,.\, l / C \,.\, J$$

Assuming the torque T and shear modulus C to be same for all the sections, we have Angle of twist for hollow shaft *AB*,

$$\theta_1 = T \,.\, l_1 / C \,.\, J_1$$

Similarly, angle of twist for solid shaft *BC*,

$$\theta_2 = T \,.\, l_2 / C \,.\, J_2$$

and angle of twist for solid shaft *CD*,

$$\theta_3 = T \,.\, l_3 / C \,.\, J_3$$

Since the angle of twist is same for each sectiont,herefore

$$\theta_1 = \theta_2$$

$$J_3 = \frac{\pi}{32}(d_3)^4 = \frac{\pi}{32}(87.5)^4 = 5.75 \times 10^6 \text{ mm}^4$$

We also know that angle of twist,

$$\theta = T.l / C.J$$

Assuming the torque T and shear modulus C to be same for all the sections, we have

Angle of twist for hollow shaft AB,

$$\theta_1 = T.l_1 / C.J_1$$

Similarly, angle of twist for solid shaft BC,

$$\theta_2 = T.l_2 / C.J_2$$

and angle of twist for solid shaft CD,

Machine part of a jet engine.

$$\theta_3 = T.l_3 / C.J_3$$

Since the angle of twist is same for each sectiont,herefore

$$\theta_1 = \theta_2$$

$$\frac{T.l_1}{C.J_1} = \frac{T.l_2}{C.J_2} \quad \text{or} \quad \frac{l_1}{l_2} = \frac{J_1}{J_2} = \frac{8.32 \times 10^6}{9.82 \times 10^6} = 0.847 \qquad ...(i)$$

Also

$$\theta_1 = \theta_3$$

$$\frac{T.l_1}{C.J_1} = \frac{T.l_3}{C.J_3} \quad \text{or} \quad \frac{l_1}{l_3} = \frac{J_1}{J_3} = \frac{8.32 \times 10^6}{5.75 \times 10^6} = 1.447 \qquad ...(ii)$$

We know that $l_1 + l_2 + l_3 = L = 3.5 \text{ m} = 3500 \text{ mm}$

$$l_1\left(1 + \frac{l_2}{l_1} + \frac{l_3}{l_1}\right) = 3500$$

$$l_1\left(1 + \frac{1}{0.847} + \frac{1}{1.447}\right) = 3500$$

$$l_1 \times 2.8717 = 3500 \quad \text{or } l_1 = 3500 / 2.8717 = 1218.8 \text{ mm } \textbf{Ans.}$$

From equation (i),

$$l_2 = l_1 / 0.847 = 1218.8 / 0.847 = 1439 \text{ mm } \textbf{Ans.}$$

and from equation (ii),

$$l_3 = l_1 / 1.447 = 1218.8 / 1.447 = 842.2 \text{ mm } \textbf{Ans.}$$

Value of the applied torque

We know that the maximum shear stress in the hollow portion,

$$\tau = 47.5 \text{ MPa} = 47.5 \text{ N/mm}^2$$

For a hollow shaft, the applied torque,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{16} \times 47.5 \left[\frac{(100)^4 - (62.5)^4}{100}\right]$$

$$= 7.9 \times 10^6 \text{ N-mm} = 7900 \text{ N-m } \textbf{Ans.}$$

Total angle of twist

When the shafts are connected in series, the total angle of twist is equal to the sum of angle of twists of the individual shafts. Mathematically, the total angle of twist,

$$\theta = \theta_1 + \theta_2 + \theta_3$$

$$= \frac{T.l_1}{C.J_1} + \frac{T.l_2}{C.J_2} + \frac{T.l_3}{C.J_3} = \frac{T}{C}\left[\frac{l_1}{J_1} + \frac{l_2}{J_2} + \frac{l_3}{J_3}\right]$$

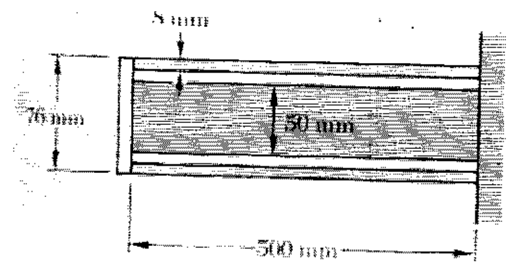
$$= \frac{7.9 \times 10^6}{82.5 \times 10^3}\left[\frac{1218.8}{8.32 \times 10^6} + \frac{1439}{9.82 \times 10^6} + \frac{842.2}{5.75 \times 10^6}\right]$$

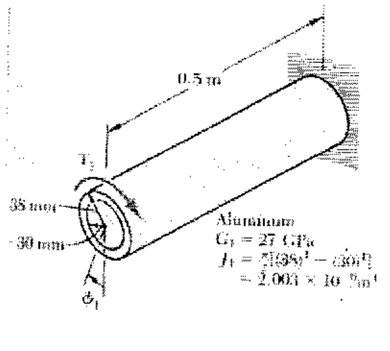
$$= \frac{7.9 \times 10^6}{82.5 \times 10^3 \times 10^6}[146.5 + 146.5 + 146.5] = 0.042 \text{ rad}$$

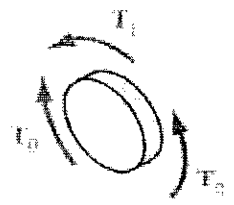
$$= 0.042 \times 180 / \pi = 2.406° \textbf{ Ans.}$$

5.4 Bending Stress in Straight Beams

A steel shaft and an aluminum tube are connected to a fixed support and to a rigid disk as shown in the cross section. Knowing that the initial stresses are zero, determine the maximum torque T_0 which may be applied to the disk if the allowable stresses are 120 MPa in the steel shaft and 70 MPa in the aluminum tube. Use $G = 80$ GPa for steel and $G = 27$ GPa for aluminum.







Statics. *Free Body of Disk.* Denoting by T_1 the torque exerted by the tube on the disk and by T_2 the torque exerted by the shaft, we find

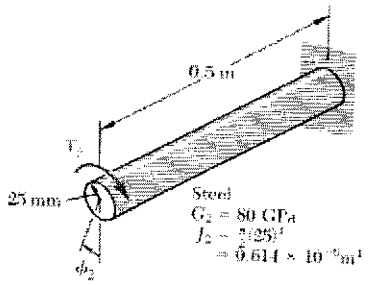
$$T_0 = T_1 + T_2 \qquad (1)$$

Deformations. Since both the tube and the shaft are connected to the rigid disk, we have

$$\phi_1 = \phi_2 : \qquad \frac{T_1 L_1}{J_1 G_1} = \frac{T_2 L_2}{J_2 G_2}$$

$$\frac{T_1(0.5 \text{ m})}{(2.003 \times 10^{-6} \text{ m}^4)(27 \text{ GPa})} = \frac{T_2(0.5 \text{ m})}{(0.614 \times 10^{-6} \text{ m}^4)(80 \text{ GPa})}$$

$$T_2 = 0.908 T_1 \qquad (2)$$



Shearing Stresses. We shall *assume* that the requirement $\tau_{alum} \leq 70$ MPa is critical. For the aluminum tube, we have

$$T_1 = \frac{\tau_{alum} J_1}{c_1} = \frac{(70 \text{ MPa})(2.003 \times 10^{-6} \text{ m}^4)}{0.038 \text{ m}} = 3690 \text{ N} \cdot \text{m}$$

Using Eq. (2), we compute the corresponding value T_2 and then find the maximum shearing stress in the steel shaft.

$$T_2 = 0.908 T_1 = 0.908(3690) = 3350 \text{ N} \cdot \text{m}$$

$$\tau_{steel} = \frac{T_2 c_2}{J_2} = \frac{(3350 \text{ N} \cdot \text{m})(0.025 \text{ m})}{0.614 \times 10^{-6} \text{ m}^4} = 136.4 \text{ MPa}$$

We note that the allowable steel stress of 120 MPa is exceeded; our assumption was *wrong*. Thus the maximum torque T_0 will be obtained by making $\tau_{steel} = 120$ MPa. We first determine the torque T_2.

$$T_2 = \frac{\tau_{steel} J_2}{c_2} = \frac{(120 \text{ MPa})(0.614 \times 10^{-6} \text{ m}^4)}{0.025 \text{ m}} = 2950 \text{ N} \cdot \text{m}$$

From Eq. (2), we have

$$2950 \text{ N} \cdot \text{m} = 0.908 T_1 \qquad T_1 = 3250 \text{ N} \cdot \text{m}$$

Using Eq. (1), we obtain the maximum permissible torque

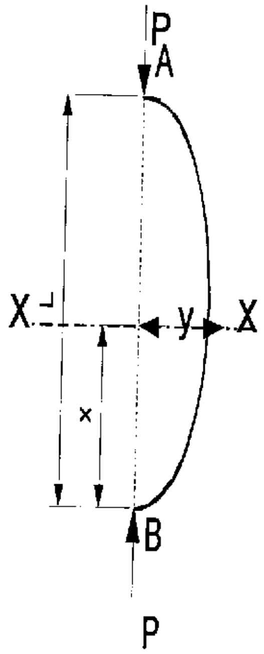
$$T_0 = T_1 + T_2 = 3250 \text{ N} \cdot \text{m} + 2950 \text{ N} \cdot \text{m}$$

$$T_0 = 6.20 \text{ kN} \cdot \text{m} \blacktriangleleft$$

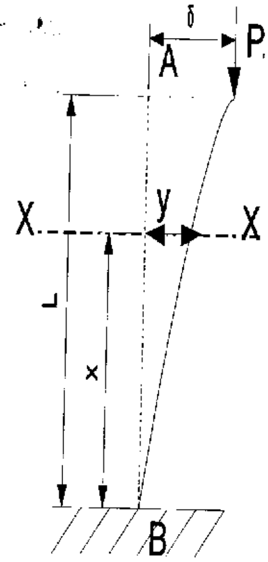
COLUMN

DERIVATIONS OF EULER'S FORMULA FOR DIFFERENT END CONDITIONS

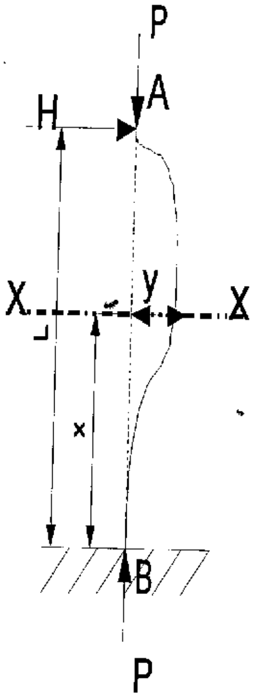
BOTH ENDS ARE HINGED OR PINNED



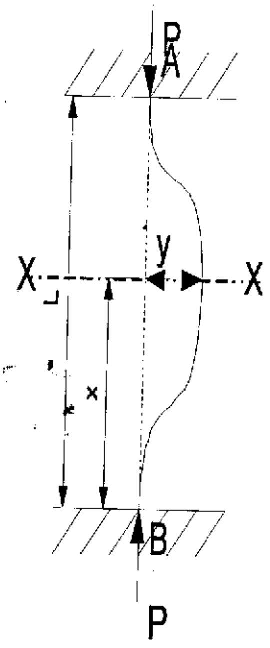
ONE END FIXED & OTHER END FREE



ONE END FIXED & OTHER END PINNED



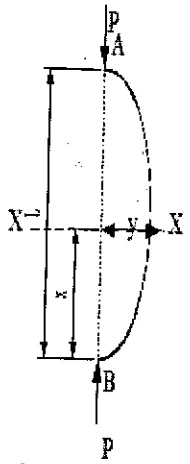
BOTH ENDS ARE FIXED



Unit - II

1) Derive an expression for Euler's buckling load for a column with

(a) Both the ends hinged



From bending moment equation

$$\Rightarrow M = EI \frac{d^2y}{dx^2}$$

$$\Rightarrow M = -Py$$

$$\Rightarrow EI \frac{d^2y}{dx^2} = -Py \qquad \left(\alpha = \sqrt{P/EI}\right)$$

$$\Rightarrow EI \frac{d^2y}{dx^2} + Py = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} + \left(P/EI\right) y = 0$$

$$y = C_1 \cos(\alpha x) + C_2 \sin(\alpha x)$$

$$y = C_1 \cos(x\sqrt{P/EI}) + C_2 \sin(x\sqrt{P/EI}) \quad -(1)$$

(i) At A

$x=0, \quad y=0$;

equation (i) substituting we get

$= C_1 \cos(0) + C_2 \sin(0)$

$= \quad C_1 = 0$

(ii) At 'B'

$x=L \quad y=0$

equat(i) substituting x.y we get

$\Rightarrow C_2 \sin(L\sqrt{P/EI})$

$C_2 = 0$

$\sin(L\sqrt{(P/EI)}) = 0$

$\sin(l\sqrt{P/EI}) = 0$

$L \cdot (\sqrt{P/EI}) = 0, \pi, 2\pi \cdots n\pi$

$L\sqrt{P/EI} = \pi$

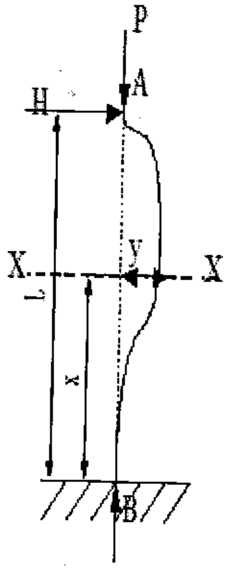
$$\Rightarrow \boxed{P = \frac{\pi^2 EI}{L^2}}$$

E = Young's modulus

I = Moment of Inertia

L = length of the beam

b) One end is fixed and the other is pinned (or) hinged ?



H = Horizontal reaction at B due to 'm' at A

From bending moment equation

$$M = EI \frac{d^2y}{dx^2}$$

$$M = -Py + H(L-x)$$

$$-Py + H(l-x) = EI \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} + \left(\frac{P}{EI}\right)y = \frac{H(l-x)}{EI}$$

$$\frac{d^2y}{dx^2} + \left(\frac{P}{EI}\right)y = \left(\frac{P}{EI}\right)\left(\frac{H(l-x)}{P}\right)$$

Solution of the Differential equation

$$y = C_1 \cos(P/EI)x + C_2 \sin((P/EI)x) + \frac{H(L-x)}{P} \quad \text{— (1)}$$

$x=0, \ y=0$

equation (1) substituting x and y we get

$$c_1 = -\frac{H(l)}{P}$$

$$c_1 = -\frac{Hl}{P}$$

(ii) At

$x=0 \quad \frac{dy}{dx}=0$

substituting in equation (1) we get

$$\frac{dy}{dx} = c_1 \sin(\sqrt{P/EI})(\sqrt{P/EI}) + c_2 \cos(x\sqrt{P/EI}) - (\sqrt{P/EI}) - \frac{H}{P} \quad (3)$$

$$\Rightarrow c_2(\sqrt{P/EI}) - \frac{H}{P}$$

$$c_2 = \frac{H}{P}\left(\sqrt{\frac{EI}{P}}\right)$$

equation (1) substituting above values

$$(1) \Rightarrow y = \frac{-Hl}{P}\cos(x\sqrt{P/EI}) + \frac{H}{P}\left(\sqrt{\frac{EI}{P}}\right)\sin(x\sqrt{P/EI}) + \frac{H(l-x)}{P} \quad (4)$$

iii) at 'B'

$x=L \quad y=0$

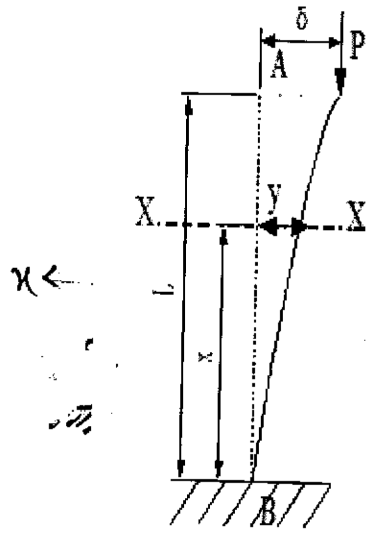
substituting in equation (1) x, y values we get

$$L\sqrt{P/EI} = 4.5^c = \sqrt{2}\pi$$

$$\tan(L\sqrt{P/EI}) = L\sqrt{P/EI}$$

$$\boxed{P = \frac{2\pi^2 EI}{L^2}}$$

(C) One end is fixed and the other is free ?



From Bending moment equation

$$M = EI \frac{d^2y}{dx^2}$$

$$M = (a - y) P$$

$$EI \frac{d^2y}{dx^2} = P(a-y)$$

$$\Rightarrow EI \frac{d^2y}{dx^2} - P(a-y) = 0$$

$$\Rightarrow EI \frac{d^2y}{dx^2} + Py = Pa$$

$$\frac{d^2y}{dx^2} + \left(\frac{P}{EI}\right) y = (P/EI)\, a$$

$$\therefore \frac{d^2y}{dx^2} + \alpha^2 y = \alpha^2 \cdot a$$

$$y = c_1 \cos(\alpha x) + c_2 \sin(\alpha x) + a$$

$$\Rightarrow y = c_1 \cos(\sqrt{P/EI}\, x) + c_2 \sin(\sqrt{P/EI}\, x) + a \quad \text{—(1)}$$

$x=0, \; y=0$

substituting in equation (1) x and y values.

we get

$(1) = 0$

$\Rightarrow C_1 + 0 + a = 0$

$\Rightarrow C_1 = -a$

(ii) At A $x = 0$ $\frac{dy}{dx} = 0$

substituting in equation (1)

we get

$(1) \Rightarrow \frac{dy}{dx} = -C_1 (\sin(\sqrt{P/EI}\, x)) \sqrt{P/EI} + C_2 \cos(\sqrt{P/EI}\, x) \sqrt{P/EI}$ —— (2)

$(2) \Rightarrow 0$

$= C_2 \sqrt{P/EI}$

$C_2 = 0$

$P/EI \neq 0$

$\therefore y = -a \cos(\sqrt{P/EI}\, x) + a$ —— (3)

(iii) At 'B'

$x = L \quad y = a$

$\Rightarrow (3) = 0$

$= -a \cos(L\sqrt{P/EI}) + a$

$\therefore \cos(L\sqrt{P/EI}) = 0$

$L\sqrt{P/EI} = \frac{\pi}{2}, \frac{3\pi}{2}$

$L\sqrt{P/EI} = \frac{\pi}{2}$

$$P = \frac{\pi^2 EI}{4l^2}$$

Problem: A Solid Round bar 3m long ... diameter is Used as a Strut with both Ends hinged. Determine the crippling (or Collapsing) Load. Take $E = 2.0 \times 10^5\ N/mm^2$ And also Determine the crippling load, when the given strut is Used with the following Conditions:

i) One End of the strut is fixed and the Other End is free

ii) Both the Ends of Strut are fixed

iii) One End is fixed and Other is hinged

Sol: Given: The data from problem is

Length of bar, $l = 3m = 3000\ mm$

Diameter of bar, $d = 5cm = 50\ mm$

Young's modulus $E = 2.0 \times 10^5\ N/mm^2$

moment of Inertia $I = \frac{\pi}{64} \times 5^4 = 30.68\ cm^4$

$= 30.68 \times 10^4\ mm^4$

Let P = Crippling load.

Let i) Crippling load when One End is fixed and Other End is free

Using equation $P = \frac{\pi^2 EI}{4l^2}$

$$= \frac{\pi^2 \times 2\times10^5 \times 30.68 \times 10^4}{4\times 3000^2}$$

$$= 16822\ N.$$

Alternate Method:

The crippling load for any type of End Condition is given by equation

$$P = \frac{\pi^2 EI}{L_e^2} \qquad \text{——— (1)}$$

where L_e = Effective length

The effective length (L_e) when One End is fixed and Other End is free from.

$$L_e = 2l = 2\times 3000 = 6000\ mm$$

Substituting the value of L in equation (1) we get,

$$P = \frac{\pi^2 \times 2\times10^5 \times 30.68 \times 10^4}{6000^2} = 16822\ N$$

ii) Crippling load when both ends are fixed:

Using equation $P = \frac{4\pi^2 EI}{l^2}$

$$= \frac{4\pi^2 \times 2\times10^5 \times 30.68 \times 10^4}{3000^2}$$

$$= 269152\ N = 269.152\ kN.$$

Alternate method:

Using equation $P = \frac{\pi^2 EI}{L_e^2}$

L_e = Effective length

$= l/2$

$= \frac{3000}{2}$

$= 1500\ mm$

$$P = \frac{\pi^2 \times 2.0 \times 10^5 \times 30.68 \times 10^4}{1500^2}$$

$$P = 269152\ N.$$

iii) Crippling load when One End is fixed and the other is hinged :

Useing eqn. $P = \frac{2\pi^2 E I}{l^2}$.

$$= \frac{2 \times \pi^2 \times 2.0 \times 10^5 \times 30.68 \times 10^4}{3000^2}$$

$$= 134576 \text{ N}$$

Alternate Method :

Using eqn. $P = \frac{\pi^2 E I}{L_e^2}$.

L_e = EFFECTIVE LENGTH.

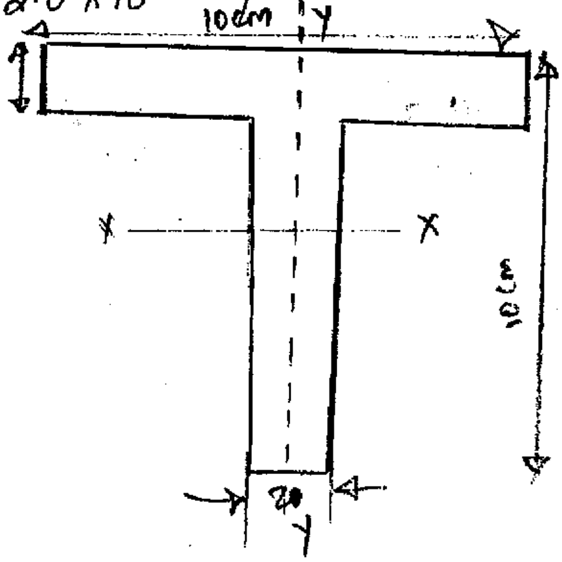
$= l/\sqrt{2}$ (when One End is fixed and the other is hinged)

$$P = \frac{\pi^2 \times 2.0 \times 10^5 \times 30.68 \times 10^4}{\left[\frac{3000}{\sqrt{2}}\right]^2}$$

$P = 134576$ N.

problem: Determine the crippling load for a T-section of dimensions 10cm × 10cm × 2cm and of length 5m when it is used as strut with both of its ends hinged. Take Young's modulus

$E = 2.0 \times 10^5 \ N/mm^2$



Sol:

Given: Dimensions of T-section = 10cm × 10cm × 2cm

Actual length, l = 5m = 5000 mm

Young's modulus, $E = 2.0 \times 10^5 \ N/mm^2$

y_1 = Distance of C.G of area a_1 from the bottom end = 8 + 1 = 9cm

For the web, we have $a_2 = 8 \times 2 = 16cm^2$

y_2 = Distance of C.G of area a_2 from bottom end = 8/2 = 4 cm.

Using the relation, $\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$

$$= \frac{20 \times 9 + 16 \times 4}{20 + 16}$$

$$= \frac{180 + 64}{36}$$

$$= 6.777 \text{ cm}$$

Moment of inertia of the section about the axis X-X,

$$I_{xx} = \left(\frac{10 \times 8^3}{12} + 20 \times 2.223^2\right) + \left(\frac{2 \times 8^3}{12} + 16 \times 2.777^2\right)$$

$$= (6.667 + 98.834) + (85.333 + 123.387) = 314.221 \text{ cm}^4$$

Moment of Inertia of the section about the axis Y-Y

$$I_{yy} = \frac{2 \times 10^3}{12} + \frac{8 \times 2^3}{12}$$

$$= 166.67 + 5.33$$

$$= 172 \text{ cm}^2.$$

Least value of moment of inertia is about Y-Y axis.

$$I = 172 \times 10^4 \text{ mm}^4$$

Since the struct is hinged at both of its end

$\therefore$ Effective length, $L_e = l = 5000$ mm

Let P = Crippling load.

Using equation we get

$$P = \frac{\pi^2 E I}{L_e^2}$$

$$= \frac{\pi^2 \times 2.0 \times 10^5 \times 172 \times 10^4}{5000^2}$$

$$= 135805.7 \text{ N}.$$

DERIVATIONS OF EULER'S FORMULA FOR DIFFERENT END CONDITIONS

END CONDITIONS	***Both Ends Are Hinged or Pinned or Free***	***One End Fixed & Other End Free***	***One End Fixed & Other End Pinned***	***Both Ends are Fixed***
Bending Moment at X-X	$EI\, d^2y/dx^2 = -Py$	$EI\, d^2y/dx^2 = P(\delta - y)$	$EI\, d^2y/dx^2 = -Py + H(L-x)$	$EI\, d^2y/dx^2 = M_0 - Py$
Solution	$y = C_1\cos(x\sqrt{P/EI})$ $+ C_2\sin(x\sqrt{P/EI})$	$y = C_1\cos(x\sqrt{P/EI})$ $+ C_2\sin(x\sqrt{P/EI}) + \delta$	$y = C_1\cos(x\sqrt{P/EI})$ $+ C_2\sin(x\sqrt{P/EI}) + H/P(L-x)$	$y = C_1\cos(x\sqrt{P/EI})$ $+ C_2\sin(x\sqrt{P/EI}) + M_0 - P$
Conditions	(i) $x=0, y=0$ (ii) $x=L, y=0$	(i) $x=0, y=0$ (ii) $x=L, dy/dx=0$ (iii) $x=L, y=\delta$	(i) $x=0, y=0$ (ii) $x=L, dy/dx=0$ (iii) $x=L, y=0$	(i) $x=0, y=0$ (ii) $x=L, dy/dx=0$ (iii) $x=L, y=0$
Critical Load	$P_e = \pi^2 EI / L^2$	$P_e = \pi^2 EI / 4L^2$	$P_e = 2\pi^2 EI / L^2$	$P_e = 4\pi^2 EI / L^2$
Equivalent Length	$Le = L$	$Le = 2L$	$Le = L / \sqrt{2}$	$Le = L / 2$

Radius of Gyration= $r = \sqrt{I/A}$, Slenderness Ratio = $\lambda = L_e / r$, Euler's Critical Load = $P_e = \pi^2 EI / L_e^2$

16.4 Euler's Column Theory

The first rational attempt, to study the stability of long columns, was made by Mr . Euler.

* The columns which have lengths less than 8 times their diamete,rare called ***short columns*** (see also Art 16.8).

** The columns which have lengths more than 30 times their diameter are called ***long columns.***

16.5 Assumptions in Euler's Column Theory

The following simplifying assumptions are made in Euler's column theory :

1. Initially the column is perfectly straight, and the load applied is truly axial.,
2. The cross-section of the column is uniform throughout its length.
3. The column material is perfectly elastic, homogeneous and isotropic, and thus obeys Hooks' law.
4. The length of column is very large as compared to its cross-sectional dimensions.
5. The shortening of column, due to direct compression (being very small) is neglected.
6. The failure of column occurs due to buckling alone.
7. The weight of the column itself is neglected.

16.6 Euler's Formula

According to Euler's theory, the crippling or buckling load (W_{cr}) under various end conditions is represented by a general equation,

16.5 Assumptions in Euler's Column Theory

16.6 Euler's Formula

$$W_{cr} = \frac{C\pi^2 E I}{l^2} = \frac{C\pi^2 E A k^2}{l^2} \qquad \ldots (\because I = A.k^2)$$

$$= \frac{C\pi^2 E A}{(l/k)^2}$$

where
E = Modulus of elasticity or Young's modulus for the material of the column,
A = Area of cross-section,
k = Least radius of gyration of the cross-section,
l = Length of the column, and
C = Constant, representing the end conditions of the column or end fixity coefficient.

The following table shows the values of end fixity coefficient (C) for various end conditions.

Table 16.1. Values of end fixity coefficient (C).

S. No.	*End conditions*	*End fixity coefficient (C)*
1.	Both ends hinged	1
2.	Both ends fixed	4
3.	One end fixed and other hinged	2
4.	One end fixed and other end free	0.25

16.7 Slenderness Ratio

In Euler's formula, the ratio l/k is known as ***slenderness ratio.*** It may be defined as the ratio of the effective length of the column to the least radius of gyration of the section.

16.8 Limitations of Euler's Formula

We have discussed in Art. 16.6 that the general equation for the crippling load is

$$W_{cr} = \frac{C\,\pi^2 E\,A}{(l/k)^2}$$

∴ Crippling stress,

$$\sigma_{cr} = \frac{W_{cr}}{A} = \frac{C\,\pi^2 E}{(l/k)^2}$$

Sometimes, the columns whose slenderness ratio is more than 80, are known as ***long columns***, and those whose slenderness ratio is less than 80 are known as ***short columns***. It is thus obvious that the Euler's formula holds good only for long columns.

16.9 Equivalent Length of a Column

16.10 Rankine's Formula for Columns

We have already discussed that Euler's formula gives correct results only for very long columns. Though this formula is applicable for columns, ranging from very long to short ones, yet it does not give reliable results. Prof. Rankine, after a number of experiments, gave the following empirical formula for columns.

$$\frac{1}{W_{cr}} = \frac{1}{W_C} + \frac{1}{W_E} \qquad \ldots(i)$$

where
W_{cr} = Crippling load by Rankine's formula,
W_C = Ultimate crushing load for the column $= \sigma_c \times A$,
W_E = Crippling load, obtained by Euler's formula $= \dfrac{\pi^2 E\,I}{L^2}$

A little consideration will show, that the value of W_C will remain constant irrespective of the fact whether the column is a long one or short one. Moreove,rin the case of short columns, the value of W_E will be very high, therefore the value of $1/W_E$ will be quite negligible as compared to $1/W_C$. It is thus obvious, that the Rankine' s formula will give the value of its crippling load (*i.e.* W_{cr}) approximately equal to the ultimate crushing load *i*(.*e.* W_C). In case of long columns, the value of W_E will be very small, therefore the value of $1/W_E$ will be quite considerable as compared to $1/W_C$. It is thus obvious, that the Rankine' s formula will give the value of its crippling load (*i.e.* W_{cr}) approximately equal to the crippling load by Euler's formula (*i.e.* W_E). Thus, we see that Rankine's formula gives a fairly correct result for all cases of columns, ranging from short to long columns.

From equation (*i*), we know that

$$\frac{1}{W_{cr}} = \frac{1}{W_C} + \frac{1}{W_E} = \frac{W_E + W_C}{W_C \times W_E}$$

$$\therefore \quad W_{cr} = \frac{W_C \times W_E}{W_C + W_E} = \frac{W_C}{1 + \dfrac{W_C}{W_E}}$$

Now substituting the value of W_C and W_E in the above equation, we have

$$W_{cr} = \frac{\sigma_c \times A}{1 + \dfrac{\sigma_c \times A \times L^2}{\pi^2 E\,I}} = \frac{\sigma_c \times A}{1 + \dfrac{\sigma_c}{\pi^2 E} \times \dfrac{A L^2}{A.k^2}} \qquad \ldots (\because I = A.k^2)$$

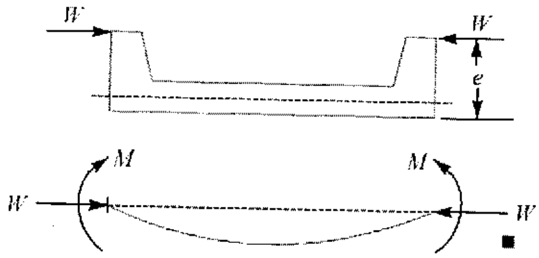
$$= \frac{\sigma_c \times A}{1 + a\left(\dfrac{L}{k}\right)^2} = \frac{\text{Crushing load}}{1 + a\left(\dfrac{L}{k}\right)^2}$$

where
σ_c = Crushing stress or yield stress in compression,
A = Cross-sectional area of the column,
a = Rankine's constant $= \dfrac{\sigma_c}{\pi^2 E}$,

16.12 Long Columns Subjected to Eccentric Loading

In the previous articles, we have discussed the ef fect of loading on long columns. We have always referred the cases when the load acts axially on the column (*i.e.* the line of action of the load coincides with the axis of the column). But in actual practice it is not always possible to have an axial load on the column, and eccentric loading takes place. Here we shall discuss the effect of eccentric loading on the Rankine's and Euler's formula for long columns.

Consider a long column hinged at both ends and subjected to an eccentric load as shown in Fig. 16.5.



We have already discussed that when a column is subjected to an eccentric load, the maximum intensity of compressive stress is given by the relation

$$\sigma_{max} = \frac{W}{A} + \frac{M}{Z}$$

The maximum bending moment for a column hinged at both ends and with eccentric loading is given by

$$M = W.e.\sec \frac{l}{2}\sqrt{\frac{W}{EI}} = W.e.\sec \frac{l}{2k}\sqrt{\frac{W}{E.A}} \qquad \ldots (\because I = A.k^2)$$

$$\therefore \quad \sigma_{max} = \frac{W}{A} + \frac{W.e.\sec \frac{l}{2k}\sqrt{\frac{W}{E.A}}}{Z}$$

$$= \frac{W}{A} + \frac{W.e.y_c.\sec \frac{l}{2k}\sqrt{\frac{W}{E.A}}}{A.k^2} \qquad \ldots (\because Z = I/y_c = A.k^2/y_c)$$

$$= \frac{W}{A}\left[1 + \frac{e.y_c}{k^2}\sec \frac{l}{2k}\sqrt{\frac{W}{E.A}}\right]$$

$$= \frac{W}{A}\left[1 + \frac{e.y_c}{k^2}\sec \frac{L}{2k}\sqrt{\frac{W}{E.A}}\right]$$

UNIT-I

STRESS AND STRAIN

Simple Stress & Strain:-

* Introduction to Stress & Strain.
* Elastic Limit.
* Hook's Law.
* Poisson's Ratio.
* Modulus of Elasticity.
* Modulus of Rigidity.
* Bulk modulus.
* Bars of varying Sections.
* Temperature stresses.
* Relation between Elastic Constants.
* Strain Energy.

The main objective of the study of Mechines of Solids is to Provide the future engineer with the means of analyzing and designing various Machines and load bearing structures. Analysis and Design of a given structure means the determination of stresses and deformations.

Why we need to study this:-

Study of internal effects (stresses and Strains) caused by external Loads (Forces and moments) acting on deformable body/structure.

Strength :-

Determined by Stress at failure.

Deformation :-

Determined by strain.

Definition :-

Resistance (Internal resistance) offered by the material (body) Per unit cross- sectional area against deformation is called **STRESS**.

$$\sigma = P/A$$

where P = applied Load

A = Area of C.S.

Unit of Stress :-

1 Pascal = $1\ N/m^2$, kN/m^2, MN/m^2, GN/m^2

$1\ MPa = 1\ N/mm^2$

STRESS

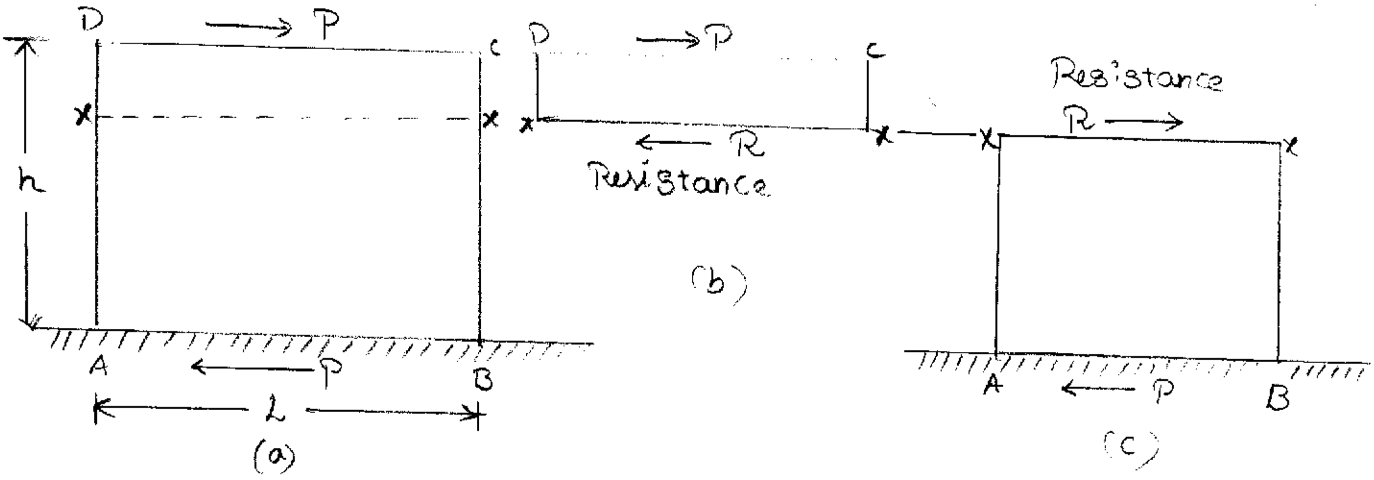
- Direct Stress / simple stress
  - Shear Stress
  - Normal stress
    - Tensile stress
    - Compensive stress
- Indirect Stress
  - Torsional Stress
  - Bending stress
- Combined Stress

Shear Stress :-

Stress induced in a body, when subjected to two equal and opposite forces which are acting tangentially across the resisting as a result of which the body tends to shear off across that section.

Hence shear stress τ $= \frac{\text{Resisting force}}{\text{Resisting area}}$

$= P\,(L \times 1) = P/A$ (for unit depth)



It is defined as deformation per unit length.
It is ratio of change in length to original length.
Strain is a dimensionless quantity.

- → Tensile Strain.
- → Compressive strain
- → Volumetric strain.
- → Shear strain

The ratio of change in dimensions of the body to its original dimensions is known as strain.

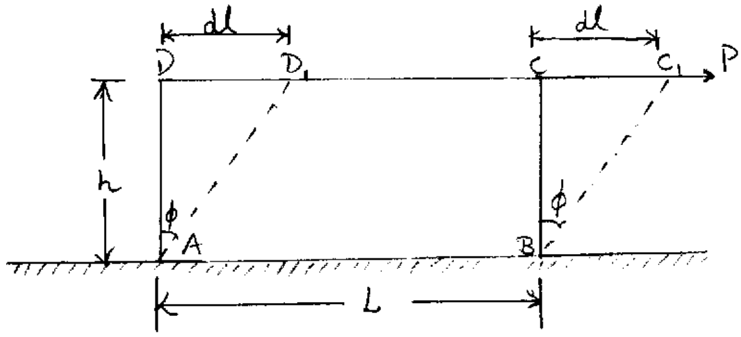
Shear Strain (φ) :-

As the face AB is fixed, the rectangular section ABCD will be distorted to ABC_1D_1 such that new vertical face AD_1 makes an angle φ with the initial face AD.

* As φ is very small.

$\varphi = \tan \varphi = DD_1/AD = dt/h.$

Hence shear strain $= dt/h.$



(a)

Types of stress we are studying :-

Stress	Strain
Tensile Stress	Tensile Strain
Compressive Stress	Compressive Strain
Shear Stress	Shear Strain

Elasticity & Elastic Limit:-

The Property of a material by virtue of which it undergoes deformation when subjected to an external force and regains it's original size and shape upon the removal of external force is called elasticity.

The stress corresponding to the limiting value of external force upon and with in which the deformation disappears completely upon the removal of external force is called elastic limit. A material is said to be elastic if it returns to it's original size when load is removed.

Hook's Law:-

Stress required to produce a strain of unity.

The modulus of elasticity is a measure of the stiffness of the material.

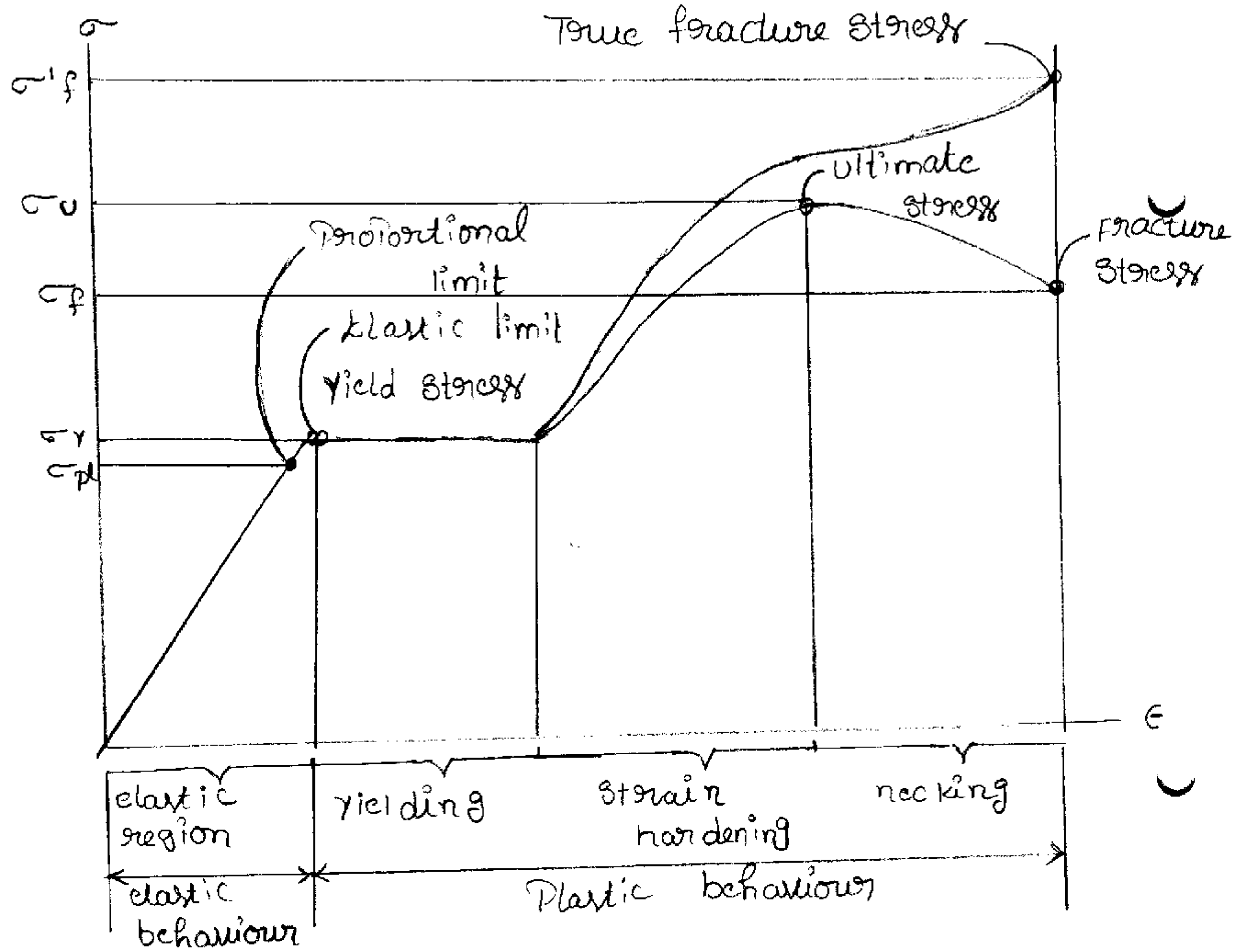
Stress - Strain Curve for mild steel (Ductile Material):-

Standard tensile test involves subjecting a circular bar of uniform across section to a gradually increasing tensile load until the failure across.

Tensile test is carried out to compare the strengths of various material.

Change in length of selected gauge length of bar is recorded by extensometers.

A graph is plotted with load vs extension or stress vs strain.



<u>Proportional limit</u> :- It is the highest stress at which stress is directly proportional to strain.

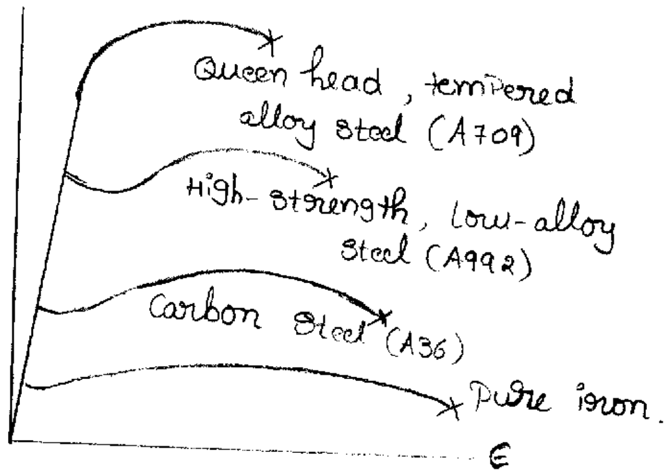
<u>Elastic Limit</u> :- It is the greatest stress the material can with stand with out any measurble permanent strain on removal of load.

Yield Strength:-

It is the stress required to produce a small-specified amount of plastic deformation.

Ultimate tensile strength:-

Or, more simply, the tensile strength, is the maximum engineering stress level reached in a tension t



Factor of safety:-

Structural members or machines must be designed such that the working stresses are less than the ultimate strength of the material.

And this ratio is known as factor of safety.

Factor of safety considerations:-

* Uncertainty in material properties

$F.S$ = factor of safety.

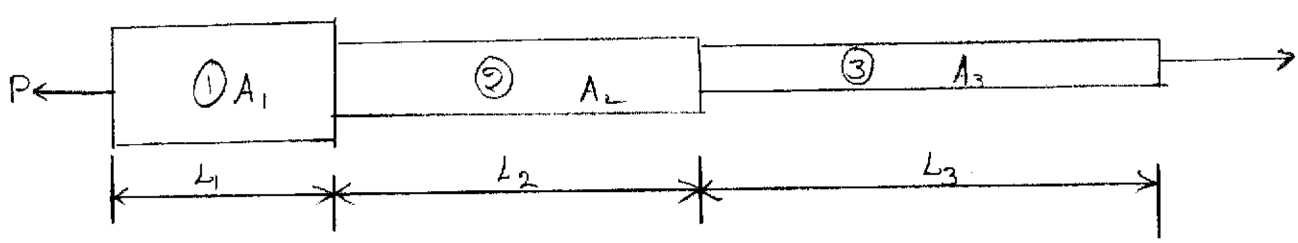
$$F.S = \frac{\sigma_u}{\sigma_{all}} = \frac{\text{ultimate stress}}{\text{allowable stress}}.$$

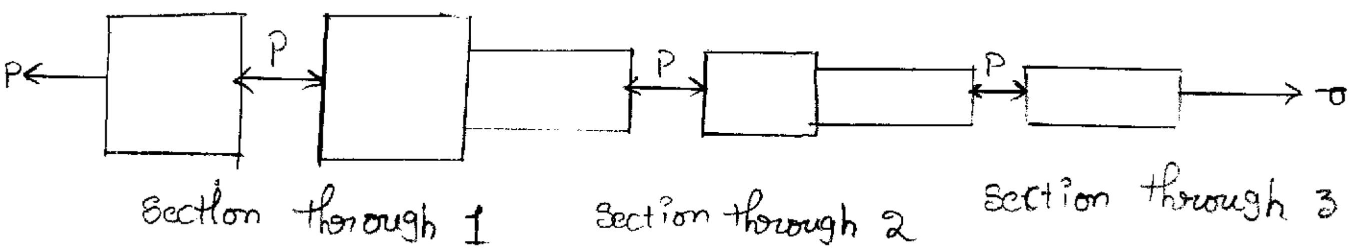
* uncertainty of loadings.
* uncertainty of analyses.
* Number of loading cycles.
* Types of failures.
* Maintaince requirement and deterioration effects.
* risk to life and Property.
* Influence on machine functions.

1. f or steel - 1.85
2. f or Concrete - 3
3. f or timber - 4 to 6.

Bars with Cross-sections varying in steps:-

figure shows the forces acting on the Cross-sections of the 3 Portions. It is obvious that to maintain equilibrium the load acting on each Portion is P only.

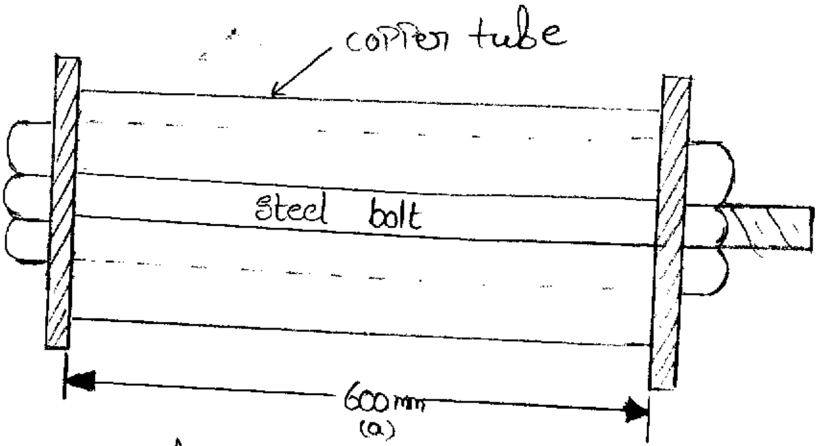


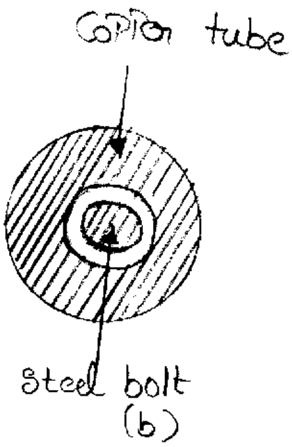


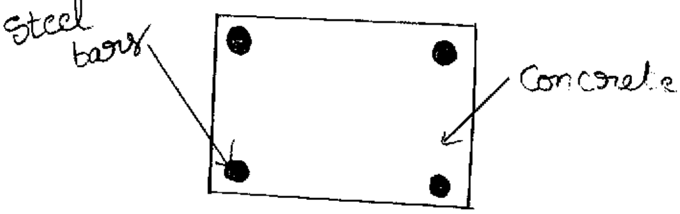
Portion	Stress	Strain	Extansion
1	$P_1 = \frac{P}{A_1}$	$e_1 = \frac{P_1}{E} = \frac{P}{A_1 E}$	$\Delta_1 = \frac{PL_1}{A_1 E}$
2	$P_2 = \frac{P}{A_2}$	$e_2 = \frac{P_2}{E} = \frac{P}{A_2 E}$	$\Delta_2 = \frac{PL_2}{A_2 E}$
3.	$P_3 = \frac{P}{A_3}$	$e_3 = \frac{P_3}{E} = \frac{P}{A_3 E}$	$\Delta_3 = \frac{PL_3}{A_3 E}$

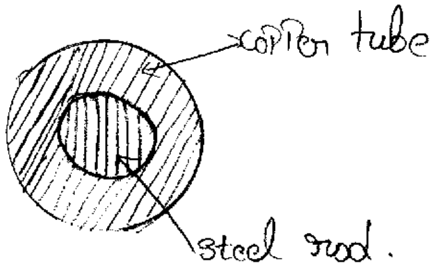
$$\Delta = \Delta_1 + \Delta_2 + \Delta_3 = \frac{PL_1}{A_1 E} + \frac{PL_2}{A_2 E} + \frac{PL_3}{A_3 E}$$

Composite Bars:-









As both the materials deforms axially by same value strain in both materials (or change in length) are same.

$$\delta L_s = \delta L_c = e$$

$$\sigma_s / E_s = \sigma_c / E_c \quad (\because e = \delta L / L) \longrightarrow (1)$$

Load is shared between the two materials.

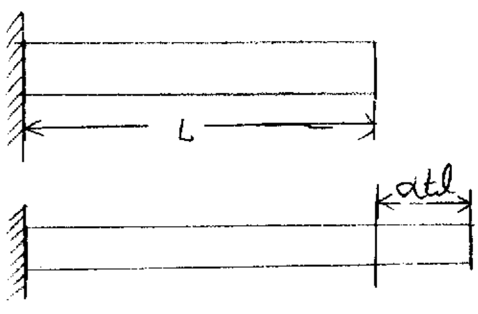
$$P_s + P_c = P$$

$$\text{i.e. } P = \sigma_s * A_s + \sigma_c * A_c \longrightarrow (2)$$

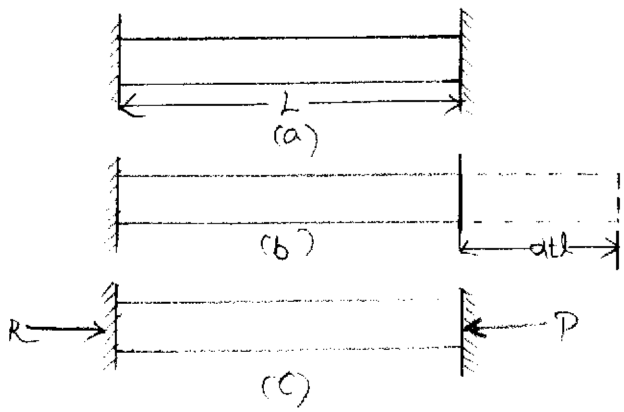
Thermal stress:-

Every material expands when temperature rises and contracts when temperature falls. It is established experimentally that the change in length δL is directly proportional to the length of the member L and change in temperature t.

The constant of proportionality a is called coefficient of thermal expansion and is defined as change in unit length of material due to unit change in temperature.



Free expansion permitted



Prevented deformation

If the expansion of the member is freely permitted as shown in temperature stresses are induced in the material.

If the free expansion is prevented fully or partially the temperature stresses are induced in the bar, by the support forces.

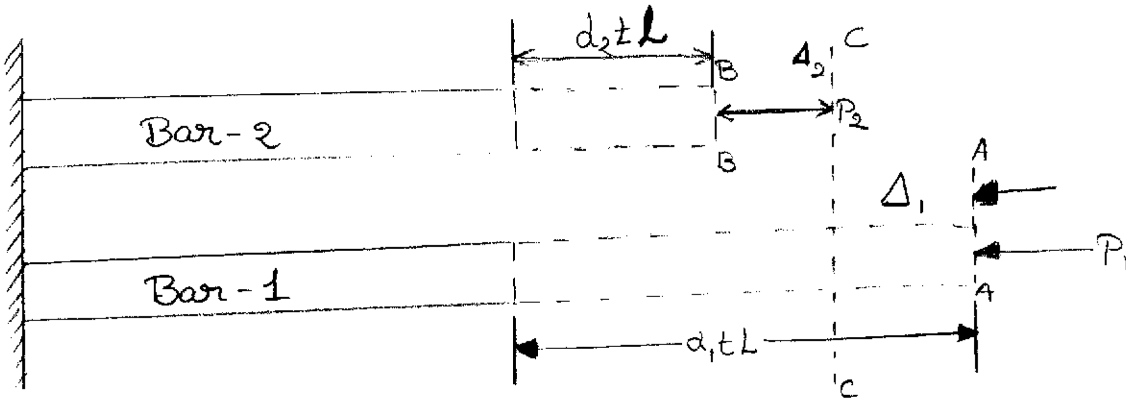
Thermal stresses in Composite bars:-

When temperature rises the two materials of the compound bar experiance different free expansion. Thus one bar develops tensile force and another develops the compressive force.

Consider the compound bar shown in fig. Let a_1, a_2 be coefficient of thermal expasion and E_1, E_2 be moduli of elasticity of the two materials respectively. If rise in temperature is 't'.

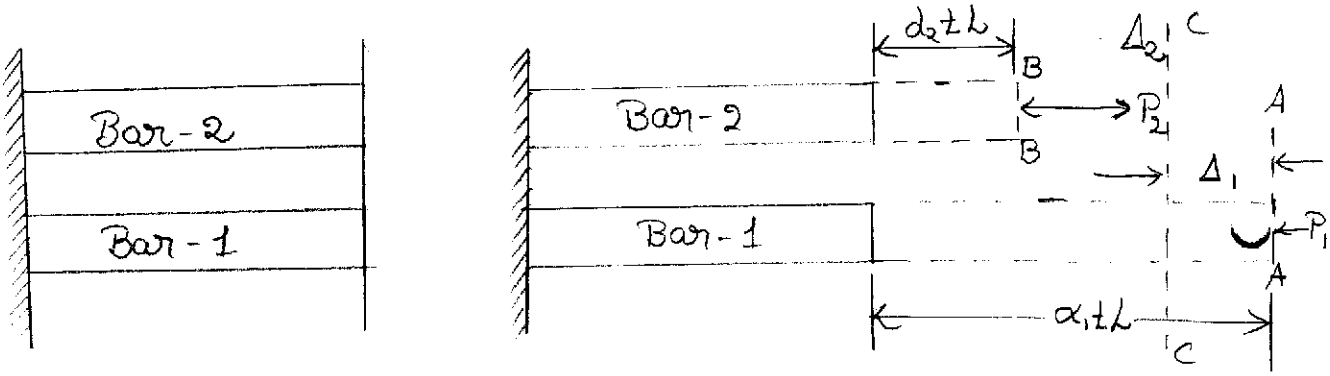
* Free expansion of bar 1 = $a_1 tL$
* Free expansion of bar 2 = $a_2 tL$

Let $a_1 > a_2$. Hence the position of the two bars, if the free expansions are permitted are at AA and BB as shown.



Since the two bars are rigidly connected at the ends, the final position of the end will be somewhere between AA and BB, say at CC.

It means Bar-1 will experience compressive force P_1 which contracts it by $\Delta 1$ and Bar-2 experiance tensile force P_2 which will expand it by $\Delta 2$



For equillibrium, from fig $a1\ tL - \Delta_1 = a_2 tL + \Delta 2$

$$\therefore \Delta_1 + \Delta 2 = \alpha_1 tL - \alpha_2 tL = (\alpha_1 - \alpha_2) tL.$$

If the cross-sectional areas of the bars are A_1 and A_2, we get

$$\frac{PL}{A_1 E_1} + \frac{PL}{A_2 E_2} = (\alpha_1 - \alpha_2) tL.$$

Remember above is case of Parallel bars and length in this case can be eliminated as it is same for both materials.

For bar is in series the case will be

$$\frac{PLs}{AsEs} + \frac{PLc}{AcEc} = t\,(\alpha_s L_s + \alpha_c L_c)$$

Hoop Stresses:-

The internal or external Pressure applied to thin cylinders is resisted by stresses developed in the circumferential direction of the cylinder. This type of stress is called Hoop stress.

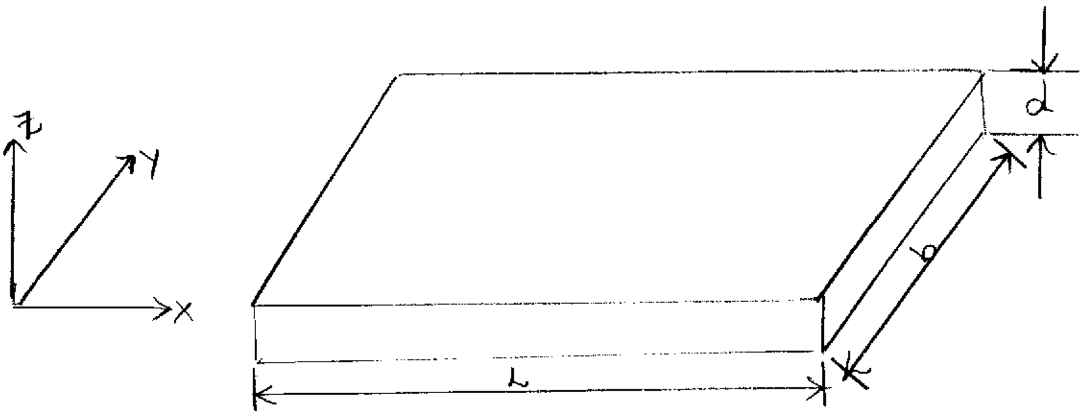
Elastic Constant in isotropic Materials:-

Elasticity modulus (E)
Poisson's Ratio (n)
Shear modulus (G)
Bulk modulus (K)

Any direct stresses produces a strain in its own direction and opposite strain in every direction at right angles to it. For most of metals its value is between 0.25 to 0.33. Its value for steel is 0.3 and for concrete 0.15.

Volumetric strain is sum of strains in three mutually Perpendicular Conditions.

i.e. Consider a bar of length L. breadth b and depth d as shown in fig.



In general for any shape volumetric strain may be taken as sum of strain in three mutually perpendicular.

What is Strain Energy?

When a body is subjected to gradual, sudden or impact load, the body deforms and work is done upon it. If the elastic limit is not exceed, this work is stored in the body. This work done or energy stored in the body is called strain energy.

Energy is stored in the body during deformation process and this energy is called "Strain Energy".

Resilience :-

Total strain energy stored in a body is called resilience.

Proof Resilience:-

Maximum strain energy which can be stored in a body is called Proof resilience.

Modulus of Resilience:-

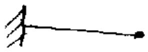
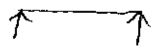
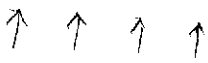
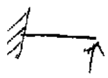
Maximum strain energy which can be stored in a body per unit volume at elastic limit is called Modulus of Resilience.

Unit - II

Shear Force & Bending Moment

Beam :- It is a structural horizontal member subjected to a system forces at right angles (transverse direction) to tan its longitudinal axis.

Types of beams :-

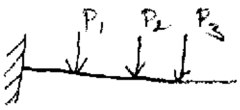
1. Cantilevel beam  one end is fixed
2. simply supported beam  two ends are simply
3. over hanging beam 
4. Fixed beam  two ends are fixed
5. Continеous beams  more than two supported
6. Prapped Cantileves beam  one end is fixed other is simply supported

Types of loads :-

1. **Point load / Concentrated load :-**

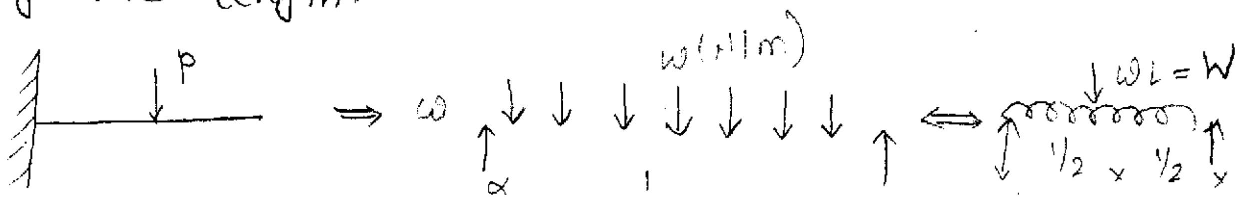
A Point load is one which is considered to act at a Point. i.e., Concentrated at a Point.

($\downarrow_A$) (W) (Newtons)



2. **Uniformly distributed load (UDL) / Rectangular loads :-**

It is one which is spread over a beam in as such a manner the rate of loading ω is uniform along the length.

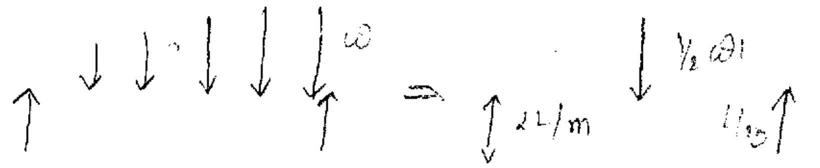


Total load = ωL = Area of Rectangle (ω) N/m → rate of load means load/unit length

Note:-
The total load is concentrated at centre of UDL

3. Uniformly varing loads (UVL):-

(ω) N/m
In which the load is zero at one end and increase uniformly to other end.



Total load = Area of Δ^{le} = $\frac{1}{2}\omega L$
It is concentrated at centre of Δ^{le}

Shear force:-

units: N

The algebraic sum of the vertical forces at any section of a beam to the right (or) left of the section.

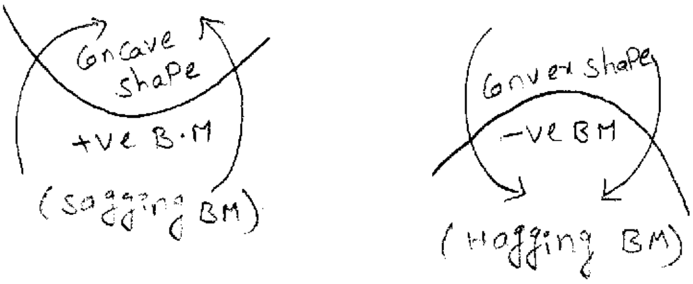
SF Diagram:-
It is one which shows the variation of shear force along the length of the beam.

SF Consideration:-
→ If we are choosing right side, (R)
down ward force (+ve) & upward forces (-ve)

→ If we are choosing left side, (L)
down ward forces (-ve) & upward forces (+ve)

Bending moment

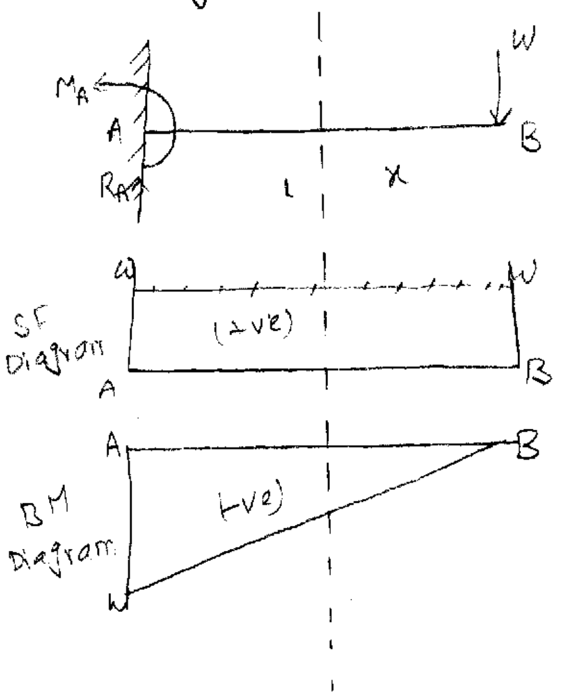
The algebraic sum of the moments of all the forces acting to right (or) left of the sections. is called Bending moment at that section.



downward force (↓) (-ve B.M)

Upward force (↑) (+ve B.M)

⇒ A Cantilever beam of length (L) carrying a point Load (W) at the free end. Draw the SF & BM diagram.



⇒ At equilibrium

Sum of forces = 0

i.e., $\Sigma F = 0$

$R_A = W \longrightarrow (1)$

Sum of moments = 0

i.e., $\Sigma M_A = 0$

$M_A = WL \longrightarrow (2)$

⇒ Draw $x' - x'$

Consider either L/R Portions

Let us consider, Right Portion.

Bending moments.

$M_n = -Wx$

$M_{x=0} = M_B = 0$

$M_{x=l} = M_A = -WL$

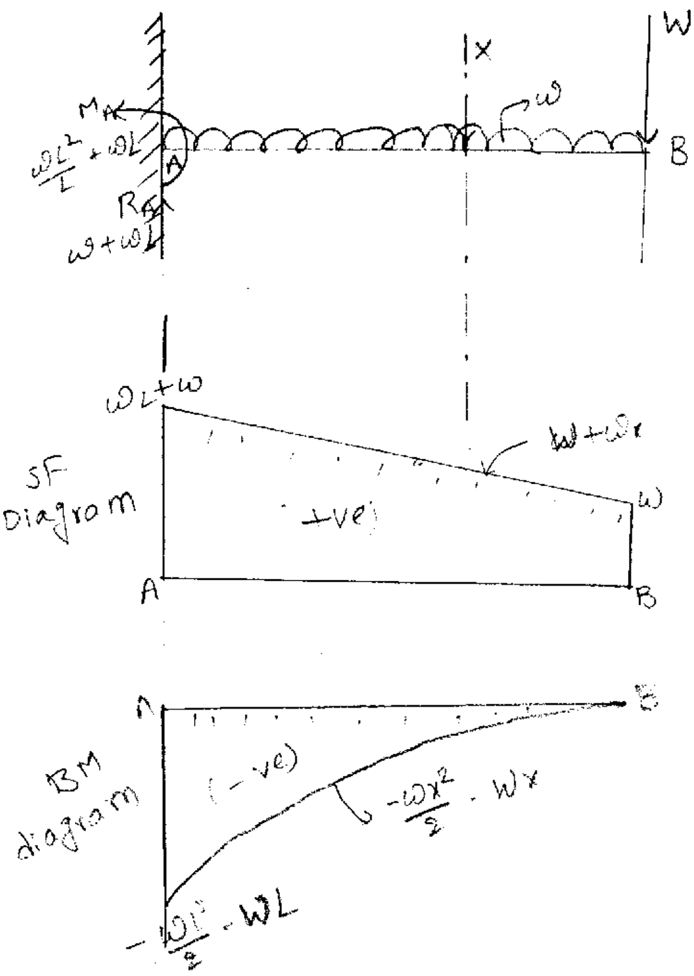
Shear force,

$SF_x = W$

$SF_{x=0} = SF_B = W$

$SF_{x=L} = SF_A = W$

4) A Cantilever beam of length (L) carrying a UDL (ω) per unit length run over the entire length and a Point load (W) at the free end.



① Sum of force $\Sigma F=0$

$\Rightarrow R_A = \omega L + W$

Sum of moments $\Sigma M=0$

$\Rightarrow M_A = \omega L(L/2) + WL$

$= \frac{\omega L^2}{2} + WL$

② (x-x) section = x ⟶ 0 to L

Shear force

$SF_x = \omega x + W$

$SF_{x=0} = W = SF_B$

$SF_{x=L} = \omega L + W = SF_A$

Bending moment

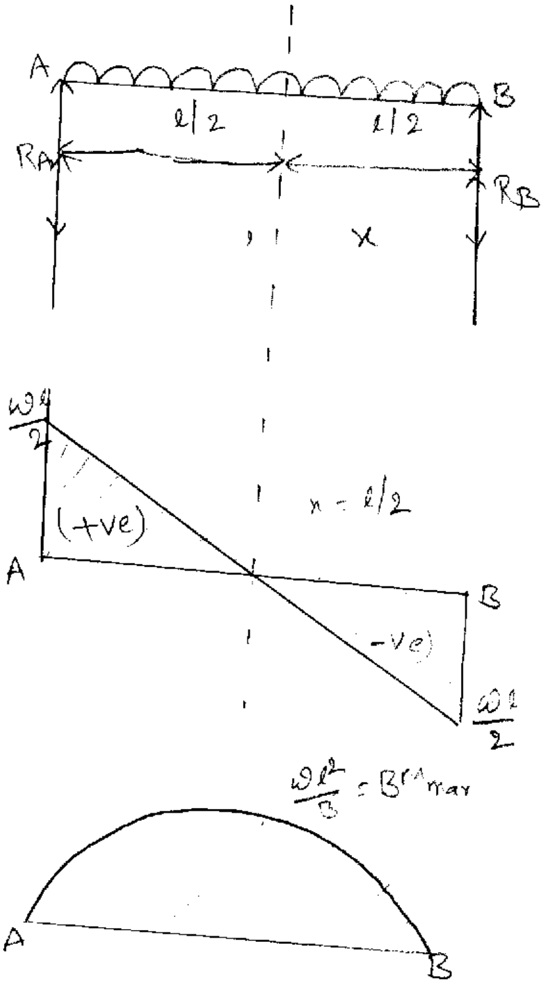
$M_x = -\omega x\left(\frac{x}{2}\right) - Wx$

$= -\frac{\omega x^2}{2} - Wx$

$M_{x=0} = M_B = 0$

$M_{x=L} = M_A = \frac{-\omega L^2}{2} - WL$

11) A SSB of length 'l' Carrying a UDL (ω) over entire span Draw SF and BM diagrams.



① Sum of forces $\Sigma F = 0$. $R_A + R_B = \omega L$

$\rightarrow R_A = R_B = \frac{\omega L}{2}$ {∵ symmetric}

② (x-x) section for AB : $x = 0$ to l.

Shear force,

$SF_x = -R_B + \omega x$

$SF_x = -\frac{\omega l}{2} + \omega x$

$SF_{x=0} = \frac{-\omega l}{2} = SF_B$

$SF_{x=l} = \frac{\omega l}{2} = SF_A$

Bending moment

$M_x = R_B x - \omega x(x/2)$

$M_x = \frac{\omega l x}{2} - \frac{\omega x^2}{2}$

$M_{x=0} = 0 = SF_B$

$M_{x=L} = SF_A = 0$

Consider, $M_x = \frac{\omega l x}{2} - \frac{\omega x^2}{2}$

$\frac{dM_x}{dx} = 0.$

i.e., $\frac{\omega L}{2} - \frac{\omega}{2}(2x) = 0$

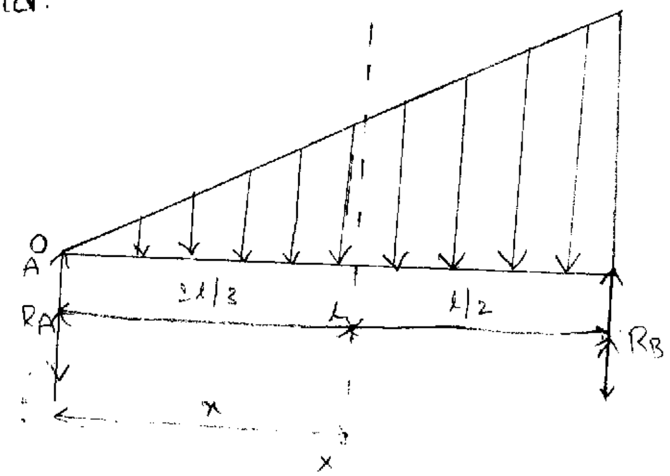
$$L - 2x = 0$$

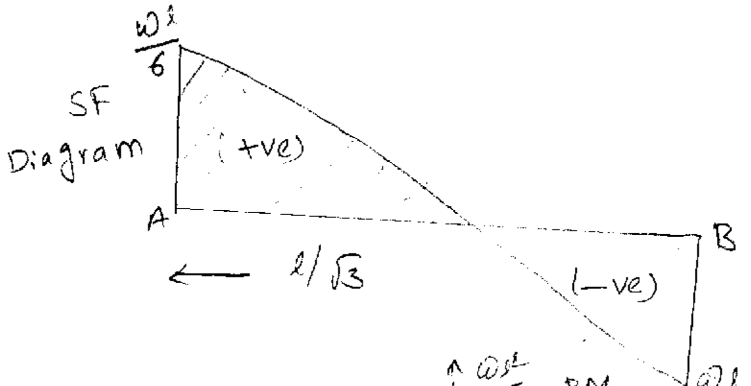
$$x = l/2 \quad \{\because \text{BM is max. at } x = l/2\}$$

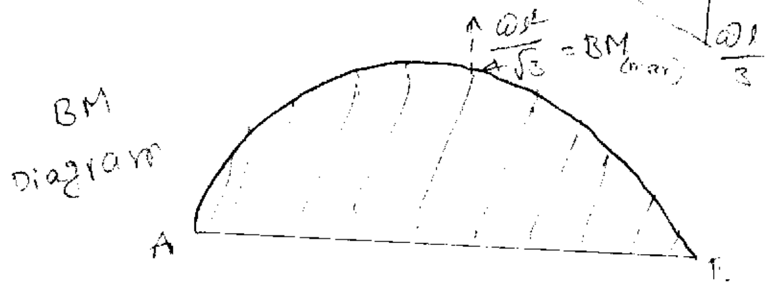
at $M_{x=l/2} = \frac{\omega l^2}{4} - \frac{\omega l^2}{8}$

$$\boxed{M_{x=l/2} = \frac{\omega l^2}{8} = M_{max}}$$

12) A SSB carrying a load whose intensity varies uniformly from 0 at one end and ω/unit run at the said span other end.







① Sum of force $\Sigma F = 0$

$$\Rightarrow R_A + R_B = \frac{1}{2}\omega L$$

Sum of moments $\Sigma M = 0$

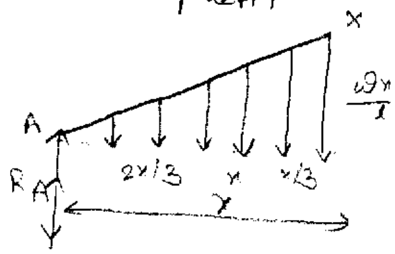
$$\Sigma M_A = 0$$

$$R_B(l) - \frac{1}{2}\omega l\left(\frac{2l}{3}\right) = 0$$

$$\boxed{R_B = \frac{\omega l}{3}}, \quad R_A = \frac{1}{2}\omega l - \frac{\omega l}{3}$$

$$\boxed{R_A = \frac{\omega l}{6}}$$

② (x-x) Section for AB ; $x = 0$ to l
|Left|



Shear force

$$SF_x = R_A - \frac{1}{2}\left|\frac{\omega x}{l}\right| x$$

$$= \frac{\omega l}{6} - \frac{\omega x^2}{2l}$$

$$SF_{x=0} = SF_A = \frac{\omega l}{6}$$

$$SF_{x=l} = SF_B = \frac{-\omega l}{3}$$

Bending moment;

$$M_x = \frac{-1}{2} \cdot \frac{\omega x}{l} \cdot x \cdot \frac{x}{3} + R_A x$$

$$= \frac{-\omega x^3}{6} + \frac{\omega l x}{6}$$

$$M_{x=0} = M_A = 0$$

$$M_{x=l} = M_B = -\frac{\omega l^2}{3} \cdot 0.$$

Consider, $M_x = \frac{\omega l x}{6} - \frac{\omega x^3}{6l}$

$$\frac{dM_x}{dx} = 0$$

i.e., $\frac{\omega l}{6} - \frac{\omega}{6l}(3x^2) = 0$

$$l^2 = 3x^2$$

$$x = \sqrt{\frac{l^2}{3}}$$

$$\boxed{x = l/\sqrt{3}}$$

{∵ The B.M is max. at $x = l/\sqrt{3}$ from end A}

$$\therefore \text{at } M_{x=l/\sqrt{3}} = \frac{\omega l^2}{6\sqrt{3}} - \frac{\omega l^3}{18\sqrt{3}\,l} = \frac{\omega l^2}{6\sqrt{3}} - \frac{\omega l^2}{18\sqrt{3}}$$

$$= \frac{2\omega l^2}{18\sqrt{3}} = \frac{\omega l^2}{9\sqrt{3}}$$

$$\boxed{M_{max} = \frac{\omega l^2}{9\sqrt{3}}}$$

Properties of SF and BM diagram:-

1) The slope of SF = UDL value [Intensity of UDL]
= Rate of loading (ω).

2) The slope of BM, S.F at that section

3) At Point loads there is abrupt change in SF

4) At couple there is abrupt change in BM

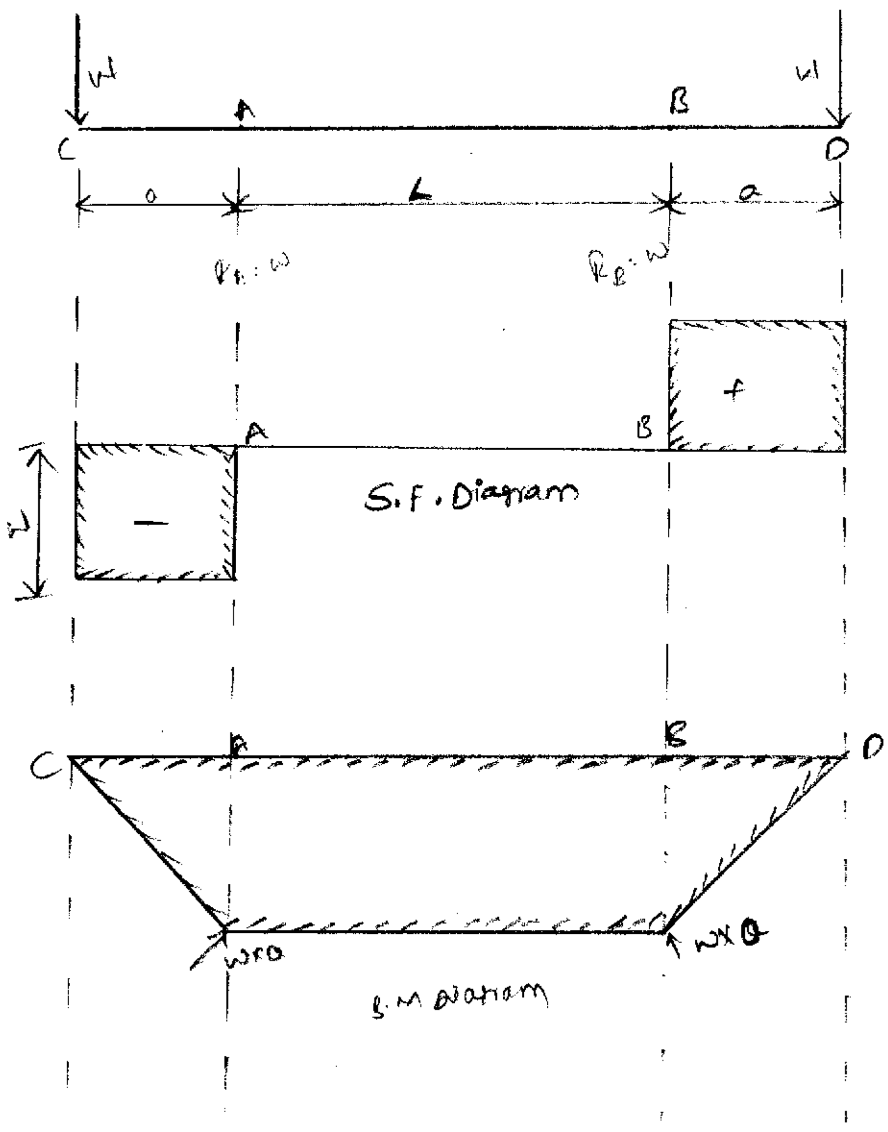
5) Area of SF = BM difference between sections.

6) The total load on a beam b/w two sections is equal to the difference between the SF b/w the sections.

*. FLEXURAL STRESSES (or) BENDING STRESSES IN BEAMS.

→ **Bending stress:** The stress induced by bending moment are known as bending stress.

→ **Simple bending:** If a length of a beam is subjected to a constant bending moment & shear force is zero. then the stress set up in that length of the beam are known as bending stress and that length of the beam is said to be in pure bending.



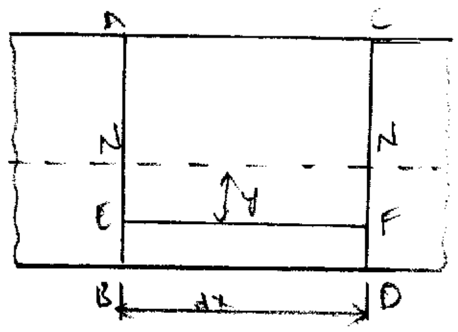
There is No shear force b/w A & B but bending moment b/w A & B is constant. this condition of beam b/w A & B is known as pure bending.

→ ASSUMPTIONS MADE IN THE THEORY OF SIMPLE BENDING

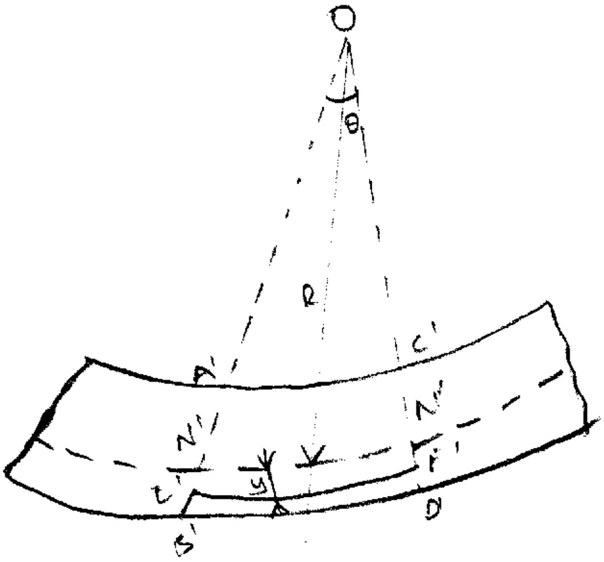
1) the material of the beam is homogeneous and isotropic
2) the value of young's modulus of elasticity is same in tension and compression
3) The transverse section which were plane before bending remain plane after bending also.
4) The beam is initially straight and all longitudinal filaments bend into circular arcs with a common centre of a curvature.
5) The Radius of curvature is large compounded with the dimension of the cross-section.

*. Theory of Simple Bending (or) Derivation of Bending equations.

A Small Length δx of a beam subjected to a simple bending as shown in the fig. And (a) is due to action of bending. the part of length δx will be deformed as shown in the fig.



(a) fig.



(b)

Neutral layer (or) surface (N-N);-

A layer which is neither shotend nor elongated is know as neutral layer.

Neutral axis (N-A);

The line of intersection of neutral layer on a cross-section of beam is known as "neutral axis".

-) Due to decrease in length of the layers above N-N, these layers will be subjected to compressive stresses.

-) Due to the increases in length of the layers above N-N, thes layers will be subjected to tensile stresses.

-) the amount By which a layer increases (or) decreases in length depends upon the position of the layer. w.r.t. N-N, this theory of bending is known as theory of Simple bending.

Let

R = Radius of neutral layer "N-N"

θ = angle is subjected at "O" By A'B' and C'D produce

Y = Distance from the neutral layer.

original length of the layer = EF = OX = NN = N'N'

from the above fig (b). N'N' = Rθ

increase in length of the EF = E'F' − EF = (R+y)θ − Rθ = yθ

→ Strain in the layer EF = e_{EF} = increase in length / original length

$= y\theta / R\theta = y/R$

→ $e_{EF} \propto y$

$\sigma = E \times y/R = E/R \times y$

$\sigma/y = E/R$

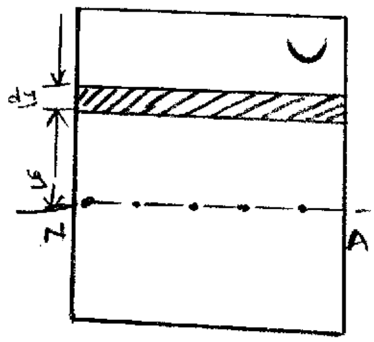
→ $\sigma \propto y$.

→ Cross section of the beam:

force on the layer = stress on layer × Area of layer

$= \sigma \times dA$

$= E/R \times y \times dA$



Total force on the beam section

$= \int E/R \times y \times dA$

$= E/R \int y \times dA$

But for pure bending, Total force = 0

$\therefore E/R \int y \times dA = 0$

$\int y \times dA = 0$

-) y dA Represents the moment of Area dA about actual axis. that is centroidal axis of a section give the positi of Neutral axis.

Force on laye = $E/R \times y \times dA$

moment of this force about N.A.

$= \text{Force on layer} \times y$

$= E/R \times y \times dA \times y$

$= E/R \times y^2 \times dA$.

-) Total moment of the forces on the section of the beam

$M = \int E/R \times y^2 \times dA$

$= E/R \int y^2 \times dA$

$M = E/R \times I$

$\therefore \Rightarrow \quad \frac{M}{I} = \frac{\sigma}{y} = E/R$.

The above eqetn is known as Bending Equation. And it is Applicable at bending moment is maximum.

= 0 >

problem:

1. A timber beam of rectangular section of length 8m is simply supported. The beam carries a U.D.L of 12kN/m run over the entire length and a point load of 10kN at 3m from the left support. If the depth is two times the width and the stress in the timber is not to exceed 8 N/mm². Find the suitable dimensions of the section.

1: given data:

Length $(L) = 8m$

U.D.L $(w) = 12kN/m = 12000 N/m$

point load (W) = 10kN = 10000N

Depth of beam = 2 × width of beam

∴ $d = 2b$

Stress $\sigma_{max} = 8 N/mm^2$

taking moments about A, we get

$R_B \times 8 = 12000 \times 8 \times 4 + 10000 \times 3$

$= \frac{12000 \times 32 + 3000}{8}$

$R_B = 51750 N$

R_A = Total load − R_B

$= (12000 \times 8 + 10000) - 51750$

$= 54250 N$

∴ Now

S.F. at A = $+R_A = +542500 N$

S.F. just L.H.S at C $= 54250 - 12000 \times 3$

$= +18250 N.$

S.F. Just R.H.S of C $= 18250 - 10000 = 8250 N$

S.F. at B $= -R_B = -51750 N.$

the S.F is changing sign b/w section CB and hence at some section in C and B the S.F. will be zero.

Let S.F. is zero at x metre from B.

Equating the S.F. at this section to zero, we have.

$12000 \times x - R_B = 0$

$12000 \times x - 51750 = 0$

$$x = \frac{51750}{12000} = 4.3125\ m$$

→ Bending moment = $M = R_B \times 4.3125 - 12000 \times 4.3125 \times \frac{4.3125}{2}$

$= 51750 \times 4.3125 - 111585.9375$

$= 111585.9375\ Nm$

$= 111585.9375 \times 1000\ Nmm$

$$Z = \frac{bd^2}{6} = \frac{b \times (2b)^2}{6} = \frac{2b^3}{3}$$

$M = \sigma_{max} \cdot Z$

$$111585.9375 \times 1000 = 8 \times \frac{2b^3}{3}$$

$b = (20.9223 \times 10^6)^{1/3}$

$= 275.5\ mm$

$d = 2 \times 275.5$

$d = 551\ mm.$

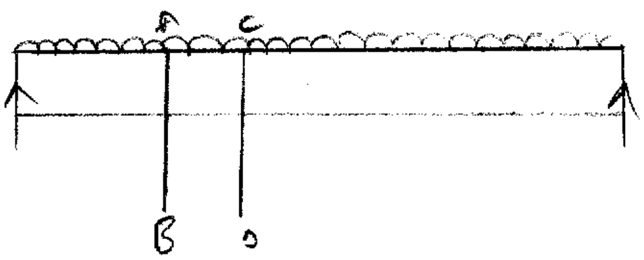
*) SHEAR STRESSES IN BEAMS

→ **Shear Stress:**

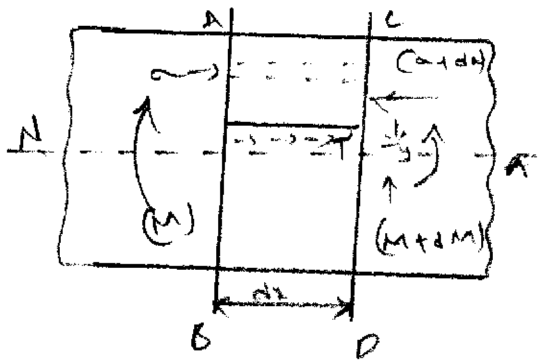
The stress induced by shear force are known as "Bending stress".

*) theorm or Derivation:

$$\tau = F \times \frac{A\bar{y}}{I \times b}$$

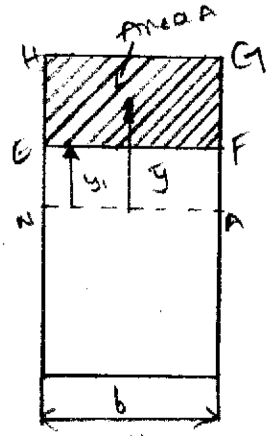


(a)

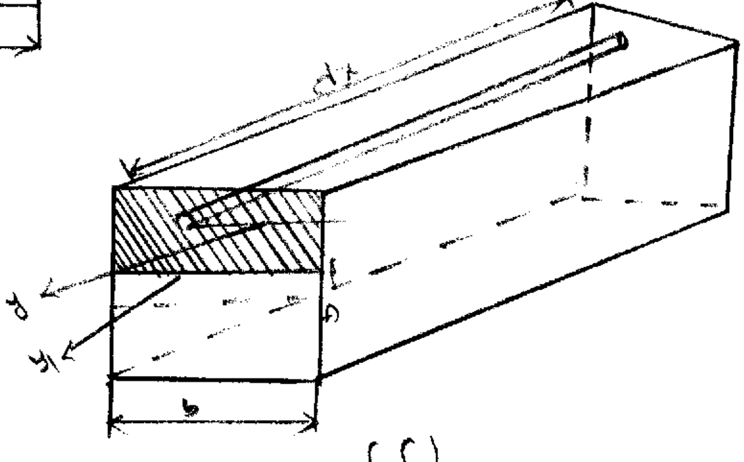


(b)

(d)



(d)



(C)

The fig (a) shows a simply Suppdted Beam Carrying a U.D.L
the fig (b) shows two sections AB & CD at a distance dx apart
the fig (c) & (d) shows the cross section of the Beam.

Let at section AB, F = shear force & M = Bending moment
Let at section CD, F+dF = shear force &
M+dM = Bending moment.

Let dA = Area of elemental cylinder from neutral axis
y = Distance of elemental cylinder from neutral axis.
σ = intensity of Bending stress on the elemental cylinder on the section AB.
σ+dσ = Intensity of Bending stress on the end of the element cylinder on the section CD

Bending stress on the end of elemental cylinder on the section AB will be.

$$\sigma = \frac{M}{I} \times y$$

Bending stress on the end of elemental cylinder on the section CD, will be.

$$\sigma + d\sigma = \frac{(M+dM)}{I} \times y$$

force on the end of the elemental cylinder on the section AB

= Stress x Area of elemental cylinder =

$$= \sigma \times dA = \frac{M}{I} \times y \times dA$$

Similarly, force on the elemental cylinder on the section CD

$$= (\sigma + d\sigma) = \frac{M}{I} \times y \times dA$$

∴ Net unbalanced force on the elemental cylinder

$$= \frac{dM}{I} \times y \times dA.$$

$\therefore$ Total unbalanced force $= \int \frac{dM}{I} \times y \times dA$

$$= \frac{dM}{I} \int y \times dA$$

$$= \frac{dM}{I} \times A \times \bar{y}$$

where

A = Area of the section above the level EF.

$\bar{y}$ = Distance of the C.G of the area of A from the neutral axis.

$\therefore$ Shear Resistance at the level EF = total unbalanced force

$$= \frac{dM}{I} \times A \times \bar{y}$$

Let τ = intensity of horizontal shear at the level EF

b = width of beam at the level EF

$\therefore$ Shear force due to τ = Shear stress × shear area

$$= \tau \times b \times dx$$

Equating the two values of shear force given by eqtn.

$$\therefore \tau \times b \times dx = \frac{dM}{I} \times A \times \bar{y}$$

$$\therefore \tau = \frac{dM}{dx} \cdot \frac{A\bar{y}}{I \times b} = F \times \frac{A\bar{y}}{I \times b}.$$

$$\therefore \tau = F \times \frac{A\bar{y}}{I \times b}.$$

= o =

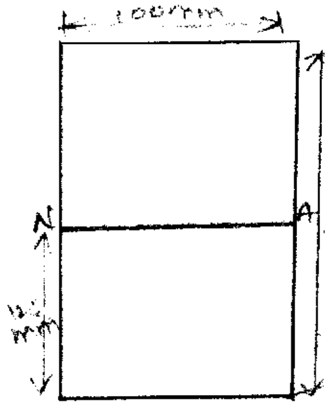
problem:

Q3. A Rectangular beam 100mm wide and 250mm deep is subjected to a max shear force of 50kN.

Determine i) Average shear stress.

ii) Max. shear stress

iii) shear stress at a distance of 25mm above the neutral axis.



Sol:

Given data:

Width $b = 100\,mm$

Depth $d = 250\,mm$

Max shear force $F = 50\,kN = 50{,}000\,N$.

i) Avg. shear stress $\tau_{avg} = \frac{F}{Area} = \frac{50{,}000}{100 \times 250} = 2\,N/mm^2$.

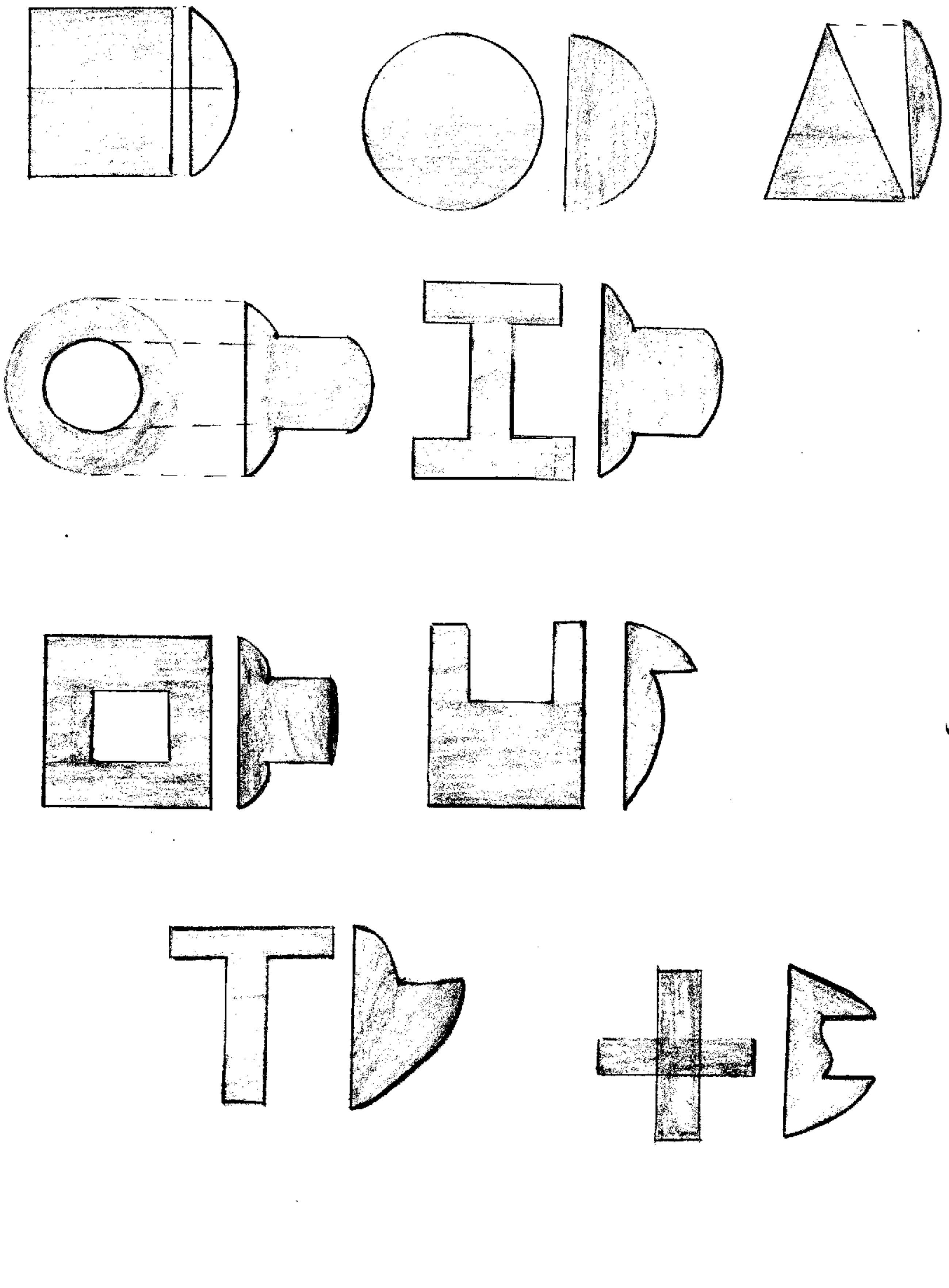
ii) Max. shear stress $= \tau_{max} = 1.5 \times \tau_{avg} = 3\,N/mm^2$

iii) The shear stress at a distance y from N.A is

$$= \tau = \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right) = \frac{50000}{2I}\left(\frac{(250)^2}{4} - (25)^2\right) = 2.88\,N/mm^2$$

-o-

*). Shear stress Distribution for Beams of various sections :

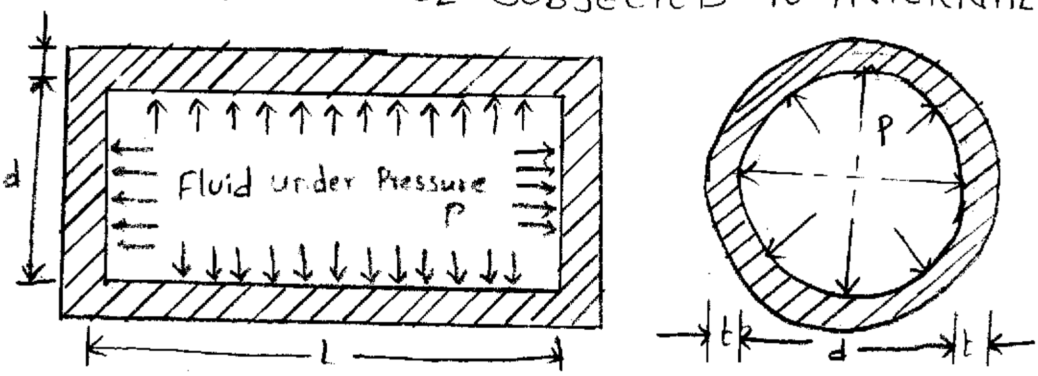


UNIT - V

THIN CYLINDERS

1. Derive the expression for Hoop stress and longitudinal stress in a thin cylinder shells subjected to internal fluid Pressure? (OR) show that in thin cylinder shells subjected to internal fluid Pressure, the Hoop stress is twice the longitudinal stress?

Ans: THIN CYLINDRICAL VESSEL SUBJECTED TO INTERNAL PRESSURE:

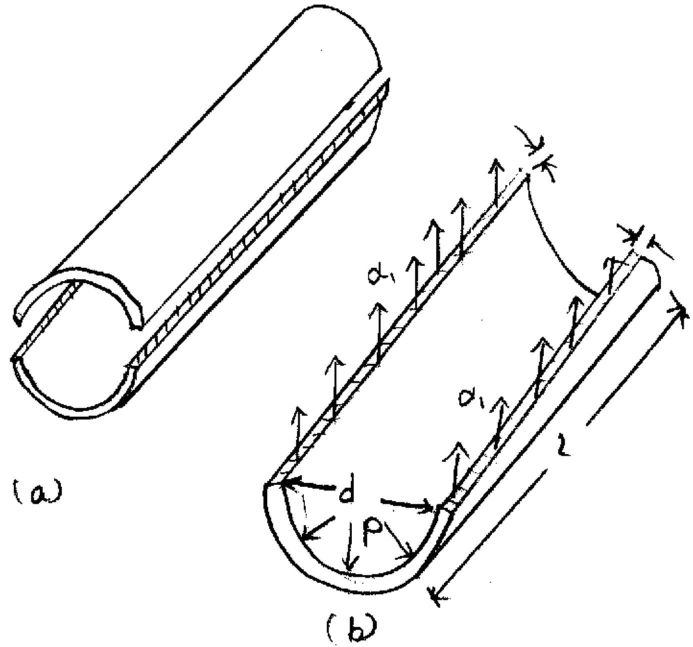


The figure shows a thin cylindrical vesel in which a fluid under Pressure is stored.

Let d = internal diameter of the thin cylinder
t = thickness of the wall of the cylinder
P = internal Pressure of the fluid
l = Length of the cylinder

EXPRESSION FOR CIRCUMFERENTIAL STRESS (HOOP STRESS):-

Let P = internal Pressure of fluid
d = internal diameter of the cylinder
t = Thickness of the wall of the cylinder
σ_1 = circumferential or hoop stress in the material



Force due to fluid Pressure = P × Area on which P is acting

$= P \times (d \times L)$ ⟶ (i)

[∵ P is acting on projected area $d \times L$]

Force due to circumferential stress

$= \sigma_1 \times$ Area on which σ_1 is acting

$= \sigma_1 \times (L \times t + L \times t)$

$= \sigma_1 \times 2Lt = 2\sigma_1 \times L \times t$ ⟶ (ii)

Equating (i) and (ii), we get

$$P \times d \times L = 2\sigma_1 \times L \times t$$

$$\sigma_1 = \frac{Pd}{2t} \text{ (cancelling } L\text{)}.$$

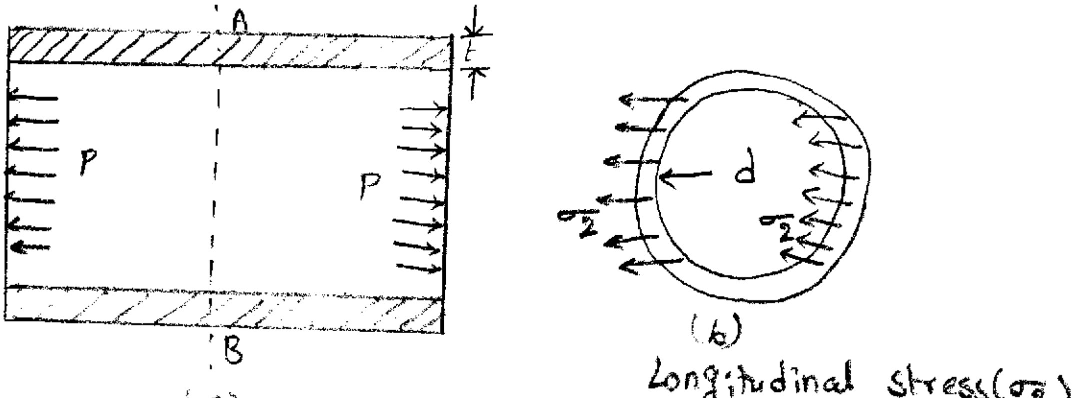
EXPRESSION FOR LONGITUDINAL STRESS:-

Let P = Internal Pressure of fluid

t = Thickness of the wall of the cylinder

d = Internal diameter of the cylinder

σ_2 = Longitudinal stress in the material



(a) (b)

Longitudinal stress(σ_2) develop

Resisting force $= \sigma_2 \times$ Area on which σ_2 is acting

$= \sigma_2 \times \pi d \times t$

∴ Hence in the limiting case

Force due to fluid Pressure = Resisting force

$$P \times \frac{\pi}{4} d^2 = \sigma_2 \times \pi d \times t$$

$$\sigma_2 = \frac{P \times \frac{\pi}{4} d^2}{\pi d \times t} = \frac{Pd}{4t}$$

$$\sigma_2 = \frac{Pd}{2 \times 2t} = \frac{1}{2} \times \sigma_1$$

Longitudinal stress = Half of circumferential stress

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\frac{Pd}{2t} - \frac{Pd}{4t}}{2} = \frac{Pd}{8t}$$

→ Derive the Expression for effect of internal pressure on the dimensions of a thin cylindrical shell?

Ans: EXPRESSION FOR EFFECT OF INTERNAL PRESSURE ON THE DIMENSIONS OF A THIN CYLINDRICAL SHELL:

$$\sigma_1 = \frac{Pd}{2t} \quad \& \quad \sigma_2 = \frac{P \times d}{4t}$$

Let e_1 = circumferential strain

e_2 = Longitudinal strain

But change in volume (δv) = final volume − original volume

original volume(v) = Area of cylindrical sheet

$$= \frac{\pi}{4} d^2 \times L$$

Then circumferential strain,

$$e_1 = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E}$$

final volume = (final area of cross section) × final length

$$= \frac{\pi}{4}[d+\delta d]^2 \times [L+\delta L]$$

$$= \frac{\pi}{4}[d^2+(\delta d)^2+2d\delta d] \times [L+\delta L]$$

$$= \frac{\pi}{4}[d^2L+(\delta d)^2L+2dL\delta d+\delta L d^2+\delta L(\delta d)^2+2d\,\delta d\,\delta L]$$

Neglecting the smaller quantities such as $(\delta d)^2L$, $\delta L(\delta d)^2$ and $2d\delta d\delta L$, we get

$$\text{Final volume} = \frac{\pi}{4}[d^2L+2dL\delta d+\delta L d^2]$$

∴ change in volume (δv)

$$= \frac{\pi}{4}[d^2L+2dL\delta d+\delta L d^2] - \frac{\pi}{4}d^2 \times L$$

$$= \frac{\pi}{4}[2dL\delta d+\delta L d^2]$$

∴ volumetric strains

$$= \frac{\delta v}{v} = \frac{\frac{\pi}{4}[2dL\delta d+\delta L d^2]}{\frac{\pi}{4}d^2 \times L}$$

$$= \frac{2\delta d}{d} + \frac{\delta L}{L} = 2e_1+e_2 \quad \left[\because \frac{\delta d}{d}=e_1, \frac{\delta L}{L}=e_2\right]$$

$$= 2\times\frac{Pd}{2Et}\left[1-\frac{\mu}{2}\right] + \frac{Pd}{2Et}\left[\frac{1}{2}-\mu\right]$$

$$= \frac{Pd}{2Et}\left[2-\frac{2\mu}{2}+\frac{1}{2}-\mu\right] \quad \text{(substituting } e_1 \text{ and } e_2 \text{ values)}$$

$$= \frac{Pd}{2Et}\left[2+\frac{1}{2}-\mu-\mu\right] = \frac{Pd}{2Et}\left(\frac{5}{2}-2\mu\right)$$

Also change in volume (δv) = $v(2e_1+e_2)$.

→ A cylindrical vessel is 1.5m diameter and 4m long is closed at ends by rigid plates. It is subjected to an internal pressure of 3 N/mm². If the maximum principal stress is not to exceed 150 N/mm², find the thickness of the shell. Assume $E = 2\times10^5$ N/mm² and Poisson's ratio = 0.25. Find the changes in diameter, length and volume of the shell?

Ans:-

Dia., $d = 1.5m = 1500$ mm

Length $L = 4m = 4000$ mm

Internal Pressure, $P = 3$ N/mm²

max. Principal stress = 150 N/mm²

Max. Principal stress means the circumferential stress

∴ Circumferential stress, $\sigma_1 = 150$ N/mm²

Volume of $E = 2\times10^5$ N/mm².

Poisson's ratio, $\mu = 0.25$

Let

t = thickness of the shell,

δd = change in diameter

δL = change in length, and

δV = change in volume.

$$\delta d = \frac{Pd^2}{2t\times E}\left[1 - \frac{1}{2}\times\mu\right]$$

$$= \frac{3\times1500^2}{2\times15\times2\times10^5}\left[1-\frac{1}{2}\times0.25\right] = 0.984 \text{ mm.}$$

$$\frac{\delta V}{V} = \frac{P\times d}{2E\times t}\left[\frac{5}{2} - 2\times\mu\right]$$

$$= \frac{3\times1500}{2\times2\times10^5\times15}\left[\frac{5}{2} - 2\times0.25\right] = \frac{3\times1500\times2}{4\times10^5\times15}$$

$$\delta V = \frac{3}{2000}\times V = \frac{3}{2000}\left[\frac{\pi}{4}\times d^2\times L\right]$$

$$= \frac{3}{2000}\times\left(\frac{\pi}{4}\times1500^2\times4000\right) = 10602875 \text{ mm}^3.$$

$$\sigma_1 = \frac{P \times d}{2t}$$

$$t = \frac{P \times d}{2 \times \sigma_1} = \frac{3 \times 1500}{2 \times 150} = 15\text{mm}.$$

$$\delta L = \frac{P \times d \times L}{2t \times E}\left(\frac{1}{2} - \mu\right)$$

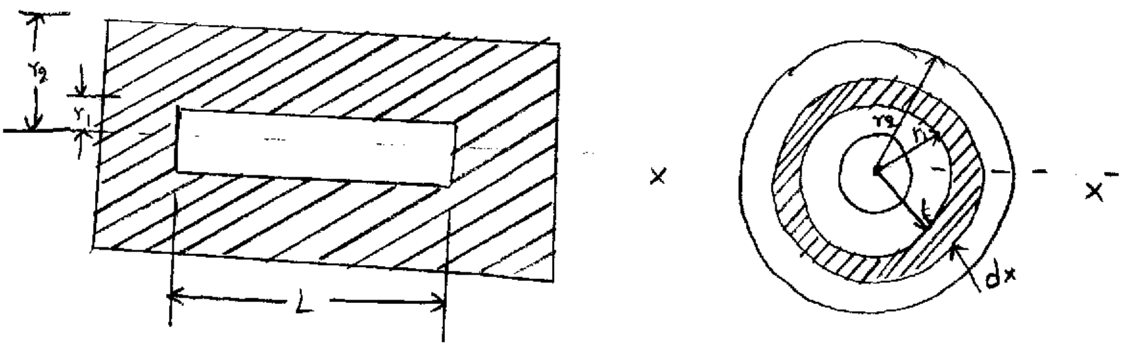
$$= \frac{3 \times 1500 \times 4000}{2 \times 15 \times 2 \times 10^5}\left(\frac{1}{2} - 0.25\right)$$

$$= 0.75\text{mm}$$

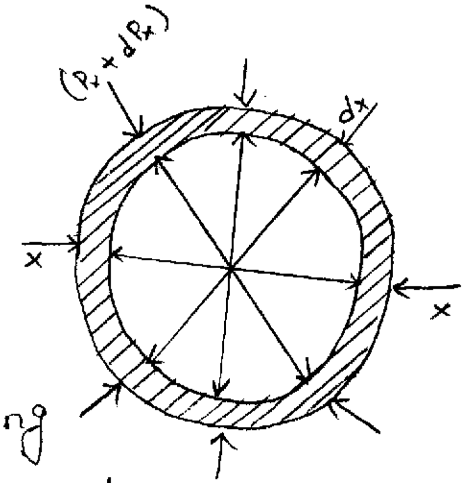
THICK CYLINDERS

→ Derive the expressions for the stresses in a thick cylindrical shell subjected to an internal fluid pressure? (or) Derive the Lame's equations?

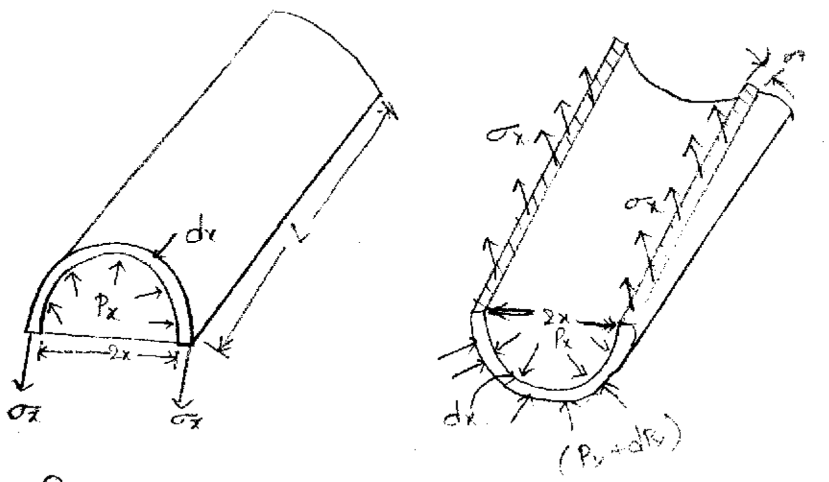
Ans:-



Let r_2 = External radius of the cylinder
r_1 = Internal radius of the cylinder and
L = Length of cylinder



Let P_x = Radial Pressure on the inner surface of the ring
$P_x + dP_x$ = Radial Pressure on the outer surface of the ring
σ_x = Hoop Stress induced in the ring



Brusting force

$= P_x(2xL) - (P_x + dP_x) \times 2(x+dx) \cdot L$

$= 2L\,[P_x \cdot x - (P_x \cdot x + P_x \cdot dx + x \cdot dP_x + dP_x \cdot dx)]$

$= 2L\,[-P_x \cdot dx - x \cdot dP_x]$ (Neglecting $dP_x \cdot dx$)

$= -2L\,[P_x dx + x \cdot dP_x]$

Resisting force = Hoop stress x Area on which it acts $= \sigma_x \times 2dx \cdot L$

$\sigma_x \times 2dx \cdot L = -2L(P_x \cdot dx + x \cdot dP_x)$

$\sigma_x = -P_x - x\dfrac{dP_x}{dx}$ ⟶ (iii)

The longitudinal strain(e_2) at this point is given by,

$e_2 = \dfrac{\sigma_2}{E} - \dfrac{\mu\sigma_x}{E} + \dfrac{\mu P_x}{E}$

But longitudinal strain is constant

$\dfrac{\sigma_2}{E} - \dfrac{\mu\sigma_3}{E} + \dfrac{\mu P_x}{E} = \text{Constant}$

But σ_2 is also constant, and for the material of the cylinder E and μ are constant

$\therefore \sigma_x - P_x = \text{Constant}$

$= 2a$ where a is constant

$\sigma_x = P_x + 2a$ ⟶ (iv)

Equating the two values of σ_x given by eqn's (iii) & (iv) we get

$P_x + 2a = -P_x - x\dfrac{dP_x}{dx}$

$x \cdot \dfrac{dP_x}{dx} = -P_x - P_x - 2a = -2P_x - 2a$

$$\frac{dP_x}{dx} = -\frac{2P_x}{x} - \frac{2a}{x} = \frac{-2(P_x+a)}{x}$$

$$\frac{dP_x}{(P_x+a)} = -\frac{2dx}{x}$$

Integrating the above equation, we get

$$\log_e(P_x+a) = -2\log_e^x + \log_e^b$$

The above equation can also be written as

$$\log_e(P_x+a) = -\log_e x^2 + \log_e b$$

$$= \log_e \frac{b}{x^2}$$

$$P_x + a = \frac{b}{x^2}$$

$$P_x = \frac{b}{x^2} - a$$

Substituting the values of P_x in eqn (iv), we get

$$\sigma_x = \frac{b}{x^2} - a + 2a = \frac{b}{x^2} + a$$

These eqn's are called Lame's eqn's.

Boundary Condition's.

(i) at $x = r_1$, $P_x = P_0$

(ii) at $x = r_2$, $P_x = 0$ or atmosphere Pressure.

2) Find the thickness of a metal necessary for a cylindrical shell of internal diameter 160 mm to withstand an internal Pressure of 8 N/mm². The maximum hoop stress in the section is to exceed 35 N/m²?

S:-

Internal dia. = 160 mm

∴ Internal radius, $r_1 = \frac{160}{2} = 80$ mm

Internal Pressure = 8 N/mm²

This means at $x = 80$ mm, $P_x = 8$ N/mm²

Maximum hoop stress, $\sigma_x = 35$ N/mm²

$$P_x = \frac{b}{x^2} - a \longrightarrow (i)$$

$$\sigma_x = \frac{b}{x^2} + a \longrightarrow (ii)$$

substituting $x = 80$ mm and $P_x = 8$ N/mm² in eqn (i), we get

$$8 = \frac{b}{80^2} - a = \frac{b}{6400} - a \longrightarrow (iii)$$

Substituting $x = 80mm$ and $\sigma_x = 35\ N/mm^2$ in eqn (ii), we get

$$35 = \frac{b}{80^2} + a = \frac{b}{6400} + a \longrightarrow (iv)$$

Substituting eqn (iii) from eqn (iv), we get

$$27 = 2a \quad \text{or} \quad a = \frac{27}{2} = 13.5$$

Substituting the value of a in eqn (iii), we get

$$8 = \frac{b}{6400} - 13.5$$

$$b = (8 + 13.5) \times 6400 = 21.5 \times 6400$$

Substituting the values of 'a' and 'b' in eqn (i),

$$P_x = \frac{21.5 \times 6400}{x^2} - 13.5$$

But at the outer surface, the pressure is zero, Hence at $x = r_2$, $P_x = 0$. Substituting these in the above equation, we get

$$0 = \frac{21.5 \times 6400}{r_2^2} - 13.5$$

$$r_2^2 = \frac{21.5 \times 6400}{13.5} \quad \text{or} \quad r_2 = \sqrt{\frac{21.5 \times 6400}{13.5}} = 100.96\ mm$$

∴ Thickness of the shell, $t = r_2 - r_1$

$$= 100.96 - 80 = 20.96\ mm.$$

3 Determine the hoop stresses in a thick Compound cylinder?

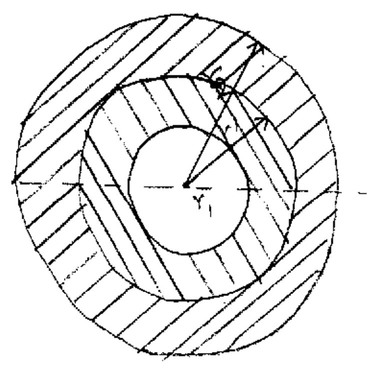
Ans:- Let

r_2 = outer radius of Compound cylinder

r_1 = Inner radius of Compound Cylinder

r^* = Radius at the junction of the two cylinder

p^* = Radius Pressure at the junction of the two cylinders.



(i) For outer cylinder

The lame's equation's at a radius x for outer cylinder are given by

$$P_x = \frac{b_1}{x^2} - a_1 \longrightarrow (i) \qquad \sigma_x = \frac{b_1}{x^2} + a_1 \longrightarrow (ii)$$

where a_1, b_1 are constants for outer cylinder.

at $x = r_2$, $P_x = 0$. And at $x = r^*$, $P_x = p^*$

Substituting these conditions in eqn (i), we get

$$0 = \frac{b_1}{r_2^2} - a_1 \longrightarrow (iii) \qquad p^* = \frac{b_1}{r^{*2}} - a_1 \longrightarrow (iv)$$

From equations (iii) and (iv), the constants a_1 and b_1 can be determined. These values substituted in equation (ii) And thes hoop stresses in the outer cylinder due to shrinking can obtained.

(ii) For inner cylinder

The lame's equations for inner cylinder at a radius x are given by

$$P_x = \frac{b_2}{x^2} - a_2, \quad \sigma_x = \frac{b_2}{x^2} + a_2$$

where a_2, b_2 are constants for inner cylinder

At $x = r_1$, $P_x = 0$ as fluid under pressure is not admitted into the inner at $x = r^*$, $P_x = p^*$

substituting these values in the above value of P_x we get

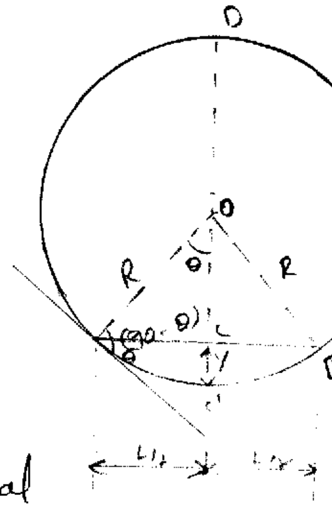
$$0 = \frac{b_2}{r_0^2} - a_2 \longrightarrow (v) \quad \text{and} \quad p^* = \frac{b_2}{r^{*2}} - a_2 \longrightarrow (vi)$$

$$P_x = \frac{B}{x^2} - A \longrightarrow (vii) \quad \text{and} \quad \sigma_x = \frac{B}{x^2} + A \longrightarrow (viii)$$

UNIT-6

→ DEFLCTION OF BEAMS

*) Expression for slope and deflection of a beam subjected to uniform bending moment.



Ans:

Let

R = Radius of curvature of the deflection beam,

y = Deflection of Beam at the centre.

I = moment of inertia of the beam section.

E = young's modulus of the beam material

θ = slope of the beam at the end.

Hence

$\frac{dy}{dx} = \tan\theta = \theta.$

$AC = BC = (L/2)$ Eq

$AC \times CB = DC \times CC'$

$L/2 \times L/2 = (2R - y) \times y$

$L^2/4 = 2Ry - y^2$

Hence neglecting y^2, we get

$L^2/4 = 2Ry$

$y = L^2/8R$

for Bending eqn, we have

$\frac{M}{I} = E/R$

$R = \frac{E \times I}{M}$ Eq

$y = \frac{L^2}{8 \times \frac{EI}{M}}$

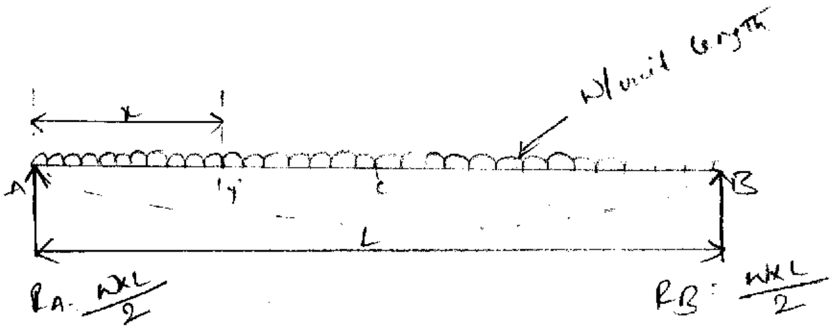
$y = \frac{ML^2}{8EI}$

for triangle AOB, we know that

$\sin\theta = AC/AO = \frac{(L/2)}{R} = L/2R \Rightarrow \theta = L/2R = L/2 \times \frac{EI}{M} = \frac{M \times L}{2EI}$

-) Find the deflection of a simple supported beam of length "L". Carrying a uniform distributed load of w. per unit length.

Sol:



$R_A = R_B = \frac{W \times L}{2}$ & $M_x = R_A \times x - W \times x \times x/2 = \frac{W.L}{2}.x - \frac{W.x^2}{2}$

Bending moment at any section is

$$M = EI\frac{d^2y}{dx^2}$$

equating the 2 values of B.M,

$$EI\frac{d^2y}{dx^2} = \frac{W.L}{2}(x) - \frac{W.x^2}{2}$$

Integrating the above eq. we get.

$$EI\frac{dy}{dx} = \frac{W.L}{2}.(x^2/2) - W/2.(x^3/3) + C_1 \quad \text{---}(i)$$

Once again integrate the above eqn, we get

$$EI.\frac{dy}{dx} = \frac{W.L}{4}.x^3/3 - W/6.x^4/4 + C_1x + C_2 \quad \text{---}(ii)$$

The Boundary Conditions are

i) at $x=0$, $y=0$ and ii) at $x=L$, $y=0$

Substituting the 1st Boundary condition i.e, $x=0$, $y=0$ in equation ii) we get.

$$0 = 0 - 0 + 0 + C_2 \text{ (or) } C_2 = 0$$

Subtracting the 2nd Boundary condition i.e.

at $x = L$, $y = 0$ in eqn ii) we get

$$0 = \frac{w.L}{4}.\left(\frac{L^3}{3}\right) - \frac{w}{6}.\frac{L^4}{4} + C_1.L.$$

$$= \frac{w.L^4}{12} - \frac{w.L^4}{24} + C_1 L$$

$$C_1 = \frac{WL^3}{12} + \frac{WL^3}{24} = -\frac{WL^3}{24}$$

Subtracting the value C_1 in eqn (i) & ii)

$$EI\frac{dy}{dx} = \frac{wL}{4}.x^2 - \frac{w}{6}x^3 - \frac{WL^3}{24} \quad --- (iii)$$

&

$$EI.y = \frac{w.L}{12}x^3 - \frac{w}{24}.x^4 + \left(-\frac{WL^3}{24}\right)x + 0$$

(or)

$$EIy = \frac{w.L}{12}x^3 - \frac{w}{24}x^4 - \frac{WL^3}{24}.x \quad --- (iv)$$

Let θ_A = slope at support A. this is equal to $\left(\frac{dy}{dx}\right)$ at A.

Let θ_A = Slope at support A. it is equal to $\left(\frac{dy}{dx}\right)$ at A.

θ_B = slope at support B. $= \left(\frac{dy}{dx}\right)_{at\, B}$

At A, $x = 0$, & $\frac{dy}{dx} = \theta_A$.

Substituting these values in eqn(iii) we get

$$E.I.\theta_A = \frac{WL}{4} \times 0 - \frac{W}{6} \times 0 - \frac{WL^3}{24}$$

$$= \frac{WL^3}{24} = -\frac{WL^2}{24}$$

$$\theta_A = \frac{-WL^2}{24EI}$$

Similarly $\theta_B = \frac{-WL^2}{24EI}$

$$EI \cdot y_c = \frac{w \cdot L}{12} \cdot (L/2)^3 - w/_{24} (L/2)^4 - \frac{wL^3}{24} \cdot (L/2)$$

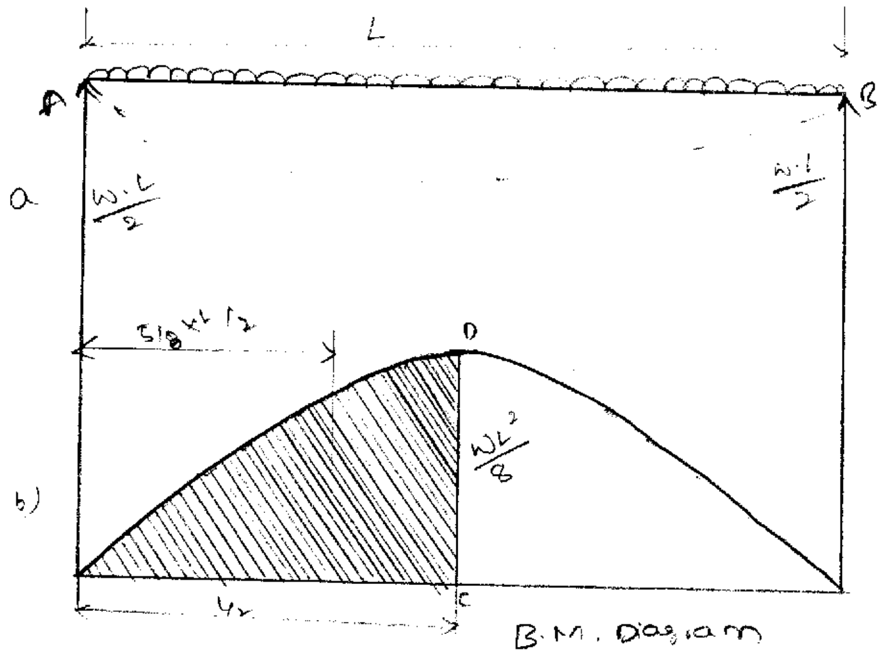
$$= \frac{w \cdot L^4}{96} - \frac{wL^4}{384} - \frac{wL^4}{48} = -\frac{5 w \cdot L^4}{384}$$

$$y_C = -\frac{5}{384} \cdot \frac{wL^4}{EI}$$

$$= \frac{-5}{384} \cdot \frac{W \cdot L^3}{EI}$$

($\because$) $w \cdot L = W =$ Total load)

*) Find the slope and Deflection of a simply Supported Beam AB of Length L and carrying a U.D.L of w per unit length over the entire Span By Using MOHR'S theorem.



B.M. Diagram

i) Now Using Mohr's theorem for slope, we get

$$\text{slope at } A = \frac{\text{Area of B.M. diagram b/w A \& C}}{EI}$$

B.M. diagram b/w A & C

= Area of parabola ACD

$= \text{Area of parabola ACD}$

$= 2/3 \times AC \times CD$

$= 2/3 \times L/2 \times \frac{WL^2}{8} = \frac{W.L^3}{24}$

$A = \frac{W.L^3}{24.EI}$

Slope at $A = \frac{W.L^3}{24.EI}$

Now using Mohr's theorem for deflection.

$$y = \frac{A\bar{x}}{EI}$$

$\bar{x}$ = Distance of C.G. of Area A from A.

$= 5/8 \times AC = 5/8 \times \frac{L}{2} = \frac{5L}{16}$.

$$y = \frac{\frac{W.L^3}{24} \times \frac{5L}{16}}{EI} =$$

$$= \frac{5}{384} \cdot \frac{W.L^4}{EI}.$$

=0=

) Explain about MOMENT AREA METHOD?

A:

Consider an element PQ of small length 'dx at a distance x from B.

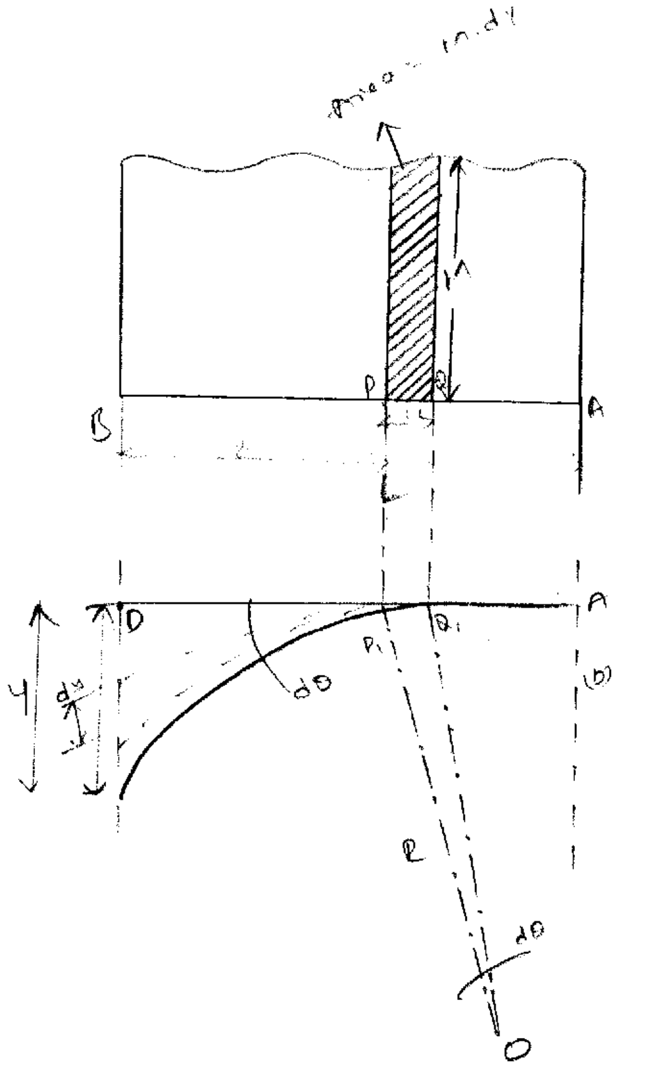
→ the Corresponding points on the deflected beam are $P_1 Q_1$

Let R = Radius of Curvature of deflected part $P_1 Q_1$

$d\theta$ = Angle subtended by the arc $P_1 Q_1$ at the Centre O.

$P_1 C$ = Tangent at point P_1

$Q_1 D$ = Tangent at point Q_1.



For the deflected part P_1Q_1 of the beam, we have

$$P_1Q_1 = R.d\theta$$

$$P_1Q_1 = dx$$

$$dx = R.d\theta$$

But

$$d\theta = \frac{dx}{R}$$

∴ But for a loaded beam, we have

$$\frac{M}{I} = E/R \text{ (or) } R = \frac{EI}{M}$$

Substituting the values of R in eq (i).

$$d\theta = \frac{dx}{\left(\frac{EI}{M}\right)} = \frac{M\,dx}{EI}$$

Since the slope at point A is assumed zero, hence total slope at B is obtained by integrating the above eqtn b/w the limits 0 and L

$$\theta = \int_0^L \frac{M.dx}{EI} = 1/EI \int_0^L M.dx$$

But M. dx represents the area of B.M. diagram of length dx.

Hence $\int_0^L M.dx$ represents the area of B.M. diagram b/w. A & B.

$\therefore$ $\theta = 1/EI$ θ = slope at B = θ_B

But slope at B,

$$\theta_B = \frac{\text{Area of B.M.-diagram b/w A and B}}{EI}$$

If the slope at A is not zero then, we have total change of slope b/w B and A is equal to the Area of B. M. diagram b/w B and A divided by the flexural rigidity EI.

(or) $$\theta_B - \theta_A = \frac{\text{Area of B.M b/w A \& B}}{EI}$$

Now the deflection, due to bending of the portion P_1Q_1 given by

$$dy = x.d\theta$$

Substituting the value of $d\theta$ from eqtn (ii), we get

$$dy = x.\frac{M.dx}{EI} \quad --- (iii)$$

Since the deflection at A is assumed to be zero, hence the total deflection 'δ' is obtained by integrating the above eqn b/w the limits zero and L.

$$y = \int_0^L \frac{xM.dx}{EI}$$

$$\therefore = 1/EI \int_0^L xM.dx$$

But $x \times M.dx$ represents the moment of Area of the B.M. diagram of length dx about point B.

$$\therefore \quad y = 1/EI \times A \times \bar{x} = \frac{A\bar{x}}{EI}$$

Roll No. : Sheet No. :

Shaft series and parallel :-

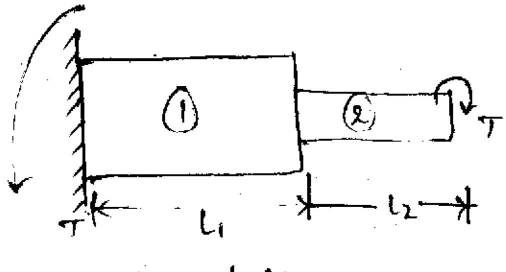
When two shafts of different diameters are connected together to form one shaft it is then known as composite shaft. If the driving torque is applied at one end and the resisting torque at the other end then the shafts are said to be connected in series as shown in fig (a) In such cases each shaft transmitts the same torque and the total angle of twist is equal to the sum of the angle of twist of two shafts

mathematicall total angle of twist

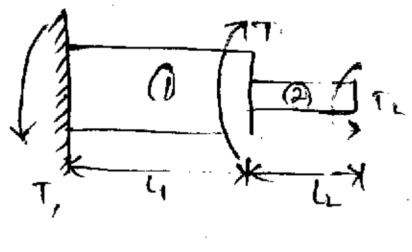
$$\theta = \theta_1 + \theta_2 = \frac{T \cdot L_1}{C_1 J_1} + \frac{T L_2}{C_2 J_2}$$

If the shafts are made of the same material then

$C_1 = C_2 = C$ $\Rightarrow$ $\theta = \frac{T \cdot L_1}{C J_1} + \frac{T L_2}{C J_2} = \frac{T}{C}\left(\frac{L_1}{J_1} + \frac{L_2}{J_2}\right)$



(a) shaft in series



(b) Shaft in parallel

When the driving torque (T) is applied at the junction of the two shafts and the resisting torques T_1 and T_2 at the other ends of the shafts then the shafts are said to be connected in parallel as shown fig In such cases the angle of twist is same for both the shafts i.e.

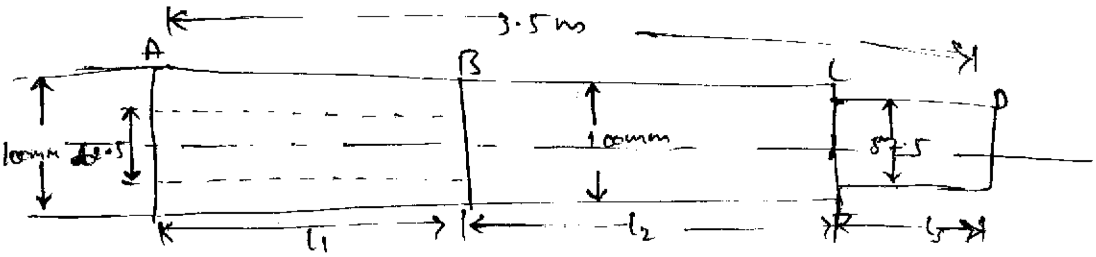
$$\theta_1 = \theta_2$$

$$\frac{T_1 l_1}{C_1 J_1} = \frac{T_2 l_2}{C_2 J_2} \quad (\text{or}) \quad \frac{T_1}{T_2} = \frac{l_1}{l_2} \times \frac{C_1}{C_2} \times \frac{J_1}{J_2}$$

If the shafts are made of the same material.

Example:- A steel shaft ABCD having a total length of 3.5m consists of three lengths having different sections as follows AB is hollow having outside and Inside diameters of 100mm and 62.5 mm respectively and BC and CD are called solid. BC has a dia of 100mm and CD has a diameter of 87.5 mm, If the angle of twist is the same for each section determine the length of each section find the value of the applied torque and the total angle of twist is the same for each section determine the length of the each section. find the value of the applied torque and the angle of twist. If the maximum shear stress in the hollow portion is 47.5 mpa and shear modulus C = 82.5 Gpa

Sol:- Given $l = 3.5m$; $d_o = 100$ mm ; $d_i = 62.5$ mm ; $d_2 = 100$mm ; $d_3 = 87.5$ mm ; $\tau = 47.5$mpa $= 47.5\ N/mm^2$; $C = 82.5$ GPa $= 82.5 \times 10^3\ N/mm^2$



Length of each Section

Let l_1 l_2 and l_3 = length of section AB, BC and CD

we know that polar moment of inertia of hollow shaft AB

$$J_1 = \frac{\pi}{32}\left[(d_0)^4 - (d_1)^4\right] = \frac{\pi}{32}\left[(100)^4 - (62.5)^4\right] = 8.32 \times 10^6 mm^4$$

polar moment of inertia of the solid shaft BC

$$J_2 = \frac{\pi}{32}(d_2)^4 = \frac{\pi}{32}(100)^4 = 9.82 \times 10^6 mm^4$$

and polar moment of inertia of solid shaft CD

$$J_3 = \frac{\pi}{32}(87.5)^4 = 5.75 \times 10^6 mm^4$$

we also known that angle of twist

$$\theta = T \cdot l / C \cdot J$$

Assuming the torque T and shear modulus C to be same for all sections we have Angle of twist for hallow shaft AB

$$\theta_1 = T \cdot L_1 / C J_1$$

Similarly angle of twist for solid shaft BC

$$\theta_2 = T \cdot l_2 / C \cdot J_2$$

and angle of twist for solid shaft CD

$$\theta_3 = T \cdot l_3 / C \cdot J_3$$

Since the angle of twist is for each section

$$\therefore \theta_1 = \theta_2$$

$$\frac{\pi}{32}(d_3)^4 = \frac{\pi}{32}(87.5) = 5.75 \times 10^6 mm^4$$

we also know that angle of twist

$$\theta = T \cdot l / C J$$

Assuming the torque T and shear modulus C to be same for all the sections we have Angle of twist for hallow shaft AB.

$\theta_1 = T \cdot l_1 / C \cdot J_1$

Similarly angle of twist for solid shaft BC

$\theta_2 = T \cdot l_2 / C \cdot J_2$

and angle of twist for solid shaft CD

$\theta_3 = T \cdot l_3 / C J_3$

Since the angle of twist is same for each section

$\therefore \theta_1 = \theta_2$

$\frac{T l_1}{C J_1} = \frac{T l_2}{C J_2}$ (or) $\frac{l_1}{l_2} = \frac{J_1}{J_2} = \frac{8.32 \times 10^6}{9.82 \times 10^6} = 0.847 \rightarrow (1)$

$\theta_1 = \theta_3 \Rightarrow \frac{T \cdot l_1}{C J_1} = \frac{T l_3}{C J_3} = \frac{8.32 \times 10^6}{5.75 \times 10^6} = 1.447 \rightarrow (2)$

we know that $l_1 + l_2 + l_3 = l = 3.5\,m = 3500\,mm$

$l_1 \left(1 + \frac{l_2}{l_1} + \frac{l_3}{l_1} \right) = 3500$

$\Rightarrow l_1 \left[1 + \frac{1}{0.847} + \frac{1}{1.447} \right] = 3500 \Rightarrow l_1 \times 2.8717 = 3500$

from (1) $l_2 = l_3 / 0.847 = 1218.8 / 0.847 = 1439\,mm$

$l_3 = l_1 / 1.447 = 1218.8 / 1.447 = 842.2\,mm$

value of applied torque.

we know that the maximum shear stress in hollow portion $\tau = 47.5\,MPa = 47.5\,N/mm^2$

$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o} \right] = \frac{\pi}{16} \times 47.5 \left[\frac{(100)^4 - (62.5)^4}{100} \right]$

$T = 7.9 \times 10^6\,N\text{-}mm = 7900\,Nm$

Total angle of twist :- when the shafts are connected in series the total angle of twist is equal to the sum of angle twists of the individual shafts mathematically the total angle twist

$$\theta = \theta_1 + \theta_2 + \theta_3$$

$$\frac{T l_1}{C J_1} + \frac{T l_2}{C J_2} + \frac{T l_3}{C J_3} = \frac{T}{C}\left[\frac{l_1}{J_1} + \frac{l_2}{J_2} + \frac{l_3}{J_3}\right]$$

$$= \frac{7.9\times10^6}{82.5\times10^3}\left[\frac{1218.8}{8.32\times10^6} + \frac{1439}{9.82\times10^6} + \frac{842.2}{5.75\times10^6}\right]$$

$$= \frac{7.9\times10^6}{82.5\times10^3\times10^6}(146.5 + 146.5 + 146.5) = 0.042 \text{ rad}$$

$$= 0.042\times180/\pi = 2.406^\circ \text{ Ans}$$

→ Euler's Column theory :

The first Rotational attempt, to study the stability of long columns, was made by Mr. Euler.

- The column which have length less then 8 times their diame called "short column".
- The column which have length less then 30 times their diame are called "long length (or) column".

→ Euler's Formula :

According to Euler's theory. the crippling (or) Buckling (W_{cr}) under various end conditions is Represen by a General eqtn.

$$W_{cr} = \frac{C\pi^2 EI}{l^2} = \frac{C\pi^2 EAk^2}{l^2}$$

$$= \frac{C\pi EA}{(l/k)^2}$$

E = modulus of elasticity

A = Area of cross-section.

k = least Radius of gyration of the cross-section.

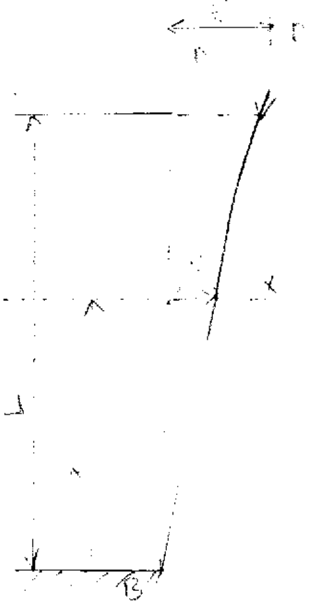
l = length of the column,

C = Constant, Representing the end conditions of the column.

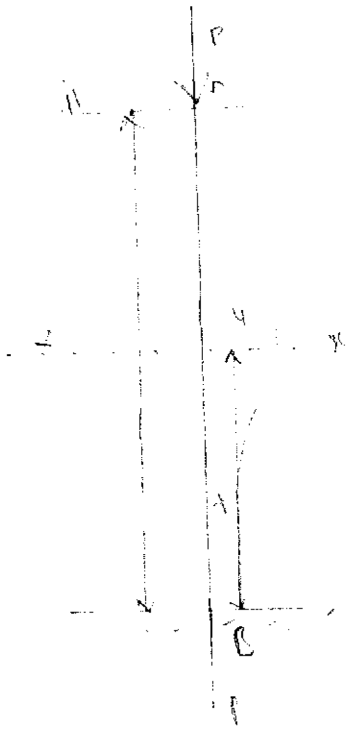
COLUMNS

Both ends are Hinged (or) pinned

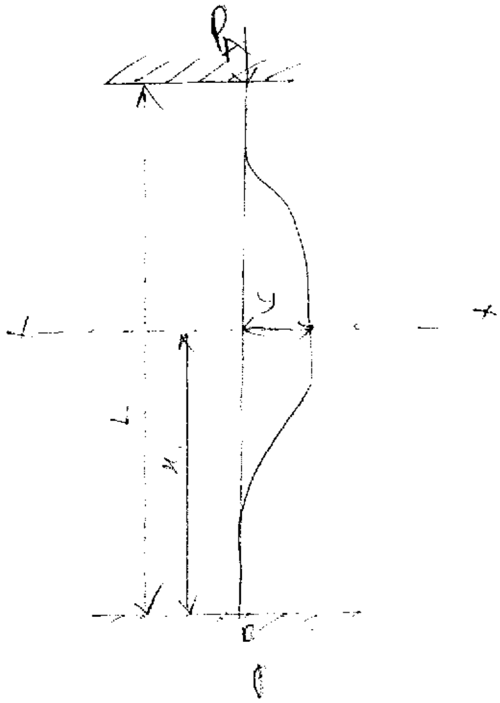
One end fixed & other end Free



One end fixed & other end pinned.

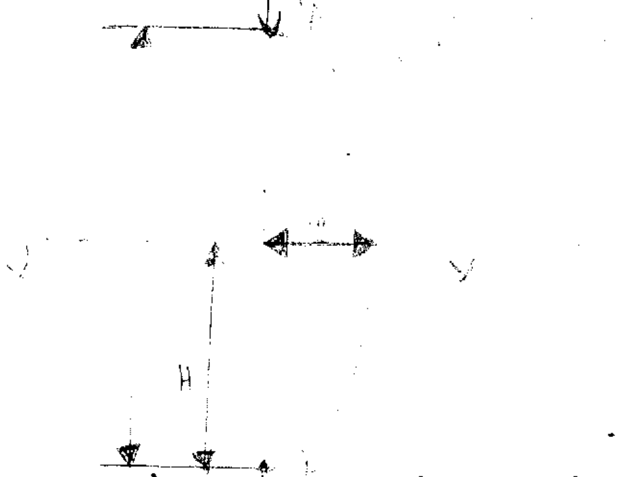


Both END's are Fixed.



1) Derive an expression for Euler's bucking load for a column

a) Both the ends hinged :-



From Bending moment equation.

$\rightarrow \quad M = EI \frac{d^2y}{dx^2}$,

$\rightarrow \quad M = -Py.$

$\rightarrow \quad EI \frac{d^2y}{dx^2} = -Py \quad [\alpha = \sqrt{P/EI}]$

$\rightarrow \quad EI \frac{d^2y}{dx^2} + Py = 0$

$\rightarrow \frac{d^2y}{dx^2} + \left(\frac{P}{EI}\right) y = 0$

$y = C_1 \cos(\alpha x) + C_2 \sin(\alpha x).$

$y = C_1 \cos(x\sqrt{P/EI}) + C_2 \sin(x\sqrt{P/EI}) \rightarrow$

i) At 'A'.

$x = 0; \; y = 0.$

equation (1) substituting we get.

$= C_1 \cos(0) + C_2 \sin(0)$

$= \quad C_1 = 0.$

ii) At 'B'.

$x = L, \quad y = 0$.

equation(1) substituting x.y we get.

$\Rightarrow \quad C_2 \sin\left(L\sqrt{P/EI}\right)$

$C_2 = 0$.

$\sin\left(L\sqrt{P/EI}\right) = 0$

$\sin\left(l\sqrt{P/EI}\right) = 0$

$L \cdot \left(\sqrt{\frac{P}{EI}}\right) = 0, \pi, 2\pi \ldots\ldots n\pi$.

$L \cdot \left(\sqrt{P/EI}\right) = \pi$

$$\Rightarrow \boxed{P = \frac{\pi^2 EI}{L^2}}$$

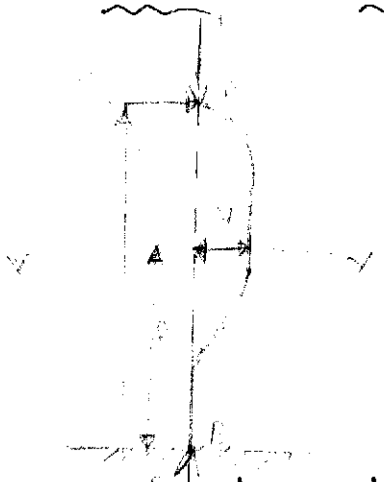
E = Young's modulus.

I = moment of inertia.

L = length of the beam.

2) Derive the equation for;

a) one end is fixed and the other is pinned (or) hinged ?



H = Horizontal Reaction at B due to 'Mo' at A.

From B.M equation;

$$M = EI \frac{d^2y}{dx^2}.$$

$$M = -Py + H(L-x)$$

$$-Py + H(l-x) = EI \frac{d^2y}{dx^2}.$$

$$\frac{d^2y}{dx^2} + \left(\frac{P}{EI}\right)y = \frac{H(l-x)}{EI}.$$

$$\frac{d^2y}{dx^2} + \left(\frac{P}{EI}\right)y = \left(\frac{P}{EI}\right)\left(\frac{H(l-x)}{P}\right)$$

solution on the differential equation.

$$y = C_1 \cos(P/EI)x + C_2 \sin((P/EI)x) + \frac{H(l-x)}{P}$$

i) At A;

$$x=0;\ y=0.$$

solution (1) substituting x and y we get.

$$C_1 = \frac{-H(l)}{P}.$$

$$C_1 = -\frac{Hl}{P}.$$

ii) At

$x = 0 \quad \frac{dy}{dx} = 0.$

substituting on equation (1) we get.

$$\frac{dy}{dx} = C_1 \sin(\sqrt{P/EI})(\sqrt{P/EI}) + C_2 \cos(x\sqrt{P/EI}) - \left(\frac{\sqrt{P}}{EI}\right) - \frac{H}{P} \rightarrow (2)$$

$$\Rightarrow C_2(\sqrt{P/EI}) = \frac{H}{P}.$$

$$C_2 = \frac{H}{P}\left(\sqrt{\frac{EI}{P}}\right).$$

equation (1); substituting above value;

$$(1) \Rightarrow y = -\frac{Hl}{P}\cos(x\sqrt{P/EI}) + \frac{H}{P}\left(\frac{\sqrt{EI}}{P}\right)\sin\left(x\sqrt{\frac{P}{EI}}\right) + \frac{H(l-x)}{P} \rightarrow (3)$$

iii) at 'B';

$x = 2 \quad y = 0.$

substituting equation (1) x, y values we get.

$$L\sqrt{P/EI} = 4.5^6 = \sqrt{2}\pi$$

$$\tan(L\sqrt{P/EI}) = L\sqrt{P/EI}$$

$$\therefore P = \frac{2\pi^2 EI}{L^2}$$

→ Rankine's Formula for columns

We have already discussed the Euler's formula gives results only for very long columns. through the formula is applicable ranging from very long to short ones. yet it does not give reliable results.

$$1/W_{cr} = \frac{1}{W_c} + \frac{1}{W_E}$$

W_{cr} = Crippling load by Rankine's formula.

W_c = Ultimate crushing load for the column

$\sigma_c \times A$.

W_E = Crippling load, obtained by Euler's formula

$= \frac{\pi^2 EI}{L^2}$

Here we know that

$$1/W_{cr} = \frac{1}{W_c} + \frac{1}{W_E} = \frac{W_E + W_c}{W_c \times W_E}$$

$$W_{cr} = \frac{W_c \times W_E}{W_c + W_E} = \frac{W_c}{1 + \frac{W_c}{W_E}}$$

Now steady the value of W_c and W_E in the above eqtn.

$$W_{cr} = \frac{\sigma_c \times A}{1 + \frac{\sigma_c \times A \times L^2}{\pi^2 EI}} = \frac{\sigma_c \times A}{1 + \frac{\sigma_c}{\pi^2 E} \times \frac{AL^2}{AK^2}}$$

$$= \frac{\sigma_c \times A}{1 + a\left(\frac{L}{K}\right)^2} = \frac{\text{crushing load}}{1 + a\left(\frac{L}{K}\right)^2}.$$

→ Assumptions in Euler's column theory.

1. Initially the column is perfectly straight, and the load applied is truly axial.

2. The cross-section of the column is uniform throughout it's length.

3, The column material is perfectly elastic, homogeneous and isotropic, and then obeys Hook's law.

4) The length of column is very large as compared to its cross-sectional dimensions.

5) The failure of column occurs due to buckling alone.

6) The weight of the column itself is neglected.

7, The shortening of column, due to direct compression is neglected.

Where

σ_c = Crushing stress (or) yield stress in compression

A = Cross-sectional Area of the column.

a = Rankine's constant $= \frac{\sigma_c}{\pi^2 E}$.

Limitations of Euler's formula:

We have discussed in that the general eqn. for the crippling load is

$$W_{cr} = \frac{C \pi^2 E A}{(l/k)^2}$$

Crippling stress

$$\sigma_{cr} = \frac{W_{cr}}{A} = \frac{C \pi^2 E}{(l/k)^2}$$

Some times, the column whose slenderness Ratio is more than 80, are known "long columns".

Whose slenderness ratio is less than 80 are known as "short columns". It is thus obvious that the Euler's Formula holds good only for long columns.

NARASARAOPETA ENGINEERING COLLEGE

(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

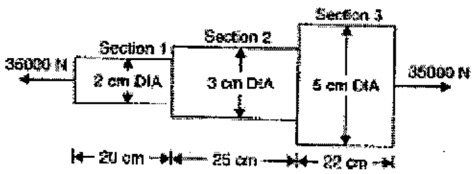
MID & ASSIGNMENT EXAMINATION QUESTION PAPERS WITH SCHEME AND SOLUTIONS

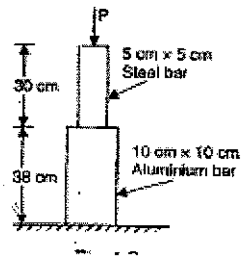
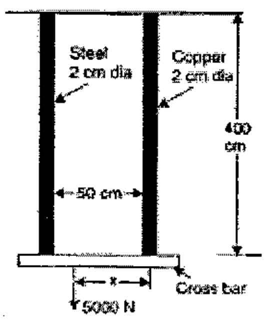
NARASARAOPET ENGINEERING COLLEGE (AUTONOMOUS): NARASARAOPET

DEPARTMENT OF MECHANICAL ENGINEERING

II B.TECH I - SEMESTER ASSIGNMENT TEST – I, November – 2022

SUBJECT: MECHANICS OF SOLIDS	**DATE:** 11-11-2022
DURATION: 30 MIN	**MAX MARKS:** 10

Q. No	Questions	Course Outcome (CO)	Knowledge Level as Per Bloom's Taxonomy	Marks
1	1.A tensile test was conducted on a mild steel bar. The following data was obtained from the test: (i) Diameter of the steel bar= 3cm (ii) Gauge length of the bar= 20cm (iii) Load at elastic limit= 250kN (iv) Extension at a load of 150 kN= 0.21mm (v) Maximum load= 380kN (vi) Total extension= 60mm (vii) Diameter of the rod at the failure Determine: (a) the Young's modulus,= 2.25cm (b) the stress at elastic limit, (c) the percentage elongation, and (d) the percentage decrease in area.	CO1	Analyzing (K4)	5
2	An axial pull of 35000 N is acting on a bar consisting of three lengths as shown in Fig. 1.6 (b). If the Young's modulus = 2.1 x 10^5 N/mm², determine : (i) stresses in each section and (ii) total extension of the bar.  	CO1	Analyzing (K4)	5
3	A member formed by connecting a steel bar to an aluminium bar is shown in Fig. 1.7. Assuming that the bars are prevented from buckling sideways, calculate the magnitude of force P that will cause the total length of the member to decrease 0.25 mm. The values of elastic modulus for steel and aluminium are 2.1 x 10^5 N/mm2 and 7 x 10^4 N/mm2 respectively.	CO1	Analyzing (K4)	5

				
4	The bar shown in Fig. 1.8 is subjected to a tensile load of 160 kN. If the stress in the middle portion is limited to 150 N/mm2, determine the diameter of the middle portion. Find also the length of the middle portion if the total elongation of the bar is to be 0.2 mm. Young's modulus is given as equal to 2.1 x 10^5 N/mm².	CO1	Evaluating (K5)	5
5	A steel rod of 3 cm diameter is enclosed centrally in a hollow copper tube of external diameter 5 cm and internal diameter of 4 cm. The composite bar is then subjected to an axial pull of 45000 N. If the length of each bar is equal to 15 cm, determine : (i) The stresses in the rod and tube, and (ii) Load carried by each bar. Take E for steel = 2.1 x 10^5 N/mm2 and for copper = 1.1 x 10^5 N/mm².	CO1	Evaluating (K5)	5
6	A compound tube consists of a steel tube 140 mm internal diameter and 160 mm external diameter and an outer brass tube 160 mm internal diameter and 180 mm external diameter. The two tubes are of the same length. The compound tube carries an axial load of 900 kN. Find the stresses and the load carried by each tube and the amount it shortens. Length of each tube is 140 mm. Take E for steel as 2 x 10^5 N/mm2 and for brass as 1 x 10^5 N/mm2.	CO1	Evaluating (K5)	5
7	Two vertical rods one of steel and the other of copper are each rigidly fixed at the top and 50 cm apart. Diameters and lengths of each rod are 2 cm and 4 m respectively. A cross bar fixed to the rods at the lower ends carries a load of 5000 N such that the cross bar remains horizontal even after loading. Find the stress in each rod and the position of the load on the bar. Take E for steel = 2 x 10^5 N/mm2 and E for copper = 1 x 10^5 N/mm2. 	CO1	Evaluating (K5)	5

II B.Tech I Sem - I Assignment

(1) Given data — 2 Marks
Formulas — 1 mark
calculation & Solution — 2 marks

(2) Given data — 1 Mark
Diagram — 1 mark
calculation & Solution — 3 marks

(3) Given data — 1 mark
Diagram — 1 mark
Calculation & Solution — 3 marks

(4) Given data — 1 mark
Diagram — 1 mark
Calculation & Solution — 3 marks

(5) Given data — 1 mark
Diagram — 1 mark
Calculation & Solution — 3 marks

(6) Given data — 1M
Diagram — 1M
Calculation & Solution — 3M

NARASARAOPET ENGINEERING COLLEGE (AUTONOMOUS): NARASARAOPET
DEPARTMENT OF MECHANICAL ENGINEERING
II B.TECH I - SEMESTER ASSIGNMENT TEST – II, December – 2022

SUBJECT: MECHANICS OF SOLIDS	**DATE:**03-12-2022
DURATION: 30 MIN	**MAX MARKS:** 10

Q. No	Questions	Course Outcome (CO)	Knowledge Level as Per Bloom's Taxonomy	Marks
1	A cantilever of length 2 metre fails when a load of 2 kN is applied at the free end. If the section of the beam is 40 mm x 60 mm, find the stress at the failure.?	CO3	Analyzing (K4)	5
2	A rectangular beam 200 mm deep and 300 mm wide is simply supported over a span of 8 m. What uniformly distributed load per metre the beam may carry, if the bending stress is not to exceed 120 N/mm^2.?	CO3	Analyzing (K4)	5
3	A rectangular beam 300 mm deep is simply supported over a span of 4 metres. Determine the uniformly distributed load per metre which the beam may carry if the bending stress should not exceed 120 N/mm^2. Take I = 8 x 10^6mm^4 .?	CO3	Analyzing (K4)	5
4	A square beam 20 mm x 20 mm in section and 2 m long is supported at the ends. The beam fails when a point load of 400 N is applied at the centre of the beam. What uniformly distributed load per metre length will break a cantilever of the same material 40 mm wide, 60 mm deep and 3 m long?	CO3	Evaluating (K5)	5
5	A beam is simply supported and carries a uniformly distributed load of 40 kN/m run over the whole span. The section of the beam is rectangular having depth a 500 mm. If the maximum stress in the material of the beam is 120 N/mm^2 and moment of inertia of the section is 7 x 108 mm, find the span of the beam.?	CO3	Evaluating (K5)	5
6	A timber beam of rectangular section of length 8 m is simply supported. The beam carries a U.D.L. of 12 kN/m run over the entire length and a point load of 10 kN at 3 metre from the left support. If the depth is two times the width and the stress in the timber is not to exceed 8 N/mm2, find the suitable dimensions of the section.?	CO3	Evaluating (K5)	5

MOS – Assignment – II

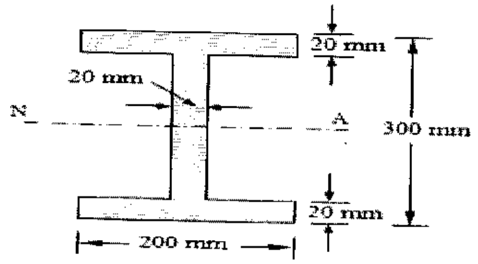
1. Diagram – 2 M
   calcution SFD – 1.5 M
   BMD – 1.5 M
2. Diagram – 2 M
   calcution & solution – 3 M
3. Diagram – 2 M
   calcution & solution – 3 M
4. Diagram – 2 M
   calcution & solution – 3 M
5. Diagram – 2 M
   calcution & solution – 3 M
6. Diagram – 2 M
   calcution & solution – 3 M

NARASARAOPET ENGINEERING COLLEGE (AUTONOMOUS): NARASARAOPET

DEPARTMENT OF MECHANICAL ENGINEERING

II B.TECH I-SEMESTER MID EXAMINATION-II, DECEMBER-2022

SUBJECT: MECHANICS OF SOLIDS	**DATE:** 23/12/2022
DURATION: 90 MIN	**MAX MARKS:**25

Q. No	Questions	Course Outcome (CO)	Knowledge Level as Per Bloom's Taxonomy	Marks
1.	A beam of an I-section shown in Fig. is simply supported over a span of 4 m. Find the uniformly distributed load the beam can carry if the bending stress is not to exceed 100 N/mm2 . 	**CO3**	Analyzing (K4)	**10**
2.	A compound cylinder is made by shrinking a cylinder of external diameter 300mm and internal diameter of 250mm over another cylinder of external diameter 250mm and internal diameter 200mm. the radial pressure at the junction after shrinking is 8 N/mm^2. **Find** the final stresses set up across the section, when the compound cylinder is subjected to an internal fluid pressure of 84.5 N/mm^2?	**CO4**	Applying (K3)	**10**
3.	Make a neat sketch of a circular shaft subjected to a twisting moment? **Derive** the torsion formula? What assumptions are taken while deriving formula for a circular shaft?	**CO6**	Analyzing (K4)	**05**

MOS – I mid

(1) Diagram – 2 M

Formulae – 2 M

Calculation & Solution – 6 M

(2) Given data – 2 M

Formula – 2 M

Calculation & Solution – 6 M

(3) Given data – 1 M

Formula – 1 M

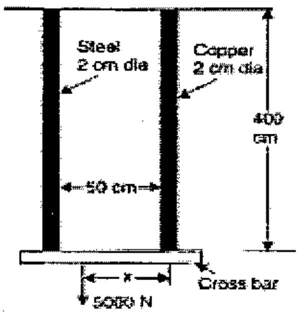
Calculation & Solution – ~~6 M~~ 3 M

NARASARAOPET ENGINEERING COLLEGE (AUTONOMOUS): NARASARAOPET

DEPARTMENT OF MECHANICAL ENGINEERING

II B.TECH I-SEMESTER MID EXAMINATION-I, November -2022

SUBJECT: MECHANICS OF SOLIDS	**DATE: 11-11-2022**
DURATION: 90 MIN	**MAX MARKS:25**
SECTION: A	

Q. No	Questions	Course Outcome (CO)	Knowledge Level as Per Bloom's Taxonomy	Marks
1.	Two vertical rods one of steel and the other of copper are each rigidly fixed at the top and 50 cm apart. Diameters and lengths of each rod are 2 cm and 4 m respectively. A cross bar fixed to the rods at the lower ends carries a load of 5000 N such that the cross bar remains horizontal even after loading. Find the stress in each rod and the position of the load on the bar. Take E for steel = 2 x 10^5 N/mm2 and E for copper = 1 x 10^5 N/mm2. 	CO1	Analyzing (K4)	10
2.	A beam AB, 10m long has supports at its ends A and B. It carries a point load of 5KN at 3m from A and a point load of 5KN at 7m from A and a uniformly distributed load of 1KN/m between the two point loads. Draw SF and BM Diagrams for the beam?	CO2	Evaluating (K5)	10
3.	Derive the Bending momentum equation with neat sketch and write the assumptions made?	CO2	Applying (K3)	05

MOS – II mid

(1) Given data – 1M

diagram – 1M

Formula – 2M

Calculated soln – 6M

(2) Diagram – 2M

SFD – 2M

BMD – 2M

Calculation of soln – 4M

(3) Assumptions – 2M

Diagram – 2M

Discription – 6M

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DEPARTMENT OF MECHANICAL ENGINEERING

UNIT WISE IMPORTANT QUESTIONS

IMPORTANT QUESTIONS

S NO	QUESTION	KNOWLEDGE LEVEL	CO
	UNIT I		
1	Explain about Stress- Strain diagram of (Mild Steel) ductile materials under tensile test?	K1	CO1
2	Derive the relation among the three elastic constants (Modulus of Elasticity and Shear Modulus, and 'Bulk Modulus)?	K2	CO1
3	Explain about different types of Stresses and Strains?	K1	CO1
4	Define Strain Energy and derive the expressions for stress due to gradually applied loads, suddenly applied loads and Impact loads?	K2	CO1
5	A reinforced concrete column 300mmX300mm in section. The column in reinforced with 8 steel bars of 20mm diameter. The column carries a load of 360KN. Find the stresses in the concrete and steel bars? Take E (Steel) = $2.1X10^5$ N/mm^2, E (Concrete) = $1.4X10^4$ N/mm^2?	K3	CO1
	UNIT 2		
1	Explain about following the terms: i) Shear Force ii) Shear Force Diagram iii) Bending Moment iv) bending Moment Diagram v) Pont of Contra Flexure.	K1	CO2
2	A cantilever of length L carrying a point load W at free end and a U.D.L. ω per unit run over the whole length. Draw S.F. and B.M. diagrams?	K2	CO2
3	A simply supported beam of length L carrying a U.D.L. of w per unit run over the whole span. Draw S.F. and B.M. diagrams?	K2	CO2
4	A beam AB, 10m long has supports at its ends A&B. It carries a point load of 5KN at 3m from A and a point load of 5KN at 7m from A and a uniformly distributed load of 1KN/m between the two point loads. Draw SF and BM Diagrams for the beam?	K3	CO2
	UNIT 3		
1	Derive the Bending Moment equation and mention the assumptions made in the theory of simple bending?	K2	CO3
2	A cast iron bracket subjected to bending has the cross-section of I-	K2&	CO3

	form with unequal flanges. The total depth of the section is 280mm and the metal is 40mm thick throughout. The top flange is 200mm wide and the bottom flange is 120mm wide. Find the position of the neutral axis and moment of inertia of the section about the neutral axis and the maximum bending moment that should be imposed on this section if the tensile stress in the top flange is not to exceed 20N/mm2. What is then the value of the maximum compressive stress in the bottom flange?	K3	
3	Prove that the shear stress at any point in the cross section of a beam which is subjected to a shear force F, is given by $\tau = F \times \frac{A\bar{y}}{I \times b}$	K2	CO3
4	Derive an expression for the shear stress at any point in a circular section of a beam, which is subjected to a shear force F. And prove that the maximum shear stress in a circular section of a beam is 4/3 times the average shear stress.	K2	CO3
	UNIT 4		
1	Derive an expression for the slope and deflection of a beam subjected to uniform bending moment (Bending into Circular ARC)?	K2	CO4
2	Find the deflection of a simply supported beam of length L carrying a uniform distributed load of w per unit length? (Double Integration Method)	K3	CO4
3	Find the deflection of a simply supported beam of length L carrying a point load W at a distance 'a' from left support and at a distance 'b' from right supports by using MACAULAY'S METHOD?	K3	CO4
4	Find the slope and deflection of a simply supported beam AB of length L and carrying a U.D.L. of w per unit length over the entire span by any method.	K2	CO4
	UNIT 5		
1	Derive the Expression for Hoop stress and longitudinal stress in a thin cylinder shells subjected to internal fluid pressure?	K2	CO5
2	Derive the Expression for effect of internal pressure on the dimensions of a thin cylindrical shell?	K2	CO5

IMPORTANT QUESTIONS

3	A cylindrical vessel is 1.5m diameter and 4m long is closed at ends by rigid plates. It is subjected to an internal pressure of $3N/mm^2$. If the maximum principal stress is not to exceed 150 N/mm^2, find the thickness of the shell. Assume $E = 2 \times 10^5$ N/mm^2 and poisson's ratio = 0.25. Find the changes in diameter, length and volume of the shell?	K3	CO5
4	Derive the expressions for the stresses in a thick cylindrical shell subjected to an internal fluid pressure? (OR) Derive the Lame's equations?	K2	CO5
5	A compound cylinder is made by shrinking a cylinder of external diameter 300mm and internal diameter of 250mm over another cylinder of external diameter 250mm and internal diameter 200mm. the radial pressure at the junction after shrinking is 8 N/ mm2. Find the final stresses set up across the section, when the compound cylinder is subjected to an internal fluid pressure of 84.5 N/mm^2?	K3	CO5

NEC NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

PREVIOUS QUESTION PAPERS

NEC NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

Subject Code: R20ME2105

II B.Tech. - I Semester Regular Examinations, February-2022

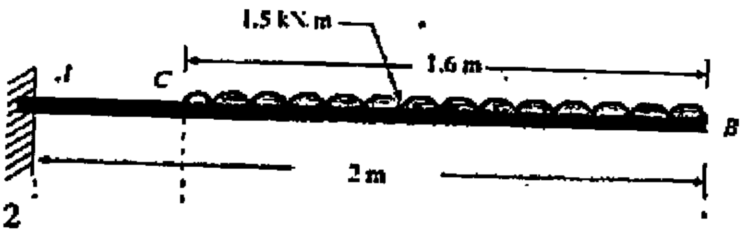
MECHANICS OF SOLIDS

(ME)

Time: 3 hours **Max. Marks: 70**

Note: Answer All FIVE Questions.

All Questions Carry Equal Marks (5 X 14 =70M)

QNo		Questions	KL	CO	Ma
		Unit-I			
1	a	i) Define Bulk modulus. Calculate the change in volume of a cubical block of side 120 mm subjected to a hydrostatic pressure of 70 MPa. Take Poisson's ratio 0.28 and young's modulus 200 GPa.	2	1	7M
		ii)Derive the expression for strain energy stored in a body when the impact load is applied?	2	1	7M
		OR			
	b	i) Define strain energy and complimentary strain energy. Derive an expression for strain energy in a body subjected to axial stress.	1	1	7M
		ii) Derive the relationship between youngs modulus and modulus of rigidity	3	1	7M
		Unit-II			
2	a	i)A cantilever beam AB, 2 m long carries a uniformly distributed load of 1.5 kN/m over a length of 1.6 m from the free end. Draw shear force and bending moment diagrams for the beam.  Fig 2	3	2	7M
		ii) Establish relation between load, shear force and bending moment	3	2	7M
		OR			
	b	Construct shear force diagram and bending moment diagrams for a beam ABE, 3L/2 m long, which is supported at A and B, 'L' m long. The beam carries a concentrated load of 2W at L/4 distance from left support A, and point load W/2 at E. It also carries an upward point load of W at a distance of L/4 from support B.	3	2	14M
		Unit-III			
3	a	i) A rectangular beam 300 mm deep is simply supported over a span of 4 meters. What uniformly distributed load the beam may carry, if the bending stress is not to exceed 120 MPa. Take $I = 225 \times 10^6\ mm^4$.	4	3	7M
		ii) Explain the following: A) Shear force and bending moment in a beam. B) Hogging and sagging moments. C) Point of contra flexure.	1	3	7M
		OR			
	b	Determine and draw the shear stress variation along the depth of an I section beam having a uniform thickness of 10 mm, for the web and flanges. The total height of the section is 200 mm and overall width of each flange is 100 mm. The shear force is 250 kN.	3	3	14M

		Unit-IV				
4	a	i) How can you find slope and deflection in beams using moment area method?	2	4	7M
		ii) A cantilever of length 3 m is carrying a UDL of 10 kN/m over a length of 2 m from fixed end. Find the maximum slope and deflection. Assume $EI = 4 \times 10^{12}$ Nmm^2	3	4	7M
		OR			
	b	i) A cantilever beam is 2 m long and has a flexural rigidity of 25MN-m^2. It carries a point load of 3 kN at mid length and a u.d.l of 2 kN/m along its entire length. Calculate the deflection and slope at the free end by Macaulay's method.	3	4	10 M
		ii) Write down Mohr's theorems for slope and deflection of beams.	2	4	4 M
		Unit-V			
5	a	Derive the stresses in thin cylindrical vessel with neat sketches	3	5	14 M
		OR			
	b	i) Calculate minimum wall thickness of a thin cylinder 1 m in diameter if it is to withstand an internal pressure of 2 N/mm^2 and hoop stress not to exceed 40 N/mm^2. Also find change in diameter. E = 210 GPa; Poisson's ratio = 0.3.	3	5	7M
		ii)A thick metallic cylindrical shell of 150 mm internal diameter is required to withstand an internal pressure of 8 N/mm^2. Find the necessary thickness of the shell, if the permissible tensile stress in the section is 20 N/mm^2.	3	5	7M

KL: Blooms Taxonomy Knowledge Level CO: Course Outcome M:Marks

NEC NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

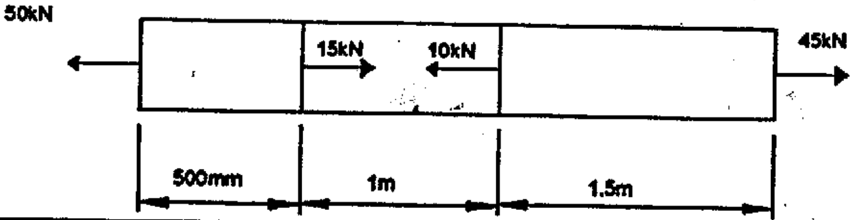
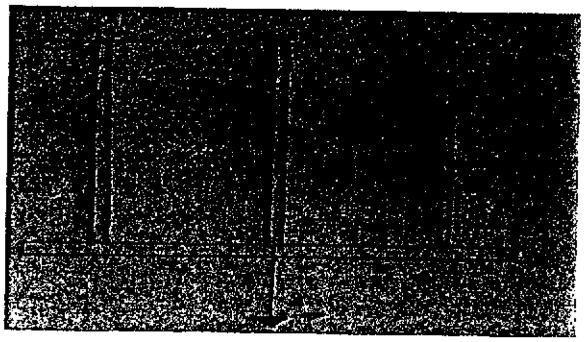
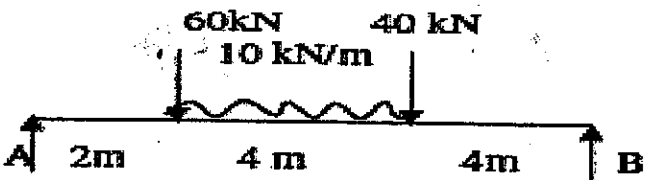
Subject Code: R20ME2105

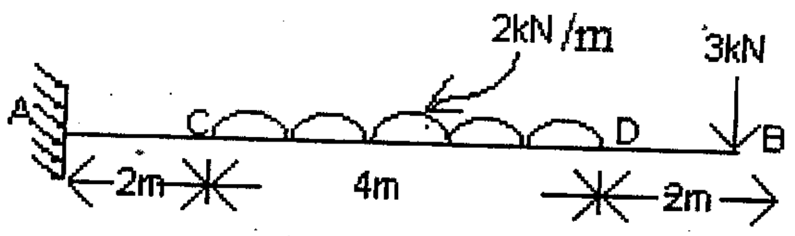
II B.Tech. - I Semester Regular & Supple Examinations, December-2022

MECHANICS OF SOLIDS
(ME)

Time: 3 hours **Max. Marks: 70**

Note: Answer All **FIVE** Questions.
All Questions Carry Equal Marks (5 X 14 =70M)

QNo.		Questions	K L	CO	Ma
		Unit-I			
1		A steel bar of cross-section 500 mm^2 is acted upon by the forces shown in figure. Find the total elongation of the bar. (For steel, E=200 GN/ m^2) 	K3	1	7M
	a	Three vertical rods equal in length and each 12 mm in diameter are equispaced in a vertical plane and together support a load of 10000 N the rods being so adjusted as to share the load equally. If now an additional load of 10000 N be added determine the stress in each rod. The middle rod is of copper and outer rods are of steel. Take E_s=2X10^5 N/mm^2 and E_c=1X10^5 N/mm^2. 	K4	1	7M
		OR			
		Define the terms strain energy derive strain energy for gradually applied load?	K3	1	7M
	b	A uniform metal bar has a cross sectional area of 7cm^2 and length of 18m. With the elastic limit of 160 MN/m^2, what will be its proof resilience? Determine also the maximum value of an applied load which may be suddenly applied without exceeding elastic limit.	K3	1	7M
		Unit-II			
2	a	Draw the S.F and B.M diagrams of the beam shown in figure 1.  Figure 1	K3	2	7M

		OR			
	b	Determine the shear force and bending moment diagrams for the cantilever loaded as shown in Figure. 	K3	2	14
		Unit-III			
3	a	Derive the fundamental law of pure bending	K3	3	7M
		Derive section modulus for a hollow circular section.	K3	3	7M
		OR			
	b	A beam of I-section is having overall depth as 500mm and over all width as 190 mm. The thickness of flanges is 25mm where as the thickness of the web is 15mm. the moment of inertia about N.A is given as 6.4×10^{8} mm^{4}. If the section carries a shear force of 40KN, calculate the maximum shear stress. Also sketch the bending stress across the section	K4	3	14M
		Unit-IV			
4	a	A beam of length 20m is simply supported at its ends and carries two point loads of 6kN and 12kN at a distance of 8m and 12m from left end respectively. Calculate i) Deflection under each load. ii) Maximum deflection. Take $E=2\times10^{6}$ N/mm^{2} and $I= 1\times10^{9}$ mm^{4}.	K4	4	14M
		OR			
	b	A cantilever beam AB of length 4m carries a point load of 100kN at free end and another point load 100kN at 2m from the free end. If $E = 10^{5}$ N/mm^{2} and $I=10^{8}$ mm^{4} for the cantilever then determine the slope and deflection at the free end by Moment area method.	K4	4	14M
		Unit-V			
5	a	Calculate the bursting pressure for cold drawn seamless steel tubing of 60 mm inside diameter with 2 mm thickness. The ultimate strength of steel is 380 N/mm^{2}.	K3	5	5M
		Find the thickness of metal necessary for a steel cylindrical shell of internal diameter 200 mm to withstand an internal pressure of 50 N/mm^{2}. The maximum hoop stress is not to exceed 150 N/mm^{2}.	K2	5	9M
		OR			
	b	What are the assumptions in the theory of pure torsion?	K2	5	7M
		Motor drives a solid circular shaft transmitting 30 kW to a gear. If the allowable shear stress in the shaft is 50 N/mm2. Find the diameter of the shaft. i) if it runs at 500 rpm ii) if it runs at 3000 rpm.	K2	5	7M

KL: Blooms Taxonomy Knowledge Level CO: Course Outcome M:Marks