NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

COURSE FILE

Academic year	**: 2022-23**
Department	**: ME**
Course Name	**: B.Tech**
Student's Batch	**: 2022-23**
Regulation	**: R20**
Year and Semester	**: III B.Tech II Semester**
Name of the Subject	**:** DYNAMICS OF MACHINERY
Subject Code	**:** R20ME3203
Faculty In charge	**:** Dr. M. Venkanna Babu

Signature of Faculty **Head of the Department**

NEC NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

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NARASARAOPETA
ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

INSTITUTE VISION AND MISSION

NARASARAOPETA
NEC ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

INSTITUTE VISION AND MISSION

VISION:

To emerge as a **Centre of excellence** in technical education with a blend of effective **student centric teaching learning** practices as well as **research** for the transformation of **lives and community**.

MISSION:

1. Provide the best class infrastructure to explore the field of engineering and research.
2. Build a passionate and a determined team of faculty with student centric teaching, imbibing experiential and innovative skills.
3. Imbibe lifelong learning skills, entrepreneurial skills and ethical values in students for addressing societal problems.

PRINCIPAL

NARASARAOPETA
NEC ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

DEPARTMENT VISION AND MISSION

NARASARAOPETA
NEC ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

DEPARTMENT VISION AND MISSION

VISION:

To strive for making competent **Mechanical Engineering Professionals** to cater the real time needs of Industry and **Research** Organizations of high repute with **Entrepreneurial Skills and Ethical Values.**

MISSION:

M1. To train the students with State of Art Infrastructure to make them industry ready professionals and to promote them for higher studies and research.

M2. To employ committed faculty for developing competent mechanical engineering graduates to deal with complex problems.

M3. To support the students in developing professionalism and make them socially committed mechanical engineers with morals and ethical values.

HOD-ME

NARASARAOPETA
ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

PROGRAM EDUCATIONAL OBJECTIVES (PEOs)

AND

PROGRAM SPECIFIC OUTCOMES (PSOs)

NARASARAOPETA
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DEPARTMENT OF MECHANICAL ENGINEERING

PROGRAM EDUCATIONAL OBJECTIVES (PEOs)

PEO 1: Excel in profession with sound knowledge in mathematics and applied sciences

PEO 2: Demonstrate leadership qualities and team spirit in achieving goals

PEO 3: Pursue higher studies to ace in research and develop as entrepreneurs.

PROGRAM SPECIFIC OUTCOMES (PSOs)

PSO1. The students will be able to apply knowledge of modern tools in manufacturing enabling to conquer the challenges of Modern Industry.

PSO2. The students will be able to design various thermal engineering systems by applying the principles of thermal sciences.

PSO3. The students will be able to design different mechanisms and machine components of transmission of power and automation in modern industry.

HOD-ME

NARASARAOPETA
ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

PROGRAM OUTCOMES
(POs)

NARASARAOPETA
NEC ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

PROGRAM OUTCOMES (POs):

Engineering Graduates will be able to:

1. Engineering knowledge: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.

2. Problem analysis: Identify, formulate, review research literature, and analyse complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.

3. Design/development of solutions: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

4. Conduct investigations of complex problems: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

5. Modern tool usage: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modelling to complex engineering activities with an understanding of the limitations.

6. The engineer and society: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

7. Environment and sustainability: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

8. Ethics: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

9. Individual and team work: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

10. Communication: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

11. Project management and finance: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

12. Life-long learning: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

HOD-ME

NARASARAOPETA
NEC ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

BLOOM'S TAXONOMY LEVELS

NARASARAOPETA ENGINEERING COLLEGE (AUTONOMOUS) NARSARAOPET

Bloom's Taxonomy Action Verbs

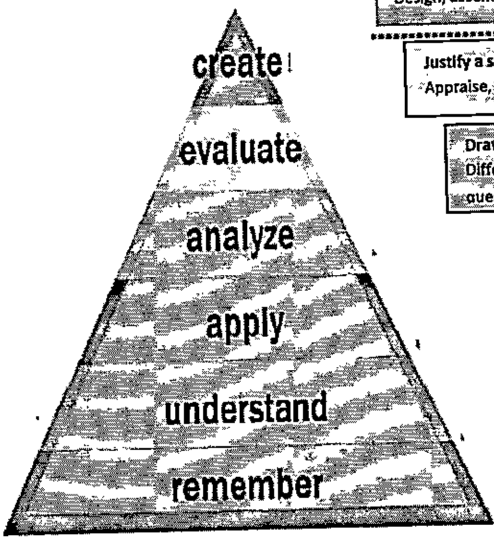
Definitions	Knowledge	Comprehension	Application	Analysis	Synthesis	Evaluation
Bloom's Definition	Remember Previously Learned Information	Demonstrate an understanding of the facts.	Apply knowledge to actual situations.	Break down objects or ideas into simpler parts and find evidence to support generalizations.	Compile component ideas into a new whole or propose alternative solutions.	Make and defend judgement based on internal evidence or external criteria.
Verbs	• Arrange • Define • Describe • Duplicate • Identify • Label • List • Match • Memorize • Name • Order • Outline • Recognize • Relate • Recall • Repeat • Reproduce • Select • state	• Classify • Convert • Defend • Describe • Discuss • Distinguish • Estimate • Explain • Express • Extend • Generalized • Give example(s) • Identify • Indicate • Infer • Locate • Paraphrase • Predict • Recognize • Rewrite • Review • Select • Summarize • Translate	• Apply • Change • Choose • Compute • Demonstrate • Discover • Dramatize • Employ • Illustrate • Interpret • Manipulate • Modify • Operate • Practice • Predict • Prepare • Produce • Relate • Schedule • Show • Sketch • Solve • Use • Write	• Analyze • Appraise • Breakdown • Calculate • Category • Compare • Contrast • Criticize • Diagram • Differentiate • Discriminate • Distinguish • Examine • Experiment • Identify • Illustrate • Infer • Model • Outline • Point out • Question • Relate • Select • Separate • Subdivide • Test	• Arrange • Assemble • Categorize • Collect • Combine • Comply • Compose • Construct • Create • Design • Develop • Devise • Explain • Formulate • Generate • Plan • Prepare • Rearrange • Reconstruct • Relate • Reorganize • Revise • Rewrite • Set up • Summarize • Synthesize • Tell • Write	• Appraise • Argue • Assess • Attach • Choose • Compare • Conclude • Contrast • Defend • Descry • Discriminate • Estimate • Evaluate • Explain • Judge • Justify • Interpret • Relate • Predict • Rate • Select • Sum • Support • Value

REVISED Bloom's Taxonomy Action Verbs

Definitions	I. Remembering	II. Understanding	III. Applying	IV. Analyzing	V. Evaluating	VI. Creating
Bloom's Definition	Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers.	Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas.	Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations.	Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria.	Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions.
Verbs	• Choose • Define • Find • How • Label • List • Match • Name • Omit • Recall • Relate • Select • Show • Spell • Tell • What • When • Where • Which • Who • Why	• Classify • Compare • Contrast • Demonstrate • Explain • Extend • Illustrate • Infer • Interpret • Outline • Relate • Rephrase • Show • Summarize • Translate	• Apply • Build • Choose • Construct • Develop • Experiment with • Identify • Interview • Make use of • Model • Organize • Plan • Select • Solve • Utilize	• Analyze • Assume • Categorize • Classify • Compare • Conclusion • Contrast • Discover • Dissect • Distinguish • Divide • Examine • Function • Inference • Inspect • List • Motive • Relationships • Simplify • Survey • Take part in • Test for • Theme	• Agree • Appraise • Assess • Award • Choose • Compare • Conclude • Criteria • Criticize • Decide • Deduct • Defend • Determine • Disprove • Estimate • Evaluate • Explain • Importance • Influence • Interpret • Judge • Justify • Mark • Measure • Opinion • Perceive • Prioritize • Prove • Rate • Recommend • Rule on • Select • Support • Value	• Adapt • Build • Change • Choose • Combine • Compile • Compose • Construct • Create • Delete • Design • Develop • Discuss • Elaborate • Estimate • Formulate • Happen • Imagine • Improve • Invent • Make up • Maximize • Minimize • Modify • Original • Originate • Plan • Predict • Propose • Solution • Solve • Suppose • Test • Theory

derson, L. W., & Krathwohl, D. R. (2001). A taxonomy for learning, teaching, and assessing, Abridged Edition. Boston, MA: Allyn and Bacon.

Bloom's Taxonomy



Produce new or original work
Design, assemble, construct, conjecture, develop, formulate, author, investigate

Justify a stand or decision
Appraise, argue, defend, judge, select, support, value, critique, weigh

Draw connections among ideas
Differentiate, organize, relate, compare, contrast, distinguish, examine, experiment, question, test

Use information in new situations
Execute, implement, solve, use, demonstrate, interpret, operate, schedule, sketch

Explain ideas or concepts
Classify, describe, discuss, explain, identify, recognize, report, select, translate

Recall facts and basic concepts
Define, duplicate, list, memorize, repeat, state

NEC NARASARAOPETA ENGINEERING COLLEGE
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DEPARTMENT OF MECHANICAL ENGINEERING

COURSE OUTCOMES (COs)

NEC **NARASARAOPETA ENGINEERING COLLEGE**

(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

III B.TECH – II SEMESTER

R20 REGULATION

COURSE OUTCOMES

Course Name: DYNAMICS OF MACHINERY	Course Code: C323
CO	**After successful completion of this course, the students will be able to:**
C323.1	**Analyse** the effect of precession motion on the stability of motor cycles, aero planes and ships, under gyroscope
C323.2	**Compute** the frictional losses and transmission in clutches and Brakes.
C323.3	**Analyze** the stability of different types of governors under dynamic analysis and study
C323.4	**Analyze** balancing of rotating and reciprocating masses for primary and secondary forces by analytical and graphical methods
C323.5	**Analyse** balancing of reciprocating masses for primary and secondary forces by analytical and graphical methods

NARASARAOPETA
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(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

COURSE INFORMATION SHEET

Narasaraopeta Engineering College
(Autonomous)
Yallmanda (Post), Narasaraopet- 522601
Department of Mechanical Engineering

COURSE INFORMATION SHEET

PROGRAMME: **B.TECH MECHANICAL ENGINEERING**	
COURSE: **DYNAMICS OF MACHINERY**	Semester : **VI** CREDITS: **3**
COURSE CODE: **R20ME3203** REGULATION: **R20**	COURSE TYPE (CORE /ELECTIVE / BREADTH/ S&H): **CORE**
COURSE AREA/DOMAIN: **DESIGN**	PERIODS: **6** Per Week.

COURSE PRE-REQUISITES:

C.CODE	COURSE NAME	DESCRIPTION	SEM
R20CC1107	Engineering Mechanics	Knowledge of free body diagram, kinematics and dynamics, impulse and momentum.	I
R20ME2105	Mechanics of Solids	Knowledge of stress, strain, shear force and bending moment, center of gravity and torsion.	III
R20ME2202	Kinematics of Machinery	Knowledge of Kinematics of Mechanisms, Velocity and Acceleration Diagrams and Mechanics of Gears and Gear Trains	IV

COURSE OUTCOMES:

S.NO	COURSE OUTCOME STATEMENT
CO1	**Analyse** the effect of precession motion on the stability of motor cycles, aero planes and ships, under gyroscope
CO2	**Compute** the frictional losses and transmission in clutches and Brakes
CO3	**Analyse** the stability of different types of governors under dynamic analysis and study the difference between governor and flywheel
CO4	**Analyse** balancing of rotating masses for forces by analytical and graphical methods
CO5	**Analyse** balancing of reciprocating masses for primary and secondary forces by analytical and graphical methods

SYLLABUS:

UNIT	DETAILS
I	**GYROSCOPE AND GYROSCOPIC EFFECTS:** Introduction, Precessional Angular Motion, Gyroscopic Couple, Effect of precession motion on the stability of moving vehicles such as, Aero plane and Naval ship, Four wheel vehicle moving in a curved path, Two wheel vehicle Taking a Turn.
II	**FRICTION CLUTCHES:** Friction clutches- Single disc or plate clutch, Multiple disc clutch, Cone clutch, Centrifugal clutch. **BRAKES:** Types of Brakes, Single Block or Shoe Brake, Simple Band Brake, Differential Band Brake, Internal Expanding Brakes.

III	**GOVERNERS:** Types of Governors, Terms used in governors, Watt governor, Porter governor, Proell governor, Hartnell governor, Hartung governor, Sensitiveness of governors, **FLYWHEELS:** Functions, Differences between flywheel and governor, turning moment diagrams, flywheel analysis for I-C Engines and presses.
IV	**BALANCING OF ROTATING MASSES:** Static balancing, Dynamic balancing, Balancing of a single rotating mass by a single mass rotating in the same plane, Balancing of a single rotating mass by two masses rotating in different planes, Balancing of several masses rotating in the same plane by analytical and graphical methods, Balancing of several rotating in different planes.
V	**BALANCING OF RECIPROCATING MASSES:** Unbalanced force, Primary and Secondary unbalanced forces of reciprocating masses, Partial Balancing of unbalanced Primary force in a reciprocating engine, Partial balancing of Locomotives, Variation of Tractive force, Swaying Couple, Hammer Blow, Balancing of V-engines.

TEXT BOOKS	
T	**BOOK TITLE/AUTHORS/PUBLISHER**
T1	Theory of Machines, Thomas Bevan, Pearson education publications.
T2	Theory of machines, SS Rattan, Tata McGraw Hill publications.
REFERENCE BOOKS	
R	**BOOK TITLE/AUTHORS/PUBLISHER**
R1	Theory of Machines, W.G.Green, Blackie publications.
R2	Mechanism and Machine Theory / JS Rao and RV Dukkipati / New Age
R3	Theory of Machines / Shigley / MGH2.
R4	Theory of Machines, R.S. Khurmi & J.K.Gupta, S. Chand Publications

TOPICS BEYOND SYLLABUS/ADVANCED TOPICS:

S.NO	DESCRIPTION	Associated PO & PSO
1	Introduction to Whirling of Shaft and Vibrations.	PO1,PO2,PO3, PO4, PSO3

WEB SOURCE REFERENCES:

1	https://nptel.ac.in/courses/112/104/112104114//
2	https://nptel.ac.in/courses/112/104/112104121/

DELIVERY/INSTRUCTIONAL METHODOLOGIES:

✓ Chalk & Talk	✓ PPT	☐ Active Learning
✓ Web Resources	✓ Students Seminars	☐ Case Study
☐ Blended Learning	☐ Quiz	☐ Tutorials
☐ Project based learning	✓ NPTEL/MOOCS	☐ Simulation
✓ Flipped Learning	☐ Industrial Visit	✓ Model Demonstration
☐ Brain storming	☐ Role Play	✓ Virtual Labs

MAPPING CO'S WITH PO'S:

CO	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12	PSO 1	PSO 2	PSO 3
CO1	2	2	3	1	-	-	-	-	1	-	-	-	-	-	3
CO2	2	2	3	1	-	-	-	-	-	-	-	-	-	-	3
CO3	2	2	3	1	-	-	-	-	-	-	-	-	-	-	3
CO4	2	3	3	2	2	-	-	-	-	-	-	-	-	-	3
CO5	2	3	3	2	2	-	-	-	-	-	-	-	-	-	3
Average	2	2.4	3	1.4	2	-	-	-	-	-	-	-	-	-	3

MAPPING COURSE WITH POs & PSOs:

Course	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3
C3103	2	2.4	3	1.4	2	-	-	-	-	-	-	-	-	-	3

<table>
<tr><th colspan="3">Course Outcome Assessment Methods</th><th colspan="2">Weightages</th><th rowspan="6">Final Course Outcome (100%)</th></tr>
<tr><td rowspan="4">Direct Assessment</td><td rowspan="3">Cumulative Internal Examinations (CIE)</td><td>Descriptive Test</td><td rowspan="3">30%</td><td rowspan="4">90%</td></tr>
<tr><td>Objective Test</td></tr>
<tr><td>Assignment Test</td></tr>
<tr><td colspan="2">Semester End Examinations (SEE)</td><td>70%</td></tr>
<tr><td>Indirect Assessment</td><td colspan="3">Course End Survey</td><td>10%</td></tr>
</table>

Rubrics for overall attainment of course outcomes:
If 50% of the students crossed 50% of the marks: Attainment Level 1
If 60% of the students crossed 50% of the marks: Attainment Level 2
If 70% of the students crossed 50% of the marks: Attainment Level 3

Course Instructor

Course Coordinator **Module Coordinator** **Head of the Department**

ANNEXURE I:

(A) PROGRAM OUTCOMES (POs) Engineering Graduates will be able to:

1. Engineering knowledge: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.

2. Problem analysis: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.

3. Design/development of solutions: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

4. Conduct investigations of complex problems: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

5. Modern tool usage: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.

6. The engineer and society: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

7. Environment and sustainability: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

8. Ethics: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

9. Individual and team work: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

10. Communication: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

11. Project management and finance: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

12. Life-long learning: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

(B) PROGRAM SPECIFIC OUTCOMES (PSOs):

1. The students will be able to understand the modern tools of machining which gives them good expertise on advanced manufacturing methods.
2. The students are able to design different heat transfer devices with emphasis on combustion and power production.
3. The students are able to design different mechanisms and machine components suitable to automation industry.

Cognitive levels as per Revised Blooms Taxonomy:

Cognitive Domain	Level	Key words
Remembering	K1	Choose, Define, Find, How, Label, List, Match, Name, Omit, Recall, Relate, Select, Show, Spell, Tell, What, When, Where, Which, Who, Why.
Understanding	K2	Classify, Compare, Contrast, Demonstrate, Explain, Extend, Illustrate, Infer, Interpret, Outline, Relate, Rephrase, Show, Summarize, Translate.
Applying	K3	Apply, Build, Choose, Construct, Develop, Experiment With, Identify, Interview, Make Use of, Model, Organize, Plan, Select, Solve, Utilize.
Analyzing	K4	Analyze, Assume, Categorize, Classify, Compare, Conclusion, Contrast, Discover, Dissect, Distinguish, Divide, Examine, Function, Inference, Inspect, List, Motive, Relationships, Simplify, Survey, Take part in, Test for, Theme
Evaluating	K5	Agree, Appraise, Assess, Award, Choose, Compare, Conclude, Criteria, Criticize, Decide, Deduct, Defend, Determine, Disprove, Estimate, Evaluate, Explain, Importance, Influence, Interpret, Judge, Justify, Mark, Measure, Opinion, Perceive, Prioritize, Prove, Rate, Recommend, Rule on, Select, Support, Value
Creating	K6	Adapt, Build, Change, Choose, Combine, Compile, Compose, Construct, Create, Delete, Design, Develop, Discuss, Elaborate, Estimate, Formulate, Happen, Imagine, Improve, Invent, Make up, Maximize, Minimize, Modify, Original, Originate, Plan, Predict, Propose, Solution, Solve, Suppose, Test, Theory

UNIT WISE SAMPLE ASSESSMENT QUESTIONS

COURSE OUTCOMES:

S.NO	COURSE OUTCOME STATEMENT
CO1	**Analyse** the effect of precession motion on the stability of motor cycles, aero planes and ships, under gyroscope
CO2	**Compute** the frictional losses and transmission in clutches and Brakes
CO3	**Analyse** the stability of different types of governors under dynamic analysis and study the difference between governor and flywheel
CO4	**Analyse** balancing of rotating masses for forces by analytical and graphical methods
CO5	**Analyse** balancing of reciprocating masses for primary and secondary forces by analytical and graphical methods

S. NO:	QUESTION	KNOWLEDGE LEVEL	CO
	UNIT I		
1	Differentiate 'Natural Precession' from 'Forced Precession'.	K2	CO1
2	What is the effect of the gyroscopic couple on the stability of a four-wheeler while negotiating a curve?	K2	CO1
3	What is the principle of gyroscope?	K2	CO1
4	Analyze Precessional Angular Motion and Gyroscopic Couple?	K4	CO1
	UNIT 2		
1	Describe with a neat sketch the working of a single plate friction clutch.	K3	CO2
2	What are the various types of the brakes?	K3	CO2
3	Describe with the help of a neat sketch the principles of operation of an internal expanding shoe. Derive the expression for the braking torque	K3	CO2
	UNIT 3		
1	Explain the difference in the construction features of a Watt governor, Porter governor, and Proell governor.	K3	CO3
2	Differences between flywheel and governor?	K2	CO3
3	Explain about the turning moment diagrams	K3	CO3
	UNIT 4		
1	What is static balancing?	K2	CO4
2	Analyze the Dynamic balancing?	K5	CO4
3	Explain the method of balancing of different masses revolving in the same plane	K3	CO4
4	How the different masses rotating in different planes are balanced?		
	UNIT 5		
1	Explain the terms: variation of tractive force, swaying couple, and hammer blow	K5	CO5
2	Write a short note on primary and secondary balancing.	K3	CO5
3	Compare Partial Balancing of unbalanced Primary force in a reciprocating engine and Partial balancing of Locomotives,	K3	CO5

		and Y is 400mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.			
		OR			
	b	Explain the method of balancing of different masses revolving in the same plane.	K3	CO4	[7M]
		How the different masses rotating in different planes are balanced?	K3	CO4	[7M]
5		**Unit - V**			
	a	Analyze the Balancing of V- engines	K4	CO5	[7M]
		Write a short note on primary and secondary balancing.	K3	CO5	[7M]
		OR			
	b	A four cylinder vertical engine has cranks 150 mm long. The planes of rotation of the first, second and fourth cranks are 400 mm, 200 mm and 200 mm respectively from the third crank and their reciprocating masses are 50 kg, 60 kg and 50 kg respectively. Find the mass of the reciprocating parts for the third cylinder and the relative angular positions of the cranks in order that the engine may be in complete primary balance	K4	CO5	[14M]

NEC NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

III B.Tech II Semester Regular Examinations

Sub Code: R20ME3203

SUBJECT NAME: DYNAMICS OF MACHINERY

(ME)

MODEL PAPER-II

Time: 3 hours

Max. Marks: 70

Note: Answer All **FIVE** Questions.
All Questions Carry Equal Marks (5 X 14 = 70M)

Q. NO		QUESTION	KL	CO	MARKS
		Unit - I			
	a	Explain about the terminology of an Aeroplane and the effect of Gyroscopic Couple on an Aeroplane?	K3	CO1	[7M]
		Explain the gyroscopic effect on four wheeled vehicles.	K3	CO1	[7M]
1		**OR**			
	b	An aeroplane makes a complete half circle of 50m radius, towards left, when flying at 200km per hr. The rotary engine and propeller of the plane has a mass of 400kg and a radius of gyration of 0.3m. The engine rotates at 200 r.p.m. clockwise when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it?	K4	CO1	[14M]
		Unit - II			
	a	Describe with a neat sketch the working of a multi plate friction clutch.	K3	CO2	[7M]
2		A car engine has its rated output of 12 kW. The maximum torque developed is 100 N-m. The clutch used is of single plate type having two active surfaces. The axial pressure is not to exceed 85 KN/m2.The external diameter of the friction plate is 1.25 times the internal diameter. Determine the dimensions of the friction plate and the axial force exerted by the springs. Coefficient of friction =	K4	CO2	[7M]

NARASARAOPETA
NEC ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

ACADEMIC CALENDAR

NEC NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

ACADEMIC CALENDAR
(B.Tech. 2020 Admitted Batch, Academic Year 2022-23)

2020 Batch 3rd Year 1st Semester			
Description	**From Date**	**To Date**	**Duration**
Commencement of Class Work	25-07-2022		7 Weeks
1st Spell of Instructions	25-07-2022	10-09-2022	
Assignment Test-I	15-08-2022	20-08-2022	
I Mid examinations	12-09-2022	17-09-2022	1 Week
2nd Spell of Instructions	19-09-2022	05-11-2022	7 Weeks
Assignment Test-II	10-10-2022	15-10-2022	
II Mid examinations	07-11-2022	12-11-2022	1 Week
Preparation & Practicals	14-11-2022	19-11-2022	1Week
Semester End Examinations	21-11-2022	03-12-2022	2 Weeks
2020 Batch 3rd Year 2nd Semester			
Commencement of Class Work	05-12-2022		7 Weeks
1st Spell of Instructions	05-12-2022	21-01-2023	
Assignment Test-I	26-12-2022	31-12-2022	
I Mid examinations	23-01-2023	28-01-2023	1 Week
2nd Spell of Instructions	30-01-2023	18-03-2023	7 Weeks
Assignment Test-II	20-02-2023	25-02-2023	
II Mid examinations	20-03-2023	25-03-2023	1 Week
Preparation & Practicals	27-03-2023	01-04-2023	1 Week
Semester End Examinations	03-04-2023	15-04-2023	2 Weeks
Commencement of 4th Year 1st Sem Class Work	05-06-2023		

PRINCIPAL

NEC NARASARAOPETA ENGINEERING COLLEGE

(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

TIME TABLE

NARASARAOPETA ENGINEERING COLLEGE
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DEPARTMENT OF MECHANICAL ENGINEERING

SYLLABUS COPY

III B.TECH II-SEMESTER	L	T	P	INTERNAL MARKS	EXTERNAL MARKS	TOTAL MARKS	CREDITS
	2	1	0	30	70	100	3
Code: R20ME3203	DYNAMICS OF MACHINERY						

COURSE OBJECTIVES:

1. To understand the effect gyroscopic couple in motor cycles, aeroplanes and ships.
2. To analyze the forces in clutches and brakes involving friction.
3. To analyze the speed and stability of governors and fly wheels
4. To learn analytical and graphical methods for calculating balancing of rotary and reciprocating masses.

COURSE OUTCOMES:

After successful completion of this course, the students will be able to:

CO1 **Analyse** the effect of precession motion on the stability of motor cycles, aero planes and ships, under gyroscope

CO2 **Compute** the frictional losses and transmission in clutches and Brakes.

CO3 **Analyze** the stability of different types of governors under dynamic analysis and study the difference between governor and flywheel

CO4 **analyze** balancing of rotating and reciprocating masses for primary and secondary forces by analytical and graphical methods

CO5 **Analyse** balancing of reciprocating masses for primary and secondary forces by analytical and graphical methods

UNIT – I:

GYROSCOPE AND GYROSCOPIC EFFECTS: Introduction, Precessional Angular Motion, Gyroscopic Couple, Effect of precession motion on the stability of moving vehicles such as, Aero plane and Naval ship, Four wheel vehicle moving in a curved path, Two wheel vehicle Taking a Turn.

UNIT – II:

FRICTION CLUTCHES: Friction clutches- Single disc or plate clutch, Multiple disc clutch, Cone clutch, Centrifugal clutch.

BRAKES: Types of Brakes, Single Block or Shoe Brake, Simple Band Brake, Differential Band Brake, Internal Expanding Brakes.

UNIT – III

GOVERNERS: Types of Governors, Terms used in governors, Watt governor, Porter governor, Proell governor, Hartnell governor, Hartung governor, Sensitiveness of governors,

FLYWHEELS: Functions, Differences between flywheel and governor, turning moment diagrams, flywheel analysis for I-C Engines and presses.

UNIT- IV

BALANCING OF ROTATING MASSES: Static balancing, Dynamic balancing, Balancing of a single rotating mass by a single mass rotating in the same plane, Balancing of a single rotating mass by two masses rotating in different planes, Balancing of several masses rotating in the same plane by analytical and graphical methods, Balancing of several rotating in different planes.

UNIT- V

BALANCING OF RECIPROCATING MASSES: Unbalanced force, Primary and Secondary unbalanced forces of reciprocating masses, Partial Balancing of unbalanced Primary force in a reciprocating engine, Partial balancing of Locomotives, Variation of Tractive force, Swaying Couple, Hammer Blow, Balancing of V-engines.

TEXT BOOKS:

1. Theory of Machines, Thomas Bevan, Pearson education publications.
2. Theory of machines, SS Rattan, Tata McGraw Hill publications.

REFERENCES:

1. Theory of Machines, W.G.Green, Blackie publications.
2. Mechanism and Machine Theory / JS Rao and RV Dukkipati / New Age
3. Theory of Machines / Shigley / MGH2.
4. Theory of Machines, R.S. Khurmi & J.K.Gupta, S. Chand Publications

WEB REFERENCES:

1. https://nptel.ac.in/courses/112/104/112104114//
2. https://nptel.ac.in/courses/112/104/112104121/

NEC NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

LESSON PLAN

NEC	**Narasaraopeta Engineering College** (Autonomous) Yallmanda (Post), Narasaraopet - 522601

DEPARTMENT OF MECHANICAL ENGINEERING
LESSON PLAN

Course Code	Course Title (Regulation)	Sem	Branch	Contact Periods/ Week	Sections
R20ME203	DYNAMICS OF MACHINERY (R20)	VI	Mechanical Engineering	6	A & B

S.NO	COURSE OUTCOME STATEMENT
CO	**After successful completion of this course, the students will be able to:**
CO1	**Analyse** the effect of precession motion on the stability of motor cycles, aero planes and ships, under gyroscope
CO2	**Compute** the frictional losses and transmission in clutches and Brakes
CO3	**Analyse** the stability of different types of governors under dynamic analysis and study the difference between governor and flywheel
CO4	**Analyse** balancing of rotating masses for forces by analytical and graphical methods
CO5	**Analyse** balancing of reciprocating masses for primary and secondary forces by analytical and graphical methods

Unit No	Outcome		Topics/Activity	Ref Text book	Total Periods	Delivery Method
1	CO 1. **Analyse** the effect of precession motion on the stability of motor cycles, aero planes and ships, under gyroscope		**UNIT-1: GYROSCOPE AND GYROSCOPIC EFFECTS**			
		1.1	Introduction to Dynamics	T1, T2, R1,R3	2	Chalk, Talk, PPT
		1.2	Precessional Angular Motion, Gyroscopic Couple	T1, T2, R1,R3	2	Chalk, Talk, PPT
		1.3	Effect of precession motion on the stability of Aero plane taking a Turn.	T1, T2, R1,R3	2	Chalk, Talk, PPT, Web Resources
		1.4	Effect of precession motion on the stability of Naval ship taking a Turn.	T1, T2, R1,R3	2	Chalk, Talk, PPT, Web Resources
		1.5	Effect of precession motion on the stability of Four wheel vehicle moving in a curved path.	T1, T2, R1,R3	2	Chalk, Talk, Case Study,
		1.6	Effect of precession motion on the stability of Two wheel vehicle Taking a Turn.	T1, T2, R1,R3	2	Chalk, Talk, PPT, Case Study,
2	CO 2. **Compute**		**UNIT-2: FRICTION CLUTCHES AND BRAKES**			
		2.1	Introduction to Friction clutches	T1, T2,	2	Chalk, Talk,

	the frictional losses and transmission in clutches and Brakes		.	R1,R3		PPT,
		2.2	Single disc or plate clutch and Multiple disc clutch.	T1, T2, R1,R3	2	Chalk, Talk, PPT, Students Seminar,
		2.3	Cone clutch, Centrifugal clutch.	T1, T2, R1,R3	2	Chalk, Talk, PPT, Model Demonstration
		2.4	Types of Brakes.	T1, T2, R1,R3	2	Chalk, Talk, PPT
		2.5	Single Block or Shoe Brake, Simple Band Brakes.	T1, T2, R1,R3	2	Chalk, Talk, PPT
		2.6	Differential Band Brake, Internal Expanding Brakes	T1, T2, R1,R3	2	Chalk, Talk, PPT, Model Demonstration
3	CO 3. **Analyse** the stability of different types of governors under dynamic analysis and study the difference between governor and flywheel	**UNIT-3: GOVERNERS AND FLYWHEELS:**				
		3.1	Types of Governors, Terms used in governors.	T1, T2, R1,R3	2	Chalk, Talk, PPT
		3.2	Watt governor, Porter governor,	T1, T2, R1,R3	2	Chalk, Talk, PPT
		3.3	Proell governor, Hartnell governor, Hartung governor, Sensitiveness of governors.	T1, T2, R1,R3	4	Chalk, Talk, PPT, MOOCS
		3.4	Functions, Differences between flywheel and governor.	T1, T2, R1,R3	2	Chalk, Talk, PPT, Model Demonstration
		3.5	Turning moment diagrams, flywheel analysis for I-C Engines and presses	T1, T2, R1,R3	2	Chalk, Talk, PPT, Model Demonstration
MID I EXAMINATION DURING SIXTH WEEK						
4	CO 4. **Analyse** balancing of rotating masses for forces by analytical and graphical methods	**UNIT 4: BALANCING OF ROTATING MASSES**				
		4.1	Static balancing, Dynamic balancing.	T1, T2, R1,R3	2	Chalk, Talk, PPT, Virtual Labs
		4.2	Balancing of a single rotating mass by a single mass rotating in the same plane.	T1, T2, R1,R3	2	Chalk, Talk, PPT, Virtual Labs
		4.3	Balancing of a single rotating mass by two masses rotating in different planes..	T1, T2, R1,R3	2	Chalk, Talk, PPT, Virtual Labs
		4.4	Balancing of several masses rotating in the same plane by analytical and graphical methods.	T1, T2, R1,R3	2	Chalk, Talk, Quiz, Virtual Labs
		4.5	Balancing of several rotating in different planes	T1, T2, R1,R3	4	Chalk, Talk, PPT, Virtual Labs
5	CO 5. **Analyse** balancing of reciprocating masses for	**UNIT 5. BALANCING OF RECIPROCATING MASSES**				
		5.1	Unbalanced force, Primary and Secondary unbalanced forces of reciprocating masses,	T1, T2, R1,R3	4	Chalk, Talk, PPT,
		5.2	Partial Balancing of unbalanced Primary force in a reciprocating engine.	T1, T2, R1,R3	2	Chalk, Talk, PPT,

	primary and secondary forces by analytical and graphical methods	5.3	Partial balancing of Locomotives,	T1, T2, R1,R3	2	Chalk, Talk, PPT,
		5.4	Variation of Tractive force, Swaying Couple, Hammer Blow,	T1, T2, R1,R3	2	Chalk, Talk, PPT,
		5.5	Balancing of V-engines.	T1, T2, R1,R3	2	Chalk, Talk, PPT,
				Total	60	
MID II EXAMINATION DURING EIGHTEENTH WEEK						
END EXAMINATIONS						

TEXT BOOKS	
T	**BOOK TITLE/AUTHORS/PUBLISHER**
T1	Theory of Machines, Thomas Bevan, Pearson education publications.
T2	Theory of machines, SS Rattan, Tata McGraw Hill publications.

REFERENCE BOOKS	
R	**BOOK TITLE/AUTHORS/PUBLISHER**
R1	Theory of Machines, W.G.Green, Blackie publications.
R2	Mechanism and Machine Theory / JS Rao and RV Dukkipati / New Age
R3	Theory of Machines / Shigley / MGH2.

Principal **HOD** **Faculty**

NARASARAOPETA
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DEPARTMENT OF MECHANICAL ENGINEERING

CO-POs & CO-PSOs MAPPING (COURSE ARTICULATION MATRIX)

NEC NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING
COURSE ARTICULATION MATRIX
R20-REGULATION
III B.Tech II SEMESTER

Explanation of Course Articulation Matrix Table to be ascertained:

- Course Articulation Matrix correlates the individual COs of a course with POs and PSOs.
- The Course Outcomes are mapped with POs and PSOs in the scale of 1 to 3.
- The strength of correlation is indicated as 3 for **Substantial (High)** correlation, 2 for **Moderate (Medium)** correlation, and 1 for **Slight (Low)** correlation.

Course Code: C323	Course Name: DYNAMICS OF MACHINERY														
COs	POs & PSOs														
	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3
C323.1	3	2	2	-	-	-	-	-	-	-	-	-	-	-	3
C323.2	3	3	-	-	-	-	-	-	-	-	-	-	-	-	3
C323.3	3	3	-	-	-	-	-	-	-	-	-	-	-	-	3
C323.4	3	3	3	2	-	-	-	-	-	-	-	-	-	-	3
C323.5	3	3	3	2	-	-	-	-	-	-	-	-	-	-	3
C323	3.00	2.80	2.67	2.00	-	-	-	-	-	-	-	-	-	-	3.00

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DEPARTMENT OF MECHANICAL ENGINEERING

WEB REFERENCES

NARASARAOPETA ENGINEERING COLLEGE

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DEPARTMENT OF MECHANICAL ENGINEERING

STUDENT'S ROLL LIST

NARASARAOPETA ENGINEERING COLLEGE : NARASARAOPET

(AUTONOMOUS)

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5	20471A0305	BOYAPATI PAVAN KUMAR
6	20471A0306	DADDANALA VEERANJIREDDY
7	20471A0307	DERANGILA PARDHU GANESH
8	20471A0308	DOPPALAPUDI S S NAGA RAVITEJA
9	20471A0309	EEDARA MOHAN SAI
10	20471A0310	GANESH SAI PAVAN
11	20471A0312	GERA KOTESWARA RAO
12	20471A0313	KARASALA PRASANTH
13	20471A0314	KARASANI PAVAN KUMAR REDDY
14	20471A0315	KATTA MAHESWAR
15	20471A0317	KESARI DHANUNJAYA REDDY
16	20471A0318	KOMARAGIRI SASIKUMAR
17	20471A0319	KOMERA SIVA NAGARAJU
18	20471A0320	KOTHA GOPI
19	20471A0321	KUNDURTHI NAVEEN
20	20471A0323	MADANU JOSEPH VINAY KUMAR
21	20471A0324	MADDUMALA RAMAKRISHNA
22	20471A0325	MAGANTI SASI PAVAN
23	20471A0326	MAKKENA SAMBASIVA RAO
24	20471A0327	MIRIYALA SASHANK
25	20471A0328	NALLA ABHIRAM CHOWDARY
26	20471A0329	NUTHAKKI RAKESH
27	20471A0330	ARAVAPALLI SAI SRINIVAS
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35	20471A0339	RAMAVATHU BADDUNAIK
36	20471A0341	SHAIK APPAPURAM MAHABOOB SUBHANI
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39	20471A0345	SHAIK LAL AHAMAD BASHA
40	20471A0346	SHAIK MAHAMMAD FAREED
41	20471A0348	SHAIK MANISHA
42	20471A0349	SHAIK PARVEZ
43	20471A0350	SHAIK SADHIK
44	20471A0352	TIPPIREDDY AMARNATHREDDY
45	20471A0353	VADLAVALLI GANESH
46	20471A0354	VEERAGANDHAM VENKATA MANIKANTA
47	20471A0356	ADAKA GOPIRAJU
48	20471A0357	ATCHYUTHA PAVAN KUMAR
49	20471A0358	BALLE RAMANJANEYULU
50	20471A0359	BANDARU SAI GANESH
51	20471A0360	BERAM NARENDRA REDDY
52	20471A0361	CHEBROLU MANIKANTA SAI NITHIN
53	20471A0362	CHENNAMSETTY GOPI
54	20471A0363	GANGULA SUNNY
55	20471A0364	GANJI HANUMA KOTI GANESH
56	20471A0365	GANNNAVARAPU JAYA SRIKANTH
57	20471A0366	GUTTIKONDA AYYAPPA REDDY
58	20471A0367	MADDINENI AJAY
59	20471A0368	MANNEPALLI VEERA NARASIMHA
60	20471A0369	MARAGANI NAGA THIRUMALA RAO

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94	21475A0327	GOLLA SUNDARA SAMRAJYA SUGNAN
95	21475A0328	CHATTA VENKATRAMAIAH
96	21475A0329	KSHATRIYA JITHENDRA SINGH
97	21475A0330	BOMMALI BALA SIVA YOGENDRA SAI NANDU
98	21475A0331	REVALLA SAI
99	21475A0332	BANDI SRINIVAS
100	21475A0333	GURRAM SIVA GANESH
101	21475A0334	EMANI LEELA SHANKAR
102	21475A0335	KUPPALA SRINU

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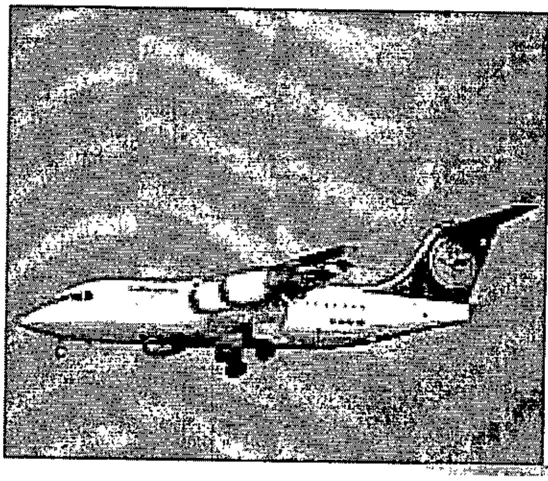
DEPARTMENT OF MECHANICAL ENGINEERING

HAND WRITTEN/PRINTED LECTURE NOTES



14

Gyroscopic Couple and Precessional Motion

Features

1. *Introduction.*
2. *Precessional Angular Motion.*
3. *Gyroscopic Couple.*
4. *Effect of Gyroscopic Couple on an Aeroplane.*
5. *Terms Used in a Naval Ship.*
6. *Effect of Gyroscopic Couple on a Naval Ship during Steering.*
7. *Effect of Gyroscopic Couple on a Naval Ship during Pitching.*
8. *Effect of Gyroscopic Couple on a Navalship during Rolling.*
9. *Stability of a Four Wheel drive Moving in a Curved Path.*
10. *Stability of a Two Wheel Vehicle Taking a Turn.*
11. *Effect of Gyroscopic Couple on a Disc Fixed Rigidly at a Certain Angle to a Rotating Shaft.*

14.1. Introduction

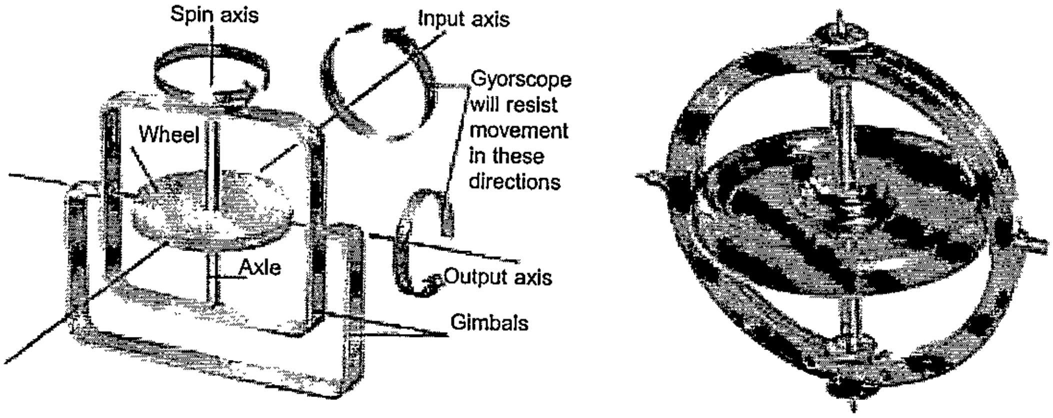
We have already discussed that,

1. When a body moves along a curved path with a uniform linear velocity, a force in the direction of centripetal acceleration (known as centripetal force) has to be applied externally over the body, so that it moves along the required curved path. This external force applied is known as *active force.*

2. When a body, itself, is moving with uniform linear velocity along a circular path, it is subjected to the centrifugal force* radially outwards. This centrifugal force is called *reactive force.* The action of the reactive or centrifugal force is to tilt or move the body along radially outward direction.

Note: Whenever the effect of any force or couple over a moving or rotating body is to be considered, it should be with respect to the reactive force or couple and not with respect to active force or couple.

* Centrifugal force is equal in magnitude to centripetal force but opposite in direction.



Gyroscopic inertia prevents a spinning top from falling sideways.

14.2. Precessional Angular Motion

We have already discussed that the angular acceleration is the rate of change of angular velocity with respect to time. It is a vector quantity and may be represented by drawing a vector diagram with the help of right hand screw rule (see chapter 2, Art. 2.13).

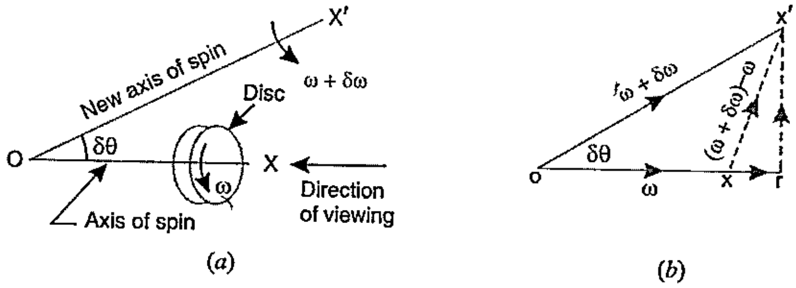


Fig. 14.1. Precessional angular motion.

Consider a disc, as shown in Fig. 14.1 (*a*), revolving or spinning about the axis OX (known as **axis of spin**) in anticlockwise when seen from the front, with an angular velocity ω in a plane at right angles to the paper.

After a short interval of time δt, let the disc be spinning about the new axis of spin OX' (at an angle $\delta\theta$) with an angular velocity $(\omega + \delta\omega)$. Using the right hand screw rule, initial angular velocity of the disc (ω) is represented by vector ox; and the final angular velocity of the disc $(\omega + \delta\omega)$ is represented by vector ox' as shown in Fig. 14.1 (*b*). The vector xx' represents the change of angular velocity in time δt *i.e.* the angular acceleration of the disc. This may be resolved into two components, one parallel to ox and the other perpendicular to ox.

Component of angular acceleration in the direction of ox,

$$\alpha_t = \frac{xr}{\delta t} = \frac{or - ox}{\delta t} = \frac{ox' \cos \delta\theta - ox}{\delta t}$$

$$= \frac{(\omega + \delta\omega)\ \cos\delta\theta - \omega}{\delta t} = \frac{\omega\cos\delta\theta + \delta\omega\ \cos\delta\theta - \omega}{\delta t}$$

Since $\delta\theta$ is very small, therefore substituting $\cos \delta\theta = 1$, we have

$$\alpha_t = \frac{\omega + \delta\omega - \omega}{\delta t} = \frac{\delta\omega}{\delta t}$$

In the limit, when $\delta t \to 0$,

$$\alpha_t = \underset{\delta t \to 0}{\text{Lt}} \left(\frac{\delta\omega}{\delta t}\right) = \frac{d\omega}{dt}$$

Component of angular acceleration in the direction perpendicular to *ox*,

$$\alpha_c = \frac{rx'}{\delta t} = \frac{ox' \sin\delta\theta}{\delta t} = \frac{(\omega + \delta\omega)\sin\delta\theta}{\delta t} = \frac{\omega\sin\delta\theta + \delta\omega.\sin\delta\theta}{\delta t}$$

Since $\delta\theta$ in very small, therefore substituting $\sin\delta\theta = \delta\theta$, we have

$$\alpha_c = \frac{\omega.\delta\theta + \delta\omega.\delta\theta}{\delta t} = \frac{\omega.\delta\theta}{\delta t}$$

...(Neglecting $\delta\omega.\delta\theta$, being very small)

In the limit when $\delta t \to 0$,

$$\alpha_c = \underset{\delta t \to 0}{\text{Lt}} \frac{\omega.\delta\theta}{\delta t} = \omega \times \frac{d\theta}{dt} = \omega.\omega_P \qquad ...\left(\text{Substituting } \frac{d\theta}{dt} = \omega_P\right)$$

$\therefore$ Total angular acceleration of the disc

$$= \text{vector } xx' = \text{vector sum of } \alpha_t \text{ and } \alpha_c$$

$$= \frac{d\omega}{dt} + \omega \times \frac{d\theta}{dt} = \frac{d\omega}{dt} + \omega.\omega_P$$

where $d\theta/dt$ is the angular velocity of the axis of spin about a certain axis, which is perpendicular to the plane in which the axis of spin is going to rotate. This angular velocity of the axis of spin (*i.e.* $d\theta/dt$) is known as ***angular velocity of precession*** and is denoted by ω_P. The axis, about which the axis of spin is to turn, is known as ***axis of precession.*** The angular motion of the axis of spin about the axis of precession is known as ***precessional angular motion.***

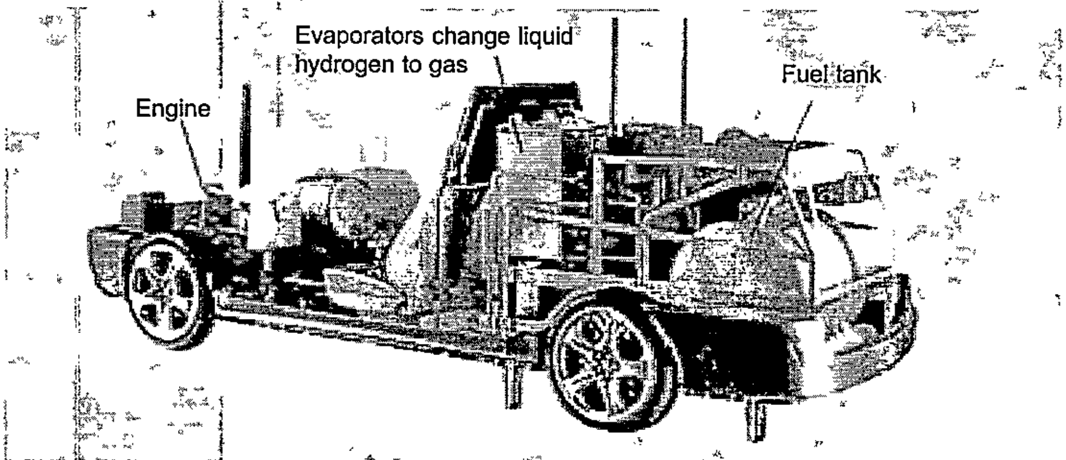
Notes: 1. The axis of precession is perpendicular to the plane in which the axis of spin is going to rotate.

2. If the angular velocity of the disc remains constant at all positions of the axis of spin, then $d\theta/dt$ is zero; and thus α_c is zero.

3. If the angular velocity of the disc changes the direction, but remains constant in magnitude, then angular acceleration of the disc is given by

$$\alpha_c = \omega.d\theta/dt = \omega.\omega_P$$

The angular acceleration, α_c is known as ***gyroscopic acceleration.***



This experimental car burns hydrogen fuel in an ordinary piston engine. Its exhaust gases cause no pollution, because they contain only water vapour.

Note : This picture is given as additional information and is not a direct example of the current chapter.

14.3. Gyroscopic Couple

Consider a disc spinning with an angular velocity ω rad/s about the axis of spin OX, in anticlockwise direction when seen from the front, as shown in Fig. 14.2 (*a*). Since the plane in which the disc is rotating is parallel to the plane YOZ, therefore it is called ***plane of spinning.*** The plane XOZ is a horizontal plane and the axis of spin rotates in a plane parallel to the horizontal plane about an axis OY. In other words, the axis of spin is said to be rotating or processing about an axis OY. In other words, the axis of spin is said to be rotating or processing about an axis OY (which is perpendicular to both the axes OX and OZ) at an angular velocity ω_P rap/s. This horizontal plane XOZ is called ***plane of precession*** and OY is the ***axis of precession.***

Let $\quad I$ = Mass moment of inertia of the disc about OX, and

$\quad \omega$ = Angular velocity of the disc.

∴ Angular momentum of the disc

$$= I.\omega$$

Since the angular momentum is a vector quantity, therefore it may be represented by the vector $\vec{ox}$, as shown in Fig. 14.2 (*b*). The axis of spin OX is also rotating anticlockwise when seen from the top about the axis OY. Let the axis OX is turned in the plane XOZ through a small angle $\delta\theta$ radians to the position OX', in time δt seconds. Assuming the angular velocity ω to be constant, the angular momentum will now be represented by vector ox'.

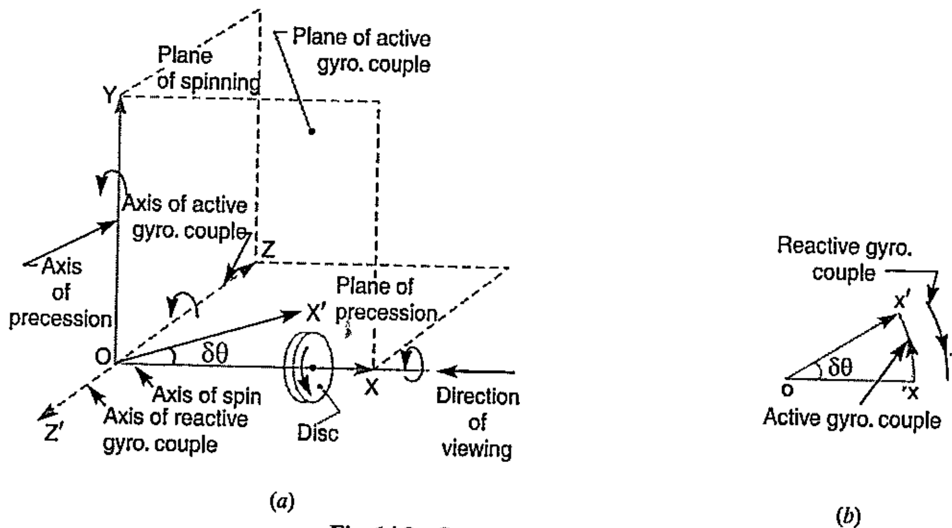


Fig. 14.2. Gyroscopic couple.

∴ Change in angular momentum

$$= \vec{ox'} - \vec{ox} = \vec{xx'} = \vec{ox}.\delta\theta \qquad \text{...(in the direction of } \vec{xx'}\text{)}$$

$$= I.\,\omega.\delta\theta$$

and rate of change of angular momentum

$$= I.\omega \times \frac{\delta\theta}{dt}$$

Since the rate of change of angular momentum will result by the application of a couple to the disc, therefore the couple applied to the disc causing precession,

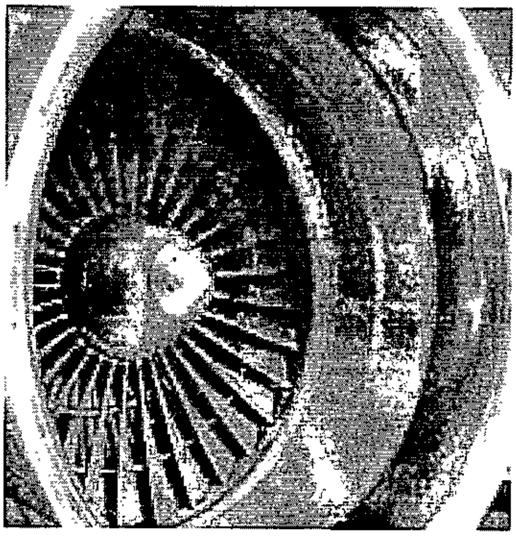
$$C = \underset{\delta t \to 0}{\text{Lt}}\ I.\omega \times \frac{\delta\theta}{\delta t} = I.\omega \times \frac{d\theta}{dt} = I.\omega.\omega_P \qquad \ldots\left(\because \frac{d\theta}{dt} = \omega_P\right)$$

where ω_p = Angular velocity of precession of the axis of spin or the speed of rotation of the axis of spin about the axis of precession OY.

In S.I. units, the units of C is N-m when I is in kg-m^2.

It may be noted that

1. The couple $I.\omega.\omega_p$, in the direction of the vector xx' (representing the change in angular momentum) is the ***active gyroscopic couple,*** which has to be applied over the disc when the axis of spin is made to rotate with angular velocity ω_p about the axis of precession. The vector xx' lies in the plane XOZ or the horizontal plane. In case of a very small displacement $\delta\theta$, the vector xx' will be perpendicular to the vertical plane XOY. Therefore the couple causing this change in the angular momentum will lie in the plane XOY. The vector xx', as shown in Fig. 14.2 (*b*), represents an anticlockwise couple in the plane XOY. Therefore, the plane XOY is called the ***plane of active gyroscopic couple*** and the axis OZ perpendicular to the plane XOY, about which the couple acts, is called the axis of active gyroscopic couple.


Above picture shows an aircraft propeller. These rotors play role in gyroscopic couple.

2. When the axis of spin itself moves with angular velocity ω_p, the disc is subjected to ***reactive couple*** whose magnitude is same (*i.e.* $I.\ \omega.\omega_p$) but opposite in direction to that of active couple. This reactive couple to which the disc is subjected when the axis of spin rotates about the axis of precession is known as ***reactive gyroscopic couple.*** The axis of the reactive gyroscopic couple is represented by OZ' in Fig. 14.2 (*a*).

3. The gyroscopic couple is usually applied through the bearings which support the shaft. The bearings will resist equal and opposite couple.

4. The gyroscopic principle is used in an instrument or toy known as ***gyroscope.*** The gyroscopes are installed in ships in order to minimize the rolling and pitching effects of waves. They are also used in aeroplanes, monorail cars, gyrocompasses etc.

Example 14.1. *A uniform disc of diameter 300 mm and of mass 5 kg is mounted on one end of an arm of length 600 mm. The other end of the arm is free to rotate in a universal bearing. If the disc rotates about the arm with a speed of 300 r.p.m. clockwise, looking from the front, with what speed will it precess about the vertical axis?*

Solution. Given: d = 300 mm or r = 150 mm = 0.15 m ; m = 5 kg ; l = 600 mm = 0.6 m ; N = 300 r.p.m. or $\omega = 2\pi \times 300/60 = 31.42$ rad/s

We know that the mass moment of inertia of the disc, about an axis through its centre of gravity and perpendicular to the plane of disc,

$$I = m.r^2/2 = 5(0.15)^2/2 = 0.056 \text{ kg-m}^2$$

and couple due to mass of disc,

$$C = m.g.l = 5 \times 9.81 \times 0.6 = 29.43 \text{ N-m}$$

Let ω_p = Speed of precession.

We know that couple (C),

$$29.43 = I.\omega.\omega_p = 0.056 \times 31.42 \times \omega_p = 1.76\ \omega_p$$

$$\therefore \quad \omega_p = 29.43/1.76 = 16.7 \text{ rad/s } \textbf{Ans.}$$

Example 14.2. *A uniform disc of 150 mm diameter has a mass of 5 kg. It is mounted centrally in bearings which maintain its axle in a horizontal plane. The disc spins about it axle with a constant speed of 1000 r.p.m. while the axle precesses uniformly about the vertical at 60 r.p.m. The directions of rotation are as shown in Fig. 14.3. If the distance between the bearings is 100 mm, find the resultant reaction at each bearing due to the mass and gyroscopic effects.*

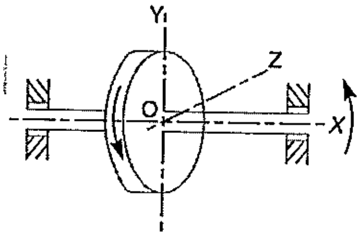


Fig. 14.3

Solution. Given: d = 150 mm or r = 75 mm = 0.075 m ; m = 5 kg ; N = 1000 r.p.m. or $\omega = 2\pi \times 1000/60 = 104.7$ rad/s (anticlockwise); N_P = 60 r.p.m. or $\omega_P = 2\pi \times 60/60 = 6.284$ rad/s (anticlockwise); x = 100 mm = 0.1 m

We know that mass moment of inertia of the disc, about an axis through its centre of gravity and perpendicular to the plane of disc,

$$I = m.r^2/2 = 5\,(0.075)^2/2 = 0.014 \text{ kg m}^2$$

∴ Gyroscopic couple acting on the disc,

$$C = I.\,\omega.\,\omega_P = 0.014 \times 104.7 \times 6.284 = 9.2 \text{ N-m}$$

The direction of the reactive gyroscopic couple is shown in Fig.14.4 (*b*). Let F be the force at each bearing due to the gyroscopic couple.

$$\therefore \quad F = C/x = 9.2/0.1 = 92 \text{ N}$$

The force F will act in opposite directions at the bearings as shown in Fig. 14.4 (*a*). Now let R_A and R_B be the reaction at the bearing A and B respectively due to the weight of the disc. Since the disc is mounted centrally in bearings, therefore,

$$R_A = R_B = 5/2 = 2.5 \text{ kg} = 2.5 \times 9.81 = 24.5 \text{ N}$$

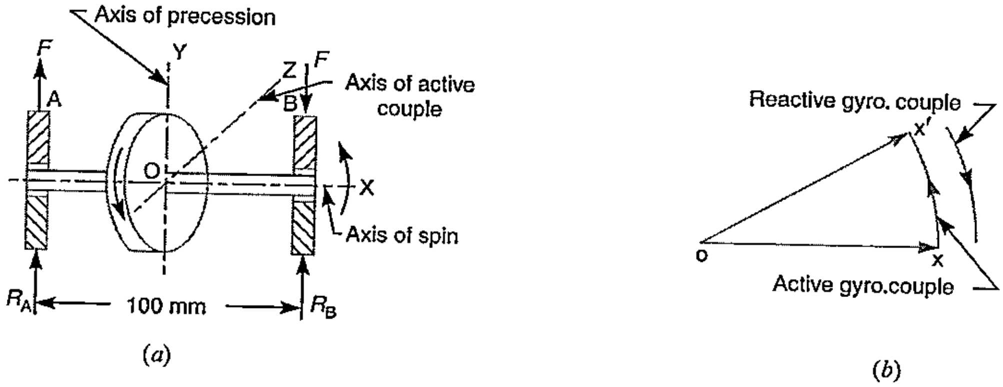


Fig. 14.4

Resultant reaction at each bearing

Let R_{A1} and R_{B1} = Resultant reaction at the bearings A and B respectively.

Since the reactive gyroscopic couple acts in clockwise direction when seen from the front, therefore its effect is to increase the reaction on the left hand side bearing (*i.e.* A) and to decrease the reaction on the right hand side bearing (*i.e.* B).

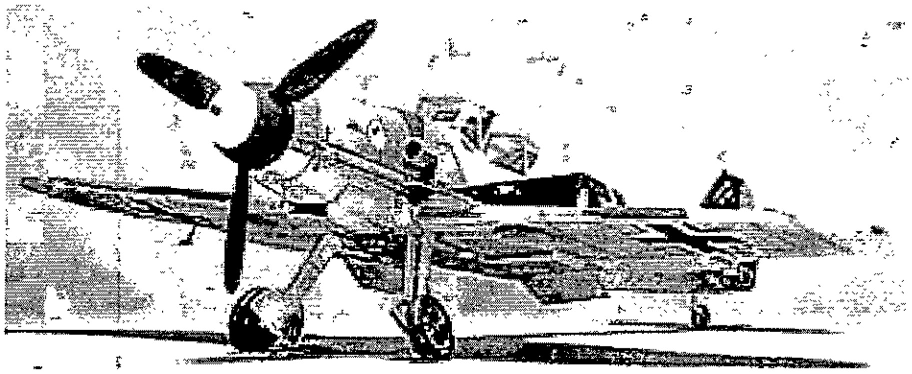
$$\therefore \quad R_{A1} = F + R_A = 92 + 24.5 = 116.5 \text{ N (upwards)} \quad \textbf{Ans.}$$

and

$$R_{B1} = F - R_B = 92 - 24.5 = 67.5 \text{ N (downwards)} \quad \textbf{Ans.}$$

14.4. Effect of the Gyroscopic Couple on an Aeroplane

The top and front view of an aeroplane are shown in Fig 14.5 (*a*). Let engine or propeller rotates in the clockwise direction when seen from the rear or tail end and the aeroplane takes a turn to the left.



Let ω = Angular velocity of the engine in rad/s,

m = Mass of the engine and the propeller in kg,

k = Its radius of gyration in metres,

I = Mass moment of inertia of the engine and the propeller in kg-m^2 $= m.k^2$,

v = Linear velocity of the aeroplane in m/s,

R = Radius of curvature in metres, and

ω_P = Angular velocity of precession $= \frac{v}{R}$ rad/s

$\therefore$ Gyroscopic couple acting on the aeroplane,

$$C = I.\omega.\omega_P$$

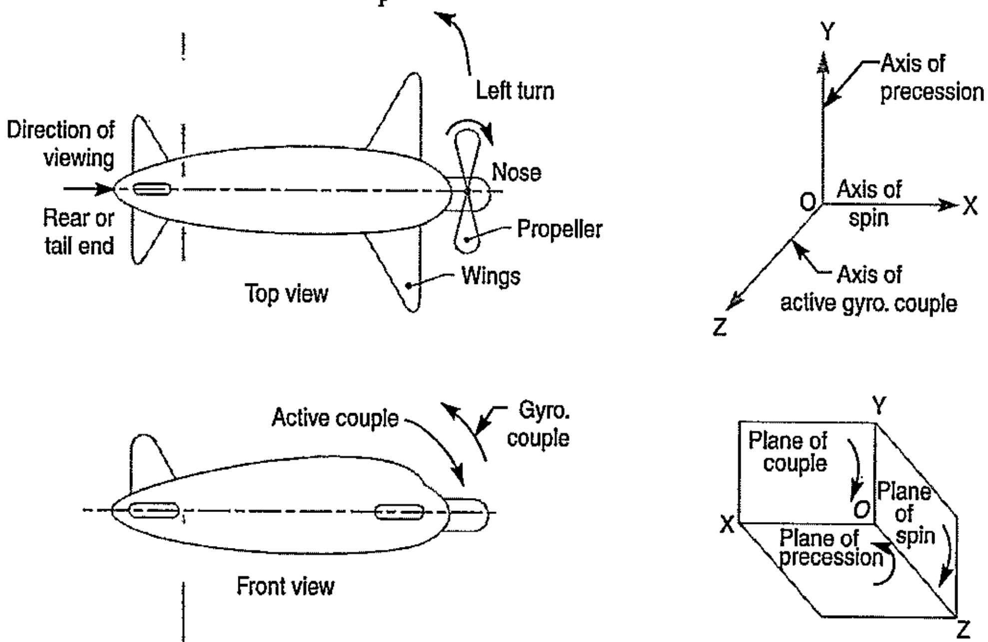


Fig. 14.5. Aeroplane taking a left turn.

Before taking the left turn, the angular momentum vector is represented by ox. When it takes left turn, the active gyroscopic couple will change the direction of the angular momentum vector from ox to ox' as shown in Fig. 14.6 (*a*). The vector xx', in the limit, represents the change of angular momentum or the active gyroscopic couple and is perpendicular to ox. Thus the plane of active gyroscopic couple XOY will be perpendicular to xx', *i.e.* vertical in this case, as shown in Fig 14.5 (*b*). By applying right hand screw rule to vector xx', we find that the direction of active gyroscopic couple is clockwise as shown in the front view of Fig. 14.5 (*a*). In other words, for left hand turning, the active gyroscopic couple on the aeroplane in the axis OZ will be clockwise as shown in Fig. 14.5 (*b*). The reactive gyroscopic couple (equal in magnitude of active gyroscopic couple) will act in the opposite direction (*i.e.* in the anticlockwise direction) and the effect of this couple is, therefore, to **raise the nose** and **dip the tail** of the aeroplane.

(*a*) Aeroplane taking left turn. (*b*) Aeroplane taking right turn.

Fig. 14.6. Effect of gyroscopic couple on an aeroplane.

Notes : **1.** When the aeroplane takes a **right turn** under similar conditions as discussed above, the effect of the reactive gyroscopic couple will be to **dip the nose** and **raise the tail** of the aeroplane.

2. When the engine or propeller rotates in **anticlockwise direction** when viewed from the rear or tail end and the aeroplane takes a **left turn**, then the effect of reactive gyroscopic couple will be to **dip the nose** and **raise the tail** of the aeroplane.

3. When the aeroplane takes a **right turn** under similar conditions as mentioned in note 2 above, the effect of reactive gyroscopic couple will be to **raise the nose** and **dip the tail** of the aeroplane.

4. When the engine or propeller rotates in **clockwise direction** when viewed from the front and the aeroplane takes a left turn, then the effect of reactive gyroscopic couple will be to **raise the tail** and **dip the nose** of the aeroplane.

5. When the aeroplane takes a **right turn** under similar conditions as mentioned in note 4-above, the effect of reactive gyroscopic couple will be to **raise the nose** and **dip the tail** of the aeroplane.

Example 14.3. *An aeroplane makes a complete half circle of 50 metres radius, towards left, when flying at 200 km per hr. The rotary engine and the propeller of the plane has a mass of 400 kg and a radius of gyration of 0.3 m. The engine rotates at 2400 r.p.m. clockwise when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it.*

Solution. Given : $R = 50$ m ; $v = 200$ km/hr $= 55.6$ m/s ; $m = 400$ kg ; $k = 0.3$ m ; $N = 2400$ r.p.m. or $\omega = 2\pi \times 2400/60 = 251$ rad/s

We know that mass moment of inertia of the engine and the propeller,

$$I = m.k^2 = 400(0.3)^2 = 36 \text{ kg-m}^2$$

and angular velocity of precession,

$$\omega_p = v/R = 55.6/50 = 1.11 \text{ rad/s}$$

We know that gyroscopic couple acting on the aircraft,

$$C = I.\ \omega.\ \omega_p = 36 \times 251.4 \times 1.11 = 100\ 46 \text{ N-m}$$

$$= 10.046 \text{ kN-m } \textbf{Ans.}$$

We have discussed in Art. 14.4 that when the aeroplane turns towards left, the effect of the gyroscopic couple is to lift the nose upwards and tail downwards. **Ans.**

14.5. Terms Used in a Naval Ship

The top and front views of a naval ship are shown in Fig 14.7. The fore end of the ship is called *bow* and the rear end is known as *stern* or *aft.* The left hand and right hand sides of the ship, when viewed from the stern are called *port* and *star-board* respectively. We shall now discuss the effect of gyroscopic couple on the naval ship in the following three cases:

1. Steering, 2. Pitching, and 3. Rolling.

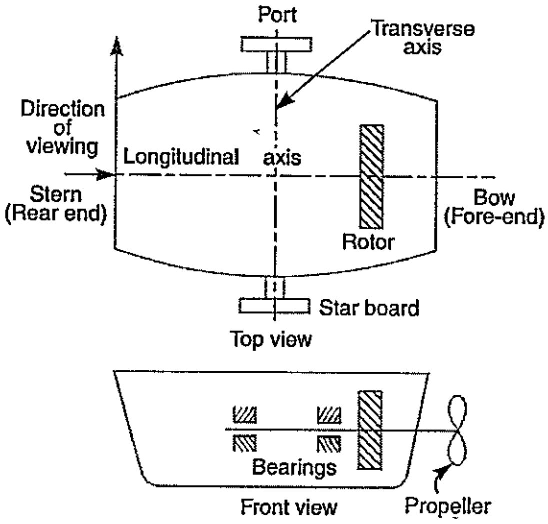


Fig. 14.7. Terms used in a naval ship.

14.6. Effect of Gyroscopic Couple on a Naval Ship during Steering

Steering is the turning of a complete ship in a curve towards left or right, while it moves forward. Consider the ship taking a left turn, and rotor rotates in the clockwise direction when viewed from the stern, as shown in Fig. 14.8. The effect of gyroscopic couple on a naval ship during steering taking left or right turn may be obtained in the similar way as for an aeroplane as discussed in Art.14.4.

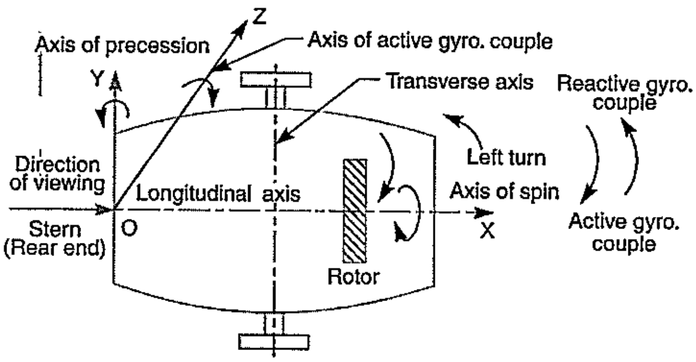


Fig. 14.8. Naval ship taking a left turn.

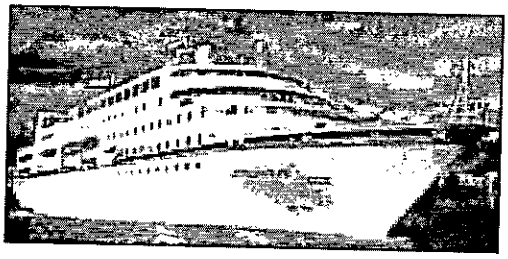
When the rotor of the ship rotates in the clockwise direction when viewed from the stern, it will have its angular momentum vector in the direction ox as shown in Fig. 14.9 (*a*). As the ship steers to the left, the active gyroscopic couple will change the angular momentum vector from ox to ox'. The vector xx' now represents the active gyroscopic couple and is perpendicular to ox. Thus the plane of active gyroscopic couple is perpendicular to xx' and its direction in the axis OZ for left hand turn is clockwise as shown in Fig. 14.8. The reactive gyroscopic couple of the same magnitude will act in the opposite direction (*i.e.* in anticlockwise direction). The ***effect of this reactive gyroscopic couple is to raise the bow and lower the stern.***

Notes: 1. When the ship steers to the right under similar conditions as discussed above, the effect of the reactive gyroscopic couple, as shown in Fig. 14.9 (*b*), will be to **raise the stern and lower the bow.**

(*a*) Streeing to the left (*b*) Streeing to the right

Fig. 14.9. Effect of gyroscopic couple on a naval ship during steering.

2. When the rotor rates in the anticlockwise direction, when viewed from the stern and the ship is steering to the left, then the effect of reactive gyroscopic couple will be to **lower the bow and raise the stern.**



3. When the ship is steering to the right under similar conditions as discussed in note 2 above, then the effect of reactive gyroscopic couple will be to **raise the bow and lower the stern.**

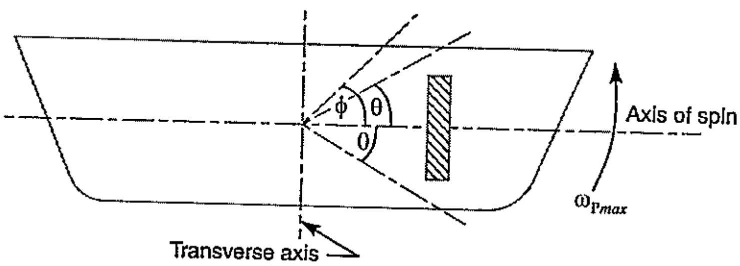
4. When the rotor rotates in the clockwise direction when viewed from the bow or fore end and the ship is steering to the left, then the effect of reactive gyroscopic couple will be to **raise the stern and lower the bow.**

5. When the ship is steering to the right under similar conditions as discussed in note 4 above, then the effect of reactive gyroscopic couple will be to **raise the bow and lower the stern.**

6. The effect of the reactive gyroscopic couple on a boat propelled by a turbine taking left or right turn is similar as discussed above.

14.7. Effect of Gyroscopic Couple on a Naval Ship during Pitching

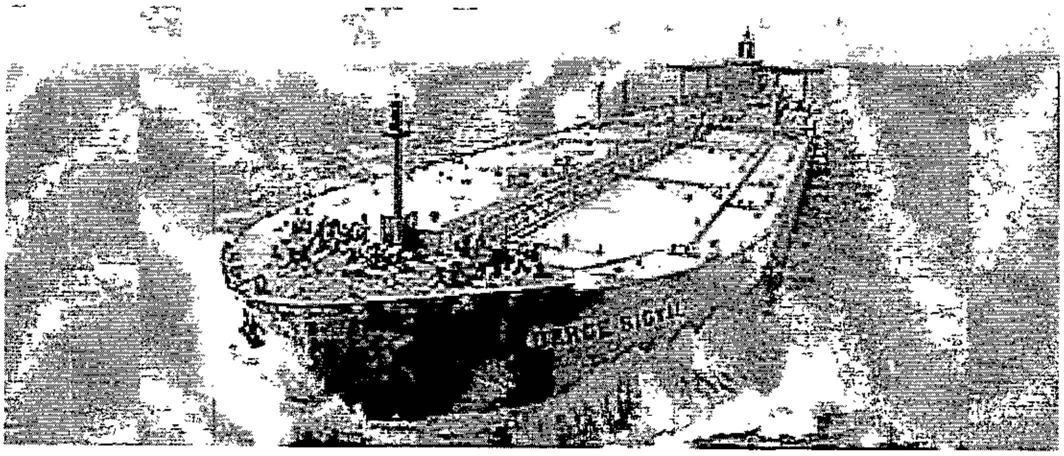
Pitching is the movement of a complete ship up and down in a vertical plane about transverse axis, as shown in Fig. 14.10 (*a*). In this case, the transverse axis is the axis of precession. The pitching of the ship is assumed to take place with simple harmonic motion *i.e.* the motion of the axis of spin about transverse axis is simple harmonic.



(*a*) Pitching of a naval ship

(*b*) Pitching upward (*c*) Pitching downward

Fig. 14.10. Effect of gyroscopic couple on a naval ship during pitching.



Gryroscopic couple plays its role during ship's turning and pitching.

∴ Angular displacement of the axis of spin from mean position after time t seconds,

$$\theta = \phi \sin \omega_1 . t$$

where ϕ = Amplitude of swing *i.e.* maximum angle turned from the mean position in radians, and

ω_1 = Angular velocity of S.H.M.

$$= \frac{2\pi}{\text{Time period of S.H.M. in seconds}} = \frac{2\pi}{t_p} \text{ rad/s}$$

Angular velocity of precession,

$$\omega_P = \frac{d\theta}{dt} = \frac{d}{dt}(\phi \sin \omega_1 . t) = \phi\, \omega_1 \cos \omega_1\, t$$

The angular velocity of precession will be maximum, if $\cos \omega_1 . t = 1$.

∴ Maximum angular velocity of precession,

$$\omega_{Pmax} = \phi . \omega_1 = \phi \times 2\pi / t_p \qquad \text{...(Substituting } \cos \omega_1 . t = 1)$$

Let I = Moment of inertia of the rotor in kg-m^2, and

ω = Angular velocity of the rotor in rad/s.

∴ Mamimum gyroscopic couple,

$$C_{max} = I.\ \omega.\ \omega_{Pmax}$$

When the pitching is upward, the effect of the reactive gyroscopic couple, as shown in Fig. 14.10 (*b*), will try to move the ship toward star-board. On the other hand, if the pitching is downward, the effect of the reactive gyroscopic couple, as shown in Fig. 14.10 (*c*), is to turn the ship towards port side.

Notes : 1. The effect of the gyroscopic couple is always given on specific position of the axis of spin *i.e.* whether it is pitching downwards or upwards.

2. The pitching of a ship produces forces on the bearings which act horizontally and perpendicular to the motion of the ship.

3. The maximum gyroscopic couple tends to shear the holding-down bolts.

4. The angular acceleration during pitching,

$$\alpha = \frac{d^2\theta}{dt^2} = -\phi(\omega_1)^2 \sin \omega_1 t \qquad \ldots\left(\text{Differentiating } \frac{d\theta}{dt} \text{ with respect to } t\right)$$

The angular acceleration is maximum, if $\sin \omega_1 t = 1$.

∴ Maximum angular acceleration during pitching,

$$\alpha_{max} = (\omega_1)^2$$

14.8. Effect of Gyroscopic Couple on a Naval Ship during Rolling

We know that, for the effect of gyroscopic couple to occur, the axis of precession should always be perpendicular to the axis of spin. If, however, the axis of precession becomes parallel to the axis of spin, there will be no effect of the gyroscopic couple acting on the body of the ship.

In case of rolling of a ship, the axis of precession (*i.e.* longitudinal axis) is always parallel to the axis of spin for all positions. Hence, there is no effect of the gyroscopic couple acting on the body of a ship.

Example 14.4. *The turbine rotor of a ship has a mass of 8 tonnes and a radius of gyration 0.6 m. It rotates at 1800 r.p.m. clockwise, when looking from the stern. Determine the gyroscopic couple, if the ship travels at 100 km/hr and steer to the left in a curve of 75 m radius.*

Solution. Given: $m = 8$ t $= 8000$ kg ; $k = 0.6$ m ; $N = 1800$ r.p.m. or $\omega = 2\pi \times 1800/60 = 188.5$ rad/s ; $v = 100$ km/h $= 27.8$ m/s ; $R = 75$ m

We know that mass moment of inertia of the rotor,

$$I = m.k^2 = 8000\,(0.6)^2 = 2880 \text{ kg-m}^2$$

and angular velocity of precession,

$$\omega_P = v / R = 27.8 / 75 = 0.37 \text{ rad/s}$$

We know that gyroscopic couple,

$$C = I.\omega.\omega_P = 2880 \times 188.5 \times 0.37 = 200\,866 \text{ N-m}$$

$$= 200.866 \text{ kN-m} \quad \textbf{Ans.}$$

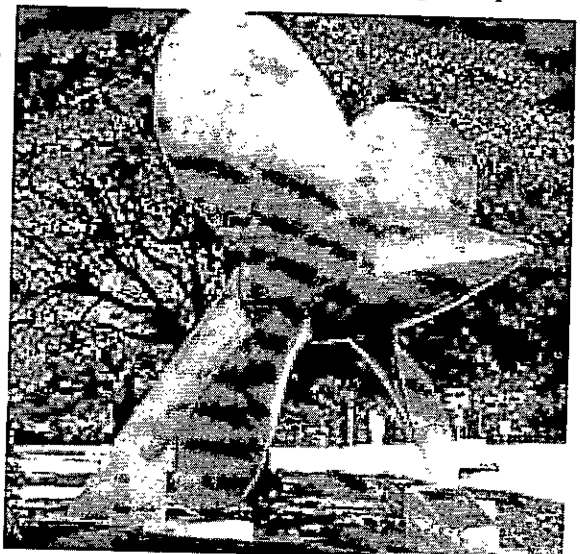
We have discussed in Art. 14.6, that when the rotor rotates in clockwise direction when looking from the stern and the ship steers to the left, the effect of the reactive gyroscopic couple is to raise the bow and lower the stern.

Example 14.5. *The heavy turbine rotor of a sea vessel rotates at 1500 r.p.m. clockwise looking from the stern, its mass being 750 kg. The vessel pitches with an angular velocity of 1 rad/s. Determine the gyroscopic couple transmitted to the hull when bow is rising, if the radius of gyration for the rotor is 250 mm. Also show in what direction the couple acts on the hull?*

Solution. Given: $N = 1500$ r.p.m. or $\omega = 2\pi \times 1500/60 = 157.1$ rad/s; $m = 750$ kg; $\omega_P = 1$ rad/s; $k = 250$ mm $= 0.25$ m

We know that mass moment of inertia of the rotor,

$$I = m.k^2 = 750\,(0.25)^2 = 46.875 \text{ kg-m}^2$$



Ship's propeller shown as a separate part. A ship's propeller is located at backside (stern) of the ship below the water surface.

$\therefore$ Gyroscopic couple transmitted to the hull (*i.e.* body of the sea vessel),

$$C = I.\omega.\omega_P = 46.875 \times 157.1 \times 1 = 7364 \text{ N-m} = 7.364 \text{ kN-m}$$

We have discussed in Art. 14.7, that when the bow is rising *i.e.* when the pitching is upward, the reactive gyroscopic couple acts in the clockwise direction which moves the sea vessel towards star-board.

Example 14.6. *The turbine rotor of a ship has a mass of 3500 kg. It has a radius of gyration of 0.45 m and a speed of 3000 r.p.m. clockwise when looking from stern. Determine the gyroscopic couple and its effect upon the ship:*

1. when the ship is steering to the left on a curve of 100 m radius at a speed of 36 km/h.

2. when the ship is pitching in a simple harmonic motion, the bow falling with its maximum velocity. The period of pitching is 40 seconds and the total angular displacement between the two extreme positions of pitching is 12 degrees.

Solution. Given : $m = 3500$ kg ; $k = 0.45$ m; $N = 3000$ r.p.m. or $\omega = 2\pi \times 3000/60 = 314.2$ rad/s

1. *When the ship is steering to the left*

Given: $R = 100$ m ; $v = $ km/h $= 10$ m/s

We know that mass moment of inertia of the rotor,

$$I = m.k^2 = 3500\,(0.45)^2 = 708.75 \text{ kg-m}^2$$

and angular velocity of precession,

$$\omega_P = v/R = 10/100 = 0.1 \text{ rad/s}$$

$\therefore$ Gyroscopic couple,

$$C = I.\omega.\omega_P = 708.75 \times 314.2 \times 0.1 = 22\,270 \text{ N-m}$$
$$= 22.27 \text{ kN-m } \textbf{Ans.}$$

We have discussed in Art. 14.6, that when the rotor rotates clockwise when looking from the stern and the ship takes a left turn, the effect of the reactive gyroscopic couple is to raise the bow and lower the stern. **Ans.**

2. *When the ship is pitching with the bow falling*

Given: $t_p = 40$ s

Since the total angular displacement between the two extreme positions of pitching is 12° (*i.e.* $2\phi = 12°$), therefore amplitude of swing,

$$\phi = 12/2 = 6° = 6 \times \pi/180 = 0.105 \text{ rad}$$

and angular velocity of the simple harmonic motion,

$$\omega_1 = 2\pi / t_p = 2\pi / 40 = 0.157 \text{ rad/s}$$

We know that maximum angular velocity of precession,

$$\omega_P = \phi.\omega_1 = 0.105 \times 0.157 = 0.0165 \text{ rad/s}$$

$\therefore$ Gyroscopic couple,

$$C = I.\omega.\omega_P = 708.75 \times 314.2 \times 0.0165 = 3675 \text{ N-m}$$
$$= 3.675 \text{ kN-m } \textbf{Ans.}$$

We have discussed in Art. 14.7, that when the bow is falling (*i.e.* when the pitching is downward), the effect of the reactive gyroscopic couple is to move the ship towards port side. **Ans.**

Example 14.7. *The mass of the turbine rotor of a ship is 20 tonnes and has a radius of gyration of 0.60 m. Its speed is 2000 r.p.m. The ship pitches 6° above and 6° below the horizontal position. A complete oscillation takes 30 seconds and the motion is simple harmonic. Determine the following:*

1. Maximum gyroscopic couple, 2. Maximum angular acceleration of the ship during pitching, and 3. The direction in which the bow will tend to turn when rising, if the rotation of the rotor is clockwise when looking from the left.

Solution. Given : $m = 20$ t $= 20\ 000$ kg ; $k = 0.6$ m ; $N = 2000$ r.p.m. or $\omega = 2\pi \times 2000/60 = 209.5$ rad/s; $\phi = 6° = 6 \times \pi/180 = 0.105$ rad ; $t_p = 30$ s

1. ***Maximum gyroscopic couple***

We know that mass moment of inertia of the rotor,

$$I = m.k^2 = 20\ 000\ (0.6)^2 = 7200 \text{ kg-m}^2$$

and angular velocity of the simple harmonic motion,

$$\omega_1 = 2\pi / t_p = 2\pi/30 = 0.21 \text{ rad/s}$$

∴ Maximum angular velocity of precession,

$$\omega_{Pmax} = \phi.\omega_1 = 0.105 \times 0.21 = 0.022 \text{ rad/s}$$

We know that maximum gyroscopic couple,

$$C_{max} = I.\omega.\omega_{Pmax} = 7200 \times 209.5 \times 0.022 = 33\ 185 \text{ N-m}$$

$$= 33.185 \text{ kN-m} \quad \textbf{Ans.}$$

2. ***Maximum angular acceleration during pitching***

We know that maximum angular acceleration during pitching

$$= \phi(\omega_1)^2 = 0.105\ (0.21)^2 = 0.0046 \text{ rad/s}^2$$

3. ***Direction in which the bow will tend to turn when rising***

We have discussed in Art. 14.7, that when the rotation of the rotor is clockwise when looking from the left (*i.e.* rear end or stern) and when the bow is rising (*i.e.* pitching is upward), then the reactive gyroscopic couple acts in the clockwise direction which tends to turn the bow towards right (*i.e.* towards star-board). **Ans.**

Example 14.8. *A ship propelled by a turbine rotor which has a mass of 5 tonnes and a speed of 2100 r.p.m. The rotor has a radius of gyration of 0.5 m and rotates in a clockwise direction when viewed from the stern. Find the gyroscopic effects in the following conditions:*

1. The ship sails at a speed of 30 km/h and steers to the left in a curve having 60 m radius.

2. The ship pitches 6 degree above and 6 degree below the horizontal position. The bow is descending with its maximum velocity. The motion due to pitching is simple harmonic and the periodic time is 20 seconds.

3. The ship rolls and at a certain instant it has an angular velocity of 0.03 rad/s clockwise when viewed from stern.

Determine also the maximum angular acceleration during pitching. Explain how the direction of motion due to gyroscopic effect is determined in each case.

Solution. Given : $m = 5$ t $= 5000$ kg ; $N = 2100$ r.p.m. or $\omega = 2\pi \times 2100/60 = 220$ rad/s ; $k = 0.5$ m

1. ***When the ship steers to the left***

Given: $v = 30 \text{ km / h} = 8.33 \text{ m / s}$; $R = 60$ m

We know that angular velocity of precession,

$$\omega_P = v/R = 8.33/60 = 0.14 \text{ rad/s}$$

and mass moment of inertia of the rotor,

$$I = m.k^2 = 5000(0.5)^2 = 1250 \text{ kg-m}^2$$

∴ Gyroscopic couple,

$$C = I.\omega.\omega_P = 1250 \times 220 \times 0.14 = 38\ 500 \text{ N-m} = 38.5 \text{ kN-m}$$

We have discussed in Art. 14.6, that when the rotor in a clockwise direction when viewed from the stern and the ship steers to the left, the effect of reactive gyroscopic couple is to raise the bow and lower the stern. **Ans.**

2. *When the ship pitches with the bow descending*

Given: $\phi = 6° = 6 \times \pi/180 = 0.105$ rad/s ; $t_p = 20$ s

We know that angular velocity of simple harmonic motion,

$$\omega_1 = 2\pi / t_p = 2\pi / 20 = 0.3142 \text{ rad/s}$$

and maximum angular velocity of precession,

$$\omega_{Pmax} = \phi.\omega_1 = 0.105 \times 0.3142 = 0.033 \text{ rad/s}$$

∴ Maximum gyroscopic couple,

$$C_{max} = I.\omega.\omega_{Pmax} = 1250 \times 220 \times 0.033 = 9075 \text{ N-m}$$

Since the ship is pitching with the bow descending, therefore the effect of this maximum gyroscopic couple is to turn the ship towards port side. **Ans.**

3. *When the ship rolls*

Since the ship rolls at an angular velocity of 0.03 rad / s, therefore angular velocity of precession when the ship rolls,

$$\omega_P = 0.03 \text{ rad/s}$$

∴ Gyroscopic couple,

$$C = I.\omega.\omega_P = 1250 \times 220 \times 0.03 = 8250 \text{ N-m}$$

In case of rolling of a ship, the axis of precession is always parallel to the axis of spin for all positions, therefore there is no effect of gyroscopic couple. **Ans.**

Maximum angular acceleration during pitching

We know that maximum angular acceleration during pitching.

$$\alpha_{max} = \phi\,(\omega_1)^2 = 0.105\,(0.3142)^2 = 0.01 \text{ rad/s}^2 \textbf{ Ans.}$$

Example 14.9. *The turbine rotor of a ship has a mass of 2000 kg and rotates at a speed of 3000 r.p.m. clockwise when looking from a stern. The radius of gyration of the rotor is 0.5 m.*

Determine the gyroscopic couple and its effects upon the ship when the ship is steering to the right in a curve of 100 m radius at a speed of 16.1 knots (1 knot = 1855 m/hr).

Calculate also the torque and its effects when the ship is pitching in simple harmonic motion, the bow falling with its maximum velocity. The period of pitching is 50 seconds and the total angular displacement between the two extreme positions of pitching is 12°. Find the maximum acceleration during pitching motion.

Solution. Given : $m = 2000$ kg ; $N = 3000$ r.p.m. or $\omega = 2\pi \times 3000/60 = 314.2$ rad/s ; $k = 0.5$ m ; $R = 100$ m ; $v = 16.1$ knots $= 16.1 \times 1855 / 3600 = 8.3$ m/s

Gyroscopic couple

We know that mass moment of inertia of the rotor,

$$I = m.k^2 = 2000\,(0.5)^2 = 500 \text{ kg-m}^2$$

and angular velocity of precession,

$$\omega_P = v/R = 8.3/100 = 0.083 \text{ rad/s}$$

∴ Gyroscopic couple,

$$C = I.\omega.\omega_P = 500 \times 314.2 \times 0.083 = 13\ 040 \text{ N-m} = 13.04 \text{ kN-m}$$

We have discussed in Art. 14.6, that when the rotor rotates clockwise when looking from a stern and the ship steers to the right, the effect of the reactive gyroscopic couple is to raise the stern and lower the bow. **Ans.**

Torque during pitching

Given : t_p =50 s ; $2\,\phi = 12°$ or $\phi = 6° \times \pi/180 = 0.105$ rad

We know that angular velocity of simple harmonic motion,

$$\omega_1 = 2\pi / t_p = 2\pi / 50 = 0.1257 \text{ rad/s}$$

and maximum angular velocity of precession,

$$\omega_{Pmax} = \phi.\omega_1 = 0.105 \times 0.1257 = 0.0132 \text{ rad/s}$$

∴ Torque or maximum gyroscopic couple during pitching,

$$C_{max} = I.\omega.\omega_{P\,max} = 500 \times 314.2 \times 0.0132 = 2074 \text{ N-m } \textbf{Ans.}$$

We have discussed in Art. 14.7, that when the pitching is downwards, the effect of the reactive gyroscopic couple is to turn the ship towards port side.

Maximum acceleration during pitching

We know that maximum acceleration during pitching

$$\alpha_{max} = \phi\,(\omega_1)^2 = 0.105\,(0.1257)^2 = 0.00166 \text{ rad/s}^2 \textbf{ Ans.}$$

14.9. Stability of a Four Wheel Drive Moving in a Curved Path

Consider the four wheels *A*, *B*, *C* and *D* of an automobile locomotive taking a turn towards left as shown in Fig. 14.11. The wheels *A* and *C* are inner wheels, whereas *B* and *D* are outer wheels. The centre of gravity (*C.G.*) of the vehicle lies vertically above the road surface.

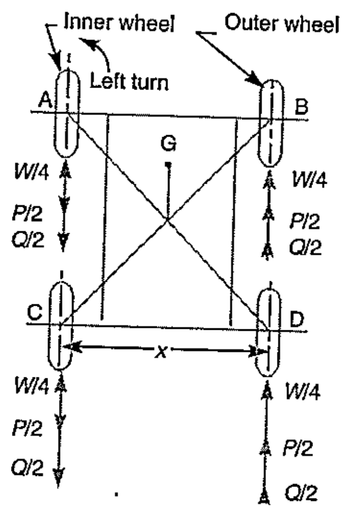


Fig. 14.11. Four wheel drive moving in a curved path.

Let m = Mass of the vehicle in kg,

W = Weight of the vehicle in newtons = $m.g$,

r_W = Radius of the wheels in metres,

R = Radius of curvature in metres ($R > r_W$),

h = Distance of centre of gravity, vertically above the road surface in metres,

x = Width of track in metres,

I_W = Mass moment of inertia of one of the wheels in kg-m^2,

ω_W = Angular velocity of the wheels or velocity of spin in rad/s,

I_E = Mass moment of inertia of the rotating parts of the engine in kg-m^2,

ω_E = Angular velocity of the rotating parts of the engine in rad/s,

G = Gear ratio = ω_E / ω_W,

v = Linear velocity of the vehicle in m/s = $\omega_W . r_W$

A little considereation will show, that the weight of the vehicle (W) will be equally distributed over the four wheels which will act downwards. The reaction between each wheel and the road surface of the same magnitude will act upwards. Therefore

Road reaction over each wheel $= W/4 = m.g/4$ newtons

Let us now consider the effect of the gyroscopic couple and centrifugal couple on the vehicle.

1. *Effect of the gyroscopic couple*

Since the vehicle takes a turn towards left due to the precession and other rotating parts, therefore a gyroscopic couple will act.

We know that velocity of precession,

$$\omega_P = v/R$$

∴ Gyroscopic couple due to 4 wheels,

$$C_W = 4\,I_W.\omega_W.\omega_P$$

and gyroscopic couple due to the rotating parts of the engine,

$$C_E = I_E.\omega_E.\omega_P = I_E.G.\omega_W.\omega_P \qquad \ldots (\because G = \omega_E/\omega_W)$$

∴ Net gyroscopic couple,

$$C = C_W \pm C_E = 4\,I_W.\omega_W.\omega_P \pm I_E.G.\omega_W.\omega_P$$
$$= \omega_W.\omega_P\,(4\,I_W \pm G.I_E)$$

The *positive* sign is used when the wheels and rotating parts of the engine rotate in the same direction. If the rotating parts of the engine revolves in opposite direction, then *negative* sign is used.

Due to the gyroscopic couple, vertical reaction on the road surface will be produced. The reaction will be vertically upwards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at the two outer or inner wheels be P newtons. Then

$$P \times x = C \quad \text{or} \quad P = C/x$$

∴ Vertical reaction at each of the outer or inner wheels,

$$P/2 = C/2x$$

Note: We have discussed above that when rotating parts of the engine rotate in opposite directions, then –ve sign is used, *i.e.* net gyroscopic couple,

$$C = C_W - C_E$$

When $C_E > C_W$, then C will be –ve. Thus the reaction will be vertically downwards on the outer wheels and vertically upwards on the inner wheels.

2. *Effect of the centrifugal couple*

Since the vehicle moves along a curved path, therefore centrifugal force will act outwardly at the centre of gravity of the vehicle. The effect of this centrifugal force is also to overturn the vehicle. We know that centrifugal force,

$$F_C = \frac{m \times v^2}{R}$$

∴ The couple tending to overturn the vehicle or overturning couple,

$$C_O = F_C \times h = \frac{m.v^2}{R} \times h$$

This overturning couple is balanced by vertical reactions, which are vertically upwards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at the two outer or inner wheels be Q. Then

$$Q \times x = C_O \quad \text{or} \quad Q = \frac{C_O}{x} = \frac{m.v^2.h}{R.x}$$

∴ Vertical reaction at each of the outer or inner wheels,

$$\frac{Q}{2} = \frac{m.v^2.h}{2R.x}$$

∴ Total vertical reaction at each of the outer wheel,

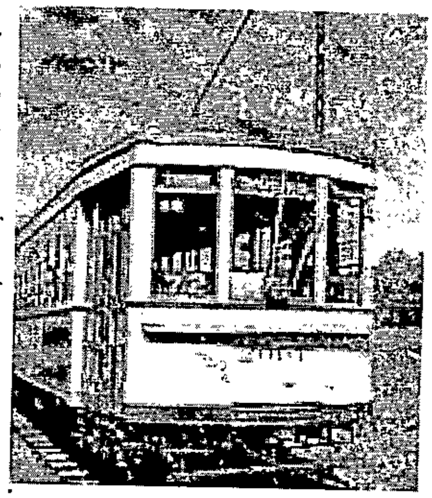
$$P_O = \frac{W}{4} + \frac{P}{2} + \frac{Q}{2}$$

and total vertical reaction at each of the inner wheel,

$$P_I = \frac{W}{4} - \frac{P}{2} - \frac{Q}{2}$$

A little consideration will show that when the vehicle is running at high speeds, P_I may be zero or even negative. This will cause the inner wheels to leave the ground thus tending to overturn the automobile. In order to have the contact between the inner wheels and the ground, the sum of $P/2$ and $Q/2$ must be less than $W/4$.

Example 14.10. *A four-wheeled trolley car of mass 2500 kg runs on rails, which are 1.5 m apart and travels around a curve of 30 m radius at 24 km / hr. The rails are at the same level. Each wheel of the trolley is 0.75 m in diameter and each of the two axles is driven by a motor running in a direction opposite to that of the wheels at a speed of five times the speed of rotation of the wheels. The moment of inertia of each axle with gear and wheels is 18 kg-m². Each motor with shaft and gear pinion has a moment of inertia of 12 kg-m². The centre of gravity of the car is 0.9 m above the rail level. Determine the vertical force exerted by each wheel on the rails taking into consideration the centrifugal and gyroscopic effects. State the centrifugal and gyroscopic effects on the trolley.*



Solution. Given : $m = 2500$ kg ; $x = 1.5$ m ; $R = 30$ m ; v = 24 km/h = 6.67 m/s ; d_W = 0.75 m or r_W = 0.375 m ; $G = \omega_E/\omega_W = 5$; I_W = 18 kg-m² ; I_E = 12 kg-m² ; h = 0.9 m

The weight of the trolley ($W = m.g$) will be equally distributed over the four wheels, which will act downwards. The reaction between the wheels and the road surface of the same magnitude will act upwards.

∴ Road reaction over each wheel $= W/4 = m.g/4 = 2500 \times 9.81/4 = 6131.25$ N

We know that angular velocity of the wheels,

$$\omega_W = v/r_W = 6.67/0.375 = 17.8 \text{ rad/s}$$

and angular velocity of precession, $\omega_P = v/R = 6.67/30 = 0.22$ rad/s

∴ Gyroscopic couple due to one pair of wheels and axle,

$$C_W = 2\,I_W.\omega_W.\omega_P = 2 \times 18 \times 17.8 \times 0.22 = 141 \text{ N-m}$$

and gyroscopic couple due to the rotating parts of the motor and gears,

$$C_E = 2\,I_E.\omega_E.\omega_P = 2\,I_E.G.\omega_W.\omega_P \qquad \text{... } (\because\ \omega_E = G.\,\omega_W)$$

$$= 2 \times 12 \times 5 \times 17.8 \times 0.22 = 470 \text{ N-m}$$

∴ Net gyroscopic couple, $C = C_W - C_E = 141 - 470 = -329$ N-m

... (–ve sign is used due to opposite direction of motor)

Due to this net gyroscopic couple, the vertical reaction on the rails will be produced. Since C_E is greater than C_W, therefore the reaction will be vertically downwards on the outer wheels and vertically upwards on the inner wheels. Let the magnitude of this reaction at each of the outer or inner wheel be $P/2$ newton.

∴ $P/2 = C/2x = 329 / 2 \times 1.5 = 109.7$ N

We know that centrifugal force, $F_C = m.v^2/R = 2500\,(6.67)^2/30 = 3707$ N

∴ Overturning couple, $C_O = F_C \times h = 3707 \times 0.9 = 3336.3$ N-m

This overturning couple is balanced by the vertical reactions which are vertically upwards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at each of the outer or inner wheels be $Q/2$ newton.

∴ $Q/2 = C_O / 2x = 3336.3 / 2 \times 1.5 = 1112.1$ N

We know that vertical force exerted on each outer wheel,

$$P_O = \frac{W}{4} - \frac{P}{2} + \frac{Q}{2} = 6131.25 - 109.7 + 1112.1 = 7142.65 \text{ N} \textbf{ Ans.}$$

and vertical force exerted on each inner wheel,

$$P_I = \frac{W}{4} + \frac{P}{2} - \frac{Q}{2} = 6131.25 + 109.7 - 1112.1 = 5128.85 \text{ N} \textbf{ Ans.}$$

Example 14.11. *A rear engine automobile is travelling along a track of 100 metres mean radius. Each of the four road wheels has a moment of inertia of 2.5 kg-m² and an effective diameter of 0.6 m. The rotating parts of the engine have a moment of inertia of 1.2 kg-m². The engine axis is parallel to the rear axle and the crankshaft rotates in the same sense as the road wheels. The ratio of engine speed to back axle speed is 3 : 1. The automobile has a mass of 1600 kg and has its centre of gravity 0.5 m above road level. The width of the track of the vehicle is 1.5 m.*

Determine the limiting speed of the vehicle around the curve for all four wheels to maintain contact with the road surface. Assume that the road surface is not cambered and centre of gravity of the automobile lies centrally with respect to the four wheels.

Solution. Given : $R = 100$ m ; $I_W = 2.5$ kg-m² ; $d_W = 0.6$ m or $r_W = 0.3$ m ; $I_E = 1.2$ kg-m²; $G = \omega_E/\omega_W = 3$; $m = 1600$ kg ; $h = 0.5$ m ; $x = 1.5$ m

The weight of the vehicle ($m.g$) will be equally distributed over the four wheels which will act downwards. The reaction between the wheel and the road surface of the same magnitude will act upwards.

∴ Road reaction over each wheel

$$= W/4 = m.g / 4 = 1600 \times 9.81/4 = 3924 \text{ N}$$

Let v = Limiting speed of the vehicle in m/s.

We know that angular velocity of the wheels,

$$\omega_W = \frac{v}{r_W} = \frac{v}{0.3} = 3.33\ v \text{ rad/s}$$

and angular velocity of precession,

$$\omega_P = \frac{v}{R} = \frac{v}{100} = 0.01\ v \text{ rad/s}$$

∴ Gyroscopic couple due to 4 wheels,

$$C_W = 4\,I_W.\omega_W.\omega_P = 4 \times 2.5 \times \frac{v}{0.3} \times \frac{v}{100} = 0.33\ v^2 \text{ N-m}$$

and gyroscopic couple due to rotating parts of the engine,

$$C_E = I_E.\omega_E.\omega_P = I_E.G.\omega_W.\omega_P$$
$$= 1.2 \times 3 \times 3.33v \times 0.01v = 0.12\ v^2 \text{ N-m}$$

∴ Total gyroscopic couple,

$$C = C_W + C_E = 0.33\ v^2 + 0.12\ v^2 = 0.45\ v^2 \text{ N-m}$$

Due to this gyroscopic couple, the vertical reaction on the rails will be produced. The reaction will be vertically upwards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at each of the outer or inner wheel be $P/2$ newtons.

∴ $$P/2 = C/2x = 0.45v^2/2 \times 1.5 = 0.15\ v^2 \text{ N}$$

We know that centrifugal force,

$$F_C = m.v^2/R = 1600 \times v^2/100 = 16\ v^2 \text{ N}$$

∴ Overturning couple acting in the outward direction,

$$C_O = F_C \times h = 16\ v^2 \times 0.5 = 8\ v^2 \text{ N-m}$$

This overturning couple is balanced by vertical reactions which are vertically upwards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at each of the outer or inner wheels be $Q/2$ newtons.

∴ $$Q/2 = C_O/2x = 8\ v^2/2 \times 1.5 = 2.67\ v^2 \text{ N}$$

We know that total vertical reaction at each of the outer wheels,

$$P_O = \frac{W}{4} + \frac{P}{2} + \frac{Q}{2} \qquad ...(i)$$

and total vertical reaction at each of the inner wheels,

$$P_I = \frac{W}{4} - \frac{P}{2} - \frac{Q}{2} = \frac{W}{4} - \left(\frac{P}{2} + \frac{Q}{2}\right) \qquad ...(ii)$$

From equation (*i*), we see that there will always be contact between the outer wheels and the road surface because $W/4$, $P/2$ and $Q/2$ are vertically upwards. In order to have contact between the inner wheels and road surface, the reactions should also be vertically upwards, which is only possible if

$$\frac{P}{2} + \frac{Q}{2} \leq \frac{W}{4}$$

i.e. $0.15\, v^2 + 2.67\, v^2 \leq 3924$ or $2.82\, v^2 \leq 3924$

$\therefore$ $v^2 \leq 3924/2.82 = 1391.5$

or $v \leq 37.3$ m/s $= 37.3 \times 3600 / 1000 = 134.28$ km/h **Ans.**

Example 14.12. *A four wheeled motor car of mass 2000 kg has a wheel base 2.5 m, track width 1.5 m and height of centre of gravity 500 mm above the ground level and lies at 1 metre from the front axle. Each wheel has an effective diameter of 0.8 m and a moment of inertia of 0.8 kg-m². The drive shaft, engine flywheel and transmission are rotating at 4 times the speed of road wheel, in a clockwise direction when viewed from the front, and is equivalent to a mass of 75 kg having a radius of gyration of 100 mm. If the car is taking a right turn of 60 m radius at 60 km/h, find the load on each wheel.*

Solution. Given : $m = 2000$ kg : $b = 2.5$ m ; $x = 1.5$ m ; $h = 500$ mm $= 0.5$ m ; $L = 1$ m ; $d_W = 0.8$ m or $r_W = 0.4$ m ; $I_W = 0.8$ kg-m² ; $G = \omega_E / \omega_W = 4$; $m_E = 75$ kg ; $k_E = 100$ mm $= 0.1$ m ; $R = 60$ m ; $v = 60$ km/h $= 16.67$ m/s

Since the centre of gravity of the car lies at 1 m from the front axle and the weight of the car ($W = m.g$) lies at the centre of gravity, therefore weight on the front wheels and rear wheels will be different.

Let W_1 = Weight on the front wheels, and

W_2 = Weight on the rear wheels.

Taking moment about the front wheels,

$$W_2 \times 2.5 = W \times 1 = m.g \times 1 = 2000 \times 9.81 \times 1 = 19\,620$$

$$\therefore \quad W_2 = 19\,620 / 2.5 = 7848 \text{ N}$$

We know that weight of the car or on the four wheels,

$$W = W_1 + W_2 = m.g = 2000 \times 9.81 = 19\,620 \text{ N}$$

or $$W_1 = W - W_2 = 19\,620 - 7848 = 11\,772 \text{ N}$$

$\therefore$ Weight on each of the front wheels

$$= W_1 / 2 = 11\,772 / 2 = 5886 \text{ N}$$

and weight on each of the rear wheels

$$= W_2 /2 = 7874 / 2 = 3924 \text{ N}$$

Since the weight of the car over the four wheels will act downwards, therefore the reaction between each wheel and the road surface of the same magnitude will act upwards as shown in Fig. 14.12.

Let us now consider the effect of gyroscopic couple due to four wheels and rotating parts of the engine.

We know angular velocity of wheels,

$$\omega_W = v/r_W = 16.67 / 0.4 = 41.675 \text{ rad/s}$$

and angular velocity of precession,

$$\omega_P = v/R = 16.67 / 60 = 0.278 \text{ rad/s}$$

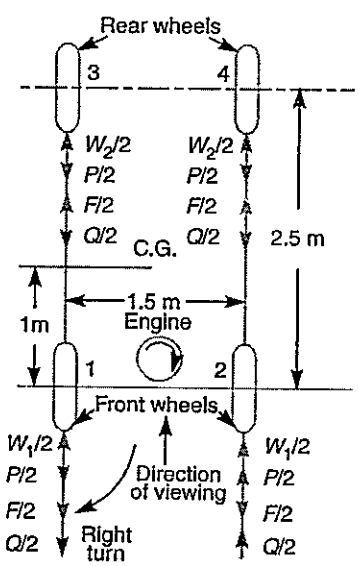


Fig. 14.12

∴ Gyroscopic couple due to four wheels,

$$C_W = 4 I_W.\omega_W.\omega_P$$
$$= 4 \times 0.8 \times 41.675 \times 0.278 = 37.1 \text{ N-m}$$

This gyroscopic couple tends to lift the inner wheels and to press the outer wheels. In other words, the reaction will be vertically downward on the inner wheels (*i.e.* wheels 1 and 3) and vertically upward on the outer wheels (*i.e.* wheels 2 and 4) as shown in Fig. 14.12. Let $P/2$ newtons be the magnitude of this reaction at each of the inner or outer wheel.

$$\therefore \quad P/2 = C_W / 2x = 37.1 / 2 \times 1.5 = 12.37 \text{ N}$$

We know that mass moment of inertia of rotating parts of the engine,

$$I_E = m_E (k_E)^2 = 75 (0.1)^2 = 0.75 \text{ kg-m}^2 \quad \ldots(\because I = m.k^2)$$

∴ Gyroscopic couple due to rotating parts of the engine,

$$C_E = I_E.\omega_E.\omega_P = m_E (k_E)^2 G.\ \omega_W.\omega_P$$
$$= 75 (0.1)^2 4 \times 41.675 \times 0.278 = 34.7 \text{ N-m}$$

This gyroscopic couple tends to lift the front wheels and to press the outer wheels. In other words, the reaction will be vertically downwards on the front wheels and vertically upwards on the rear wheels as shown in Fig. 14.12. Let $F/2$ newtons be the magnitude of this reaction on each of the front and rear wheels.

$$\therefore \quad F/2 = C_E / 2b = 34.7/2 \times 2.5 = 6.94 \text{ N}$$

Now let us consider the effect of centrifugal couple acting on the car. We know that centrifugal force,

$$F_C = m.v^2 / R = 2000 (16.67)^2 / 60 = 9263 \text{ N}$$

∴ Centrifugal couple tending to overturn the car or over turning couple,

$$C_O = F_C \times h = 9263 \times 0.5 = 4631.5 \text{ N-m}$$

This overturning couple tends to reduce the pressure on the inner wheels and to increase on the outer wheels. In other words, the reactions are vertically downward on the inner wheels and vertically upwards on the outer wheels. Let $Q/2$ be the magnitude of this reaction on each of the inner and outer wheels.

$$\therefore \quad Q/2 = C_O / 2x = 4631.5 / 2 \times 1.5 = 1543.83 \text{ N}$$

From Fig. 14.12, we see that

Load on the front wheel 1

$$= \frac{W_1}{2} - \frac{P}{2} - \frac{F}{2} - \frac{Q}{2} = 5886 - 12.37 - 6.94 - 1543.83 = 4322.86 \text{ N } \textbf{Ans.}$$

Load on the front wheel 2

$$= \frac{W_1}{2} + \frac{P}{2} - \frac{F}{2} + \frac{Q}{2} = 5886 + 12.37 - 6.94 + 1543.83 = 7435.26 \text{ N } \textbf{Ans.}$$

Load on the rear wheel 3

$$= \frac{W_2}{2} - \frac{P}{2} + \frac{F}{2} - \frac{Q}{2} = 3924 - 12.37 + 6.94 - 1543.83 = 2374.74 \text{ N } \textbf{Ans.}$$

Load on the rear wheel 4

$$= \frac{W_2}{2} + \frac{P}{2} + \frac{F}{2} + \frac{Q}{2} = 3924 + 12.37 + 6.94 + 1543.83 = 5487.14 \text{ N } \textbf{Ans.}$$

Example 14.13. *A four-wheeled trolley car of total mass 2000 kg running on rails of 1.6 m gauge, rounds a curve of 30 m radius at 54 km/h. The track is banked at 8°. The wheels have an external diameter of 0.7 m and each pair with axle has a mass of 200 kg. The radius of gyration for each pair is 0.3 m. The height of centre of gravity of the car above the wheel base is 1 m. Determine, allowing for centrifugal force and gyroscopic couple actions, the pressure on each rail.*

Solution. Given : $m = 2000$ kg ; $x = 1.6$ m ; $R = 30$ m ; $v = 54$ km / h = 15 m / s ; $\theta = 8°$; $d_W = 0.7$ m or $r_W = 0.35$ m ; $m_1 = 200$ kg ; $k = 0.3$ m ; $h = 1$ m

First of all, let us find the reactions R_A and R_B at the wheels A and B respectively. The various forces acting on the trolley car are shown in Fig. 14.13.

Resolving the forces perpendicular to the track,

$$R_A + R_B = W\cos\theta + F_C \sin\theta = m.g\cos\theta + \frac{m.v^2}{R}\sin\theta$$

$$= 2000 \times 9.81\cos 8° + \frac{2000\,(15)^2}{30} \times \sin 8°$$

$$= 19\,620 \times 0.9903 + 15\,000 \times 0.1392 = 21\,518 \text{ N}$$

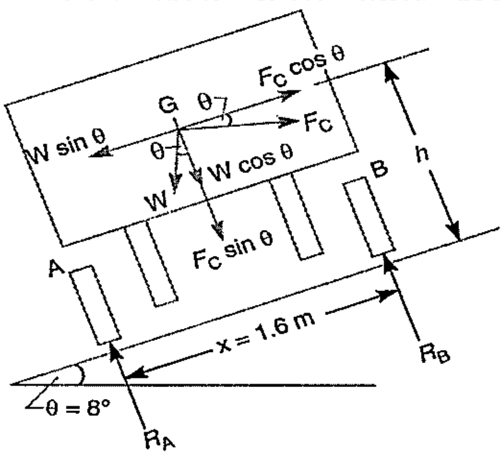


Fig. 14.13

Now taking moments about B,

$$R_A \times x = (W\cos\theta + F_C\sin\theta)\,\frac{x}{2} + W\sin\theta \times h - F_C\cos\theta \times h$$

$$\therefore \quad R_A = \left(m.g\cos\theta + \frac{m.v^2}{R}\sin\theta\right)\frac{1}{2} + \left(m.g\sin\theta - \frac{m.v^2}{R}\cos\theta\right)\frac{h}{x}$$

$$= \left(2000 \times 9.81\cos 8° + \frac{2000\,(15)^2}{30}\sin 8°\right)\frac{1}{2}$$

$$+ \left(2000 \times 9.81\sin 8° - \frac{2000\,(15)^2}{30}\cos 8°\right)\frac{1}{1.6}$$

$$= (19\,620 \times 0.9903 + 15\,000 \times 0.1392)\,\frac{1}{2}$$

$$+ (19\,620 \times 0.1392 - 15\,000 \times 0.9903)\,\frac{1}{1.6}$$

$$= (19\,430 + 2088)\frac{1}{2} + (2731 - 14\,855)\frac{1}{1.6}$$
$$= 10\,759 - 7577 = 3182 \text{ N}$$

$\therefore$ $$R_B = (R_A + R_B) - R_A = 21\,518 - 3182 = 18\,336 \text{ N}$$

We know that angular velocity of wheels,

$$\omega_W = \frac{v}{r_W} = \frac{15}{0.35} = 42.86 \text{ rad/s}$$

and angular velocity of precession,

$$\omega_P = \frac{v}{R} = \frac{15}{30} = 0.5 \text{ rad/s}$$

$\therefore$ Gyroscopic couple,

$$C = {}^{*}I\,\omega_W \cos\theta \times \omega_P = m_I.k^2.\omega_W \cos\theta.\omega_P \qquad ...(\because I = m_I.k^2)$$
$$= 200\,(0.3)^2\, 42.86 \cos 8° \times 0.5 = 382 \text{ N-m}$$

Due to this gyroscopic couple, the car will tend to overturn about the outer wheels. Let P be the force at each pair of wheels or each rail due to the gyroscopic couple,

$\therefore$ $$P = C/x = 382/1.6 = 238.75 \text{ N}$$

We know that pressure (or total reaction) on the inner rail,

$$P_I = R_A - P = 3182 - 238.75 = 2943.25 \text{ N} \textbf{ Ans.}$$

and pressure on the outer rail,

$$P_O = R_B + P = 18\,336 + 238.75 = 18\,574.75 \text{ N} \textbf{ Ans.}$$

Example 14.14. *A pair of locomotive driving wheels with the axle, have a moment of inertia of 180 kg-m². The diameter of the wheel treads is 1.8 m and the distance between wheel centres is 1.5 m. When the locomotive is travelling on a level track at 95 km/h, defective ballasting causes one wheel to fall 6 mm and to rise again in a total time of 0.1 s. If the displacement of the wheel takes place with simple harmonic motion, find : 1. The gyroscopic couple set up, and 2. The reaction between the wheel and rail due to this couple.*

Solution. Given : $I = 180$ kg-m² ; $D = 1.8$ m or $R = 0.9$ m ; $x = 1.5$ m ; $v = 95$ km / h = 26.4 m /s

1. *Gyroscopic couple set up*

We know that angular velocity of the locomotive,

$$\omega = v/R = 26.4/0.9 = 29.3 \text{ rad/s}$$

Since the defective ballasting causes one wheel to fall 6 mm and to rise again in a total time (t) of 0.1 s, therefore

$$\text{Amplitude, } A = \frac{1}{2}\text{ Fall} = \frac{1}{2}\text{ Rise} = \frac{1}{2} \times 6 = 3 \text{ mm}$$

and maximum velocity while falling,

$$v_{max} = \frac{2\pi}{t} \times A = \frac{2\pi}{0.1} \times 3 = 118.5 \text{ mm/s} = 0.1885 \text{ m/s}$$

$\therefore$ Maximum angular velocity of tilt of the axle or angular velocity of precession,

$$\omega_{P\,max} = \frac{v_{max}}{x} = \frac{0.1885}{1.5} = 0.126 \text{ rad/s}$$

---

* Angular momentum about axle = $I.\omega_W$

$\therefore$ Angular momentum about horizontal = $I.\omega_W \cos\theta$

We know that gyroscopic couple set up,

$$C = I.\omega.\omega_{P\,max} = 180 \times 29.3 \times 0.126 = 664.5 \text{ N-m } \textbf{Ans.}$$

The gyroscopic couple will act in a horizontal plane and this couple will tend to produce *swerve i.e.* it tends to turn the locomotive aside.

2. ***Reaction between the wheel and rail due to the gyroscopic couple***

We know that the reaction between the wheel and rail due to the gyroscopic couple is

$$P = C / x = 664.5 / 1.5 = 443 \text{ N } \textbf{Ans.}$$

14.10. Stability of a Two Wheel Vehicle Taking a Turn

Consider a two wheel vehicle (say a scooter or motor cycle) taking a right turn as shown in Fig. 14.14 (*a*).

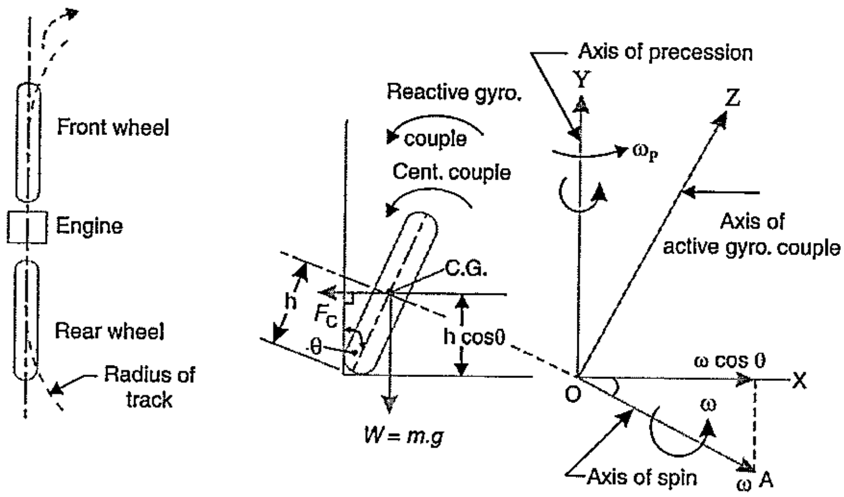
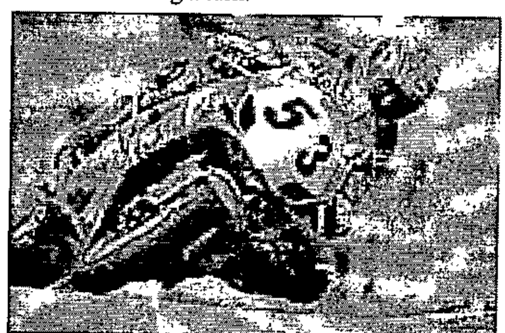


Fig. 14.14. Stability of a two wheel vehicle taking a turn.



Motorcycle taking a turn.

Let m = Mass of the vehicle and its rider in kg,

W = Weight of the vehicle and its rider in newtons = $m.g$,

h = Height of the centre of gravity of the vehicle and rider,

r_W = Radius of the wheels,

R = Radius of track or curvature,

I_W = Mass moment of inertia of each wheel,

I_E = Mass moment of inertia of the rotating parts of the engine,

ω_W = Angular velocity of the wheels,

ω_E = Angular velocity of the engine,

G = Gear ratio = ω_E / ω_W,

v = Linear velocity of the vehicle = $\omega_W \times r_W$,

θ = Angle of heel. It is inclination of the vehicle to the vertical for equilibrium.

Let us now consider the effect of the gyroscopic couple and centrifugal couple on the vehicle, as discussed below.

1. *Effect of gyroscopic couple*

We know that $v = \omega_W \times r_W$ or $\omega_W = v / r_W$

and
$$\omega_E = G.\omega_W = G \times \frac{v}{r_W}$$

$\therefore$ Total
$$(I \times \omega) = 2\, I_W \times \omega_W \pm I_E \times \omega_E$$
$$= 2\, I_W \times \frac{v}{r_W} \pm I_E \times G \times \frac{v}{r_W} = \frac{v}{r_W}(2\, I_W \pm G.I_E)$$

and velocity of precession, $\omega_P = v / R$

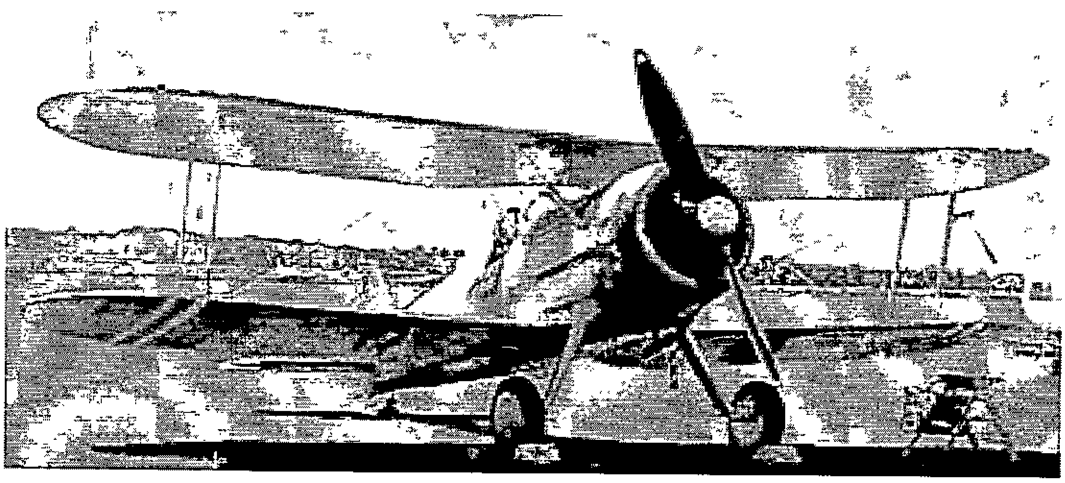
A little consideration will show that when the wheels move over the curved path, the vehicle is always inclined at an angle θ with the vertical plane as shown in Fig. 14.14 (*b*). This angle is known as *angle of heel*. In other words, the axis of spin is inclined to the horizontal at an angle θ, as shown in Fig. 14.14 (*c*). Thus the angular momentum vector $I\omega$ due to spin is represented by OA inclined to OX at an angle θ. But the precession axis is vertical. Therefore the spin vector is resolved along OX.

$\therefore$ Gyroscopic couple,

$$C_1 = I.\omega \cos\theta \times \omega_P = \frac{v}{r_W}(2\, I_W \pm G.I_E)\cos\theta \times \frac{v}{R}$$
$$= \frac{v^2}{R.r_W}(2\, I_W \pm G.I_E)\cos\theta$$

Notes : (*a*) When the engine is rotating in the same direction as that of wheels, then the ***positive*** sign is used in the above expression and if the engine rotates in opposite direction, then ***negative*** sign is used.

(*b*) The gyroscopic couple will act over the vehicle outwards *i.e.* in the anticlockwise direction when seen from the front of the vehicle. The tendency of this couple is to overturn the vehicle in outward direction.



An aircraft of 1920's model.

2. ***Effect of centrifugal couple***

We know that centrifugal force,

$$F_C = \frac{m.v^2}{R}$$

This force acts horizontally through the centre of gravity (*C.G.*) along the outward direction.

$\therefore$ Centrifugal couple,

$$C_2 = F_C \times h \cos\theta = \left(\frac{m.v^2}{R}\right) h \cos\theta$$

Since the centrifugal couple has a tendency to overturn the vehicle, therefore

Total overturning couple,

$$C_O = \text{Gyroscopic couple} + \text{Centrifugal couple}$$

$$= \frac{v^2}{R.r_W} \; 2\, I_W + G.I_E \; \cos\theta + \frac{m.v^2}{R} \times h \cos\theta$$

$$= \frac{v^2}{R}\left[\frac{2\, I_W + G.I_E}{r_W} + m.h\right]\cos\theta$$

We know that balancing couple $= m.g.h \sin\theta$

The balancing couple acts in clockwise direction when seen from the front of the vehicle. Therefore for stability, the overturning couple must be equal to the balancing couple, *i.e.*

$$\frac{v^2}{R}\left(\frac{2\, I_W + G.I_E}{r_W} + m.h\right)\cos\theta = m.g.h \sin\theta$$

From this expression, the value of the angle of heel (θ) may be determined, so that the vehicle does not skid.

Example 14.15. *Find the angle of inclination with respect to the vertical of a two wheeler negotiating a turn. Given : combined mass of the vehicle with its rider 250 kg ; moment of inertia of the engine flywheel 0.3 kg-m² ; moment of inertia of each road wheel 1 kg-m² ; speed of engine flywheel 5 times that of road wheels and in the same direction ; height of centre of gravity of rider with vehicle 0.6 m ; two wheeler speed 90 km/h ; wheel radius 300 mm ; radius of turn 50 m.*

Solution. Given : $m = 250$ kg ; $I_E = 0.3$ kg-m^2 ; $I_W = 1$ kg-m^2 ; $\omega_E = 5\,\omega_W$ or $G = \frac{\omega_E}{\omega_W} = 5$; $h = 0.6$ m ; $v = 90$ km/h $= 25$ m/s ; $r_W = 300$ mm $= 0.3$ m ; $R = 50$ m

Let $\quad \theta$ = Angle of inclination with respect to the vertical of a two wheeler.

We know that gyroscopic couple,

$$C_1 = \frac{v^2}{R \times r_W}(2\, I_W + G.I_E)\cos\theta = \frac{(25)^2}{50 \times 0.3}(2 \times 1 + 5 \times 0.3)\cos\theta$$

$$= 146 \cos\theta \text{ N-m}$$

and centrifugal couple, $\quad C_2 = \frac{m.v^2}{R} \times h\cos\theta = \frac{250\,(25)^2}{50} \times 0.6 \cos\theta = 1875 \cos\theta$ N-m

$\therefore$ Total overturning couple,

$$= C_1 + C_2 = 146\cos\theta + 1875\cos\theta = 2021\cos\theta \text{ N-m}$$

We know that balancing couple

$$= m.g.h \sin\theta = 250 \times 9.81 \times 0.6 \sin\theta = 1471.5 \sin\theta \text{ N-m}$$

Since the overturning couple must be equal to the balancing couple for equilibrium condition, therefore

$$2021 \cos \theta = 1471.5 \sin \theta$$

$\therefore$ $\tan \theta = \sin \theta / \cos \theta = 2021 / 1471.5 = 1.3734$ or $\theta = 53.94°$ **Ans.**

Example 14.16. *A gyrowheel D of mass 0.5 kg, with a radius of gyration of 20 mm, is mounted in a pivoted frame C as shown in Fig. 14.15. The axis AB of the pivots passes through the centre of rotation O of the wheel, but the centre of gravity G of the frame C is 10 mm below O. The frame has a mass of 0.30 kg and the speed of rotation of the wheel is 3000 r.p.m. in the anticlockwise direction as shown.*

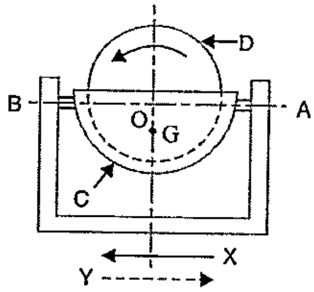


Fig. 14.15

The entire unit is mounted on a vehicle so that the axis AB is parallel to the direction of motion of the vehicle. If the vehicle travels at 15 m/s in a curve of 50 metres radius, find the inclination of the gyrowheel from the vertical, when

1. The vehicle moves in the direction of the arrow 'X' taking a left hand turn along the curve, and

2. The vehicle reverse at the same speed in the direction of arrow 'Y' along the same path.

Solution. Given : $m_1 = 0.5$ kg ; $k = 20$ mm $= 0.02$ m ; $OG = h = 10$ mm $= 0.01$ m ; $m_2 = 0.3$ kg ; $N = 3000$ r.p.m. or $\omega = 2\pi \times 3000 / 60 = 314.2$ rad/s ; $v = 15$ m/s ; $R = 50$ m

We know that mass moment of inertia of the gyrowheel,

$$I = m_1.k^2 = 0.5\,(0.02)^2 = 0.0002 \text{ kg-m}^2$$

and angular velocity of precession,

$$\omega_P = v/R = 15 / 50 = 0.3 \text{ rad/s}$$

Let θ = Angle of inclination of gyrowheel from the vertical.

1. ***When the vehicle moves in the direction of arrow X taking a left turn along the curve***

We know that gyroscopic couple about O,

$$C_1 = I\,\omega.\omega_P \cos\theta = 0.0002 \times 314.2 \times 0.3 \cos\theta \text{ N-m}$$
$$= 0.019 \cos\theta \text{ N-m (anticlockwise)}$$

and centrifugal couple about O,

$$C_2 = \frac{m_2.v^2}{R} \times h \cos\theta = \frac{0.3\,(15)^2}{50} \times 0.01 \cos\theta \text{ N-m}$$
$$= 0.0135 \cos\theta \text{ N-m (anticlockwise)}$$

$\therefore$ Total overturning couple

$$= C_1 - C_2 = 0.019 \cos\theta - 0.0135 \cos\theta$$

... (– ve sign due to opposite direction)

$$= 0.0055 \cos\theta \text{ N-m (anticlockwise)}$$

We know that balancing couple due to weight ($W_2 = m_2.g$) of the frame about O,

$$= m_2.g.h \sin\theta = 0.3 \times 9.81 \times 0.01 \sin\theta \text{ N-m}$$
$$= 0.029 \sin\theta \text{ N-m (clockwise)}$$

Since the overturning couple must be equal to the balancing couple for equilibrium condition, therefore

$$0.0055 \cos\theta = 0.029 \sin\theta$$

or $$\tan\theta = \sin\theta / \cos\theta = 0.0055 / 0.029 = 0.1896$$

$\therefore$ $$\theta = 10.74° \text{ Ans.}$$

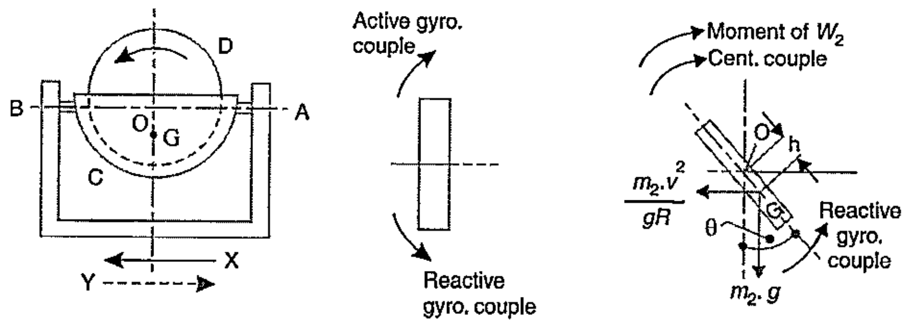


Fig. 14.16

2. *When the vehicle reverses at the same speed in the direction of arrow Y along the same path*

When the vehicle reverses at the same speed in the direction of arrow Y, then the gyroscopic and centrifugal couples (C_1 and C_2) will be in clockwise direction about O and the balancing couple due to weight ($W_2 = m_2.g$) of the frame about O will be in anticlockwise direction.

$\therefore$ Total overturning couple

$$= C_1 + C_2 = 0.019 \cos\theta + 0.0135 \cos\theta = 0.0325 \cos\theta \text{ N-m}$$

Equating the total overturning couple to the balancing couple, we have

$$0.0325 \cos\theta = 0.029 \sin\theta$$

or $$\tan\theta = \sin\theta / \cos\theta = 0.0325 / 0.029 = 1.1207$$

$\therefore$ $$\theta = 48.26° \text{ Ans.}$$

14.11. Effect of Gyroscopic Couple on a Disc Fixed Rigidly at a Certain Angle to a Rotating Shaft

Consider a disc fixed rigidly to a rotating shaft such that the polar axis of the disc makes an angle θ with the shaft axis, as shown in Fig. 14.17. Let the shaft rotates with an angular velocity ω rad/s in the clockwise direction when viewed from the front. A little consideration will show that the disc will also rotate about OX with the same angular velocity ω rad/s. Let OP be the polar axis and OD the diametral axis of the disc.

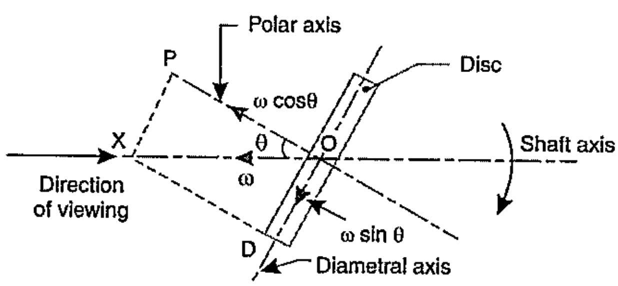


Fig. 14.17. Effect of gyroscopic couple on a disc fixed rigidly at a certain angle to a rotating shaft.

∴ Angular velocity of the disc about the polar axis OP or the angular velocity of spin

$$= \omega \cos \theta \qquad \text{... (Component of } \omega \text{ in the direction of } OP)$$

Since the shaft rotates, therefore the point P will move in a plane perpendicular to the plane of paper. In other words, precession is produced about OD.

∴ Angular velocity of the disc about the diametral axis OD or the angular velocity of precession

$$= \omega \sin \theta$$

If I_P is the mass moment of inertia of the disc about the polar axis OP, then gyroscopic couple acting on the disc,

$$C_P = I_P.\omega \cos \theta.\omega \sin \theta = \frac{1}{2} \times I_P.\omega^2 \sin 2\theta$$

$$\text{... } (\because 2 \sin \theta \cos \theta = \sin 2\theta)$$

The effect of this gyroscopic couple is to turn the disc in the anticlockwise when viewed from the top, about an axis through O in the plane of paper.

Now consider the movement of point D about the polar axis OP. In this case, OD is axis of spin and OP is the axis of precession.

∴ Angular velocity of disc about OD or angular velocity of spin

$$= \omega \sin \theta$$

and angular velocity of D about OP or angular velocity of precession

$$= \omega \cos \theta$$

If I_D is the mass moment of inertia of the disc about the diametral axis OD, then gyroscopic couple acting on the disc,

$$C_D = I_D.\omega \sin \theta.\omega \cos \theta = \frac{1}{2} \times I_D.\omega^2 \sin 2\theta$$

The effect of this couple will be opposite to that of C_P.

∴ Resultant gyroscopic couple acting on the disc,

$$C = C_P - C_D = \frac{1}{2} \times \omega^2 \sin 2\theta \, (I_P - I_D)$$

This resultant gyroscopic couple will act in the anticlockwise direction as seen from the top. In other words, the shaft tends to turn in the plane of paper in anticlockwise direction as seen from the top, as a result the horizontal force is exerted on the shaft bearings.

Notes: 1. The mass moment of inertia of the disc about polar axis OP,

$$I_P = m.r^2/2$$

and mass moment of inertia of the disc about diametral axis OD,

$$I_D = m\left(\frac{l^2}{12} + \frac{r^2}{4}\right)$$

where m = Mass of disc,

r = Radius of disc, and

l = Width of disc.

2. If the disc is thin, l may be neglected. In such a case

$$I_D = m.r^2/4$$

$$\therefore \quad C = \frac{1}{2} \times \omega^2 \sin 2\theta \left(\frac{m.r^2}{2} - \frac{m.r^2}{4}\right) = \frac{m}{8} \times \omega^2.r^2 \sin 2\theta$$

Example 14.17. *A shaft carries a uniform thin disc of 0.6 m diameter and mass 30 kg. The disc is out of truth and makes an angle of 1° with a plane at right angles to the axis of the shaft. Find the gyroscopic couple acting on the bearing when the shaft rotates at 1200 r.p.m.*

Solution. Given : $d = 0.6$ m or $r = 0.3$ m , $m = 30$ kg ; $\theta = 1°$; $N = 1200$ r.p.m. or $\omega = 2\pi \times 1200/60 = 125.7$ rad/s

We know that gyroscopic couple acting on the bearings,

$$C = \frac{m}{8} \times \omega^2 . r^2 \sin 2\theta = \frac{30}{8}(125.7)^2\ (0.3)^2\ \sin 2° = 186 \text{ N-m} \textbf{ Ans.}$$

EXERCISES

1. A flywheel of mass 10 kg and radius of gyration 200 mm is spinning about its axis, which is horizontal and is suspended at a point distant 150 mm from the plane of rotation of the flywheel. Determine the angular velocity of precession of the flywheel. The spin speed of flywheel is 900 r.p.m. **[Ans. 0.39 rad/s]**

2. A horizontal axle AB, 1 m long, is pivoted at the mid point C. It carries a weight of 20 N at A and a wheel weighing 50 N at B. The wheel is made to spin at a speed of 600 r.p.m in a clockwise direction looking from its front. Assuming that the weight of the flywheel is uniformly distributed around the rim whose mean diameter is 0.6 m, calculate the angular velocity of precession of the system around the vertical axis through C. **[Ans. 0.52 rad/s]**

3. An aeroplane runs at 600 km / h. The rotor of the engine weighs 4000 N with radius of gyration of 1 metre. The speed of rotor is 3000 r.p.m. in anticlockwise direction when seen from rear side of the aeroplane.

   If the plane takes a loop upwards in a curve of 100 metres radius, find : 1. gyroscopic couple developed; and 2. effect of reaction gyroscopic couple developed on the body of aeroplane. **[Ans. 213.5 kN-m]**

4. An aeroplane makes a complete half circle of 50 metres radius, towards left, when flying at 200 km per hour. The rotary engine and the propeller of the plane has a mass of 400 kg with a radius of gyration of 300 mm. The engine runs at 2400 r.p.m. clockwise, when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it. What will be the effect, if the aeroplane turns to its right instead of to the left ? **[Ans. 10 kN-m]**

5. Each paddle wheel of a steamer have a mass of 1600 kg and a radius of gyration of 1.2 m. The steamer turns to port in a circle of 160 m radius at 24 km / h, the speed of the paddles being 90 r.p.m. Find the magnitude and effect of the gyroscopic couple acting on the steamer. **[Ans. 905.6 N-m]**

6. The rotor of the turbine of a yacht makes 1200 r.p.m. clockwise when viewed from stern. The rotor has a mass of 750 kg and its radius of gyration is 250 mm. Find the maximum gyroscopic couple transmitted to the hull (body of the yacht) when yacht pitches with maximum angular velocity of 1 rad /s. What is the effect of this couple ? **[Ans. 5892 N-m]**

7. The rotor of a turbine installed in a boat with its axis along the longitudinal axis of the boat makes 1500 r.p.m. clockwise when viewed from the stern. The rotor has a mass of 750 kg and a radius of gyration of 300 mm. If at an instant, the boat pitches in the longitudinal vertical plane so that the bow rises from the horizontal plane with an angular velocity of 1 rad /s, determine the torque acting on the boat and the direction in which it tends to turn the boat at the instant. **[Ans. 10.6 kN-m]**

8. The mass of a turbine rotor of a ship is 8 tonnes and has a radius of gyration 0.6 m. It rotates at 1800 r.p.m. clockwise when looking from the stern. Determine the gyroscopic effects in the following cases:

   1. If the ship travelling at 100 km / h strees to the left in a curve of 75 m radius, 2. If the ship is pitching and the bow is descending with maximum velocity. The pitching is simple harmonic, the periodic time being 20 seconds and the total angular movement between the extreme positions is 10°, and 3. If the ship is rolling and at a certain instant has an angular velocity of 0.03 rad/s clockwise when looking from stern.

In each case, explain clearly how you determine the direction in which the ship tends to move as a result of the gyroscopic action. **[Ans. 201 kN-m ; 14.87 kN-m ; 16.3 kN-m]**

9. The turbine rotor of a ship has a mass of 20 tonnes and a radius of gyration of 0.75 m. Its speed is 2000 r.p.m. The ship pitches 6° above and below the horizontal position. One complete oscillation takes 18 seconds and the motion is simple harmonic. Calculate :

   1. the maximum couple tending to shear the holding down bolts of the turbine, 2. the maximum angular acceleration of the ship during pitching, and 3. the direction in which the bow will tend to turn while rising, if the rotation of the rotor is clockwise when looking from rear.

   [Ans. 86.26 kN-m ; 0.0128 rad /s^2, towards star-board]

10. A motor car takes a bend of 30 m radius at a speed of 60 km / hr. Determine the magnitudes of gyroscopic and centrifugal couples acting on the vehicle and state the effect that each of these has on the road reactions to the road wheels. Assume that :

    Each road wheel has a moment of inertia of 3 kg-m^2 and an effective road radius of 0.4 m.

    The rotating parts of the engine and transmission are equivalent to a flywheel of mass 75 kg with a radius of gyration of 100 mm. The engine turns in a clockwise direction when viewed from the front.

    The back-axle ratio is 4 : 1, the drive through the gear box being direct. The gyroscopic effects of the half shafts at the back axle are to be ignored.

    The car has a mass of 1200 kg and its centre of gravity is 0.6 m above the road wheel.

    The turn is in a right hand direction.

    If the turn has been in a left hand direction, all other details being unaltered, which answers, if any, need modification. **[Ans. 347.5 N-m : 6670 N-m]**

11. A rail car has a total mass of 4 tonnes. There are two axles, each of which together with its wheels and gearing has a total moment of inertia of 30 kg-m^2. The centre distance between the two wheels on an axle is 1.5 metres and each wheel is of 375 mm radius. Each axle is driven by a motor, the speed ratio between the two being 1 : 3. Each motor with its gear has a moment of inertia of 15 kg-m^2 and runs in a direction opposite to that of its axle. The centre of gravity of the car is 1.05 m above the rails.

    Determine the limiting speed for this car, when it rounding a curve of 240 metres radius such that no wheel leaves the rail. Consider the centrifugal and gyroscopic effects completely. Assume that no cant is provided for outer rail. **[Ans. 144 km / h]**

12. A racing car weighs 20 kN. It has a wheel base of 2 m, track width 1 m and height of C.G. 300 mm above the ground level and lies midway between the front and rear axle. The engine flywheel rotates at 3000 r.p.m. clockwise when viewed from the front. The moment of inertia of the flywheel is 4 kg-m^2 and moment of inertia of each wheel is 3 kg-m^2. Find the reactions between the wheels and the ground when the car takes a curve of 15 m radius towards right at 30 km / h, taking into consideration the gyroscopic and the centrifugal effects. Each wheel radius is 400 mm.

    [Ans. Front inner wheel = 3341.7 N ; Front outer wheel = 6309.5 N ;
    Rear inner wheel = 3690.5 N ; Rear outer wheel = 6658.3 N]

13. A four wheel trolley car of total mass 2000 kg running on rails of 1 m gauge, rounds a curve of 25 m radius at 40 km / h. The track is banked at 10°. The wheels have an external diameter of 0.6 m and each pair of an axle has a mass of 200 kg. The radius of gyration for each pair is 250 mm. The height of C.G. of the car above the wheel base is 0.95 m. Allowing for centrifugal force and gyroscopic couple action, determine the pressure on each rail. **[Ans. 4328 N ; 16 704 N]**

14. A 2.2 tonne racing car has a wheel base of 2.4 m and a track of 1.4 m from the rear axle. The equivalent mass of engine parts is 140 kg with radius of gyration of 150 mm. The back axle ratio is 5. The engine shaft and flywheel rotate clockwise when viewed from the front. Each wheel has a diameter of 0.8 m and a moment of inertia of 0.7 kg-m^2. Determine the load distribution on the wheels when the car is rounding a curve of 100 m radius at a speed of 72 km / h to the left.

15. A disc has a mass of 30 kg and a radius of gyration about its axis of symmetry 125 mm while its radius of gyration about a diameter of the disc at right angles to the axis of symmetry is 75 mm. The disc is pressed on to the shaft but due to incorrect boring, the angle between the axis of symmetry and the actual axis of rotation is 0.25°, though both these axes pass through the centre of gravity of the disc. Assuming that the shaft is rigid and is carried between bearings 200 mm apart, determine the bearing forces due to the misalignment at a speed of 5000 r.p.m. **[Ans. 1810 N]**

16. A wheel of a locomotive, travelling on a level track at 90 km / h, falls in a spot hole 10 mm deep and rises again in a total time of 0.8 seconds. The displacement of the wheel takes place with simple harmonic motion. The wheel has a diameter of 3 m and the distance between the wheel centres is 1.75 m. The wheel pair with axle has a moment of inertia of 500 kg-m^2. Determine the magnitude and the effect of gyrocouple produced in this case. **[Ans. 186.6 N-m]**

17. Each road wheel of a motor cycle has a mass moment of inertia of 1.5 kg-m^2. The rotating parts of the engine of the motor cycle have a mass moment of inertia of 0.25 kg-m^2. The speed of the engine is 5 times the speed of the wheels and is in the same sense. The mass of the motor cycle with its rider is 250 kg and its centre of gravity is 0.6 m above the ground level.

    Find the angle of heel if the cycle is travelling at 50 km / h and is taking a turn of 30 m radius. The wheel diameter is 0.6 m. **[Ans. 35.7°]**

18. A racing motor cyclist travels at 140 km/h round a curve of 120 m radius measured horizontally. The cycle and rider have mass of 150 kg and their centre of gravity lies at 0.7 m above the ground level when the motor cycle is vertical. Each wheel is 0.6 m in diameter and has moment of inertia about its axis of rotation 1.5 kg-m^2. The engine has rotating parts whose moment of inertia about their axis of rotation is 0.25 kg-m^2 and it rotates at five times the wheel speed in the same direction. Find : 1. the correct angle of banking of the track so that there is no tendency to side slip, and 2. the correct angle of inclination of the cycle and rider to the vertical. **[Ans. 52.12°; 55.57°]**

    [**Hint.** In calculating the angle of banking of the track, neglect the effect of gyroscopic couple]

DO YOU KNOW ?

1. Write a short note on gyroscope.
2. What do you understand by gyroscopic couple ? Derive a formula for its magnitude.
3. Explain the application of gyroscopic principles to aircrafts.
4. Describe the gyroscopic effect on sea going vessels.
5. Explain the effect of the gyroscopic couple on the reaction of the four wheels of a vehicle negotiating a curve.
6. Discuss the effect of the gyroscopic couple on a two wheeled vehicle when taking a turn.
7. What will be the effect of the gyroscopic couple on a disc fixed at a certain angle to a rotating shaft ?

OBJECTIVE TYPE QUESTIONS

1. A disc is spinning with an angular velocity ω rad/s about the axis of spin. The couple applied to the disc causing precession will be

   (*a*) $\frac{1}{2} I.\omega^2$ (*b*) $I.\omega^2$ (*c*) $\frac{1}{2} I.\omega.\omega_P$ (*d*) $I.\omega.\omega_P$

   where I = Mass moment of inertia of the disc, and

   ω_P = Angular velocity of precession of the axis of spin.

2. A disc spinning on its axis at 20 rad/s will undergo precession when a torque 100 N-m is applied about an axis normal to it at an angular speed, if mass moment of inertia of the disc is the 1 kg-m^2

   (*a*) 2 rad/s (*b*) 5 rad/s (*c*) 10 rad/s (*d*) 20 rad/s

3. The engine of an aeroplane rotates in clockwise direction when seen from the tail end and the aeroplane takes a turn to the left. The effect of the gyroscopic couple on the aeroplane will be

   (*a*) to raise the nose and dip the tail (*b*) to dip the nose and raise the tail

   (*c*) to raise the nose and tail (*d*) to dip the nose and tail

4. The air screw of an aeroplane is rotating clockwise when looking from the front. If it makes a left turn, the gyroscopic effect will
   (*a*) tend to depress the nose and raise the tail
   (*b*) tend to raise the nose and depress the tail
   (*c*) tilt the aeroplane
   (*d*) none of the above

5. The rotor of a ship rotates in clockwise direction when viewed from the stern and the ship takes a left turn. The effect of the gyroscopic couple acting on it will be
   (*a*) to raise the bow and stern
   (*b*) to lower the bow and stern
   (*c*) to raise the bow and lower the stern
   (*d*) to lower the bow and raise the stern

6. When the pitching of a ship is upward, the effect of gyroscopic couple acting on it will be
   (*a*) to move the ship towards port side
   (*b*) to move the ship towards star-board
   (*c*) to raise the bow and lower the stern
   (*d*) to raise the stern and lower the bow

7. In an automobile, if the vehicle makes a left turn, the gyroscopic torque
   (*a*) increases the forces on the outer wheels
   (*b*) decreases the forces on the outer wheels
   (*c*) does not affect the forces on the outer wheels
   (*d*) none of the above

8. A motor car moving at a certain speed takes a left turn in a curved path. If the engine rotates in the same direction as that of wheels, then due to the centrifugal forces
   (*a*) the reaction on the inner wheels increases and on the outer wheels decreases
   (*b*) the reaction on the outer wheels increases and on the inner wheels decreases
   (*c*) the reaction on the front wheels increases and on the rear wheels decreases
   (*d*) the reaction on the rear wheels increases and on the front wheels decreases

ANSWERS

1. (*d*)	2. (*b*)	3. (*a*)	4. (*b*)
5. (*c*)	6. (*b*)	7. (*a*)	8. (*b*)

UNIT-2

Friction

Types of Friction

1. *Static friction*. It is the friction, experienced by a body, when at rest.

2. *Dynamic friction*. It is the friction, experienced by a body, when in motion. The dynamic friction is also called *kinetic friction* and is less than the static friction. It is of the following three types :

(*a*) *Sliding friction*. It is the friction, experienced by a body, when it *slides* over another body.

(*b*) *Rolling friction*. It is the friction, experienced between the surfaces which has *balls* or *rollers* interposed between them.

(*c*) *Pivot friction*. It is the friction, experienced by a body, due to the *motion of rotation* as in case of foot step bearings.

Laws of Static Friction

Following are the laws of static friction :

1. The force of friction always acts in a direction, opposite to that in which the body tends to move.
2. The magnitude of the force of friction is exactly equal to the force, which tends the body to move.
3. The magnitude of the limiting friction (F) bears a constant ratio to the normal reaction (R_N) between the two surfaces. Mathematically

$$F/R_N = \text{constant}$$

4. The force of friction is independent of the area of contact, between the two surfaces.
5. The force of friction depends upon the roughness of the surfaces.

Laws of Kinetic or Dynamic Friction

1. The force of friction always acts in a direction, opposite to that in which the body is moving.
2. The magnitude of the kinetic friction bears a constant ratio to the normal reaction between the two surfaces. But this ratio is slightly less than that in case of limiting friction.
3. For moderate speeds, the force of friction remains constant. But it decreases slightly with the increase of speed.

Laws of Solid Friction

1. The force of friction is directly proportional to the normal load between the surfaces.
2. The force of friction is independent of the area of the contact surface for a given normal load.
3. The force of friction depends upon the material of which the contact surfaces are made.
4. The force of friction is independent of the velocity of sliding of one body relative to the other body.

Laws of Fluid Friction

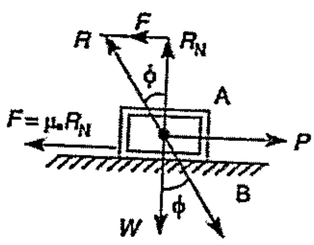
1. The force of friction is almost independent of the load.
2. The force of friction reduces with the increase of the temperature of the lubricant.
3. The force of friction is independent of the substances of the bearing surfaces.
4. The force of friction is different for different lubricants.

Coefficient of Friction

It is defined as the ratio of the limiting friction (F) to the normal reaction (R_N) between the two bodies. It is generally denoted by μ. Mathematically, coefficient of friction,

$$\mu = F/R_N$$

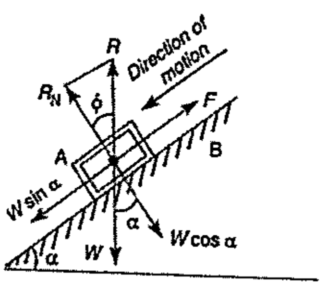
Limiting Angle of Friction



Limiting angle of friction.

$$\tan\phi = F/R_N = \mu R_N / R_N = \mu$$

Angle of Repose



Angle of repose.

$$W \sin\alpha = F = \mu . R_N = \mu . W \cos\alpha$$

$$\tan\alpha = \mu = \tan\phi \quad \text{or} \quad \alpha = \phi$$

Minimum Force Required to Slide a Body on a Rough Horizontal Plane

A body, resting on a rough horizontal plane required a pull of 180 N inclined at 30° to the plane just to move it. It was found that a push of 220 N inclined at 30° to the plane just moved the body. Determine the weight of the body and the coefficient of friction.

Solution. Given : $\theta = 30°$

Let W = Weight of the body in newtons,

R_N = Normal reaction,

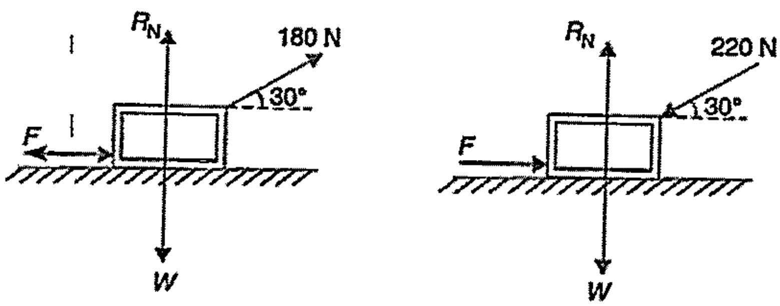
μ = Coefficient of friction, and

F = Force of friction.

First of all, let us consider a pull of 180 N. The force of friction (F) acts towards left as shown in Fig. 10.5 (*a*).

Resolving the forces horizontally,

$$F = 180 \cos 30° = 180 \times 0.866 = 156 \text{ N}$$



(*a*) (*b*)

Now resolving the forces vertically,

$$R_N = W - 180 \sin 30° = W - 180 \times 0.5 = (W - 90) \text{ N}$$

We know that $F = \mu . R_N$ or $156 = \mu (W - 90)$ ...(*i*)

Now let us consider a push of 220 N. The force of friction (F) acts towards right as shown in Fig. 10.5 (*b*).

Resolving the forces horizontally,

$$F = 220 \cos 30° = 220 \times 0.866 = 190.5 \text{ N}$$

Now resolving the forces vertically,

$$R_N = W + 220 \sin 30° = W + 220 \times 0.5 = (W + 110) \text{ N}$$

We know that $F = \mu . R_N$ or $190.5 = \mu (W + 110)$ ...(*ii*)

From equations (*i*) and (*ii*),

$$W = 1000 \text{ N}, \text{ and } \mu = 0.1714 \quad \textbf{Ans.}$$

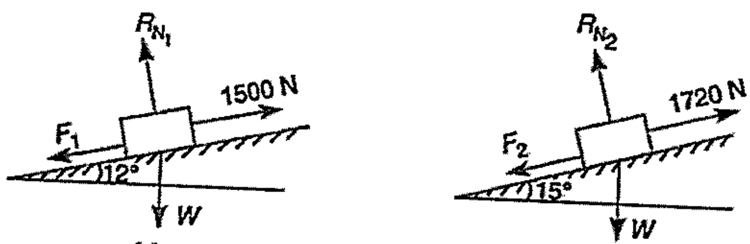
Friction of a Body Lying on a Rough Inclined Plane :

An effort of 1500 N is required to just move a certain body up an inclined plane of angle 12°, force acting parallel to the plane. If the angle of inclination is increased to 15°, then the effort required is 1720 N. Find the weight of the body and the coefficient of friction.

Solution. Given : $P_1 = 1500$ N ; $\alpha_1 = 12°$; $\alpha_2 = 15°$; $P_2 = 1720$ N

Let W = Weight of the body in newtons, and

μ = Coefficient of friction.



(a) (b)

First of all, let us consider a body lying on a plane inclined at an angle of 12° with the horizontal and subjected to an effort of 1500 N parallel to the plane as shown in Fig. 10.10 (a).

Let R_{N_1} = Normal reaction, and

F_1 = Force of friction.

We know that for the motion of the body up the inclined plane, the effort applied parallel to the plane (P_1),

$$1500 = W(\sin\alpha_1 + \mu\cos\alpha_1) = W(\sin 12° + \mu\cos 12°) \quad ...(i)$$

Now let us consider the body lying on a plane inclined at an angle of 15° with the horizontal and subjected to an effort of 1720 N parallel to the plane as shown in Fig. 10.10 (b).

Let R_{N_2} = Normal reaction, and

F_2 = Force of friction.

We know that for the motion of the body up the inclined plane, the effort applied parallel to the plane (P_2),

$$1720 = W(\sin\alpha_2 + \mu\cos\alpha_2) = W(\sin 15° + \mu\cos 15°) \quad ...(ii)$$

Coefficient of friction

Dividing equation (ii) by equation (i),

$$\frac{1720}{1500} = \frac{W(\sin 15° + \mu\cos 15°)}{W(\sin 12° + \mu\cos 12°)}$$

$$1720 \sin 12° + 1720\,\mu \cos 12° = 1500 \sin 15° + 1500\,\mu \cos 15°$$

$$\mu\,(1720 \cos 12° - 1500 \cos 15°) = 1500 \sin 15° - 1720 \sin 12°$$

$$\therefore \quad \mu = \frac{1500 \sin 15° - 1720 \sin 12°}{1720 \cos 12° - 1500 \cos 15°} = \frac{1500 \times 0.2588 - 1720 \times 0.2079}{1720 \times 0.9781 - 1500 \times 0.9659}$$

$$= \frac{388.2 - 357.6}{1682.3 - 1448.5} = \frac{30.6}{233.8} = 0.131 \text{ Ans.}$$

Weight of the body

Substituting the value of μ in equation (*i*),

$$1500 = W\,(\sin 12° + 0.131 \cos 12°)$$

$$= W\,(0.2079 + 0.131 \times 0.9781) = 0.336\,W$$

$$\therefore \quad W = 1500/0.336 = 4464 \text{ N Ans.}$$

Torque Required to Lift the Load by a Screw Jack

An electric motor driven power screw moves a nut in a horizontal plane against a force of 75 kN at a speed of 300 mm/min. The screw has a single square thread of 6 mm pitch on a major diameter of 40 mm. The coefficient of friction at the screw threads is 0.1. Estimate power of the motor.

Solution. Given : $W = 75$ kN $= 75 \times 10^3$ N ; $v = 300$ mm/min ; $p = 6$ mm ; $d_0 = 40$ mm ; $\mu = \tan\phi = 0.1$

We know that mean diameter of the screw,

$$d = d_0 - p/2 = 40 - 6/2 = 37 \text{ mm} = 0.037 \text{ m}$$

and

$$\tan\alpha = \frac{p}{\pi d} = \frac{6}{\pi \times 37} = 0.0516$$

$\therefore$ Force required at the circumference of the screw,

$$P = W \tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha.\tan\phi}\right]$$

$$= 75 \times 10^3 \left[\frac{0.0516 + 0.1}{1 - 0.0516 \times 0.1}\right] = 11.43 \times 10^3 \text{ N}$$

and torque required to overcome friction,

$$T = P \times d/2 = 11.43 \times 10^3 \times 0.037/2 = 211.45 \text{ N-m}$$

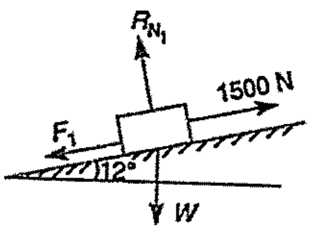
Friction of a Body Lying on a Rough Inclined Plane :

An effort of 1500 N is required to just move a certain body up an inclined plane of angle 12°, force acting parallel to the plane. If the angle of inclination is increased to 15°, then the effort required is 1720 N. Find the weight of the body and the coefficient of friction.

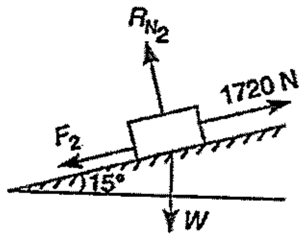
Solution. Given : $P_1 = 1500$ N ; $\alpha_1 = 12°$; $\alpha_2 = 15°$; $P_2 = 1720$ N

Let W = Weight of the body in newtons, and

μ = Coefficient of friction.



(a)



(b)

First of all, let us consider a body lying on a plane inclined at an angle of 12° with the horizontal and subjected to an effort of 1500 N parallel to the plane as shown in Fig. 10.10 (*a*).

Let R_{N_1} = Normal reaction, and

F_1 = Force of friction.

We know that for the motion of the body up the inclined plane, the effort applied parallel to the plane (P_1),

$$1500 = W(\sin\alpha_1 + \mu\cos\alpha_1) = W(\sin 12° + \mu\cos 12°) \quad \text{...}(i)$$

Now let us consider the body lying on a plane inclined at an angle of 15° with the horizontal and subjected to an effort of 1720 N parallel to the plane as shown in Fig. 10.10 (*b*).

Let R_{N_2} = Normal reaction, and

F_2 = Force of friction.

We know that for the motion of the body up the inclined plane, the effort applied parallel to the plane (P_2),

$$1720 = W(\sin\alpha_2 + \mu\cos\alpha_2) = W(\sin 15° + \mu\cos 15°) \quad \text{...}(ii)$$

Coefficient of friction

Dividing equation (*ii*) by equation (*i*),

$$\frac{1720}{1500} = \frac{W(\sin 15° + \mu\cos 15°)}{W(\sin 12° + \mu\cos 12°)}$$

Again by measurement from couple polygon,

$$-0.18\ m_A.y = \text{vector } o'd' = 3.6 \text{ kg-m}^2$$

$$-0.18 \times 20\ y = 3.6 \quad \text{or} \quad y = -1 \text{ m}$$

The negative sign indicates that the plane A is not towards left of B as assumed but it is 1 m or 1000 mm towards right of plane B. **Ans.**

Example 21.4. *A, B, C and D are four masses carried by a rotating shaft at radii 100, 125, 200 and 150 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the mass of B, C and D are 10 kg, 5 kg, and 4 kg respectively.*

Find the required mass A and the relative angular settings of the four masses so that the shaft shall be in complete balance.

Solution. Given : r_A = 100 mm = 0.1 m ; r_B = 125 mm = 0.125 m ; r_C = 200 mm = 0.2 m ; r_D = 150 mm = 0.15 m ; m_B = 10 kg ; m_C = 5 kg ; m_D = 4 kg

The position of planes is shown in Fig. 21.10 (*a*). Assuming the plane of mass A as the reference plane (*R.P.*), the data may be tabulated as below :

Table 21.4

Plane (1)	*Mass (m) kg* (2)	*Radius (r) m* (3)	*Cent. Force ÷ ω^2 (m.r) kg-m* (4)	*Distance from plane A (l) m* (5)	*Couple ÷ ω^2 (m.r.l) kg-m²* (6)
A(R.P.)	m_A	0.1	0.1 m_A	0	0
B	10	0.125	1.25	0.6	0.75
C	5	0.2	1	1.2	1.2
D	4	0.15	0.6	1.8	1.08

First of all, the angular setting of masses C and D is obtained by drawing the couple polygon from the data given in Table 21.4 (column 6). Assume the position of mass B in the horizontal direction OB as shown in Fig. 21.10 (*b*). Now the couple polygon as shown in Fig. 21.10 (*c*) is drawn as discussed below :

1. Draw vector $o'b'$ in the horizontal direction (*i.e.* parallel to OB) and equal to 0.75 kg-m², to some suitable scale.
2. From points o' and b', draw vectors $o'c'$ and $b'c'$ equal to 1.2 kg-m² and 1.08 kg-m² respectively. These vectors intersect at c'.
3. Now in Fig. 21.10 (*b*), draw OC parallel to vector $o'c'$ and OD parallel to vector $b'c'$.

By measurement, we find that the angular setting of mass C from mass B in the anticlockwise direction, *i.e.*

$$\angle BOC = 240° \textbf{ Ans.}$$

and angular setting of mass D from mass B in the anticlockwise direction, *i.e.*

$$\angle BOD = 100° \textbf{ Ans.}$$

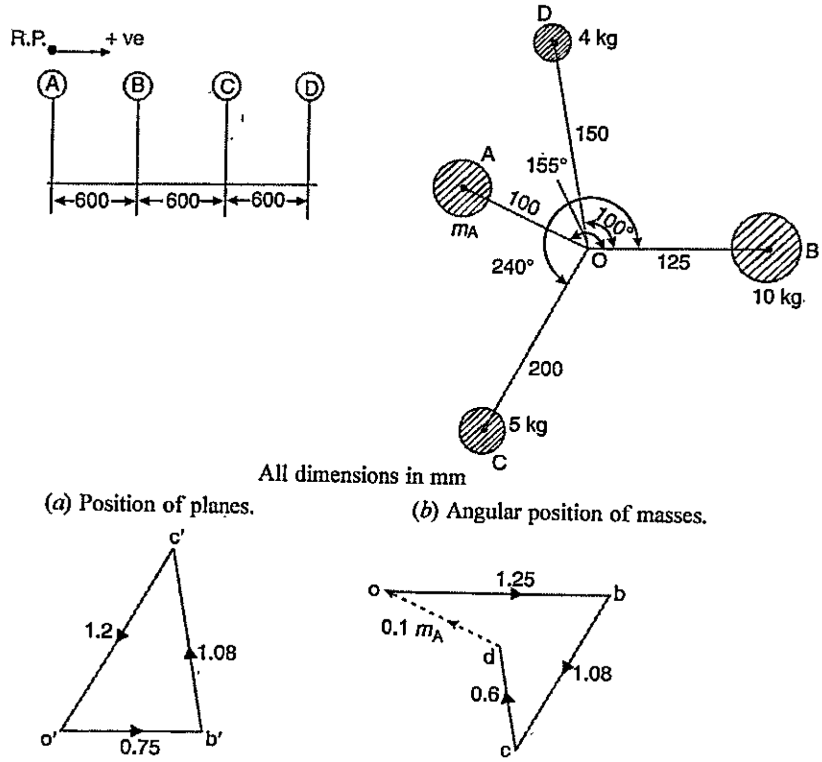
In order to find the required mass A (m_A) and its angular setting, draw the force polygon to some suitable scale, as shown in Fig. 21.10 (*d*), from the data given in Table 21.4 (column 4).

Since the closing side of the force polygon (vector *do*) is proportional to 0.1 m_A, therefore by measurement,

$$0.1\ m_A = 0.7 \text{ kg-m}^2 \quad \text{or} \quad m_A = 7 \text{ kg } \textbf{Ans.}$$

Now draw OA in Fig. 21.10 (b), parallel to vector do. By measurement, we find that the angular setting of mass A from mass B in the anticlockwise direction, *i.e.*

$$\angle BOA = 155° \textbf{ Ans.}$$



(a) Position of planes. (b) Angular position of masses.

(c) Couple polygon. (d) Force polygon.

Fig. 21.10

Example 21.5. *A shaft carries four masses in parallel planes A, B, C and D in this order along its length. The masses at B and C are 18 kg and 12.5 kg respectively, and each has an eccentricity of 60 mm. The masses at A and D have an eccentricity of 80 mm. The angle between the masses at B and C is 100° and that between the masses at B and A is 190°, both being measured in the same direction. The axial distance between the planes A and B is 100 mm and that between B and C is 200 mm. If the shaft is in complete dynamic balance, determine :*

1. The magnitude of the masses at A and D ; 2. the distance between planes A and D ; and 3. the angular position of the mass at D.

Solution. Given : $m_B = 18$ kg ; $m_C = 12.5$ kg ; $r_B = r_C = 60$ mm $= 0.06$ m ; $r_A = r_D = 80$ mm $= 0.08$ m ; $\angle BOC = 100°$; $\angle BOA = 190°$

1. *Magnitude of the masses at A and D*

Let M_A = Mass at A,

M_D = Mass at D, and

x = Distance between planes A and D.

It may be noted that equation (*i*) represents the condition for static balance, but in order to achieve dynamic balance, equations (*ii*) or (*iii*) must also be satisfied.

2. ***When the plane of the disturbing mass lies on one end of the planes of the balancing masses***

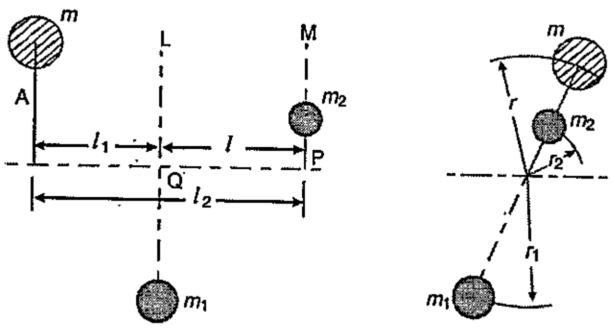


Fig. 21.3. Balancing of a single rotating mass by two rotating masses in different planes, when the plane of single rotating mass lies at one end of the planes of balancing masses.

In this case, the mass m lies in the plane A and the balancing masses lie in the planes L and M, as shown in Fig. 21.3. As discussed above, the following conditions must be satisfied in order to balance the system, *i.e.*

$$F_C + F_{C2} = F_{C1} \quad \text{or} \quad m \cdot \omega^2 \cdot r + m_2 \cdot \omega^2 \cdot r_2 = m_1 \cdot \omega^2 \cdot r_1$$

$$\therefore \quad m \cdot r + m_2 \cdot r_2 = m_1 \cdot r_1 \qquad \ldots (iv)$$

Now, to find the balancing force in the plane L (or the dynamic force at the bearing Q of a shaft), take moments about P which is the point of intersection of the plane M and the axis of rotation. Therefore

$$F_{C1} \times l = F_C \times l_2 \quad \text{or} \quad m_1 \cdot \omega^2 \cdot r_1 \times l = m \cdot \omega^2 \cdot r \times l_2$$

$$\therefore \quad m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2 \quad \text{or} \quad m_1 \cdot r_1 = m \cdot r \times \frac{l_2}{l} \qquad \ldots (v)$$

. . . [Same as equation (*ii*)]

Similarly, to find the balancing force in the plane M (or the dynamic force at the bearing P of a shaft), take moments about Q which is the point of intersection of the plane L and the axis of rotation. Therefore

$$F_{C2} \times l = F_C \times l_1 \quad \text{or} \quad m_2 \cdot \omega^2 \cdot r_2 \times l = m \cdot \omega^2 \cdot r \times l_1$$

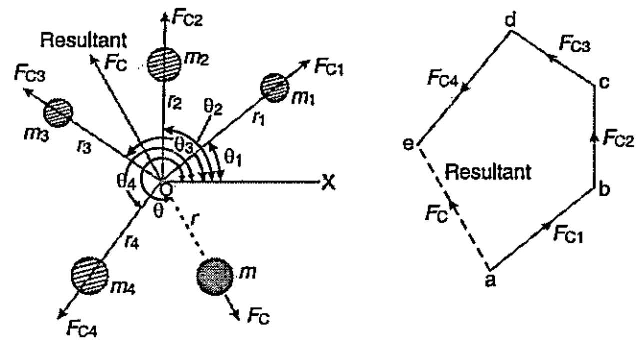
$$m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1 \quad \text{or} \quad m_2 \cdot r_2 = m \cdot r \times \frac{l_1}{l} \qquad \ldots (vi)$$

. . . [Same as equation (*iii*)]

21.5. Balancing of Several Masses Rotating in the Same Plane

Consider any number of masses (say four) of magnitude m_1, m_2, m_3 and m_4 at distances of r_1, r_2, r_3 and r_4 from the axis of the rotating shaft. Let $\theta_1, \theta_2, \theta_3$ and θ_4 be the angles of these masses with the horizontal line OX, as shown in Fig. 21.4 (*a*). Let these masses rotate about an axis through O and perpendicular to the plane of paper, with a constant angular velocity of ω rad/s.

The magnitude and position of the balancing mass may be found out analytically or graphically as discussed below :



(*a*) Space diagram. (*b*) Vector diagram.

Fig. 21.4. Balancing of several masses rotating in the same plane.

1. *Analytical method*

The magnitude and direction of the balancing mass may be obtained, analytically, as discussed below :

1. First of all, find out the centrifugal force* (or the product of the mass and its radius of rotation) exerted by each mass on the rotating shaft.

A car assembly line.

Note : This picture is given as additional information and is not a direct example of the current chapter.

---

* Since ω^2 is same for each mass, therefore the magnitude of the centrifugal force for each mass is proportional to the product of the respective mass and its radius of rotation.

Let m = Balancing mass; and

θ = The angle which the balancing mass makes with m_1.

Since the magnitude of centrifugal forces are proportional to the product of each mass and its radius, therefore

$$m_1 \cdot r_1 = 200 \times 0.2 = 40 \text{ kg-m}$$
$$m_2 \cdot r_2 = 300 \times 0.15 = 45 \text{ kg-m}$$
$$m_3 \cdot r_3 = 240 \times 0.25 = 60 \text{ kg-m}$$
$$m_4 \cdot r_4 = 260 \times 0.3 = 78 \text{ kg-m}$$

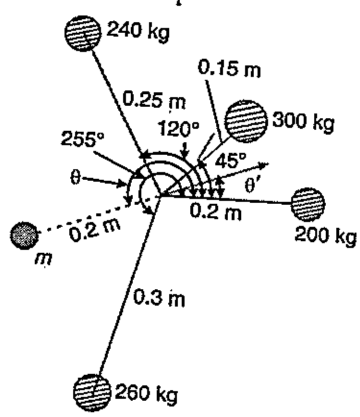


Fig. 21.5

The problem may, now, be solved either analytically or graphically. But we shall solve the problem by both the methods one by one.

1. *Analytical method*

The space diagram is shown in Fig. 21.5.

Resolving $m_1.r_1$, $m_2.r_2$, $m_3.r_3$ and $m_4.r_4$ horizontally,

$$\Sigma H = m_1 \cdot r_1 \cos\theta_1 + m_2 \cdot r_2 \cos\theta_2 + m_3 \cdot r_3 \cos\theta_3 + m_4 \cdot r_4 \cos\theta_4$$
$$= 40 \cos 0° + 45 \cos 45° + 60 \cos 120° + 78 \cos 255°$$
$$= 40 + 31.8 - 30 - 20.2 = 21.6 \text{ kg-m}$$

Now resolving vertically,

$$\Sigma V = m_1 \cdot r_1 \sin\theta_1 + m_2 \cdot r_2 \sin\theta_2 + m_3 \cdot r_3 \sin\theta_3 + m_4 \cdot r_4 \sin\theta_4$$
$$= 40 \sin 0° + 45 \sin 45° + 60 \sin 120° + 78 \sin 255°$$
$$= 0 + 31.8 + 52 - 75.3 = 8.5 \text{ kg-m}$$

$\therefore$ Resultant, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(21.6)^2 + (8.5)^2} = 23.2$ kg-m

We know that

$$m \cdot r = R = 23.2 \quad \text{or} \quad m = 23.2/r = 23.2/0.2 = 116 \text{ kg} \textbf{ Ans.}$$

and $\tan\theta' = \Sigma V / \Sigma H = 8.5/21.6 = 0.3935$ or $\theta' = 21.48°$

Since θ' is the angle of the resultant R from the horizontal mass of 200 kg, therefore the angle of the balancing mass from the horizontal mass of 200 kg,

$$\theta = 180° + 21.48° = 201.48° \textbf{ Ans.}$$

2. *Graphical method*

The magnitude and the position of the balancing mass may also be found graphically as discussed below :

1. First of all, draw the space diagram showing the positions of all the given masses as shown in Fig 21.6 (*a*).
2. Since the centrifugal force of each mass is proportional to the product of the mass and radius, therefore

$$m_1.r_1 = 200 \times 0.2 = 40 \text{ kg-m}$$
$$m_2.r_2 = 300 \times 0.15 = 45 \text{ kg-m}$$

2. Resolve the centrifugal forces horizontally and vertically and find their sums, *i.e.* ΣH and ΣV. We know that

Sum of horizontal components of the centrifugal forces,

$$\Sigma H = m_1 \cdot r_1 \cos\theta_1 + m_2 \cdot r_2 \cos\theta_2 + \ldots\ldots$$

and sum of vertical components of the centrifugal forces,

$$\Sigma V = m_1 \cdot r_1 \sin\theta_1 + m_2 \cdot r_2 \sin\theta_2 + \ldots\ldots$$

3. Magnitude of the resultant centrifugal force,

$$F_C = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

4. If θ is the angle, which the resultant force makes with the horizontal, then

$$\tan\theta = \Sigma V / \Sigma H$$

5. The balancing force is then equal to the resultant force, but in *opposite direction.*
6. Now find out the magnitude of the balancing mass, such that

$$F_C = m \cdot r$$

where m = Balancing mass, and

r = Its radius of rotation.

2. *Graphical method*

The magnitude and position of the balancing mass may also be obtained graphically as discussed below :

1. First of all, draw the space diagram with the positions of the several masses, as shown in Fig. 21.4 (*a*).
2. Find out the centrifugal force (or product of the mass and radius of rotation) exerted by each mass on the rotating shaft.
3. Now draw the vector diagram with the obtained centrifugal forces (or the product of the masses and their radii of rotation), such that *ab* represents the centrifugal force exerted by the mass m_1 (or $m_1.r_1$) in magnitude and direction to some suitable scale. Similarly, draw *bc*, *cd* and *de* to represent centrifugal forces of other masses m_2, m_3 and m_4 (or $m_2.r_2$, $m_3.r_3$ and $m_4.r_4$).
4. Now, as per polygon law of forces, the closing side *ae* represents the resultant force in magnitude and direction, as shown in Fig. 21.4 (*b*).
5. The balancing force is, then, equal to the resultant force, but in *opposite direction.*
6. Now find out the magnitude of the balancing mass (m) at a given radius of rotation (r), such that

$$m \cdot \omega^2 \cdot r = \text{Resultant centrifugal force}$$

or

$$m.r = \text{Resultant of } m_1.r_1,\ m_2.r_2,\ m_3.r_3 \text{ and } m_4.r_4$$

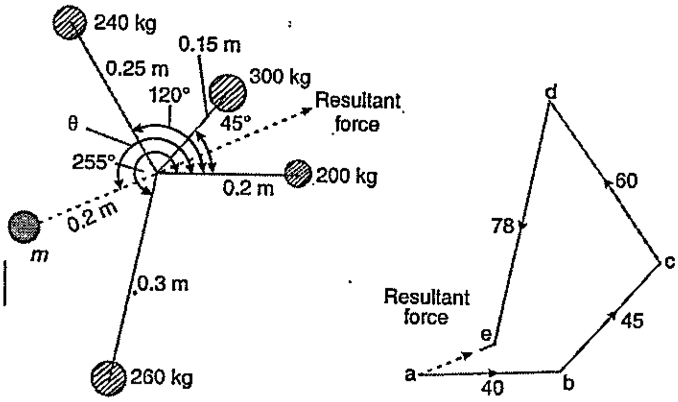
Example 21.1. *Four masses m_1, m_2, m_3 and m_4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required, if its radius of rotation is 0.2 m.*

Solution. Given : $m_1 = 200$ kg ; $m_2 = 300$ kg ; $m_3 = 240$ kg ; $m_4 = 260$ kg ; $r_1 = 0.2$ m ; $r_2 = 0.15$ m ; $r_3 = 0.25$ m ; $r_4 = 0.3$ m ; $\theta_1 = 0°$; $\theta_2 = 45°$; $\theta_3 = 45° + 75° = 120°$; $\theta_4 = 45° + 75° + 135° = 255°$; $r = 0.2$ m

$m_3.r_3 = 240 \times 0.25 = 60$ kg-m

$m_4.r_4 = 260 \times 0.3 = 78$ kg-m

3. Now draw the vector diagram with the above values, to some suitable scale, as shown in Fig. 21.6 (*b*). The closing side of the polygon *ae* represents the resultant force. By measurement, we find that *ae* = 23 kg-m.



(*a*) Space diagram. (*b*) Vector diagram

Fig. 21.6

4. The balancing force is equal to the resultant force, but ***opposite*** in direction as shown in Fig. 21.6 (*a*). Since the balancing force is proportional to *m.r*, therefore

$m \times 0.2 =$ vector $ea = 23$ kg-m or $m = 23/0.2 =$ **115 kg Ans.**

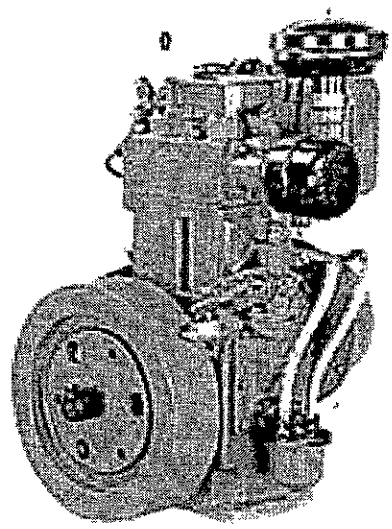
By measurement we also find that the angle of inclination of the balancing mass (*m*) from the horizontal mass of 200 kg,

$\theta = 201°$ **Ans.**

21.6. Balancing of Several Masses Rotating in Different Planes

When several masses revolve in different planes, they may be transferred to a ***reference plane*** (briefly written as ***R.P.***), which may be defined as the plane passing through a point on the axis of rotation and perpendicular to it. The effect of transferring a revolving mass (in one plane) to a reference plane is to cause a force of magnitude equal to the centrifugal force of the revolving mass to act in the reference plane, together with a couple of magnitude equal to the product of the force and the distance between the plane of rotation and the reference plane. In order to have a complete balance of the several revolving masses in different planes, the following two conditions must be satisfied :

1. The forces in the reference plane must balance, *i.e.* the resultant force must be zero.
2. The couples about the reference plane must balance, *i.e.* the resultant couple must be zero.



Diesel engine.

Let us now consider four masses m_1, m_2, m_3 and m_4 revolving in planes 1, 2, 3 and 4 respectively as shown in

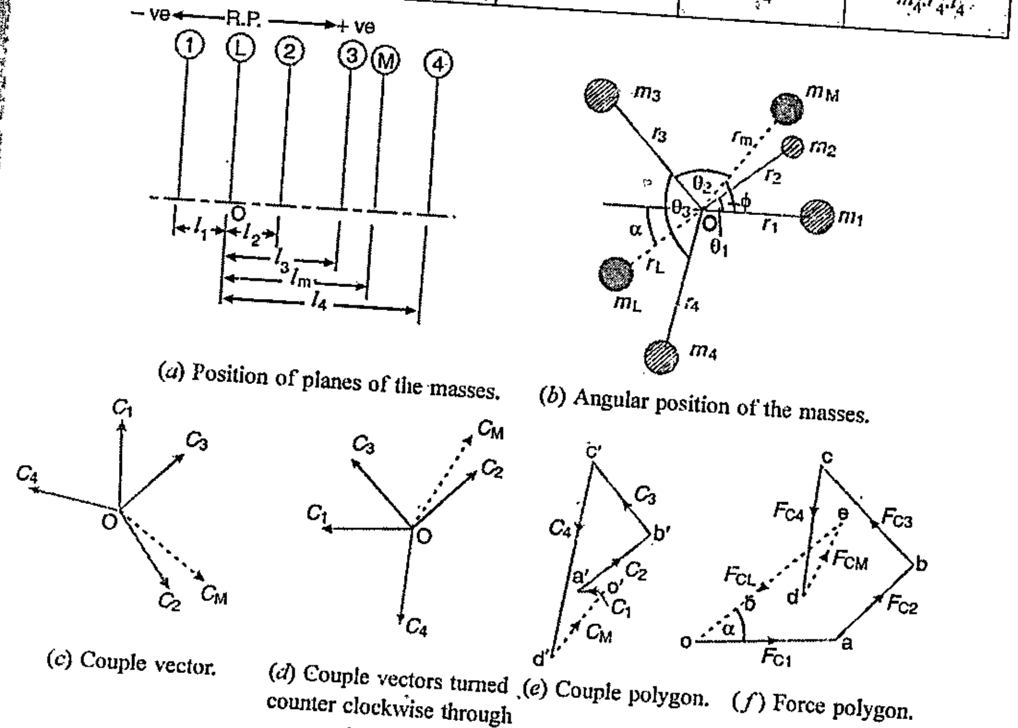
Fig. 21.7 (*a*). The relative angular positions of these masses are shown in the end view [Fig. 21.7 (*b*)]. The magnitude of the balancing masses m_L and m_M in planes L and M may be obtained as discussed below :

1. Take one of the planes, say L as the reference plane (*R.P.*). The distances of all the other planes to the left of the reference plane may be regarded as ***negative***, and those to the right as ***positive.***

2. Tabulate the data as shown in Table 21.1. The planes are tabulated in the same order in which they occur, reading from left to right.

Table 21.1

Plane (1)	*Mass (m)* (2)	*Radius(r)* (3)	*Cent.force* $\div \omega^2$ *(m.r)* (4)	*Distance from Plane L (l)* (5)	*Couple* $\div \omega^2$ *(m.r.l)* (6)
1	m_1	r_1	$m_1.r_1$	$-l_1$	$-m_1.r_1.l_1$
L(*R.P.*)	m_L	r_L	$m_L.r_L$	0	0
2	m_2	r_2	$m_2.r_2$	l_2	$m_2.r_2.l_2$
3	m_3	r_3	$m_3.r_3$	l_3	$m_3.r_3.l_3$
M	m_M	r_M	$m_M.r_M$	l_M	$m_M.r_M.l_M$
4	m_4	r_4	$m_4.r_4$	l_4	$m_4.r_4.l_4$



(*a*) Position of planes of the masses. (*b*) Angular position of the masses.

(*c*) Couple vector. (*d*) Couple vectors turned counter clockwise through a right angle. (*e*) Couple polygon. (*f*) Force polygon.

Fig. 21.7. Balancing of several masses rotating in different planes.

3. A couple may be represented by a vector drawn perpendicular to the plane of the couple. The couple C_1 introduced by transferring m_1 to the reference plane through O is propor-

tional to $m_1.r_1.l_1$ and acts in a plane through Om_1 and perpendicular to the paper. The vector representing this couple is drawn in the plane of the paper and perpendicular to Om_1 as shown by OC_1 in Fig. 21.7 (*c*). Similarly, the vectors OC_2, OC_3 and OC_4 are drawn perpendicular to Om_2, Om_3 and Om_4 respectively and in the plane of the paper.

4. The couple vectors as discussed above, are turned counter clockwise through a right angle for convenience of drawing as shown in Fig. 21.7 (*d*). We see that their relative positions remains unaffected. Now the vectors OC_2, OC_3 and OC_4 are parallel and in the same direction as Om_2, Om_3 and Om_4, while the vector OC_1 is parallel to Om_1 but in *opposite direction. Hence the ***couple vectors are drawn radially outwards for the masses on one side of the reference plane and radially inward for the masses on the other side of the reference plane.***

5. Now draw the couple polygon as shown in Fig. 21.7 (*e*). The vector $d'o'$ represents the balanced couple. Since the balanced couple C_M is proportional to $m_M.r_M.l_M$, therefore

$$C_M = m_M.r_M.l_M = \text{vector } d'o' \quad \text{or} \quad m_M = \frac{\text{vector } d'o'}{r_M.l_M}$$

From this expression, the value of the balancing mass m_M in the plane *M* may be obtained, and the angle of inclination ϕ of this mass may be measured from Fig. 21.7 (*b*).

6. Now draw the force polygon as shown in Fig. 21.7 (*f*). The vector *eo* (in the direction from *e* to *o*) represents the balanced force. Since the balanced force is proportional to $m_L.r_L$, therefore,

$$m_L.r_L = \text{vector } eo \quad \text{or} \quad m_L = \frac{\text{vector } eo}{r_L}$$

From this expression, the value of the balancing mass m_L in the plane *L* may be obtained and the angle of inclination α of this mass with the horizontal may be measured from Fig. 21.7 (*b*).

Example 21.2. *A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.*

Solution. Given : $m_A = 200$ kg ; $m_B = 300$ kg ; $m_C = 400$ kg ; $m_D = 200$ kg ; $r_A = 80$ mm $= 0.08$m ; $r_B = 70$ mm $= 0.07$ m ; $r_C = 60$ mm $= 0.06$ m ; $r_D = 80$ mm $= 0.08$ m ; $r_X = r_Y = 100$ mm $= 0.1$ m

Let m_X = Balancing mass placed in plane *X*, and

m_Y = Balancing mass placed in plane *Y*.

The position of planes and angular position of the masses (assuming the mass A as horizontal) are shown in Fig. 21.8 (*a*) and (*b*) respectively.

Assume the plane *X* as the reference plane (*R.P.*). The distances of the planes to the right of plane *X* are taken as + ve while the distances of the planes to the left of plane *X* are taken as – ve. The data may be tabulated as shown in Table 21.2.

* From Table 21.1 (column 6) we see that the couple is $-m_1.r_1.l_1$.

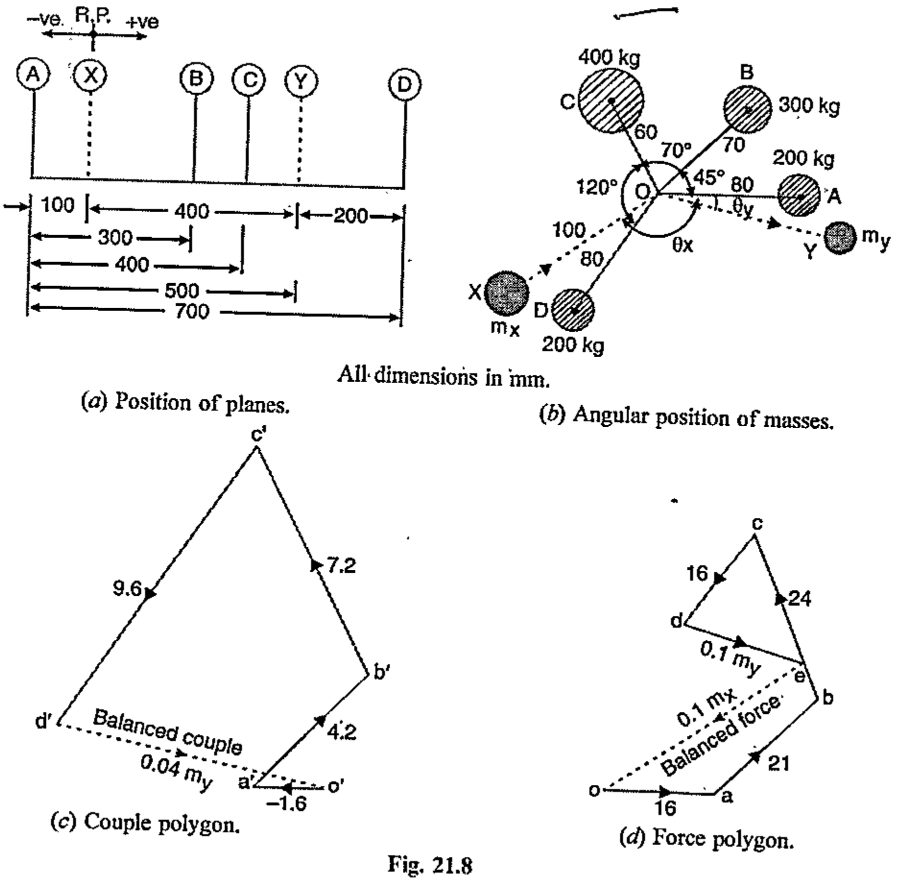
Table 21.2

Plane (1)	*Mass (m)* kg (2)	*Radius (r)* m (3)	*Cent.force* ÷ ω^2 (m.r) kg-m (4)	*Distance from Plane x(l)* m (5)	*Couple* ÷ ω^2 (m.r.l) kg-m² (6)
A	200	0.08	16	– 0.1	– 1.6
X(R.P.)	m_X	0.1	0.1 m_X	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_Y	0.1	0.1 m_Y	0.4	0.04 m_Y
D	200	0.08	16	0.6	9.6

The balancing masses m_X and m_Y and their angular positions may be determined graphically as discussed below :

1. First of all, draw the couple polygon from the data given in Table 21.2 (column 6) as shown in Fig. 21.8 (*c*) to some suitable scale. The vector $d'o'$ represents the balanced couple. Since the balanced couple is proportional to 0.04 m_Y, therefore by measurement,

$$0.04\, m_Y = \text{vector } d'o' = 7.3 \text{ kg-m}^2 \quad \text{or} \quad m_Y = 182.5 \text{ kg } \textbf{Ans.}$$



All dimensions in mm.

(*a*) Position of planes.

(*b*) Angular position of masses.

(*c*) Couple polygon.

(*d*) Force polygon.

Fig. 21.8

The angular position of the mass m_Y is obtained by drawing Om_Y in Fig. 21.8 (*b*), parallel to vector $d'o'$. By measurement, the angular position of m_Y is $\theta_Y = 12°$ in the clockwise direction from mass m_A (*i.e.* 200 kg). **Ans.**

2. Now draw the force polygon from the data given in Table 21.2 (column 4) as shown in Fig. 21.8 (*d*). The vector *eo* represents the balanced force. Since the balanced force is proportional to 0.1 m_X, therefore by measurement,

$$0.1\, m_X = \text{vector } eo = 35.5 \text{ kg-m} \quad \text{or} \quad m_X = 355 \text{ kg } \textbf{Ans.}$$

The angular position of the mass m_X is obtained by drawing Om_X in Fig. 21.8 (*b*), parallel to vector *eo*. By measurement, the angular position of m_X is $\theta_X = 145°$ in the clockwise direction from mass m_A (*i.e.* 200 kg). **Ans.**

Example 21.3. *Four masses A, B, C and D as shown below are to be completely balanced.*

	A	*B*	*C*	*D*
Mass (kg)	—	*30*	*50*	*40*
Radius (mm)	*180*	*240*	*120*	*150*

The planes containing masses B and C are 300 mm apart. The angle between planes containing B and C is 90°. B and C make angles of 210° and 120° respectively with D in the same sense. Find :

1. The magnitude and the angular position of mass A ; and

2. The position of planes A and D.

Solution. Given : r_A = 180 mm = 0.18 m ; m_B = 30 kg ; r_B = 240 mm = 0.24 m ; m_C = 50 kg ; r_C = 120 mm = 0.12 m ; m_D = 40 kg ; r_D = 150 mm = 0.15 m ; $\angle BOC = 90°$; $\angle BOD = 210°$; $\angle COD = 120°$

1. ***The magnitude and the angular position of mass A***

Let m_A = Magnitude of Mass *A*,

x = Distance between the planes *B* and *D*, and

y = Distance between the planes *A* and *B*.

The position of the planes and the angular position of the masses is shown in Fig. 21.9 (*a*) and (*b*) respectively.

Assuming the plane *B* as the reference plane (*R.P.*) and the mass *B* (m_B) along the horizontal line as shown in Fig. 21.9 (*b*), the data may be tabulated as below :

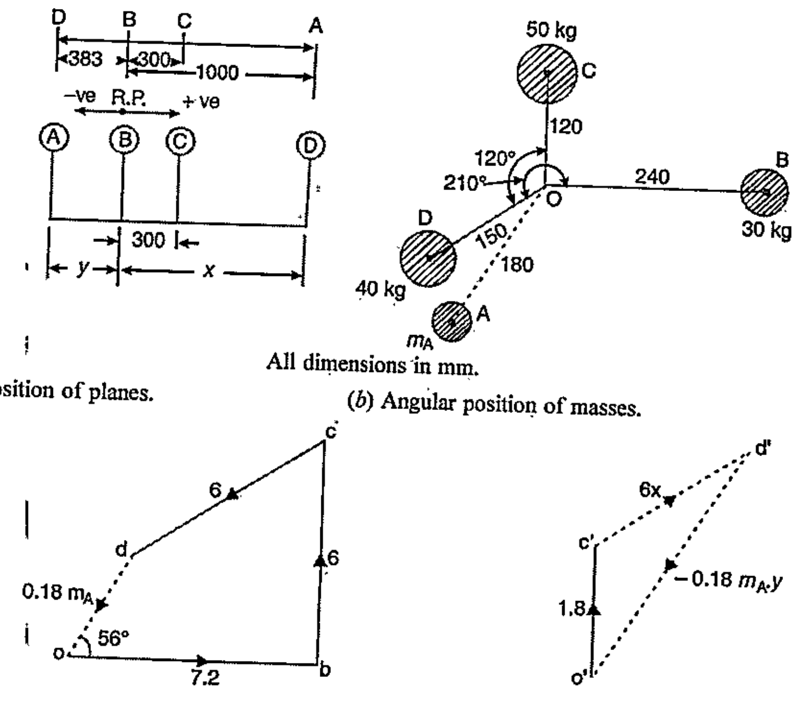
Table 21.3

Plane (1)	*Mass (m) kg* (2)	*Radius (r) m* (3)	*Cent.force ÷ ω²* *(m.r) kg-m* (4)	*Distance from plane B (l) m* (5)	*Couple ÷ ω²* *(m.r.l) kg-m²* (6)
A	m_A	0.18	$0.08\, m_A$	$-y$	$-0.18\, m_A.y$
B (*R.P*)	30	0.24	7.2	0	0
C	50	0.12	6	0.3	1.8
D	40	0.15	6	x	$6x$

The magnitude and angular position of mass *A* may be determined by drawing the force polygon from the data given in Table 21.3 (Column 4), as shown in Fig. 21.9 (*c*), to some suitable scale. Since the masses are to be completely balanced, therefore the force polygon must be a closed figure. The closing side (*i.e.* vector *do*) is proportional to $0.18\ m_A$. By measurement,

$$0.18\ m_A = \text{Vector } do = 3.6 \text{ kg-m} \quad \text{or} \quad m_A = 20 \text{ kg } \textbf{Ans.}$$

In order to find the angular position of mass *A*, draw *OA* in Fig. 21.9 (*b*) parallel to vector *do*. By measurement, we find that the angular position of mass *A* from mass *B* in the anticlockwise direction is $\angle AOB = 236°$ **Ans.**



(*a*) Position of planes.

All dimensions in mm.

(*b*) Angular position of masses.

(*c*) Force polygon.

(*d*) Couple polygon.

Fig. 21.9.

2. *Position of planes A and D*

The position of planes *A* and *D* may be obtained by drawing the couple polygon, as shown in Fig. 21.9 (*d*), from the data given in Table 21.3 (column 6). The couple polygon is drawn as discussed below :

1. Draw vector $o'c'$ parallel to *OC* and equal to 1.8 kg-m², to some suitable scale.

2. From points c' and o', draw lines parallel to *OD* and *OA* respectively, such that they intersect at point d'. By measurement, we find that

$$6x = \text{vector } c'd' = 2.3 \text{ kg-m}^2 \text{ or } x = 0.383 \text{ m}$$

We see from the couple polygon that the direction of vector $c'd'$ is opposite to the direction of mass *D*. Therefore the plane of mass *D* is 0.383 m or 383 mm towards left of plane *B* and not towards right of plane *B* as already assumed. **Ans.**

FRICTION:

whenever one block moves or tends to move tangentially with respect to the surface, on which it rests, the interlocking property of the projecting particles opposes the motion. This opposing force, which acts in the opposite direction of the movement of the upper block, is called the force of friction or simply friction.

Types of Friction: In general, the friction is of the following two types :

1. ***Static friction.*** It is the friction, experienced by a body, when at rest.

2. ***Dynamic friction.*** It is the friction, experienced by a body, when in motion. The dynamic friction is also called ***kinetic friction*** and is less than the static friction.
It is of the following three types :

(*a*) ***Sliding friction.*** It is the friction, experienced by a body , when it ***slides*** over another body.

(*b*) ***Rolling friction.*** It is the friction, experienced between the surfaces which has ***balls*** or ***rollers*** interposed between them.

(*c*) ***Pivot friction.*** It is the friction. experienced by a body, due to the ***motion of rotation*** as in case of foot step bearings.

The friction may further be classified as :

10.3. Friction Between Unlubricated Surfaces

The friction experienced between two dry and unlubricated surfaces in contact is known as ***dry*** or ***solid friction***.

10.4. Friction Between Lubricated Surfaces

When lubricant (*i.e.* oil or grease) is applied between two surfaces in contact, then the friction may be classified into the following two types depending upon the thickness of layer of a lubricant.

1. ***Boundary friction*** (***or greasy friction or non-viscous friction***). It is the friction, experienced between the rubbing surfaces, when the surfaces have a very thin layer of lubricant.

2. ***Fluid friction*** (***or film friction or viscous friction***). It is the friction, experienced between the rubbing surfaces, when the surfaces have a thick layer of the lubrhicant.

10.5. Limiting Friction

The maximum value of frictional force, which comes into play, when a body just begins to slide over the surface of the other body, is known as limiting force of friction or simply limiting friction.

10.6. Laws of Static Friction

Following are the laws of static friction :

1. The force of friction always acts in a direction, opposite to that in which the body tends to move.
2. The magnitude of the force of friction is exactly equal to the force, which tends the body to move.
3. The magnitude of the limiting friction (F) bears a constant ratio to the normal reaction (R_N) between the two surfaces. Mathematically

$$F/R_N = \text{constant}$$

4. The force of friction is independent of the area of contact, between the two surfaces.

5.The force of friction depends upon the roughness of the surfaces.

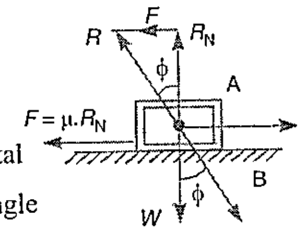
COEFFICIENT OF FRICTION: It is defined as the ratio of the limiting friction (F) to the normal reaction (RN) between the two bodies.

$$\mu = F/RN$$

ANGLE OF FRICTION:

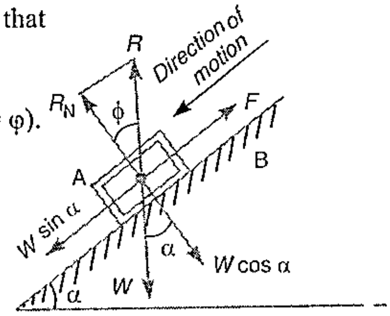
It may be defined as the angle which the resultant reaction R makes with the normal reaction RN.

tan φ = F/RN = μRN/RN = μ



ANGLE OF REPOSE:

If the angle of inclination α of the plane to the horizontal is such that the body begins to move down the plane, then the angle α is called the angle of repose. A little consideration will show that the body will begin to move down the plane when the angle of inclination of the plane is equal to the angle of friction (i.e. $\alpha = \varphi$).



10.14. Friction of a Body Lying on a Rough Inclined Plane

1. *Considering the motion of the body up the plane*

Let W = Weight of the body,

α = Angle of inclination of the plane to the horizontal.

ϕ = Limiting angle of friction for the contact surfaces,

P = Effort applied in a given direction in order to cause the body to slide with uniform velocity parallel to the plane, considering friction,

P_0 = Effort required to move the body up the plane neglecting friction,

θ = Angle which the line of action of P makes with the weight of the body W,

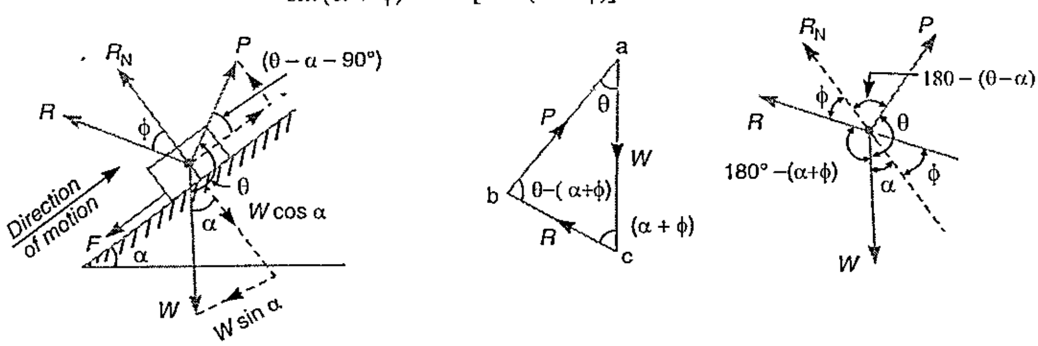
μ = Coefficient of friction between the surfaces of the plane and the body,

R_N = Normal reaction, and

R = Resultant reaction.

When friction is taken into account, a frictional force $F = \mu.R_N$ acts in the direction opposite to the motion of the body, as shown in Fig. 10.8 (*a*). The resultant reaction R between the plane and the body is inclined at an angle ϕ with the normal reaction R_N. The triangle of forces is shown in Fig. 10.8 (*b*). Now applying sine rule,

$$\frac{P}{\sin(\alpha + \phi)} = \frac{W}{\sin[\theta - (\alpha + \phi)]}$$



(*a*) (*b*) (*c*)

Fig. 10.8. Motion of the body up the plane, considering friction.

$$\therefore \quad P = \frac{W \sin(\alpha + \phi)}{\sin[\theta - (\alpha + \phi)]} \qquad ...(ii)$$

Notes : 1. When the effort applied is horizontal, then $\theta = 90°$. In that case, the equations (*i*) and (*ii*) may be written as

$$P = \frac{W \sin(\alpha + \phi)}{\sin[90^\circ - (\alpha + \phi)]} = \frac{W \sin(\alpha + \phi)}{\cos(\alpha + \phi)} = W \tan(\alpha + \phi)$$

2. When the effort applied is parallel to the plane, then $\theta = 90^\circ + \alpha$. In that case, the equations (*i*) and (*ii*) may be written as

$$P = \frac{W \sin(\alpha + \phi)}{\sin[(90^\circ + \alpha) - (\alpha + \phi)]} = \frac{W \sin(\alpha + \phi)}{\cos \phi}$$

$$= \frac{W(\sin\alpha \cos\phi + \cos\alpha \sin\phi)}{\cos\phi} = W(\sin\alpha + \cos\alpha . \tan\phi)$$

$$= W(\sin\alpha + \mu \cos\alpha) \qquad ...(\because \mu = \tan\phi)$$

2. ***Considering the motion of the body down the plane***

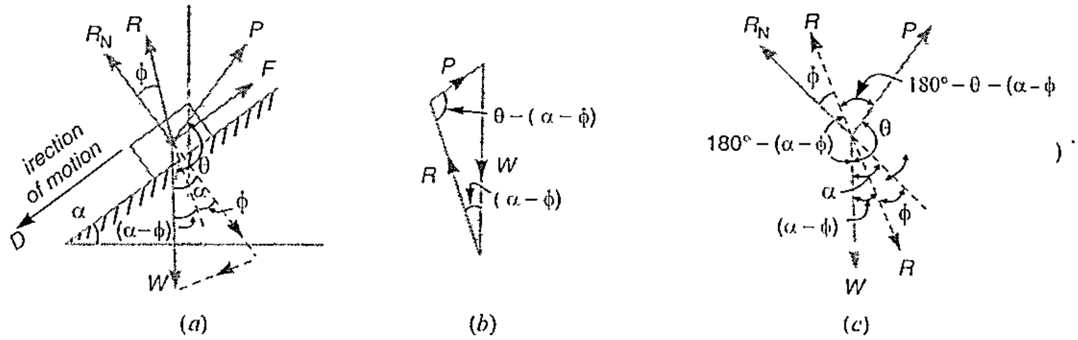


Fig. 10.9. Motion of the body down the plane. considering friction.

When the friction is taken into account, the force of friction $F = \mu.R_N$ will act up the plane and the resultant reaction R will make an angle ϕ with R_N towards its right as shown in Fig. 10.9 (*a*). The triangle of forces is shown in Fig. 10.9 (*b*). Now from sine rule,

$$\frac{P}{\sin(\alpha - \phi)} = \frac{W}{\sin[\theta - (\alpha - \phi)]}$$

or

$$P = \frac{W \sin(\alpha - \phi)}{\sin[\theta - (\alpha - \phi)]} \qquad ...(iv)$$

10.15. Efficiency of Inclined Plane

The ratio of the effort required neglecting friction (*i.e.* P_0) to the effort required considering friction (*i.e.* P) is known as efficiency of the inclined plane. Mathematically efficiency of the inclined plane,

1. ***For the motion of the body up the plane***

$$\eta = \frac{\cot(\alpha + \phi) - \cot\theta}{\cot\alpha - \cot\theta}$$

2. ***For the motion of the body down the plane***

$$\eta = \frac{\cot\alpha - \cot\theta}{\cot(\alpha - \phi) - \cot\theta}$$

Example 1. *An effort of 1500 N is required to just move a certain body up an inclined plane of angle 12°, force acting parallel to the plane. If the angle of inclination is increased to 15 °, then the effort required is 1720 N. Find the weight of the body and the coefficient of friction.*

Solution. Given : $P_1 = 1500$ N ; $\alpha_1 = 12^\circ$; $\alpha_2 = 15^\circ$; $P_2 = 1720$ N

Let W = Weight of the body in newtons, and μ = Coefficient of friction.

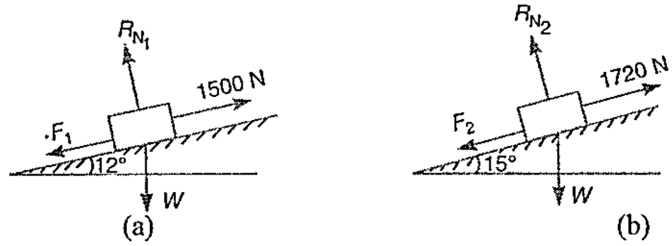


Fig. 10.10

First of all, let us consider a body lying on a plane inclined at an angle of 12° with the horizontal and subjected to an effort of 1500 N parallel to the plane as shown in Fig. 10.10 (*a*).

Let R_{N_1} = Normal reaction, and

F_1 = Force of friction.

We know that for the motion of the body up the inclined plane, the effort applied parallel to the plane (P_1),

$$1500 = W(\sin\alpha_1 + \mu\cos\alpha_1) = W(\sin 12^\circ + \mu\cos 12^\circ) \quad ...(i)$$

Now let us consider the body lying on a plane inclined at an angle of 15° with the horizontal and subjected to an effort of 1720 N parallel to the plane as shown in Fig. 10.10 (*b*).

Let R_{N_2} = Normal reaction, and

F_2 = Force of friction.

We know that for the motion of the body up the inclined plane, the effort applied parallel to the plane (P_2),

$$1720 = W(\sin\alpha_2 + \mu\cos\alpha_2) = W(\sin 15^\circ + \mu\cos 15^\circ) \quad ...(ii)$$

Coefficient of friction

Dividing equation (*ii*) by equation (*i*),

$$\frac{1720}{1500} = \frac{W(\sin 15^\circ + \mu\cos 15^\circ)}{W(\sin 12^\circ + \mu\cos 12^\circ)}$$

$$1720\sin 12^\circ + 1720\mu\cos 12^\circ = 1500\sin 15^\circ + 1500\mu\cos 15^\circ$$

$$\mu(1720\cos 12^\circ - 1500\cos 15^\circ) = 1500\sin 15^\circ - 1720\sin 12^\circ$$

$$\therefore \mu = \frac{1500\sin 15^\circ - 1720\sin 12^\circ}{1720\cos 12^\circ - 1500\cos 15^\circ} = \frac{1500\times 0.2588 - 1720\times 0.2079}{1720\times 0.9781 - 1500\times 0.9659}$$

$$= \frac{388.2 - 357.6}{1682.3 - 1448.5} = \frac{30.6}{233.8} = 0.131 \textbf{ Ans.}$$

Weight of the body

Substituting the value of μ in equation (*i*),

$$1500 = W(\sin 12^\circ + 0.131\cos 12^\circ) = W(0.2079 + 0.131\times 0.9781) = 0.336\,W$$

$$\therefore \quad W = 1500/0.336 = 4464 \text{ N } \textbf{Ans.}$$

10.16. Screw Friction

The screws, bolts, studs, nuts etc. are widely used in various machines and structures for temporary fastenings. The screw threads are mainly of two types *i.e.* V-threads and squarethreads. The V-threads are stronger and ofer more frictional resistance to motion than square threads.

The following terms are important for the study of screw :

1. ***Helix.*** It is the curve traced by a particle while moving along a screw thread.

2. ***Pitch.*** It is the distance from a point of a screw to a corresponding point on the next thread, measured parallel to the axis of the screw.

3. ***Lead.*** It is the distance, a screw thread advances axially in one turn.

4. ***Depth of thread.*** It is the distance between the top and bottom surfaces of a thread (also known as **crest** and **root** of a thread).

5. ***Single-threaded screw.*** If the lead of a screw is equal to its pitch, it is known as single threaded screw.

6. ***Multi-threaded screw.*** If more than one thread is cut in one lead distance of a screw, it is known as multi-threaded screw.

Lead = Pitch × Number of threads

7. ***Helix angle.*** It is the slope or inclination of the thread with the horizontal.

$$\tan \alpha = \frac{\text{Lead of screw}}{\text{Circumference of screw}}$$

$= p/\pi d$ ...(In single-threaded screw)

$= n.p/\pi d$ ...(In multi-threaded screw)

where α = Helix angle,

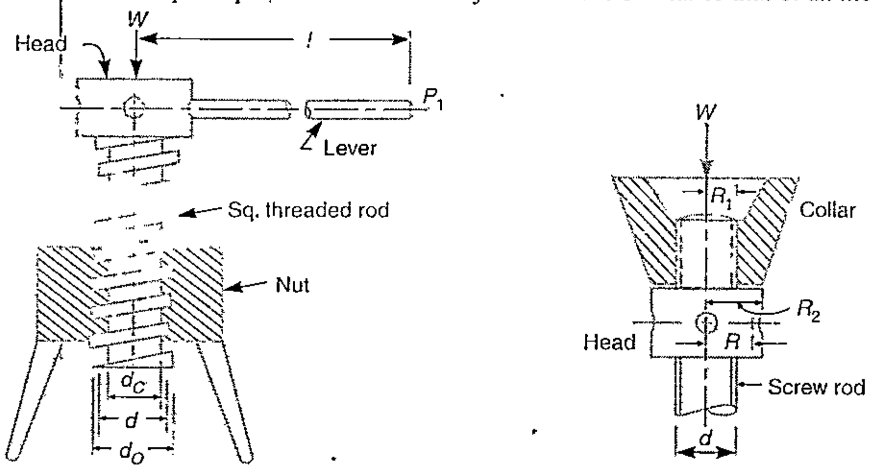
p = Pitch of the screw,

d = Mean diameter of the screw, and

n = Number of threads in one lead.

10.17. Screw Jack

The screw jack is a device, for lifting heavy loads, by applying a comparatively smaller effort at its handle. The principle, on which a screw jack works is similar to that of an inclined plane.

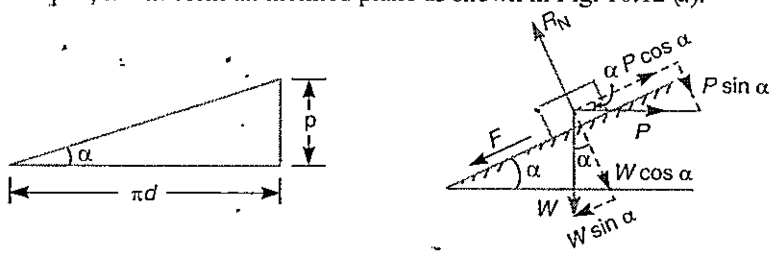


(*a*) Screw jack. (*b*) Thrust collar.

Fig. 10.11

10.18. Torque Required to Lift the Load by a Screw Jack

If one complete turn of a screw thread by imagined to be unwound, from the body of the screw and developed, it will form an inclined plane as shown in Fig. 10.12 (*a*).



(*a*) Development of a screw. (*b*) Forces acting on the screw.

Fig. 10.12

Let p = Pitch of the screw,

d = Mean diameter of the screw,

α = Helix angle,

P = Effort applied at the circumference of the screw to lift the load,

W = Load to be lifted, and

μ = Coefficient of friction, between the screw and nut = $\tan \phi$, where ϕ is the friction angle.

From the geometry of the Fig. 10.12 (*a*), we find that

$$\tan \alpha = p/\pi d$$

Since the principle on which a screw jack works is similar to that of an inclined plane, therefore the force applied on the lever of a screw jack may be considered to be horizontal as shown in Fig. 10.12 (*b*).

Since the load is being lifted, therefore the force of friction ($F = \mu.R_N$) will act downwards. All the forces acting on the screw are shown in Fig. 10.12 (*b*).

Resolving the forces along the plane,

$$P \cos \alpha = W \sin \alpha + F = W \sin \alpha + \mu.R_N \quad ...(i)$$

and resolving the forces perpendicular to the plane,

$$R_N = P \sin \alpha + W \cos \alpha \quad ...(ii)$$

Substituting this value of R_N in equation (*i*),

$$P \cos \alpha = W \sin \alpha + \mu (P \sin \alpha + W \cos \alpha)$$

$$= W \sin \alpha + \mu P \sin \alpha + \mu W \cos \alpha$$

or $$P \cos \alpha - \mu P \sin \alpha = W \sin \alpha + \mu W \cos \alpha$$

or $$P(\cos \alpha - \mu \sin \alpha) = W (\sin \alpha + \mu \cos \alpha)$$

$$\therefore \quad P = W \times \frac{\sin \alpha + \mu \cos \alpha}{\cos \alpha - \mu \sin \alpha}$$

Substituting the value of $\mu = \tan \phi$ in the above equation, we get

$$P = W \times \frac{\sin \alpha + \tan \phi \cos \alpha}{\cos \alpha - \tan \phi \sin \alpha}$$

Multiplying the numerator and denominator by $\cos \phi$,

$$P = W \times \frac{\sin \alpha \cos \phi + \sin \phi \cos \alpha}{\cos \alpha \cos \phi - \sin \alpha \sin \phi} = W \times \frac{\sin (\alpha + \phi)}{\cos (\alpha + \phi)}$$

$$= W \tan (\alpha + \phi)$$

$\therefore$ Torque required to overcome friction between the screw and nut,

$$T_1 = P \times \frac{d}{2} = W \tan (\alpha + \phi) \frac{d}{2}$$

When the axial load is taken up by a thrust collar or a flat surface, as shown in Fig. 10.11(*b*), so that the load does not rotate with the screw, then the torque required to overcome friction at the collar,

$$T_2 = \mu_1.W \left(\frac{R_1 + R_2}{2} \right) = \mu_1.W.R$$

where R_1 and R_2 = Outside and inside radii of the collar,

R = Mean radius of the collar, and

μ_1 = Coefficient of friction for the collar.

$\therefore$ Total torque required to overcome friction (*i.e.* to rotate the screw),

$$T = T_1 + T_2 = P \times \frac{d}{2} + \mu_1.W.R$$

* The **nominal diameter** of a screw thread is also known as **outside diameter** or **major diameter**.

** The **core diameter** of a screw thread is also known as inner diameter or **root diameter** or **minor diameter**.

10.19. Torque Required to Lower the Load by a Screw Jack

We have discussed in Art. 10.18, that the principle on which the screw jack works is similar to that of an inclined plane. If one complete turn of a screw thread be imagined to be unwound from the body of the screw and developed, it will form an inclined plane as shown in Fig. 10.13 (*a*).

Let p = Pitch of the screw,

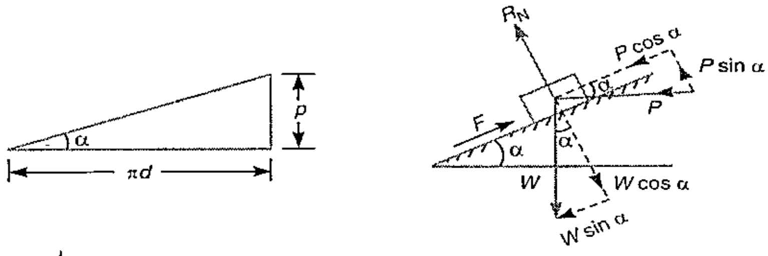
d = Mean diameter of the screw,

α = Helix angle,

P = Effort applied at the circumference of the screw to lower the load,

W = Weight to be lowered, and

μ = Coefficient of friction between the screw and nut = $\tan \phi$, where ϕ is the friction angle.



(a) (b)

Fig. 10.13

From the geometry of the figure, we find that

$$\tan\alpha = p/\pi d$$

Since the load is being lowered, therefore the force of friction ($F = \mu.R_N$) will act upwards. All the forces acting on the screw are shown in Fig. 10.13 (b).

Resolving the forces along the plane,

$$P\cos\alpha = F - W\sin\alpha = \mu.R_N - W\sin\alpha \qquad ...(i)$$

and resolving the forces perpendicular to the plane,

$$R_N = W\cos\alpha - P\sin\alpha \qquad ...(ii)$$

Substituting this value of R_N in equation (i),

$$P\cos\alpha = \mu\,(W\cos\alpha - P\sin\alpha) - W\sin\alpha$$

$$= \mu.W\cos\alpha - \mu.P\sin\alpha - W\sin\alpha$$

or $$P\cos\alpha + \mu.P\sin\alpha = \mu.W\cos\alpha - W\sin\alpha$$

or $$P(\cos\alpha + \mu\sin\alpha) = W(\mu\cos\alpha - \sin\alpha)$$

$\therefore$ $$P = W \times \frac{(\mu\cos\alpha - \sin\alpha)}{(\cos\alpha + \mu\sin\alpha)}$$

Substituting the value of $\mu = \tan\phi$ in the above equation, we get

$$P = W \times \frac{(\tan\phi\cos\alpha - \sin\alpha)}{(\cos\alpha + \tan\phi\sin\alpha)}$$

Multiplying the numerator and denominator by $\cos\phi$,

$$P = W \times \frac{(\sin\phi\cos\alpha - \sin\alpha\cos\phi)}{(\cos\alpha\cos\phi + \sin\phi\sin\alpha)} = W \times \frac{\sin(\phi - \alpha)}{\cos(\phi - \alpha)}$$

$$= W\tan(\phi - \alpha)$$

$\therefore$ Torque required to overcome friction between the screw and nut,

$$T = P \times \frac{d}{2} = W\tan(\phi - \alpha)\frac{d}{2}$$

Note: When $\alpha > \phi$, then $P = \tan(\alpha - \phi)$.

Example 2. *The mean diameter of a square threaded screw jack is 50 mm. The pitch of the thread is 10 mm. The coefficient of friction is 0.15. What force must be applied at the end of a 0.7 m long lever, which is perpendicular to the longitudinal axis of the screw to raise a load of 20 kN and to lower it?*

Solution. Given : $d = 50$ mm $= 0.05$ m ; $p = 10$ mm ; $\mu = \tan\phi = 0.15$; $l = 0.7$ m ; $W = 20$ kN $= 20 \times 10^3$ N

We know that $$\tan\alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 50} = 0.0637$$

Let P_1 = Force required at the end of the lever.

Force required to raise the load

We know that force required at the circumference of the screw

$$P = W\tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha.\tan\phi}\right]$$

$$= 20 \times 10^3 \left[\frac{0.0637 + 0.15}{1 - 0.0637 \times 0.15} \right] = 4314 \text{ N}$$

Now the force required at the end of the lever may be found out by the relation,

$$P_1 \times l = P \times d/2$$

$$\therefore \quad P_1 = \frac{P \times d}{2\,l} = \frac{4314 \times 0.05}{2 \times 0.7} = 154 \text{ N} \quad \textbf{Ans.}$$

Force required to lower the load

We know that the force required at the circumference of the screw,

$$P = W \tan(\phi - \alpha) = W \left[\frac{\tan\phi - \tan\alpha}{1 + \tan\phi . \tan\alpha} \right]$$

$$= 20 \times 10^3 \left[\frac{0.15 - 0.0637}{1 + 0.15 \times 0.0637} \right] = 1710 \text{ N}$$

Now the force required at the end of the lever may be found out by the relation,

$$P_1 \times l = P \times \frac{d}{2} \quad \text{or} \quad P_1 = \frac{P \times d}{2l} = \frac{1710 \times 0.05}{2 \times 0.7} = 61 \text{ N} \quad \textbf{Ans.}$$

10.20. Efficiency of a Screw Jack

The efficiency of a screw jack may be defined as **the ratio between the ideal effort** (*i.e.* the effort required to move the load, neglecting friction) to **the actual effort** (*i.e.* the effort required to move the load taking friction into account).

We

$$\therefore \text{ Efficiency, } \eta = \frac{\text{Ideal effort}}{\text{Actual effort}} = \frac{P_0}{P} = \frac{W \tan\alpha}{W \tan(\alpha + \phi)} = \frac{\tan\alpha}{\tan(\alpha + \phi)}$$

which shows that the efficiency of a screw jack, is independent of the load raised.

Example 3. *The pitch of 50 mm mean diameter threaded screw of a screw jack is 12.5 mm. The coefficient of friction between the screw and the nut is 0.13. Determine the torque required on the screw to raise a load of 25 kN, assuming the load to rotate with the screw Determine the ratio of the torque required to raise the load to the torque required to lower the load and also the efficiency of the machine.*

Solution. Given : $d = 50$ mm ; $p = 12.5$ mm ; $\mu = \tan\phi = 0.13$; $W = 25$ kN $= 25 \times 10^3$ N

We know that,
$$\tan\alpha = \frac{p}{\pi d} = \frac{12.5}{\pi \times 50} = 0.08$$

and force required on the screw to raise the load.

$$P = W \tan(\alpha + \phi) = W \left[\frac{\tan\phi - \tan\alpha}{1 + \tan\phi . \tan\alpha} \right]$$

$$= 25 \times 10^3 \left[\frac{0.08 + 0.13}{1 - 0.08 \times 0.13} \right] = 5305 \text{ N}$$

Torque required on the screw

We know that the torque required on the screw to raise the load,

$$T_1 = P \times d/2 = 5305 \times 50/2 = 132\,625 \text{ N-mm} \quad \textbf{Ans.}$$

Ratio of the torques required to raise and lower the load

We know that the force required on the screw to lower the load,

$$P = W \tan(\phi - \alpha) = W \left[\frac{\tan\phi - \tan\alpha}{1 + \tan\phi . \tan\alpha} \right]$$

$$= 25 \times 10^3 \left[\frac{0.13 + 0.08}{1 + 0.13 \times 0.08} \right] = 1237 \text{ N}$$

and torque required to lower the load

$$T_2 = P \times d/2 = 1237 \times 50/2 = 30\,905 \text{ N-mm}$$

$\therefore$ Ratio of the torques required,

$$= T_1 / T_2 = 132625/30925 = 4.3 \quad \textbf{Ans.}$$

Efficiency of the machine, we know that the efficiency,

$$\eta = \frac{\tan \alpha}{\tan(\alpha + \phi)} = \frac{\tan \alpha(1 - \tan \alpha . \tan \phi)}{\tan \alpha + \tan \phi} = \frac{0.08(1 - 0.08 \times 0.13)}{0.08 + 0.13}$$

$\because = 0.377 = 37.7\%$ **Ans**

10.22. Over Hauling and Self Locking Screws

10.24. Friction of a V-thread

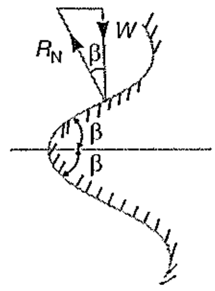


Fig. 10.14. V-thread.

Let 2β = Angle of the V-thread, and

β = Semi-angle of the V-thread.

$$\therefore \quad R_N = \frac{W}{\cos \beta}$$

and frictional force, $F = \mu . R_N = \mu \times \dfrac{W}{\cos \beta} = \mu_1 . W$

where $\dfrac{\mu}{\cos \beta} = \mu_1$, known as virtual coefficient of friction.

10.25. Friction in Journal Bearing-Friction Circle

A journal bearing forms a turning pair as shown in Fig. 10.15 (a). The fixed outer element of a turning pair is called a *bearing* and that portion of the inner element (*i.e.* shaft) which fits in the bearing is called a *journal*.

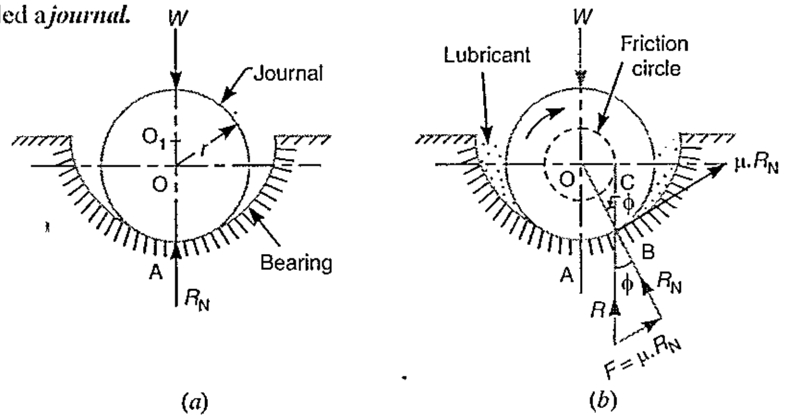


Fig. 10.15. Friction in journal bearing.

When the bearing is not lubricated (or the journal is stationary), then there is a line contact between the two elements as shown in Fig. 10.15 (a).

Now consider a shaft rotating inside a bearing in clockwise direction as shown in Fig. 10.15 (*b*). The lubricant between the journal and bearing forms a thin layer which gives rise to a greasy friction. Therefore, the reaction R does not act vertically upward, but acts at another point of pressure B.

In order that the rotation may be maintained, there must be a couple rotating the shaft.

Let ϕ = Angle between R (resultant of F and R_N) and R_N,

μ = Coefficient of friction between the journal and bearing,

T = Frictional torque in N-m, and

r = Radius of the shaft in metres.

For uniform motion, the resultant force acting on the shaft must be zero and the resultant turning moment on the shaft must be zero. In other words,

$$R = W, \text{ and } T = W \times OC = W \times OB \sin \phi = W.r \sin \phi$$

Since ϕ is very small, therefore substituting $\sin \phi = \tan \phi$

$$\therefore \quad T = W.r \tan \phi = \mu.W.r \qquad ...(\because \mu = \tan \phi)$$

If the shaft rotates with angular velocity ω rad/s, then power wasted in friction,

$$P = T.\omega = T \times 2\pi N/60 \text{ watts}$$

where N = Speed of the shaft in r.p.m.

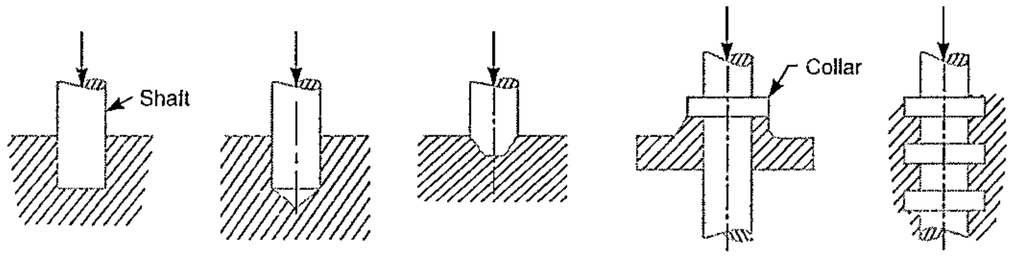
Notes : 1. If a circle is drawn with centre O and radius $OC = r \sin \phi$, then this circle is called the *friction circle* of a bearing.

10.26. Friction of Pivot and Collar Bearing

10.26. Friction of Pivot and Collar Bearing

The rotating shafts are frequently subjected to axial thrust. The bearing surfaces such as pivot and collar bearings are used to take this axial thrust of the rotating shaft.

The bearing surfaces placed at the end of a shaft to take the axial thrust are known as ***pivots.***



(*a*) Flat pivot. (*b*) Conical pivot. (*c*) Truncated pivot. (*d*) Single flat collar. (*e*) Multiple flat collar.

Fig. 10.16. Pivot and collar bearings.

Hence, in the study of friction of bearings, it is assumed that

1. The pressure is uniformly distributed throughout the bearing surface, and
2. The wear is uniform throughout the bearing surface.

10.27. Flat Pivot Bearing

When a vertical shaft rotates in a flat pivot bearing (known as **foot step bearing**), as shown in Fig. 10.17, the sliding friction will be along the surface of contact between the shaft and the bearing.

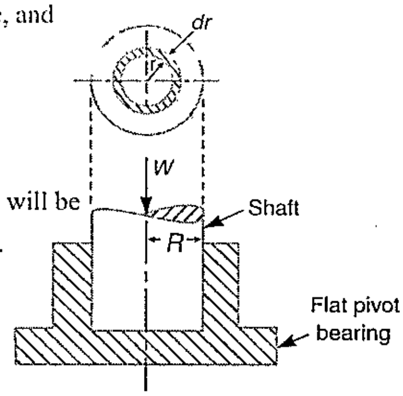


Fig. 10.17. Flat pivot or footstep bearing.

Let W = Load transmitted over the bearing surface,

R = Radius of bearing surface,

p = Intensity of pressure per unit area of bearing surface between rubbing surfaces, and

μ = Coefficient of friction.

We will consider the following two cases :

1. Considering uniform pressure:

When the pressure is uniformly distributed over the bearing area, then

$$p = \frac{W}{\pi R^2}$$

Consider a ring of radius r and thickness dr of the bearing area.

$\therefore$ Area of bearing surface. $A = 2\pi r.dr$

Load transmitted to the ring,

$$\delta W = p \times A = p \times 2\,\pi\, r.dr \qquad ...(i)$$

Frictional resistance to sliding on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W = \mu\, p \times 2\pi\, r.dr = 2\pi\, \mu.p.r.dr$$

$\therefore$ Frictional torque on the ring,

$$T_r = F_r \times r = 2\pi\, \mu\, p\, r.dr \times r = 2\,\pi\, \mu\, p\, r^2\, dr \qquad ...(ii)$$

Integrating this equation within the limits from 0 to R for the total frictional torque on the pivot bearing.

$\therefore$ Total frictional torque, $T = \int_0^R 2\pi\mu p r^2 dr = 2\pi\mu p \int_0^R r^2\, dr$

$$= 2\pi\mu\, p\left[\frac{r^3}{3}\right]_0^R = 2\,\pi\mu\, p \times \frac{R^3}{3} = \frac{2}{3} \times \pi\mu.p.R^3$$

$$= \frac{2}{3} \times \pi\mu \times \frac{W}{\pi R^2} \times R^3 = \frac{2}{3} \times \mu.W.R \qquad ...\left(\because\ p = \frac{W}{\pi R^2}\right)$$

When the shaft rotates at ω rad/s, then power lost in friction,

$$P = T.\omega = T \times 2\pi N/60 \qquad ...(\because\ \omega = 2\pi N/60)$$

2. ***Considering uniform wear***

We have already discussed that the rate of wear depends upon the intensity of pressure (p) and the velocity of rubbing surfaces (v). It is assumed that the rate of wear is proportional to the product of intensity of pressure and the velocity of rubbing surfaces (*i.e.* $p.v.$). Since the velocity of rubbing surfaces increases with the distance (*i.e.* radius r) from the axis of the bearing, therefore for uniform wear

$$p.r = C \text{ (a constant)} \quad \text{or} \quad p = C/r$$

and the load transmitted to the ring,

$$\delta W = p \times 2\pi r.dr \qquad ...[\text{From equation } (i)]$$

$$= \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

∴ Total load transmitted to the bearing

$$W = \int_0^R 2\pi\ C.dr = 2\pi\ C[r]_0^R = 2\pi C.R \quad \text{or} \quad C = \frac{W}{2\pi R}$$

We know that frictional torque acting on the ring,

$$T_r = 2\pi\mu\, p\, r^2\ dr = 2\pi\mu \times \frac{C}{r} \times r^2\ dr \qquad ...\left(\because\ p = \frac{C}{r}\right)$$

$$= 2\pi\ \mu.C.r\ dr \qquad ...(iii)$$

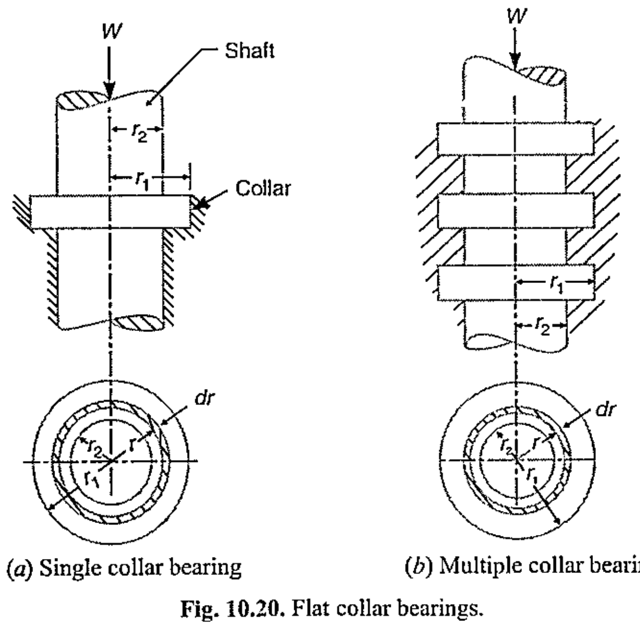
∴ Total frictional torque on the bearing,

$$T = \int_0^R 2\pi\ \mu.C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_0^R = 2\pi\mu.C \times \frac{R^2}{2} = \pi\mu.C.R^2$$

$$= \pi\mu \times \frac{W}{2\pi R} \times R^2 = \frac{1}{2} \times \mu.W.R$$

10.30. Flat Collar Bearing

We have already discussed that collar bearings are used to take the axial thrust of the rotating shafts. There may be a single collar or multiple collar bearings as shown in Fig. 10.20 (*a*) and (*b*) respectively. The collar bearings are also known as ***thrust bearings.***



(*a*) Single collar bearing (*b*) Multiple collar bearing.

Fig. 10.20. Flat collar bearings.

1. *Considering uniform pressure*

When the pressure is uniformly distributed over the bearing surface, then the intensity of pressure,

$$p = \frac{W}{A} = \frac{W}{\pi[r_1)^2 - (r_2)^2]} \qquad \text{...}(i)$$

We have seen in Art. 10.25, that the frictional torque on the ring of radius and thickness dr,

$$T_r = 2\pi\mu.p.r^2.dr$$

Integrating this equation within the limits from r_2 to r_1 for the total frictional torque on the collar.

$\therefore$ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\mu.p.r^2.dr = 2\pi\mu.p\left[\frac{r_3}{3}\right]_{r_2}^{r_1} = 2\pi\mu.p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p from equation (*i*),

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]}\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

2. *Considering unifrom wear*

We have seen in Art. 10.25 that the load transmitted on the ring, considering uniform wear is,

$$\delta W = p_r.2\pi r.dr = \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

$\therefore$ Total load transmitted to the collar,

$$W = \int_{r_2}^{r_1} 2\pi C.dr = 2\pi C[r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

We also know that frictional torque on the ring,

$$T_r = \mu.\delta W.r = \mu \times 2\pi C.dr.r = 2\pi\mu.C.r.dr$$

$\therefore$ Total frictional torque on the bearing,

$$T = \int_{r_2}^{r_1} 2\pi\mu C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\mu.C[(r_1)^2 - (r_2)^2]$$

Substituting the value of C from equation (*ii*),

$$T = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)}[(r_1)^2 - (r_2)^2] = \frac{1}{2} \times \mu.W(r_1 + r_2)$$

10.31. Friction Clutches

A friction clutch has its principal application in the transmission of power of shafts and machines which must be started and stopped frequently.

In automobiles, friction clutch is used to connect the engine to the driven shaft. In operating such a clutch, care should be taken so that the friction surfaces engage easily and gradually brings the driven shaft up to proper speed.

The friction clutches of the following types are important from the subject point of view :

1. Disc or plate clutches (single disc or multiple disc clutch),
2. Cone clutches, and
3. Centrifugal clutches.

10.32. Single Disc or Plate Clutch

A single disc or plate clutch, as shown in Fig. 10.21, consists of a clutch plate whose both sides are faced with a friction material (usually of Ferrodo).

It is mounted on the hub which is free to move axially along the splines of the driven shaft. The pressure plate is mounted inside the clutch body which is bolted to the flywheel.

Both the pressure plate and the flywheel rotate with the enginecrankshaft or the driving shaft.

The pressure plate pushes the clutch plate towards the flywheel by a set of strong springs which are arranged radially inside the body.

The three levers (also known as release levers or fingers) are carried on pivots suspended from the case of the body. These are arranged in such a manner so that the pressure plate moves away from the flywheel by the inward movement of a thrust bearing.

The bearing is mounted upon a forked shaft and moves forward when the clutch pedal is pressed.

When the clutch pedal is pressed down, its linkage forces the thrust release bearing to move in towards the flywheel and pressing the longer ends of the levers inward. The levers are forced to turn on their suspended pivot and the pressure plate moves away from the flywheel by the knife edges, there by compressing the clutch springs. This action removes the pressure from the clutch plate and thus moves back from the flywheel and the driven shaft becomes stationary.

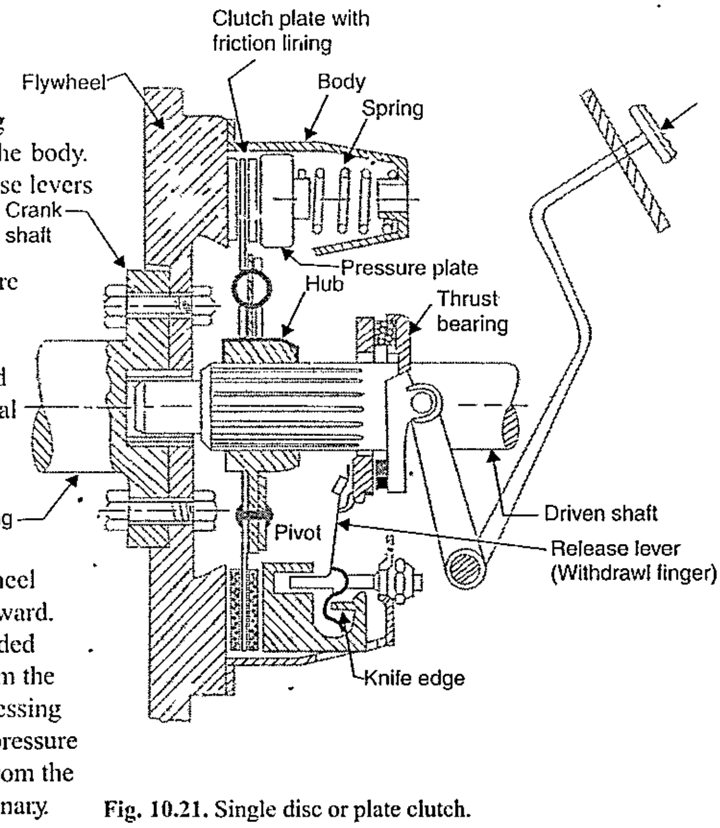


Fig. 10.21. Single disc or plate clutch.

On the other hand, when the foot is taken off from the clutch pedal, the thrust bearing moves back by the levers. This allows the springs to extend and thus the pressure plate pushes the clutch plate back towards the flywheel.

The axial pressure exerted by the spring provides a frictional force in the circumferential direction when the relative motion between the driving and driven members tends to take place. If the torque due to this frictional force exceeds the torque to be transmitted, then no slipping takes place and the power is transmitted from the driving shaft to the driven shaft.

Now consider two friction surfaces, maintained in contact by an axial thrust W, as shown in Fig. 10.22 (*a*).

Let T = Torque transmitted by the clutch,

p = Intensity of axial pressure with which the contact surfaces are held together,

r_1 and r_2 = External and internal radii of friction faces, and

μ = Coefficient of friction.

Consider an elementary ring of radius r and thickness dr as shown in Fig. 10.22 (*b*).

We know that area of contact surface or friction surface,

$$= 2\,\pi\, r.dr$$

$\therefore$ Normal or axial force on the ring,

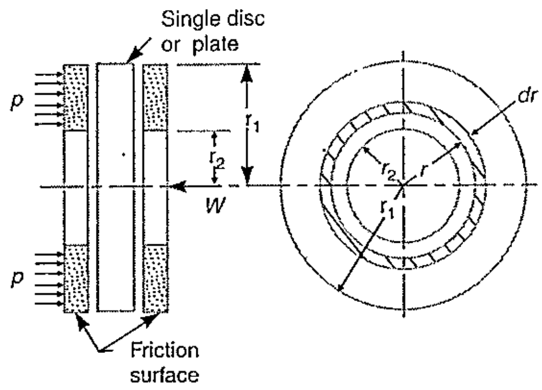
$$\delta W = \text{Pressure} \times \text{Area} = p \times 2\,\pi\, r.dr$$

and the frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W = \mu.p \times 2\,\pi\, r.dr$$

$\therefore$ Frictional torque acting on the ring.

$$T_r = F_r \times r = \mu.p \times 2\,\pi\, r.dr \times r = 2\,\pi \times \mu\,.p.r^2\,dr$$



(*a*) (*b*)

Fig. 10.22. Forces on a single disc or plate clutch.

We shall now consider the following two cases :

1. When there is a uniform pressure, and
2. When there is a uniform wear.

1. ***Considering uniform pressure***

When the pressure is uniformly distributed over the entire area of the friction face, then the intensity of pressure,

$$p = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \qquad \ldots(i)$$

where W = Axial thrust with which the contact or friction surfaces are held together

We have discussed above that the frictional torque on the elementary ring of radius r and thickness dr is

$$T_r = 2\,\pi\,\mu.p.r^2\,dr$$

Integrating this equation within the limits from r_2 to r_1 for the total frictional torque.

$\therefore$ Total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_1}^{r_2} 2\pi\mu.p.r^2.dr = 2\,\pi\mu\, p\left[\frac{r^3}{3}\right]_{r_2}^{r_1} = 2\,\pi\mu\, p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p from equation (*i*),

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \frac{(r_1)^3 - (r_2)^3}{3}$$

$$= \frac{2}{3} \times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \mu.W.R$$

where R = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

2. ***Considering uniform wear***

In Fig. 10.22, let p be the normal intensity of pressure at a distance r from the axis of the clutch. Since the intensity of pressure varies inversely with the distance, therefore

$$p.r. = C \text{ (a constant)} \quad \text{or} \quad p = C/r \qquad ...(i)$$

and the normal force on the ring,

$$\delta W = p.2\pi r.dr = \frac{C}{r} \times 2\pi C.dr = 2\pi C.dr$$

$\therefore$ Total force acting on the friction surface.

$$W = \int_{r_2}^{r} 2\pi C\,dr = 2\pi C[r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

or

$$C = \frac{W}{2\pi(r_1 - r_2)}$$

We know that the frictional torque acting on the ring,

$$T_r = 2\pi\mu.p\,r^2.dr = 2\pi\mu \times \frac{C}{r} \times r^2.dr = 2\pi\mu.C.r.dr \qquad ...(\because\ p = C/r)$$

$\therefore$ Total frictional torque on the friction surface,

$$T = \int_{r_2}^{r_1} 2\pi\mu.C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\mu.C[(r_1)^2 - (r_2)^2] = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)}\left[(r_1)^2 - (r_2)^2\right]$$

$$= \frac{1}{2} \times \mu.W(r_1 + r_2) = \mu.W.R$$

where R = Mean radius of the friction surface $= \dfrac{r_1 + r_2}{2}$

10.33. Multiple Disc Clutch

A multiple disc clutch, as shown in Fig. 10.23, may be used when a lar ge torque is to be transmitted. The inside discs (usually of steel) are fastened to the driven shaft to permit axial motion (except for the last disc). The outside discs (usually of bronze) are held by bolts and are fastened to the housing which is keyed to the driving shaft. The multiple disc clutches are extensively used in motor cars, machine tools etc.

Let n_1 = Number of discs on the driving shaft, and

n_2 = Number of discs on the driven shaft.

$\therefore$ Number of pairs of contact surfaces,

$$n = n_1 + n_2 - 1$$

and total frictional torque acting on the friction surfaces or on the clutch,

$$T = n.\mu.W.R$$

where R = Mean radius of the friction surfaces

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \qquad \text{...(For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \qquad \text{...(For uniform wear)}$$

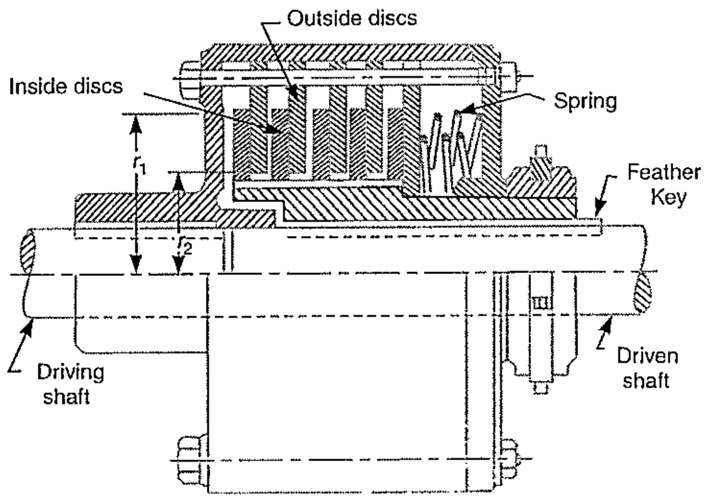


Fig. 10.23. Multiple disc clutch.

Example 4. *Determine the maximum, minimum and average pressure in plate clutch when the axial force is 4 kN. The inside radius of the contact surface is 50 mm and the outside radius is 100 mm. Assume uniform wear*

Solution. Given : $W = 4$ kN $= 4 \times 10^3$ N ; $r_2 = 50$ mm ; $r_1 = 100$ mm

Maximum pressure

Let $\quad p_{max}$ = Maximum pressure.

Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p_{max} \times r_2 = C \quad \text{or} \quad C = 50\, p_{max}$$

We know that the total force on the contact surface (W),

$$4 \times 10^3 = 2\,\pi\, C\,(r_1 - r_2) = 2\,\pi \times 50\, p_{max}\,(100 - 50) = 15\ 710\, p_{max}$$

$$\therefore \quad p_{max} = 4 \times 10^3/15\ 710 = 0.2546 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Minimum pressure

Let $\quad p_{min}$ = Minimum pressure.

Since the intensity of pressure is minimum at the outer radius (r_1), therefore

$$p_{min} \times r_1 = C \quad \text{or} \quad C = 100\, p_{min}$$

We know that the total force on the contact surface (W),

$$4 \times 10^3 = 2\,\pi\, C\,(r_1 - r_2) = 2\pi \times 100\, p_{min}\,(100 - 50) = 31\ 420\, p_{min}$$

$$\therefore \quad p_{min} = 4 \times 10^3/31\ 420 = 0.1273 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Average pressure

We know that average pressure,

$$p_{av} = \frac{\text{Total normal force on contact surface}}{\text{Cross-sectional area of contact surfaces}}$$

$$= \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{4 \times 10^3}{\pi[(100)^2 - (50)^2]} = 0.17 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Example 5. *A single plate clutch, with both sides effective, has outer and inner diameters 300 mm and 200 mm respectively. The maximum intensity of pressure at any point in the contact surface is not to exceed 0.1 N/mm². If the coefficient of friction is 0.3, determine the power transmitted by a clutch at a speed 2500 r.p.m.*

Solution. Given : $d_1 = 300$ mm or $r_1 = 150$ mm ; $d_2 = 200$ mm or $r_2 = 100$ mm ; $p = 0.1$ N/mm² ; $\mu = 0.3$; $N = 2500$ r.p.m. or $\omega = 2\pi \times 2500/60 = 261.8$ rad/s

Since the intensity of pressure (p) is maximum at the inner radius (r_2), therefore for uniform wear,

$$p.r_2 = C \quad \text{or} \quad C = 0.1 \times 100 = 10 \text{ N/mm}$$

We know that the axial thrust,

$$W = 2\,\pi\, C\,(r_1 - r_2) = 2\,\pi \times 10\,(150 - 100) = 3142 \text{ N}$$

and mean radius of the friction surfaces for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{150 + 100}{2} = 125 \text{ mm} = 0.125 \text{ m}$$

We know that torque transmitted,

$$T = n.\mu.W.R = 2 \times 0.3 \times 3142 \times 0.125 = 235.65 \text{ N-m}$$

...($\because$ $n = 2$, for both sides of plate effective)

$\therefore$ Power transmitted by a clutch,

$$P = T.\omega = 235.65 \times 261.8 = 61\ 693 \text{ W} = 61.693 \text{ kW} \textbf{ Ans.}$$

10.34. Cone Clutch

A cone clutch, as shown in Fig. 10.24, was extensively used in automobiles but now-a-days it has been replaced completely by the disc clutch.

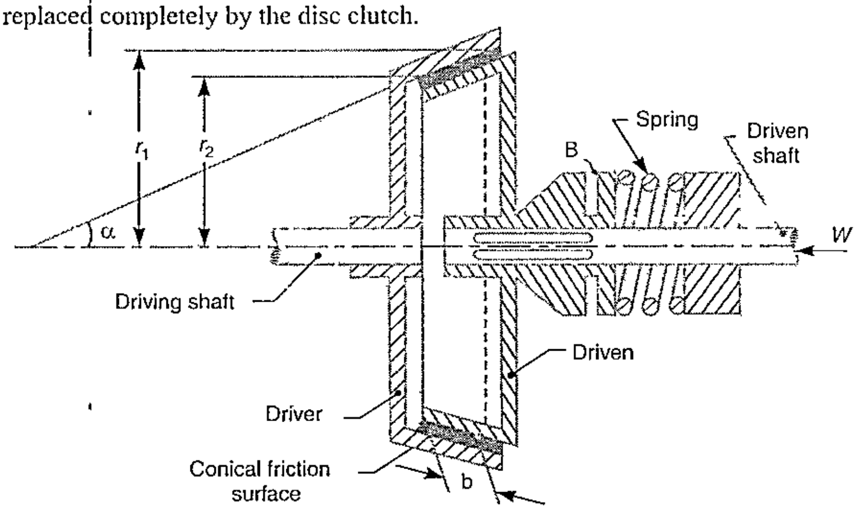


Fig. 10.24. Cone clutch.

It consists of one pair of friction surface only . In a cone clutch, the driver is keyed to the driving shaft by a sunk key and has an inside conical surface or face which exactly fits into the outside conical surface of the driven. The driven member resting on the feather key in the driven shaft, may be shifted along the shaft by a forked lever provided a*B*, in order to engage the clutch by bringing the two conical surfaces in contact. Due to the frictional resistance set up at this contact surface, the torque is transmitted from one shaft to another. In some cases, a spring is placed around the driven shaft in contact with the hub of the driven. This spring holds the clutch faces in contact and maintains the pressure between them, and the forked lever is used only for disengagement of the clutch. The contact surfaces of the clutch may be metal to metal contact, but more often the driven member is lined with some material like wood, leather cork or asbestos etc. The material of the clutch faces *(i.e.* contact surfaces) depends upon the allowable normal pressure and the coefficient of friction.

Consider a pair of friction surface as shown in Fig. 10.25 (*a*). Since the area of contact of a pair of friction surface is a frustrum of a cone, therefore the torque transmitted by the cone clutch may be determined in the similar manner as discussed for conical pivot bearings in Art. 10.28.

Let p_n = Intensity of pressure with which the conical friction surfaces are held together (*i.e.* normal pressure between contact surfaces),

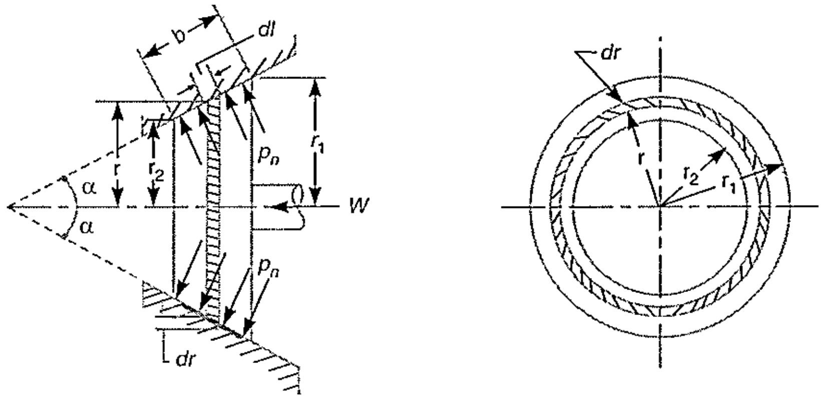
r_1 and r_2 = Outer and inner radius of friction surfaces respectively.

R = Mean radius of the friction surface = $\frac{r_1 + r_2}{2}$,

α = Semi angle of the cone (also called face angle of the cone) or the angle of the friction surface with the axis of the clutch,

μ = Coefficient of friction between contact surfaces, and

b = Width of the contact surfaces (also known as face width or clutch face).



(a) (b)

Fig. 10.25. Friction surfaces as a frustrum of a cone.

Consider a small ring of radius r and thickness dr, as shown in Fig. 10.25 (b). Let dl is length of ring of the friction surface, such that

$$dl = dr.\text{cosec } \alpha$$

∴ Area of the ring,

$$A = 2\pi\, r.dl = 2\pi r.dr \text{ cosec } \alpha$$

1. *Considering uniform pressure*

We know that normal load acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_n \times 2\,\pi\, r.dr.\text{cosec } \alpha$$

and the axial load acting on the ring,

$$\delta W = \text{Horizontal component of } \delta W_n \text{ (i.e. in the direction of } W)$$

$$= \delta W_n \times \sin\alpha = p_n \times 2\pi\, r.dr.\text{ cosec } \alpha \times \sin\alpha = 2\pi \times p_n.r.dr$$

∴ Total axial load transmitted to the clutch or the axial spring force required,

$$W = \int_{r_2}^{r_1} 2\pi\, p_n.r.dr = 2\pi\, p_n\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\, p_n\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\, p_n\left[(r_1)^2 - (r_2)^2\right]$$

$$\therefore \qquad p_n = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \qquad \text{...(i)}$$

We know that frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W_n = \mu.p_n \times 2\,\pi\, r.dr.\text{cosec } \alpha$$

∴ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p_n \times 2\,\pi\, r.dr.\text{ cosec } \alpha.r = 2\,\pi\,\mu.p_n.\text{cosec } \alpha.r^2\, dr$$

Integrating this expression within the limits from r_2 to r_1 for the total frictional torque on the clutch.

$$\therefore \text{ Total frictional torque, } T = \int_{r_2}^{r_1} 2\pi\mu.p_n.\text{cosec } \alpha.r^2.dr = 2\pi\mu\, p_n.\text{cosec } \alpha\left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\,\mu\, p_n.\text{cosec } \alpha\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p_n from equation (*i*), we get

$$T = 2\pi\,\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \text{cosec } \alpha\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu.W.\text{cosec } \alpha\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \qquad \text{..(ii)}$$

2. *Considering uniform wear*

In Fig. 10.25, let p_r be the normal intensity of pressure at a distance r from the axis of the clutch. We know that, in case of uniform wear, the intensity of pressure varies inversely with the distance.

$$\therefore \quad p_r \cdot r = C \text{ (a constant)} \quad \text{or} \quad p_r = C/r$$

We know that the normal load acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_r \times 2\pi r.dr \operatorname{cosec} \alpha$$

and the axial load acting on the ring,

$$\delta W = \delta W_n \times \sin \alpha = p_r.2\,\pi\, r.dr.\operatorname{cosec} \alpha.\sin \alpha = p_r \times 2\,\pi\, r.dr$$

$$= \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr \quad \ldots(\because\ p_r = C/r)$$

$\therefore$ Total axial load transmitted to the clutch,

$$W = \int_{r_2}^{r_1} 2\pi C.dr = 2\pi C[r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

or

$$C = \frac{W}{2\pi(r_1 - r_2)} \quad \ldots(iii)$$

We know that frictional force acting on the ring,

$$F_r = \mu.\delta W_n = \mu.p_r \times 2\,\pi\, r \times dr \operatorname{cosec} \alpha$$

and frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p_r \times 2\,\pi\, r.dr.\operatorname{cosec} \alpha \times r$$

$$= \mu \times \frac{C}{r} \times 2\pi r^2.dr.\operatorname{cosec} \alpha = 2\pi\mu.C \operatorname{cosec} \alpha \times r\, dr$$

$\therefore$ Total frictional torque acting on the clutch,

$$T = \int_{r_2}^{r_1} 2\pi\mu.C.\operatorname{cosec} \alpha.r\,dr = 2\pi\mu.C.\operatorname{cosec} \alpha \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu.C.\operatorname{cosec} \alpha \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

Substituting the value of C from equation (*i*), we have

$$T = 2\pi\mu \times \frac{W}{2\pi(r_1 - r_2)} \times \operatorname{cosec} \alpha \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

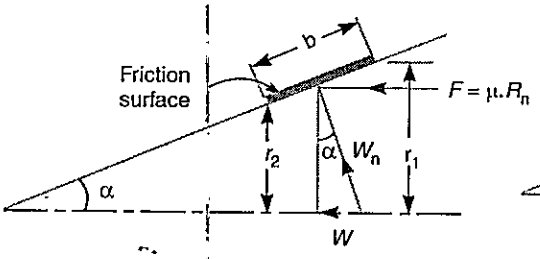
$$= \mu.W \operatorname{cosec} \alpha \left(\frac{r_1 + r_2}{2}\right) = \mu.W.R \operatorname{cosec} \alpha \quad \ldots(iv)$$

where $\quad R = \dfrac{r_1 + r_2}{2}$ = Mean radius of friction surface

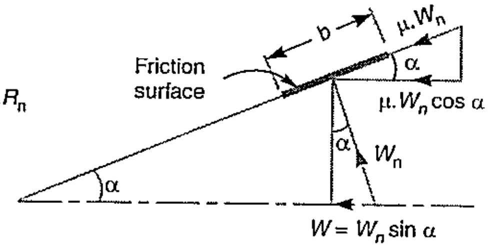
Since the normal force acting on the friction surface, $W_n = W/\sin \alpha$, therefore the equation (*iv*) may be written as

$$T = \mu.W_n.R \quad \ldots(v)$$

The forces on a friction surface, for steady operation of the clutch and after the clutch is engaged, is shown in Fig. 10.26.



(*a*) For steady operation of the clutch



(*b*) During engagement of the clutch.

Fig. 10.26. Forces on a friction surface.

$$r_1 - r_2 = b\sin\alpha;\ \text{and}\ R = \frac{r_1 + r_2}{2} \quad \text{or} \quad r_1 + r_2 = 2R$$

$\therefore$ From equation, (*i*), normal pressure acting on the friction surface,

$$p_n = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{W}{\pi(r_1 + r_2)(r_1 - r_2)} = \frac{W}{2\pi R.b.\sin\alpha}$$

or $$W = p_n \times 2\,\pi\, R.b \sin\alpha = W_n \sin\alpha$$

where W_n = Normal load acting on the friction surface $= p_n \times 2\,\pi\, R.b$

Now the equation (*iv*) may be written as,

$$T = \mu(p_n \times 2\pi R.b \sin\alpha)\ R\ \text{cosec}\ \alpha = 2\pi\mu.p_n.R^2 b$$

Example 6. *A conical friction clutch is used to transmit 90 kW at 1500 rp.m. The semi-cone angle is 20° and the coefficient of friction is 0.2. If the mean diameter of the bearing surface is 375 mm and the intensity of normal pressure is not to exceed 0.25 N/mm find the dimensions of the conical bearing surface and the axial load required.*

Solution. Given : $P = 90$ kW $= 90 \times 10^3$ W ; $N = 1500$ r.p.m. or $\omega = 2\,\pi \times 1500/60 = 156$ rad/s : $\alpha = 20°$; $\mu = 0.2$; $D = 375$ mm or $R = 187.5$ mm ; $p_n = 0.25$ N/mm²

Dimensions of the conical bearing surface

Let r_1 and r_2 = External and internal radii of the bearing surface respectively,
b = Width of the bearing surface in mm, and
T = Torque transmitted.

We know that power transmitted (P),

$$90 \times 10^3 = T.\omega = T \times 156$$

$$\therefore \quad T = 90 \times 10^3/156 = 577 \text{ N-m} = 577 \times 10^3 \text{ N-mm}$$

and the torque transmitted (T),

$$577 \times 10^3 = 2\,\pi\,\mu\, p_n.R^2.b = 2\pi \times 0.2 \times 0.25\,(187.5)^2\, b = 11\,046\, b$$

$$\therefore \quad b = 577 \times 10^3/11\,046 = 52.2 \text{ mm} \quad \textbf{Ans.}$$

We know that $r_1 + r_2 = 2R = 2 \times 187.5 = 375$ mm ...(*i*)

and $r_1 - r_2 = b \sin\alpha = 52.2 \sin 20° = 18$ mm ...(*ii*)

From equations (*i*) and (*ii*),

$r_1 = 196.5$ mm, and $r_2 = 178.5$ mm **Ans.**

Axial load required

Since in case of friction clutch, uniform wear is considered and the intensity of pressure is maximum at the minimum contact surface radius (r_2), therefore

$$p_n.r_2 = C\ (\text{a constant})\ \text{or}\ C = 0.25 \times 178.5 = 44.6 \text{ N/mm}$$

We know that the axial load required, $W = 2\pi C\,(r_1 - r_2) = 2\pi \times 44.6\,(196.5 - 178.5) = 5045$ N **Ans**

10.35. Centrifugal Clutch

The centrifugal clutches are usually incorporated into the motor pulleys. It consists of a number of shoes on the inside of a rim of the pulleyas shown in Fig. 10.28.

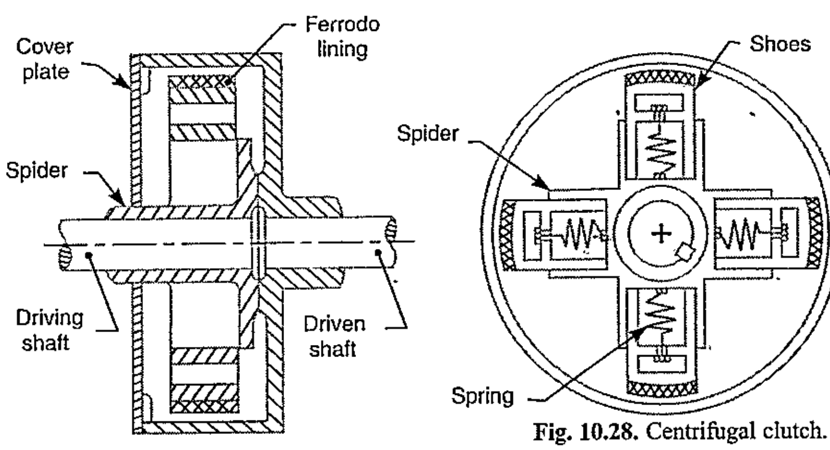


Fig. 10.28. Centrifugal clutch.

The outer surface of theshoes are covered with a friction material. These shoes, which can move radially in guides, are heldagainst the boss (or spider) on the driving shaft by means of springs. The springs exert a radially inward force which is assumed constant. The mass of the shoe, when revolving, causes it to exert a radially outward force (*i.e.* centrifugal force). The magnitude of this centrifugal force depends upon the speed at which the shoe is revolving. A little consideration will show that when the centrifugal force is less than the spring force, the shoe remains in the same position as when the driving shaft was stationary, but when the centrifugal force is equal to the spring force, the shoe is just floating. When the centrifugal force exceeds the spring force, the shoe moves outward and comes into contact with the driven member and presses against it. The force with which the shoe presses against the driven member is the difference of the centrifugal force and the spring force. The increase of speed causes the shoe to press harder and enables more torque to be transmitted.

In order to determine the mass and size of the shoes, the following procedure is adopted :

1. *Mass of the shoes*

Consider one shoe of a centrifugal clutch as shown in Fig. 10.29.

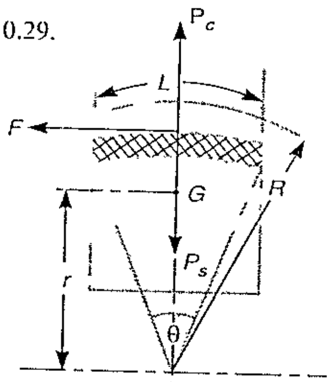


Fig. 10.29. Forces on a shoe of centrifugal clutch.

Let m = Mass of each shoe.

n = Number of shoes,

r = Distance of centre of gravity of the shoe from the centre of the spider,

R = Inside radius of the pulley rim,

N = Running speed of the pulley in r.p.m.,

ω = Angular running speed of the pulley in rad/s = $2\pi N/60$ rad/s,

ω_1 = Angular speed at which the engagement begins to take place, and

μ = Coefficient of friction between the shoe and rim.

We know that the centrifugal force acting on each shoe at the running speed.

$$*P_c = m.\omega^2.r$$

and the inward force on each shoe exerted by the spring at the speed at which engagement begins to take place,

$$P_s = m\,(\omega_1)^2\,r$$

∴ The net outward radial force (*i.e.* centrifugal force) with which the shoe presses against the rim at the running speed

$$= P_c - P_s$$

and the frictional force acting tangentially on each shoe,

$$F = \mu\,(P_c - P_s)$$

∴ Frictional torque acting on each shoe,

$$= F \times R = \mu\,(P_c - P_s)\,R$$

and total frictional torque transmitted,

$$T = \mu\,(P_c - P_s)\,R \times n = n.F.R$$

From this expression, the mass of the shoes (m) may be evaluated.

2. *Size of the shoes*

Let l = Contact length of the shoes,

b = Width of the shoes,

R = Contact radius of the shoes. It is same as the inside radius of the rim of the pulley.

θ = Angle subtended by the shoes at the centre of the spider in radians.

p = Intensity of pressure exerted on the shoe. In order to ensure reasonable life, the intensity of pressure may be taken as 0.1 N/mm².

We know that $\theta = l/R$ rad or $l = \theta.R$

$\therefore$ Area of contact of the shoe,

$$A = l.b$$

and the force with which the shoe presses against the rim

$$= A \times p = l.b.p$$

Since the force with which the shoe presses against the rim at the running speed is $(P_c - P_s)$, therefore

$$l.b.p = P_c - P_s$$

From this expression, the width of shoe (b) may be obtained.

Example 7. *A centrifugal clutch is to transmit 15 kW at 900 r.p.m. The shoes are four in number. The speed at which the engagement begins is 3/4th of the running speed. The inside radius of the pulley rim is 150 mm and the centre of gravity of the shoe lies at 120 mm from the centre of the spider. The shoes are lined with Ferrodo for which the coefficient of friction may be taken as 0.25. Determine : 1. Mass of the shoes, and 2. Size of the shoes, if angle subtended by the shoes at the centre of the spider is 60° and the pressure exerted on the shoes is 0.1 N/mm².*

Solution. Given : $P = 15$ kW $= 15 \times 10^3$ W ; $N = 900$ r.p.m. or $\omega = 25 \times 900/60 = 94.26$ rad/s ; $n = 4$; $R = 150$ mm $= 0.15$ m ; $r = 120$ mm $= 0.12$ m ; $\mu = 0.25$

Since the speed at which the engagement begins (*i.e.* ω_1) is 3/4th of the running speed (*i.e.* ω), therefore

$$\omega_1 = \frac{3}{4}\omega = \frac{3}{4} \times 94.26 = 70.7 \text{ rad/s}$$

Let T = Torque transmitted at the running speed.

We know that power transmitted (P),

$$15 \times 10^3 = T.\omega = T \times 94.26 \quad \text{or} \quad T = 15 \times 10^3/94.26 = 159 \text{ N-m}$$

1. ***Mass of the shoes***

Let m = Mass of the shoes in kg.

We know that the centrifugal force acting on each shoe,

$$P_c = m.\omega^2.r = m\,(94.26)^2 \times 0.12 = 1066\,m \text{ N}$$

and the inward force on each shoe exerted by the spring *i.e.* the centrifugal force at the engagement speed ω_1,

$$P_s = m\,(\omega_1)^2\,r = m\,(70.7)^2 \times 0.12 = 600\,m \text{ N}$$

$\therefore$ Frictional force acting tangentially on each shoe,

$$F = \mu\,(P_c - P_s) = 0.25\,(1066\,m - 600\,m) = 116.5\,m \text{ N}$$

We know that the torque transmitted (T),

$$159 = n.F.R = 4 \times 116.5\,m \times 0.15 = 70\,m \quad \text{or} \quad m = 2.27 \text{ kg} \quad \textbf{Ans.}$$

2. ***Size of the shoes***

Let l = Contact length of shoes in mm,

b = Width of the shoes in mm,

θ = Angle subtended by the shoes at the centre of the spider in radians $= 60° = \pi/3$ rad, and ...(Given)

p = Pressure exerted on the shoes in N/mm² = 0.1 N/mm² ...(Given)

We know that $l = \theta.R = \frac{\pi}{3} \times 150 = 157.1$ mm

and $l.b.p = P_c - P_s = 1066\,m - 600\,m = 466\,m$

$\therefore$ $157.1 \times b \times 0.1 = 466 \times 2.27 = 1058$

or $b = 1058/157.1 \times 0.1 = 67.3$ mm **Ans.**

EXERCISES

UNIT-III BRAKES AND DYNAMOMETERS

19.1. Introduction

A *brake* is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine. The energy absorbed by brakes is dissipated in the form of heat. (K. E)

The capacity of a brake depends upon the following factors :

1. The unit pressure between the braking surfaces,
2. The coefficient of friction between the braking surfaces,
3. The peripheral velocity of the brake drum,
4. The projected area of the friction surfaces, and
5. The ability of the brake to dissipate heat equivalent to the energy being absorbed.

19.2. Materials for Brake Lining

The material used for the brake lining should have thefollowing characteristics :

1.It should have high coefficient of friction with minimum fading. In other words, the coeffi-cient of friction should remain constant with change in temperature.

2.It should have low wear rate.

3.It should have high heat resistance.

4.It should have high heat dissipation capacity.

5.It should have adequate mechanical strength.

6.It should not be affected by moisture and oil.

Types of Brakes:

The brakes, according to the means used for transforming the energy by the braking elements,are classified as :

1. Hydraulic brakes e.g. pumps or hydrodynamic brakeand fluid agitator,
2. Electric brakes e.g. generators and eddy currentbrakes,
3. Mechanical brakes.

The mechanical brakes, according to the direction ofacting force, may be divided into the following two groups :

(a) Radial brakes: In these brakes, the force acting onthe brake drum is in radial direction.

The radial brakes may be sub-divided into external brakes and internal brakes.

According to the shape of the friction elements, these brakes may be block or shoe brakes and band brakes.

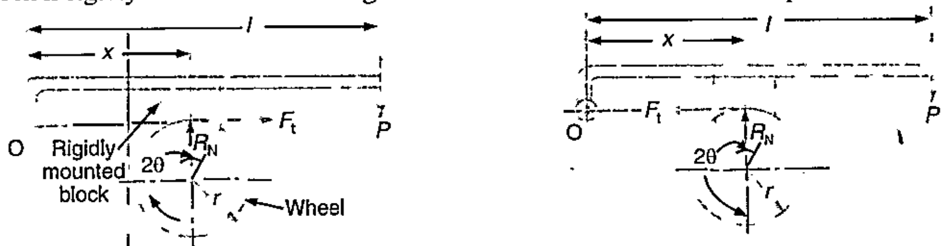
(b) Axial brakes: In these brakes, the force acting on the brake drum is in axial direction.

The axial brakes may be disc brakes and cone brakes.

19.4. Single Block or Shoe Brake

A single block or shoe brake is consists of a block or shoe which is pressed against the rim of a revolving brake wheel drum. The block is made of a softer material thanthe rim of the wheel. This type of a brake is commonly used on railway trains and tram cars.

The friction between the block and the wheel causes a tangential braking force to act on the wheel, which retard the rotation of the wheel. The block is pressed against the wheel by a force applied to one endbf a lever to which the block is rigidly fixed as shown in Fig. 19.1. The other end of the lever is pivoted on a fixed fulcrum O.



(*a*) Clockwise rotation of brake wheel (*b*) Anticlockwise rotation of brake wheel.

Fig. 19.1. Single block brake. Line of action of tangential force passes through the fulcrum of the lever

R = Contact radius of the shoes. It is same as the inside radius of the rim of the pulley.

θ = Angle subtended by the shoes at the centre of the spider in radians.

p = Intensity of pressure exerted on the shoe. In order to ensure reasonable life, the intensity of pressure may be taken as 0.1 N/mm^2.

We know that $\theta = l/R$ rad or $l = \theta.R$

$\therefore$ Area of contact of the shoe,

$$A = l.b$$

and the force with which the shoe presses against the rim

$$= A \times p = l.b.p$$

Since the force with which the shoe presses against the rim at the running speed is $(P_c - P_s)$, therefore

$$l.b.p = P_c - P_s$$

From this expression, the width of shoe (b) may be obtained.

Example 7. *A centrifugal clutch is to transmit 15 kW at 900 r.p.m. The shoes are four in number. The speed at which the engagement begins is 3/4th of the running speed. The inside radius of the pulley rim is 150 mm and the centre of gravity of the shoe lies at 120 mm from the centre of the spider. The shoes are lined with Ferrodo for which the coefficient of friction may be taken as 0.25. Determine : 1. Mass of the shoes, and 2. Size of the shoes, if angle subtended by the shoes at the centre of the spider is 60° and the pressure exerted on the shoes is 0.1 N/mm².*

Solution. Given : $P = 15$ kW $= 15 \times 10^3$ W ; $N = 900$ r.p.m. or $\omega = 25 \times 900/60 = 94.26$ rad/s ; $n = 4$; $R = 150$ mm $= 0.15$ m ; $r = 120$ mm $= 0.12$ m ; $\mu = 0.25$

Since the speed at which the engagement begins (*i.e.* ω_1) is 3/4th of the running speed (*i.e.* ω), therefore

$$\omega_1 = \frac{3}{4}\omega = \frac{3}{4} \times 94.26 = 70.7 \text{ rad/s}$$

Let T = Torque transmitted at the running speed.

We know that power transmitted (P),

$$15 \times 10^3 = T.\omega = T \times 94.26 \quad \text{or} \quad T = 15 \times 10^3/94.26 = 159 \text{ N-m}$$

1. *Mass of the shoes*

Let m = Mass of the shoes in kg.

We know that the centrifugal force acting on each shoe,

$$P_c = m.\omega^2.r = m\,(94.26)^2 \times 0.12 = 1066\,m \text{ N}$$

and the inward force on each shoe exerted by the spring *i.e.* the centrifugal force at the engagement speed ω_1,

$$P_s = m\,(\omega_1)^2\,r = m\,(70.7)^2 \times 0.12 = 600\,m \text{ N}$$

$\therefore$ Frictional force acting tangentially on each shoe,

$$F = \mu\,(P_c - P_s) = 0.25\,(1066\,m - 600\,m) = 116.5\,m \text{ N}$$

We know that the torque transmitted (T),

$$159 = n.F.R = 4 \times 116.5\,m \times 0.15 = 70\,m \quad \text{or} \quad m = 2.27 \text{ kg} \quad \textbf{Ans.}$$

2. *Size of the shoes*

Let l = Contact length of shoes in mm,

b = Width of the shoes in mm,

θ = Angle subtended by the shoes at the centre of the spider in radians $= 60° = \pi/3$ rad, and ...(Given)

p = Pressure exerted on the shoes in N/mm^2 = 0.1 N/mm^2 ...(Given)

We know that $l = \theta.\,R = \frac{\pi}{3} \times 150 = 157.1$ mm

and $l.b.p = P_c - P_s = 1066\,m - 600\,m = 466\,m$

$\therefore$ $157.1 \times b \times 0.1 = 466 \times 2.27 = 1058$

or $b = 1058/157.1 \times 0.1 = 67.3$ mm **Ans.**

EXERCISES

Let P = Force applied at the end of the lever,

R_N = Normal force pressing the brake block on the wheel,

r = Radius of the wheel,

2θ = Angle of contact surface of the block,

μ = Coefficient of friction, and

F_t = Tangential braking force or the frictional force acting at the contact surface of the block and the wheel.

If the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. In such cases, tangential braking force on the wheel,

$$F_t = \mu.R_N \quad ...(i)$$

and the braking torque, $T_B = F_t.r = \mu.R_N.r \quad ...(ii)$

Let us now consider the following three cases :

Case 1. When the line of action of tangential braking force (F_t) passes through the fulcrum O of the lever, and the brake wheel rotates clockwise as shown in Fig. 19.(a), then for equilibrium, taking moments about the fulcrum O, we have

$$R_N \times x = P \times l \quad \text{or} \quad R_N = \frac{P \times l}{x}$$

∴ Braking torque,

$$T_B = \mu.R_N.r = \mu \times \frac{P.l}{x} \times r = \frac{\mu.P.l.r}{x}$$

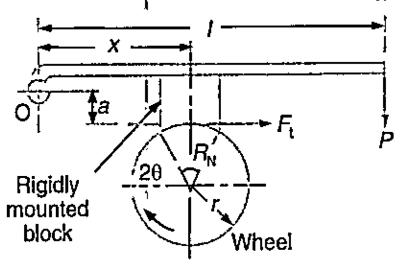
It may be noted that when the brake wheel rotates anticlockwise as shown in Fig. 19.1 (*b*), then the braking torque is same, *i.e.*

$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x}$$

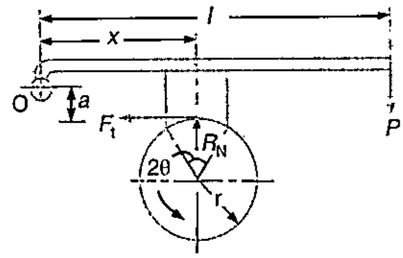
Case 2. When the line of action of the tangential braking force (F_t) passes through a distance 'a' below the fulcrum O, and the brake wheel rotates clockwise as shown in Fig. 19.2 (*a*), then for equilibrium, taking moments about the fulcrum O,

$$R_N \times x + F_t \times a = P.l \quad \text{or} \quad R_N \times x + \mu\, R_N \times a = P.l \quad \text{or} \quad R_N = \frac{P.l}{x + \mu.a}$$

and braking torque, $T_B = \mu R_N.r = \dfrac{\mu.p.l.r}{x + \mu.a}$



(*a*) Clockwise rotation of brake wheel.



(*b*) Anticlockwise rotation of brake wheel.

Fig. 19.2. Single block brake. Line of action of F_t passes below the fulcrum.

When the brake wheel rotates anticlockwise, as shown in Fig. 19.2 (*b*), then for equilibrium,

$$R_N.x = P.l + F_t.a = P.l + \mu.R_N.a \quad ...(i)$$

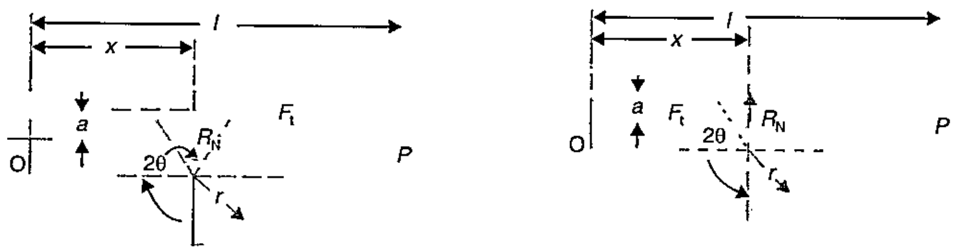
or $R_N (x - \mu.a) = P.l$ or $R_N = \dfrac{P.l}{x - \mu.a}$

and braking torque, $T_B = \mu.R_N.r = \dfrac{\mu.P.l.r}{x - \mu.a}$

Case 3. When the line of action of the tangential braking force (F_t) passes through a distance 'a' above the fulcrum O, and the brake wheel rotates clockwise as shown in Fig. 19.3 (*a*), then for equilibrium, taking moments about the fulcrum O, we have

$$R_N.x = P.l + F_t.\, a = P.l + \mu.R_N.a \quad ...(ii)$$

or $R_N (x - \mu.a) = P.l$ or $R_N = \dfrac{P.l}{x - \mu.a}$



(*a*) Clockwise rotation of brake wheel. (*b*) Anticlockwise rotation of brake wheel.

Fig. 19.3. Single block brake. Line of action of F_t passes above the fulcrum.

and braking torque, $\quad T_B = \mu.R_N.r = \dfrac{\mu.P.l.r}{x - \mu.a}$

When the brake wheel rotates anticlockwise as shown in Fig. 19.3 (*b*), then for equilibrium, taking moments about the fulcrum *O*, we have

$$R_N \times x + F_t \times a = P.l \quad \text{or} \quad R_N \times x + \mu.R_N \times a = P.l \quad \text{or} \quad R_N = \frac{P.l}{x + \mu.a}$$

and braking torque, $\quad T_B = \mu.R_N.r = \dfrac{\mu.P.l.r}{x + \mu.a}$

19.5. Pivoted Block or Shoe Brake

We have discussed in the previous article that when the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. But when the angle of contact is greater than 60°, then the unit pressure normal to the surface of contact is less at the ends than at the centre. This gives uniform wear of the brake lining in the direction of the applied force. The braking torque for a pivoted block or shoe brake (*i.e.* when $2\theta > 60°$) is given by

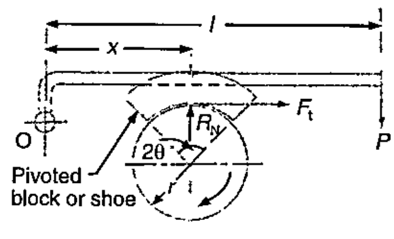


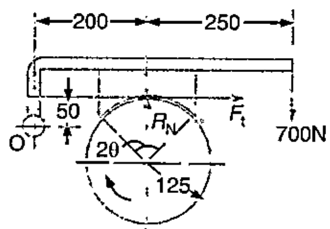
Fig. 19.4. Pivoted block or shoe brake.

$$T_B = F_t \times r = \mu'.R_N.r$$

where $\quad \mu'$ = Equivalent coefficient of friction = $\dfrac{4\mu \sin\theta}{2\theta + \sin 2\theta}$, and

μ = Actual coefficient of friction.

Example 1. *A single block brake is shown in Fig. 19.5. The diameter of the drum is 250 mm and the angle of contact is 90°. If the operating force of 700 N is applied at the end of a lever and the coefficient of friction between the drum and the lining is 0.35, determine the torque that may be transmitted by the block brake.*



All dimensions in mm.

Fig. 19.5

Solution. Given : $d = 250$ mm or $r = 125$ mm ; $2\theta = 90°$ $= \pi/2$ rad ; $P = 700$ N ; $\mu = 0.35$

Since the angle of contact is greater than 60°, therefore equivalent coefficient of friction,

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.35 \times \sin 45°}{\pi/2 + \sin 90°} = 0.385$$

Let $\quad R_N$ = Normal force pressing the block to the brake drum, and

F_t = Tangential braking force = $\mu'.R_N$

Taking moments about the fulcrum *O*, we have

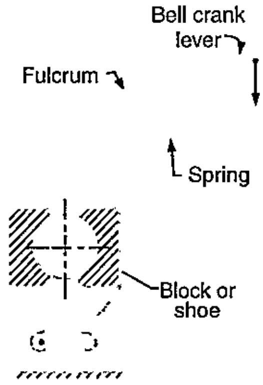
$$700(250 + 200) + F_t \times 50 = R_N \times 200 = \frac{F_t}{\mu'} \times 200 = \frac{F_t}{0.385} \times 200 = 520\,F_t$$

or $\quad 520\,F_t - 50F_t = 700 \times 450 \quad$ or $\quad F_t = 700 \times 450/470 = 670$ N

We know that torque transmitted by the block brake,

$$T_B = F_t \times r = 670 \times 125 = 8\,3750 \text{ N-mm} = 83.75\text{N-m} \textbf{ Ans.}$$

19.6. Double Block or Shoe Brake



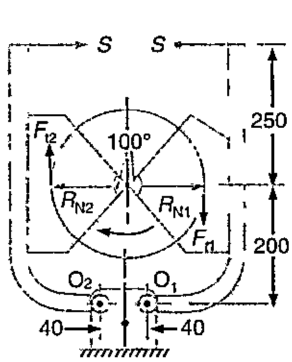
It consists of two brake blocks applied at the opposite ends of a diameter of the wheel which eliminate or reduces the unbalanced force on the shaft. The brake is set by a spring which pulls the upper ends of the brake arms together. When a force P is applied to the bellcrank lever, the spring is compressed and the brake is released.

In a double block brake, the braking action is doubled by the use of two blocks and these blocks may be operated practically by the same force which will operate one. In case of double block or shoe brake, the braking torque is given by

$$T_B = (F_{t1} + F_{t2})\, r$$

where F_{t1} and F_{t2} are the braking forces on the two blocks.

Example 2. *A double shoe brake, as shown in Fig. 19.10, is capable of absorbing a torque of 1400 N-m. The diameter of the brake drum is 350 mm and the angle of contact for each shoe is 100°. If the coefficient of friction between the brake drum and lining is 0.4 ; find 1. the spring force necessary to set the brake ; and 2. the width of the brake shoes, if the bearing pressure on the lining material is not to exceed 0.3 N/mm².*



Solution. Given : $T_B = 1400$ N-m $= 1400 \times 10^3$ N-mm ; $d = 350$ mm or $r = 175$ mm ; $2\theta = 100° = 100 \times \pi/180 = 1.75$ rad; $\mu = 0.4$; $p_b = 0.3$ N/mm²

1. ***Spring force necessary to set the brake***

Let S = Spring force necessary to set the brake

R_{N1} and F_{t1} = Normal reaction and the braking force on the right hand side shoe, and

R_{N2} and F_{t2} = Corresponding values on the left hand side shoe.

Since the angle of contact is greater than 60°, therefore equivalent coefficient of friction,

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.4 \times \sin 50°}{1.75 + \sin 100°} = 0.45$$

Taking moments about the fulcrum O_1, we have

$$S \times 450 = R_{N1} \times 200 + F_{t1}(175 - 40) = \frac{F_{t1}}{0.45} \times 200 + F_{t1} \times 135 = 579.4\, F_{t1} \quad \left(\text{Substituting } R_{N1} = \frac{F_{t1}}{\mu'}\right)$$

$$\therefore \quad F_{t1} = S \times 450 / 579.4 = 0.776\, S$$

Again taking moments about O_2, we have

$$S \times 450 + F_{t2}(175 - 40) = R_{N2} \times 200 = \frac{F_{t2}}{0.45} \times 200 = 444.4\, F_{t2} \quad \left(\text{Substituting } R_{N2} = \frac{F_{t2}}{\mu'}\right)$$

$$444.4\, F_{t2} - 135 F_{t2} = S \times 450 \quad \text{or} \quad 309.4\, F_{t2} = S \times 450$$

$$\therefore \quad F_{t2} = S \times 450 / 309.4 = 1.454\, S$$

We know that torque capacity of the brake (T_B),

$$1400 \times 10^3 = (F_{t1} + F_{t2})\, r = (0.776\, S + 1.454\, S)\, 175 = 390.25\, S$$

$$\therefore \quad S = 1400 \times 10^3 / 390.25 = 3587 \text{ N } \textbf{Ans.}$$

2. ***Width of the brake shoes***

Let b = Width of the brake shoes in mm.

We know that projected bearing area for one shoe,

$$A_b = b(2r \sin\theta) = b(2 \times 175 \sin 50°) = 268\, b \text{ mm}^2$$

Normal force on the right hand side of the shoe,

$$R_{N1} = \frac{F_{t1}}{\mu'} = \frac{0.776 \times S}{0.45} = \frac{0.776 \times 3587}{0.45} = 6186 \text{ N}$$

and normal force on the left hand side of the shoe,

$$R_{N2} = \frac{F_{t2}}{\mu'} = \frac{1.454 \times S}{0.45} = \frac{1.454 \times 3587}{0.45} = 11\,590 \text{ N}$$

We see that the maximum normal force is on the left hand side of the shoe. Therefore we shall find the width of the shoe for the maximum normal force *i.e.* R_{N2}.

We know that the bearing pressure on the lining material (p_b),

$$0.3 = \frac{R_{N2}}{A_h} = \frac{11\,590}{268\,b} = \frac{43.25}{b}$$

$\therefore$ $b = 43.25 / 0.3 = 144.2$ mm **Ans.**

19.7. Simple Band Brake

A band brake consists of a flexible band of leather, one or more ropes,or a steel lined with friction material, which embraces a part of the circumference of the drum. A band brake, as shown in Fig. 19.11, is called a ***simple band brake*** in which one end of the band is attached to a fixed pin or fulcrum of the lever while the other end is attached to the lever at a distance b from the fulcrum.

When a force P is applied to the lever at C, the lever turns about the fulcrum pin O and tightens the band on the drum and hence the brakes are applied. The friction between the band and the drum provides the braking force. The force P on the lever at C may be determined as discussed below :

Let T_1 = Tension in the tight side of the band,

T_2 = Tension in the slack side of the band,

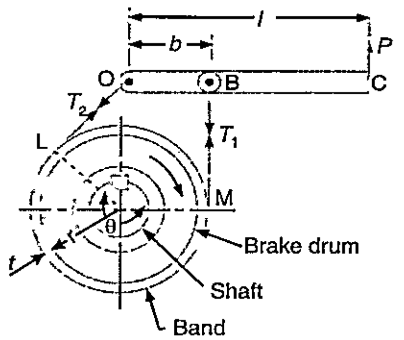
θ = Angle of lap (or embrace) of the band on the drum,

μ = Coefficient of friction between the band and the drum,

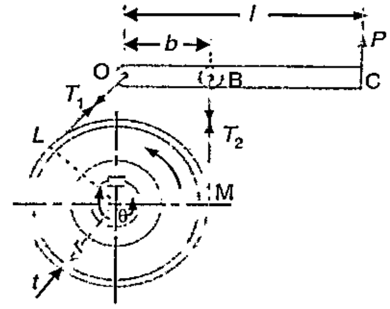
r = Radius of the drum,

t = Thickness of the band, and

r_e = Effective radius of the drum = $r + \frac{t}{2}$



(*a*) Clockwise rotation of drum.



(*b*) Anticlockwise rotation of drum.

Fig. 19.11. Simple band brake.

We know that limiting ratio of the tensions is given by the relation,

$$\frac{T_1}{T_2} = e^{\mu\theta} \qquad \text{or} \qquad 2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta$$

and braking force on the drum $= T_1 - T_2$

$\therefore$ Braking torque on the drum,

$$T_B = (T_1 - T_2)\, r \qquad \text{... (Neglecting thickness of band)}$$

$$= (T_1 - T_2)\, r_e \qquad \text{... (Considering thickness of band)}$$

Now considering the equilibrium of the lever OBC. It may be noted that when the drum rotates in the clockwise direction, as shown in Fig. 19.11 (*a*), the end of the band attached to the fulcrum O will be slack with tension T_2 and end of the band attached to B will be tight with tension T_1. On the other hand, when the drum rotates in the anticlockwise direction, as shown in Fig. 19.11 (*b*), the tensions in the band will reverse, *i.e.* the end of the band attached to the fulcrum O will be tight with tension T_1 and the end of the band attached to B will be slack with tension T_2. Now taking moments about the fulcrum O, we have

$$P.l = T_1.b \qquad \text{... (For clockwise rotation of the drum)}$$

and $$P.l = T_2.b \qquad \text{... (For anticlockwise rotation of the drum)}$$

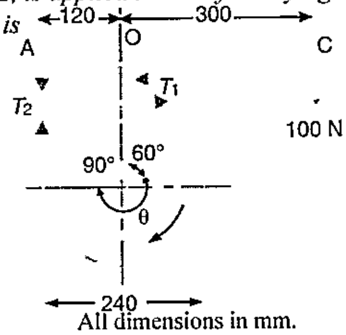
where l = Length of the lever from the fulcrum (OC), and

b = Perpendicular distance from O to the line of action of T_1 or T_2.

Example 3. *The simple band brake, as shown in Fig. 19.12, is applied to a shaft carrying a flywheel of mass 400 kg. The radius of gyration of the flywheel is 450 mm and runs at 300 r.p.m.*

If the coefficient of friction is 0.2 and the brake drum diameter is 240 mm, find :

1. *the torque applied due to a hand load of 100 N,*
2. *the number of turns of the wheel before it is brought to rest, and*
3. *the time required to bring it to rest, from the moment of the application of the brake.*



All dimensions in mm.
Fig. 19.12

Solution: Given : $m = 400$ kg ; $k = 450$ mm $= 0.45$ m ; $N = 300$ r.p.m. or $\omega = 2\pi\times300/60 = 31.42$ rad/s ; $\mu = 0.2$; $d = 240$ mm $= 0.24$ m or $r = 0.12$ m

1. *Torque applied due to hand load*

First of all, let us find the tensions in the tight and slack sides of the band *i.e.* T_1 and T_2 respectively.

From the geometry of the Fig. 19.12, angle of lap of the band on the drum,

$$\theta = 360° - 150° = 210° = 210 \times \frac{\pi}{180} = 3.666 \text{ rad}$$

We know that
$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.2 \times 3.666 = 0.7332$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.7332}{2.3} = 0.3188 \qquad \text{or } \frac{T_1}{T_2} = 2.08 \qquad \ldots (i)$$

... (Taking antilog of 0.3188)

Taking moments about the fulcrum O,

$T_2 \times 120 = 100 \times 300 = 30\ 000$ or $T_2 = 30\ 000/120 = 250$ N

$\therefore$ $T_1 = 2.08 T_2 = 2.08 \times 250 = 520$ N ... [From equation (*i*)]

We know that torque applied,

$$T_B = (T_1 - T_2)\, r = (520 - 250)\ 0.12 = 32.4 \text{ N-m } \textbf{Ans.}$$

2. *Number of turns of the wheel before it is brought to rest*

Let n = Number of turns of the wheel before it is brought to rest.

We know that kinetic energy of rotation of the drum

$$= \frac{1}{2} \times I.\omega^2 = \frac{1}{2} \times m.k^2.\omega^2 = \frac{1}{2} \times 400 (0.45)^2 (31.42)^2 = 40\ 000 \text{ N-m}$$

This energy is used to overcome the work done due to the braking torque (T_B).

$\therefore$ $40\ 000 = T_B \times 2\pi n = 32.4 \times 2\pi n = 203.6\ n$

or $n = 40\ 000 / 203.6 = 196.5$ **Ans.**

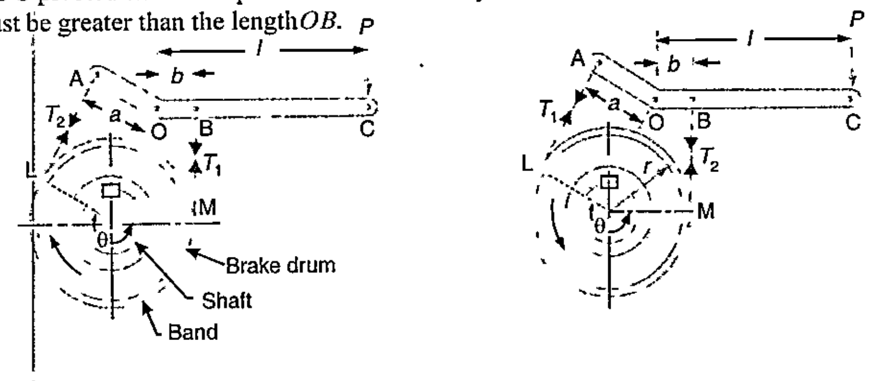
3. *Time required to bring the wheel to rest*

We know that the time required to bring the wheel to rest

$= n / N = 196.5 / 300 = 0.655$ min $= 39.3$ s **Ans**

19.8. Differential Band Brake

In a differential band brake, as shown in Fig. 19.14, the ends of the band are joined at A and B to a lever AOC pivoted on a fixed pin or fulcrum O. It may be noted that for the band to tighten, the length OA must be greater than the length OB.



The braking torque on the drum may be obtained in the similar way as discussed in simple band brake. Now considering the equilibrium of the lever AOC. It may be noted that when the drum rotates in the clockwise direction, as shown in Fig. 19.14 (*a*), the end of the band attached to A will be slack with tension T_2 and end of the band attached to B will be tight with tension T_1. On the other hand, when the drum rotates in the anticlockwise direction, as shown in Fig. 19.14 (*b*), the end of the band attached to A will be tight with tension T_1 and end of the band attached to B will be slack with tension T_2. Now taking moments about the fulcrum O, we have

$$P.l + T_1.b = T_2.a \quad \text{... (For clockwise rotation of the drum)}$$

or $$P.l = T_2.a - T_1.b \quad \text{... (i)}$$

and $$P.l + T_2.b = T_1.a \quad \text{.. (For anticlockwise rotation of the drum)}$$

or $$P.l = T_1.a - T_2.b \quad \text{... (ii)}$$

We have discussed in block brakes (Art. 19.4), that when the frictional force helps to apply the brake, it is said to be self energizing brake. In case of differential band brake, we see from equations (*i*) and (*ii*) that the moment $T_1.b$ and $T_2.b$ helps in applying the brake (because it adds to the moment $P.l$) for the clockwise and anticlockwise rotation of the drum respectively

We have also discussed that when the force P is negative or zero, then brake is self locking. Thus for differential band brake and for clockwise rotation of the drum, the condition for self locking is

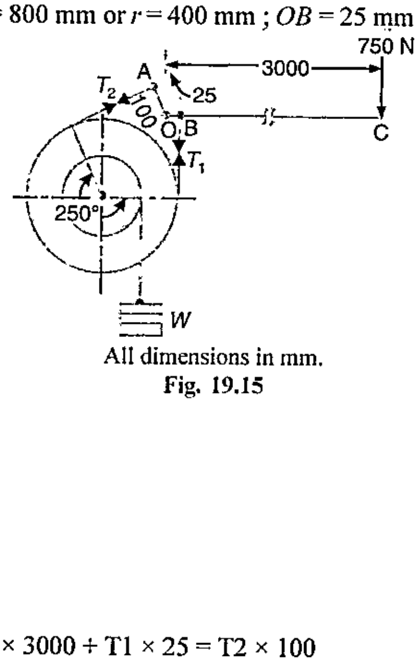
$$T_2.a \le T_1.b \quad \text{or} \quad T_2 / T_1 \le b / a$$

and for anticlockwise rotation of the drum, the condition for self locking is

$$T_1.a \le T_2.b \quad \text{or} \quad T_1 / T_2 \le b / a$$

Example 4. *In a winch, the rope supports a load W and is wound round a barrel 450 mm diameter. A differential band brake acts on a drum 800 mm diameter which is keyed to the same shaft as the barrel. The two ends of the bands are attached to pins on opposite sides of the fulcrum of the brake lever and at distances of 25 mm and 100 mm from the fulcrum. The angle of lap of the brake band is 250° and the coefficient of friction is 0.25. What is the maximum load W which can be supported by the brake when a force of 750 N is applied to the lever at a distance of 3000 mm from the fulcrum ?*

Solution. Given : $D = 450$ mm or $R = 225$ mm ; $d = 800$ mm or $r = 400$ mm ; $OB = 25$ mm ; $OA = 100$ mm ; $\theta = 250° = 250 \times \pi/180 = 4.364$ rad ; $\mu = 0.25$; $P = 750$ N ; $l = OC = 3000$ mm



All dimensions in mm.
Fig. 19.15

Since OA is greater than OB, therefore the operating force ($P = 750$ N) will act downwards.

First of all, let us consider that the drum rotates in clockwise direction.

We know that when the drum rotates in clockwise direction, the end of band attached to A will be slack with tension T_2 and the end of the band attached to B will be tight with tension T_1, as shown in Fig. 19.15. Now let us find out the values of tensions T_1 and T_2. We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 4.364 = 1.091$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{1.091}{2.3} = 0.4743 \quad \text{or}$$

T1 = 2.98 T2... (i)

Now taking moments about the fulcrum O, $750 \times 3000 + T1 \times 25 = T2 \times 100$

or $T_2 \times 100 - 2.98\, T_2 \times 25 = 2250 \times 10^3$ ... ($\because T_1 = 2.98\, T_2$)

$$25.5\, T_2 = 2250 \times 10^3 \quad \text{or} \quad T_2 = 2250 \times 10^3/25.5 = 88 \times 10^3 \text{ N}$$

and $$T_1 = 2.98 T_2 = 2.98 \times 88 \times 10^3 = 262 \times 10^3 \text{ N}$$

We know that braking torque,

$$T_B = (T_1 - T_2)\, r$$
$$= (262 \times 10^3 - 88 \times 10^3)\, 400 = 69.6 \times 10^6 \text{ N-mm} \quad \text{...(i)}$$

B III-B → 353, 58, 59, 72, 75, 77, 79, 82,
L-306, 11, 12, 19, 20,

(∵

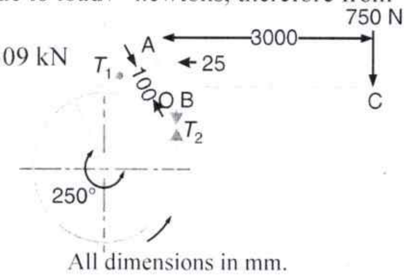
and the torque due to load W newtons,

$$T_W = W.R = W \times 225 = 225\ W \text{ N-mm} \qquad \text{... (ii)}$$

Since the braking torque must be equal to the torque due to load W newtons, therefore from equations (*i*) and (*ii*),

$$W = 69.6 \times 10^6/225 = 309 \times 10^3 \text{ N} = 309 \text{ kN}$$

Now let us consider that the drum rotates in anticlockwise direction. We know that when the drum rotates in anticlockwise direction, the end of the band attached to A will be tight with tension T_1 and end of the band attached to B will be slack with tension T_2, as shown in Fig. 19.16. The ratio of tensions T_1 and T_2 will be same as calculated above, *i.e.*



All dimensions in mm.
Fig. 19.16

$$\frac{T_1}{T_2} = 2.98 \text{ or } T_1 = 2.98\ T_2$$

Now taking moments about the fulcrum O,

$$750 \times 3000 + T_2 \times 25 = T_1 \times 100$$

or $2.98\ T_2 \times 100 - T_2 \times 25 = 2250 \times 10^3$ ... ($\because T_1 = 2.98\ T_2$)

$$273\ T_2 = 2250 \times 10^3 \quad \text{or} \quad T_2 = 2250 \times 10^3/273 = 8242 \text{ N}$$

and $T_1 = 2.98\ T_2 = 2.98 \times 8242 = 24\ 561$ N

∴ Braking torque, $T_B = (T_1 \times T_2)r$

$$= (24\ 561 - 8242)400 = 6.53 \times 10^6 \text{ N-mm} \qquad \text{...(iii)}$$

From equations (*ii*) and (*iii*),

$$W = 6.53 \times 10^6/225 = 29 \times 10^3 \text{ N} = 29 \text{ kN}$$

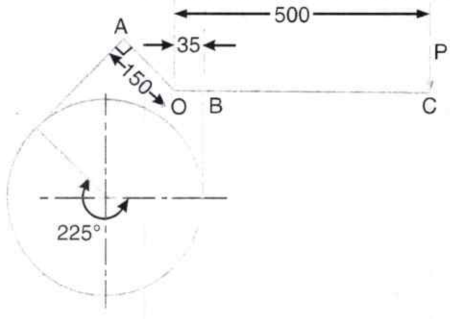
From above, we see that the maximum load (W) that can be supported by the brake is 309 kN, when the drum rotates in clockwise direction. **Ans.**

Example 5. *A differential band brake, as shown in Fig. 19.17, has an angle of contact of 225°. The band has a compressed woven lining and bears against a cast iron drum of 350 mm diameter. The brake is to sustain a torque of 350 N-m and the coeficient of friction between the band and the drum is 0.3. Find : 1. The necessary force (P) for the clockwise and anticlockwise rotation of the drum; and 2. The value of 'OA' for the brake to be self locking, when the drum rotates clockwise.*

Solution. Given: $\theta = 225° = 225 \times \pi/180 = 3.93$ rad ; $d = 350$ mm or $r = 175$ mm ; $T = 350$ N-m $= 350 \times 10^3$ N-mm

1. *Necessary force (P) for the clockwise and anticlockwise rotation of the drum*

When the drum rotates in the clockwise direction, the end of the band attached to A will be slack with tension T_2 and the end of the band attached to B will be tight with tension T_1, as shown in Fig. 19.18. First of all, let us find the values of tensions T_1 and T_2.



All dimensions in mm.
Fig. 19.17

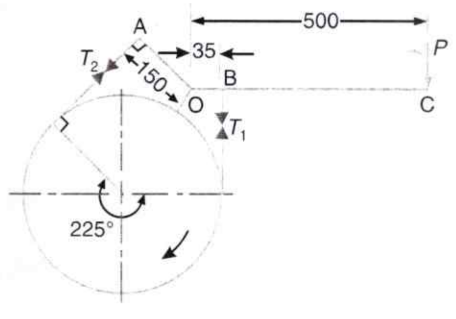


Fig. 19.18

We know that $2.3 \log\left(\dfrac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 3.93 = 1.179$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{1.179}{2.3} = 0.5126 \quad \text{or}$$

III–B, 5/9/19
58, 59, 61, 76, 77, 80, L-305, 12, 15, 19,

$$\frac{T_1}{T_2} = 3.255 \quad \text{... (Taking antilog of 0.5126) ... } (i)$$

and braking torque (T_B),

$$350 \times 10^3 = (T_1 - T_2)r = (T_1 - T_2)\, 175$$

$$\therefore \quad T_1 - T_2 = 350 \times 10^3/175 = 2000 \text{ N} \quad \text{... } (ii)$$

From equations (*i*) and (*ii*), we find that

$$T_1 = 2887 \text{ N} \;;\; \text{and } T_2 = 887 \text{ N}$$

Now taking moments about the fulcrum O, we have

$$P \times 500 = T_2 \times 150 - T_1 \times 35 = 887 \times 150 - 2887 \times 35 = 32 \times 10^3$$

$$\therefore \quad P = 32 \times 10^3/500 = 64 \text{ N } \textbf{Ans.}$$

When the drum rotates in the anticlockwise direction, the end of the band attached to A will be tight with tension T_1 and end of the band attached to B will be slack with tension T_2, as shown in Fig. 19.19. Tking moments about the fulcrum O, we have

$$P \times 500 = T_1 \times 150 - T_2 \times 35$$
$$= 2887 \times 150 - 887 \times 35$$
$$= 402 \times 10^3$$
$$P = 402 \times 10^3/500 = 804 \text{ N } \textbf{Ans.}$$

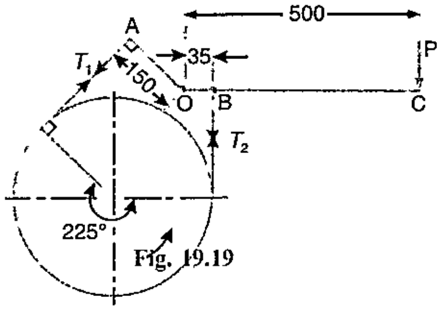


Fig. 19.19

2. *Value of 'OA' for the brake to be self locking, when the drum rotates clockwise*

The clockwise rotation of the drum is shown in Fig 19.18.

For clockwise rotation of the drum, we know that

$$P \times 500 = T_2 \times OA - T_1 \times OB$$

For the brake to be self locking, P must be equal to zero. Therefore

$$T_2 \times OA = T_1 \times OB$$

Taking moments about O,

$$200 \times 750 + T_1 \times 30 = T_2 \times 120$$

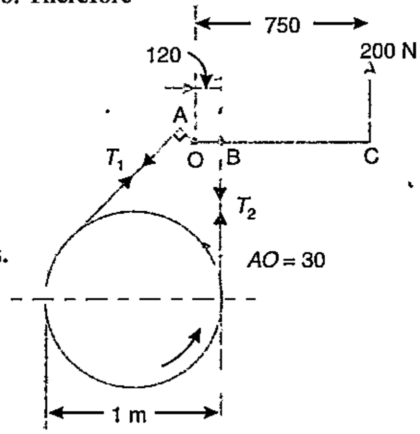
$$12\, T_2 - 3T_1 = 15\,000 \quad \ldots (i)$$

We know that $OA = \dfrac{T_1 \times OB}{T_2} = \dfrac{2887 \times 35}{887} = 114$ mm **Ans.**

$$\frac{T_1}{T_2} = \left(\frac{1+\mu \tan\theta}{1-\mu\tan\theta}\right)^n$$

$$= \left(\frac{1+0.25\tan 7.5^\circ}{1-0.25\tan 7.5^\circ}\right)^{14} = \left(\frac{1+0.25\times 0.1317}{1-.025\times 0.1317}\right)^{14}$$

$$= (1.068)^{14} = 2.512 \ldots (ii)$$



All dimensions in mm
Fig. 19.23

From equations (*i*) and (*ii*),

$$T_1 = 8440 \text{ N, and } T_2 = 3360 \text{ N}$$

We know that maximum braking torque,

$$T_B = (T_1 - T_2)\,r = (8440 - 3360)0.5 = 2540 \text{ N-m} \quad \textbf{Ans.}$$

2. *Angular retardation of the drum*

Let α = Angular retardation of the drum.

We know that braking torque (T_B),

$$2540 = I.\alpha = m.k^2.\alpha = 2000(0.5)^2\alpha = 500\alpha$$

$$\therefore \quad \alpha = 2540/500 = 5.08 \text{ rad/s}^2 \textbf{ Ans.}$$

19.10. Internal Expanding Brake

3. *Time taken by the system to come to rest*

Let t = Required time.

Since the system is to come to rest from the rated speed of 360 r.p.m., therefore

Initial angular speed, $\omega_1 = 2\pi \times 360 / 60 = 37.7$ rad/s

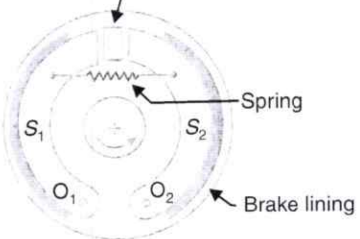
and final angular speed, $\omega_2 = 0$

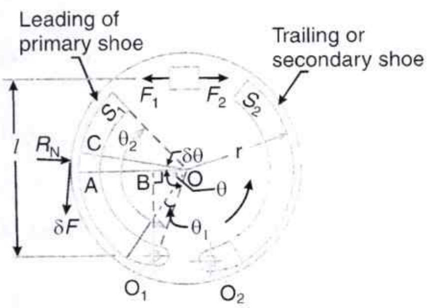
We know that $\omega_2 = \omega_1 - \alpha . t$ ... (– ve sign due to retardation)

$\therefore$ $t = \omega_1 / \alpha = 37.7 / 5.08 = 7.42$ s **Ans.**

19.10. Internal Expanding Brake

An internal expanding brake consists of two shoes S_1 and S_2 as shown in Fig. 19.24. The outer surface of the shoes are lined with some friction material (usually with Ferodo) to increase the coefficient of friction and to prevent wearing away of the metal. Each shoe is pivoted at one end about a fixed fulcrum O_1 and O_2 and made to contact a cam at the other end. When the cam rotates, the shoes are pushed outwards against the rim of the drum. The friction between the shoes and the drum produces the braking torque and hence reduces the speed of the drum. The shoes are normally held in off position by a spring as shown in Fig. 19.24. The drum encloses the entire mechanism to keep out dust and moisture. This type of brake is commonly used in motor cars and light trucks.





We shall now consider the forces acting on such a brake, when the drum rotates in the anticlockwise direction as shown in Fig. It may be noted that for the anticlockwise direction, the left hand shoe is known as *leading* or *primary shoe* while the right hand shoe is known as *trailing* or *secondary shoe.*

Let r = Internal radius of the wheel rim,

b = Width of the brake lining,

p_1 = Maximum intensity of normal pressure,

p_N = Normal pressure,

F_1 = Force exerted by the cam on the leading shoe, and

F_2 = Force exerted by the cam on the trailing shoe.

Consider a small element of the brake lining AC subtending an angle $\delta\theta$ at the centre. Let OA makes an angle θ with OO_1 as shown in Fig. 19.25. It is assumed that the pressure distribution on the shoe is nearly uniform, however the friction lining wears out more at the free end. Since the shoe turns about O_1, therefore the rate of wear of the shoe lining at A will be proportional to the radial displacement of that point.

The rate of wear of the shoe lining varies directly as the perpendicular distance from O_1 to OA, *i.e.* O_1B.

From the geometry of the figure,

$$O_1B = OO_1 \sin\theta$$

and normal pressure at A,

$$p_N \propto \sin\theta \quad \text{or} \quad p_N = p_1 \sin\theta$$

$\therefore$ Normal force acting on the element,

$$\delta R_N = \text{Normal pressure} \times \text{Area of the element}$$

$$= p_N (b.r.\delta\theta) = p_1 \sin\theta (b.r.\delta\theta)$$

III-C → 24, 29, 30, 32, 33, 35, ~~34~~, 43, 45, 47,
2818 49, 50, 58, 64, 65, 67, 71,

and braking or friction force on the element,

$$\delta F = \mu \times \delta R_N = \mu . p_1 \sin\theta (b.r.\delta\theta)$$

$\therefore$ Braking torque due to the element about O,

$$\delta T_B = \delta F \times r = \mu . p_1 \sin\theta (b.r.\delta\theta) r = \mu . p_1 b\, r^2 (\sin\theta . \delta\theta)$$

and total braking torque about O for whole of one shoe,

$$T_B = \mu\, p_1 b\, r^2 \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta = \mu\, p_1 b\, r^2 \left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$= \mu\, p_1 b r^2 (\cos\theta_1 - \cos\theta_2)$$

Moment of normal force δR_N of the element about the fulcrum O_1,

$$\delta M_N = \delta R_N \times O_1 B = \delta R_N (OO_1 \sin\theta)$$

$$= p_1 \sin\theta (b.r.\delta\theta)(OO_1 \sin\theta) = p_1 \sin^2\theta (b.r.\delta\theta) OO_1$$

$\therefore$ Total moment of normal forces about the fulcrum O_1,

$$M_N = \int_{\theta_1}^{\theta_2} p_1 \sin^2\theta (b.r.\delta\theta) OO_1 = p_1 . b.r.OO_1 \int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta$$

$$= p_1 . b.r.OO_1 \int_{\theta_1}^{\theta_2} \frac{1}{2}(1 - \cos 2\theta) d\theta \qquad \ldots\left[\because \sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)\right]$$

$$= \frac{1}{2} p_1 . b.r.OO_1 \left[\theta - \frac{\sin 2\theta}{2}\right]_{\theta_1}^{\theta_2}$$

$$= \frac{1}{2} p_1 . b.r.OO_1 \left[\theta_2 - \frac{\sin 2\theta_2}{2} - \theta_1 + \frac{\sin 2\theta_1}{2}\right]$$

$$= \frac{1}{2} p_1 . b.r.OO_1 \left[(\theta_2 - \theta_1) + \frac{1}{2}(\sin 2\theta_1 - \sin 2\theta_2)\right]$$

Moment of frictional force δF about the fulcrum O_1,

$$\delta M_F = \delta F \times AB = \delta F (r - OO_1 \cos\theta) \qquad \ldots (\because AB = r - OO_1 \cos\theta)$$

$$= \mu\, p_1 \sin\theta (b.r.\delta\theta)(r - OO_1 \cos\theta)$$

$$= \mu . p_1 . b.r (r \sin\theta - OO_1 \sin\theta \cos\theta) \delta\theta$$

$$= \mu . p_1 . b.r \left(r \sin\theta - \frac{OO_1}{2} \sin 2\theta\right) \delta\theta \qquad \ldots (\because 2 \sin\theta \cos\theta = \sin 2\theta)$$

$\therefore$ Total moment of frictional force about the fulcrum O_1,

$$M_F = \mu\, p_1 b\, r \int_{\theta_1}^{\theta_2} \left(r \sin\theta - \frac{OO_1}{2} \sin 2\theta\right) d\theta$$

$$= \mu\, p_1 b\, r \left[-r \cos\theta + \frac{OO_1}{4} \cos 2\theta\right]_{\theta_1}^{\theta_2}$$

$$= \mu\, p_1 b\, r \left[-r \cos\theta_2 + \frac{OO_1}{4} \cos 2\theta_2 + r \cos\theta_1 - \frac{OO_1}{4} \cos 2\theta_1\right]$$

$$= \mu\, p_1 b\, r \left[r(\cos\theta_1 - \cos\theta_2) + \frac{OO_1}{4} (\cos 2\theta_2 - \cos 2\theta_1)\right]$$

Now for leading shoe, taking moments about the fulcrum O_1,

$$F_1 \times l = M_N - M_F$$

and for trailing shoe, taking moments about the fulcrum O_2,

$$F_2 \times l = M_N + M_F$$

Note : If $M_F > M_N$, then the brake becomes self locking.

(P) III-B → 301, 13, 15, 24, 65, 54, 61, 6, 70, 56, 79, 62, 53, 51
66 55, 73, 60, 69 → Also.
III-A → 48, 15, 36, 33, 39, 47, 04,

Example 6. *The arrangement of an internal expanding friction brake, in which the brake shoe is pivoted at 'C' is shown in Fig. 19.26. The distance 'CO' is 75 mm, O being the centre of the drum. The internal radius of the brake drum is 100 mm. The friction lining extends over an arc AB, such that the angle AOC is 135° and angle BOC is 45°. The brake is applied by means of a force at Q, perpendicular to the line CQ, the distance CQ being 150 mm.*

The local rate of wear on the lining may be taken as proportional to the normal pressure on an element at an angle of ' θ *' with OC and may be taken as equal to* $p_1 \sin \theta$, *where* p_1 *is the maximum intensity of normal pressure.*

The coefficient of friction may be taken as 0.4 and the braking torque required is 21 N-m. Calculate the force Q required to operate the brake when 1. The drum rotates clockwise, and 2. The drum rotates anticlockwise.

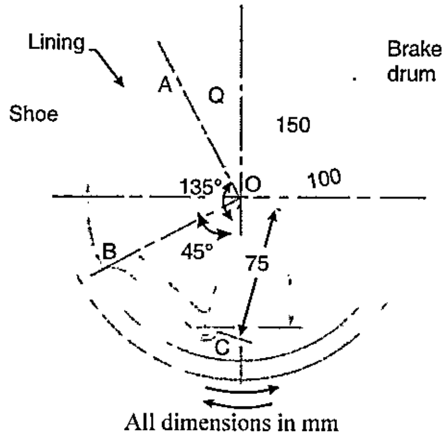


Fig. 19.26

Solution. Given : $OC = 75$ mm ; $r = 100$ mm ;

$\theta_2 = 135° = 135 \times \pi/180 = 2.356$ rad ; $\theta_1 = 45° = 45 \times \pi/180 = 0.786$ rad ; $l = 150$ mm ; $\mu = 0.4$; $T_B = 21$ N-m $= 21 \times 10^3$ N-mm

1. ***Force 'Q' required to operate the brake when drum rotates clockwise***

We know that total braking torque due to shoe (T_B),

$$21 \times 10^3 = \mu . p_1 . b . r^2 (\cos\theta_1 - \cos\theta_2)$$

$$= 0.4 \times p_1 \times b(100)^2 (\cos 45° - \cos 135°) = 5656\ p_1 . b$$

$\therefore \quad p_1 . b = 21 \times 10^3 / 5656 = 3.7$

Total moment of normal forces about the fulcrum C,

$$M_N = \frac{1}{2} p_1 . b . r . OC \left[(\theta_2 - \theta_1) + \frac{1}{2}(\sin 2\theta_1 - \sin 2\theta_2) \right]$$

$$= \frac{1}{2} \times 3.7 \times 100 \times 75 \left[(2.356 - 0.786) + \frac{1}{2}(\sin 90° - \sin 270°) \right]$$

$$= 13\ 875\ (1.57 + 1) = 35\ 660 \text{ N-mm}$$

and total moment of friction force about the fulcrum C,

$$M_F = \mu . p_1 . b . r \left[r(\cos\theta_1 - \cos\theta_2) + \frac{OC}{4} (\cos 2\theta_2 - \cos 2\theta_1) \right]$$

$$= 0.4 \times 3.7 \times 100 \left[100\ (\cos 45° - \cos 135°) + \frac{75}{4} (\cos 270° - \cos 90°) \right]$$

$$= 148 \times 141.4 = 20\ 930 \text{ N-mm}$$

Taking moments about the fulcrum C, we have

$$Q \times 150 = M_N + M_F = 35\ 660 + 20\ 930 = 56\ 590$$

$\therefore \quad Q = 56\ 590 / 150 = 377$ N **Ans.**

2. ***Force 'Q' required to operate the brake when drum rotates anticlockwise***

Taking moments about the fulcrum C, we have

$$Q \times 150 = M_N - M_F = 35\ 660 - 20\ 930 = 14\ 730$$

$\therefore \quad Q = 14\ 730/150 = 98.2$ N **Ans.**

19.11. Braking of a Vehicle

In a four wheeled moving vehicle, the brakes may be applied to

1. the rear wheels only,

2. the front wheels only, and

3. all the four wheels.

IH – C 322, 2A, 29, 42, 4A, 45, 46, 47, 48 49, 50, 53,
55, 58, 59, 61, 62, 63, 64, 65, 67, 69, 71

Example 7. *A car moving on a level road at a speed 50 km/h has a wheel base 2.8 metres, distance of C.G. from ground level 600 mm, and the distance of C.G. from rear wheels 1.2 metres. Find the distance travelled by the car before coming to rest when brakes are applied,*
1. to the rear wheels, 2. to the front wheels, and 3. to all the four wheels.
The coefficient of friction between the tyres and the road may be taken as 0.6.

Solution. Given : $u = 50$ km/h $= 13.89$ m/s ; $L = 2.8$ m ; $h = 600$ mm $= 0.6$ m ; $x = 1.2$ m ; $\mu = 0.6$

Let s = Distance travelled by the car before coming to rest.

1. When brakes are applied to the rear wheels

30/8/19
59, 44, 57

Since the vehicle moves on a level road, therefore retardation of the car,

$$a = \frac{\mu.g(L-x)}{L+\mu.h} = \frac{0.6\times9.81(2.8-1.2)}{2.8+0.6\times0.6} = 2.98 \text{ m/s}^2$$

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(13.89)^2}{2\times2.98} = 32.4 \text{ m} \quad \textbf{Ans.}$$

2. When brakes are applied to the front wheels

Since the vehicle moves on a level road, therefore retardation of the car,

$$a = \frac{\mu.g.x}{L-\mu.h} = \frac{0.6\times9.18\times1.2}{2.8-0.6\times0.6} = 2.9 \text{ m/s}^2$$

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(13.89)^2}{2\times2.9} = 33.26 \text{ m} \quad \textbf{Ans.}$$

3. When the brakes are applied to all the four wheels

Since the vehicle moves on a level road, therefore retardation of the car,

$$a = g.\mu = 9.81\times0.6 = 5.886 \text{ m/s}^2$$

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(13.89)^2}{2\times5.886} = 16.4 \text{ m} \quad \textbf{Ans.}$$

19.12. Dynamometer

A dynamometer is a brake but in addition it has a device to measure the frictional resistance. Knowing the frictional resistance, we may obtain the torque transmitted and hence the power of the engine.

19.13. Types of Dynamometers

Following are the two types of dynamometers, used for measuring the brake power of an engine.

1. Absorption dynamometers, and
2. Transmission dynamometers.

In the *absorption dynamometers*, the entire energy or power produced by the engine is absorbed by the friction resistances of the brake and is transformed into heat, during the process of measurement. But in the *transmission dynamometers*, the energy is not wasted in friction but is used for doing work.

19.14. Classification of Absorption Dynamometers

The following two types of absorption dynamometers are important from the subject point of view :

1. Prony brake dynamometer, and 2. Rope brake dynamometer.

19.15. Prony Brake Dynamometer

A simplest form of an absorption type dynamometer is a prony brake dynamometer as shown in Fig. 19.31. It consists of two wooden blocks placed around a pulley fixed to the shaft of an engine whose power is required to be measured. The blocks are clamped by means of two bolts and nuts, as shown in Fig. 19.31. A helical spring is provided between the nut and the upper block to adjust the pressure on the pulley to control its speed. The upper block has a long lever attached to it and carries a weight W at its outer end. A counter weight is placed at the other end of the lever which balances the brake when unloaded. Two stops S, S are provided to limit the motion of the lever.

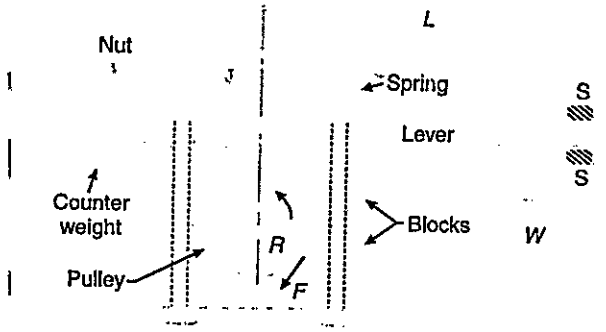


Fig. 19.31. Prony brake dynamometer.

When the brake is to be put in operation, the long end of the lever is loaded with suitable weights W and the nuts are tightened until the engine shaft runs at a constant speed and the lever is in horizontal position. Under these conditions, the moment due to the weight W must balance the moment of the frictional resistance between the blocks and the pulley.

Let W = Weight at the outer end of the lever in newtons,

L = Horizontal distance of the weight W from the centre of the pulley in metres,

F = Frictional resistance between the blocks and the pulley in newtons,

R = Radius of the pulley in metres, and N = Speed of the shaft in r.p.m.

We know that the moment of the frictional resistance or torque on the shaft,

$$T = W.L = F.R \text{ N-m}$$

Work done in one revolution = Torque × Angle turned in radians = $T \times 2\pi$ N-m

$\therefore$ Work done per minute $= T \times 2\pi N$ N-m

We know that brake power of the engine,

$$B.P. = \frac{\text{Work done per min.}}{60} = \frac{T \times 2\pi N}{60} = \frac{W.L \times 2\pi N}{60} \text{ watts}$$

19.16. Rope Brake Dynamometer

It is another form of absorption type dynamometer which is most commonly used for measuring the brake power of the engine. It consists of one, two or more ropes wound around the flywheel or rim of a pulley fixed rigidly to the shaft of an engine. The upper end of the ropes is attached to a spring balance while the lower end of the ropes is kept in position by applying a dead weight as shown in Fig. 19.32. In order to prevent the slipping of the rope over the flywheel, wooden blocks are placed at intervals around the circumference of the flywheel.

In the operation of the brake, the engine is made to run at a constant speed. The frictional torque, due to the rope, must be equal to the torque being transmitted by the engine.

Let W = Dead load in newtons,

S = Spring balance reading in newtons,

D = Diameter of the wheel in metres,

d = diameter of rope in metres, and

N = Speed of the engine shaft in r.p.m.

$\therefore$ Net load on the brake

$$= (W - S) \text{ N}$$

We know that distance moved in one revolution

$$= \pi(D + d) \text{ m}$$

$\therefore$ Work done per revolution

$$= (W - S)\pi(D + d) \text{ N-m}$$

and work done per minute

$$= (W - S)\pi(D + d)\,N \text{ N-m}$$

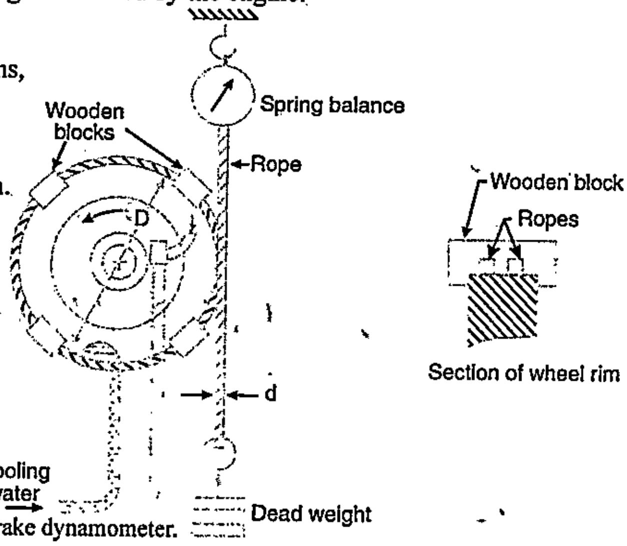


Fig. 19.32. Rope brake dynamometer.

$\therefore$ Brake power of the engine,

$$\text{B.P} = \frac{\text{Work done per min}}{60} = \frac{(W - S)\,\pi\,(D + d) N}{60} \text{ watts}$$

If the diameter of the rope (d) is neglected, then brake power of the engine,

$$\text{B.P.} = \frac{(W - S)\,\pi D\,N}{60} \text{ watts}$$

UNIT-IV BALANCING OF ROTATING MASSES

The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass, is called ***balancing of rotating masses.***

21.3. Balancing of a Single Rotating Mass By a Single Mass Rotating in the Same Plane

Consider a disturbing mass m_1 attached to a shaft rotating at ω rad/s as shown in Fig. 21.1. Let r_1 be the radius of rotation of the mass m_1.

We know that the centrifugal force exerted by the mass m_1 on the shaft,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1 \qquad \ldots (i)$$

This centrifugal force acts radially outwards and thus produces bending moment on the shaft.

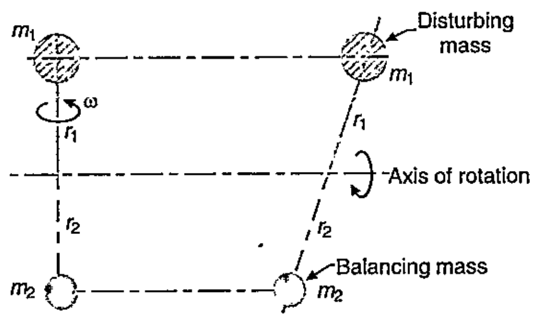


Fig. 21.1. Balancing of a single rotating mass by a single mass rotating in the same plane.

Let $\qquad r_2$ = Radius of rotation of the balancing mass m_2.

$\therefore$ Centrifugal force due to mass m_2,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2 \qquad \ldots (ii)$$

Equating equations (*i*) and (*ii*),

$$m_1 . \omega^2 \cdot r_1 = m_2 \cdot \omega^2 \cdot r_2 \quad \text{or} \quad m_1 \cdot r_1 = m_2 \cdot r_2$$

Discuss how a single revolving mass is balanced by two masses revolving in different planes ?

21.4. Balancing of a Single Rotating Mass By Two Masses Rotating in Different Planes

Two balancing masses are placed in two different planes, parallel to the plane of rotation of the disturbing mass, in such a way that they satisfy the following two conditions of equilibrium.

1. The net dynamic force acting on the shaft is equal to zero. This requires that the line of action of three centrifugal forces must be the same. In other words, the centre of the masses of the system must lie on the axis of rotation. This is the condition for S*tatic balancing.*
2. The net couple due to the dynamic forces acting on the shaft is equal to zero. In other words, the algebraic sum of the moments about any point in the plane must be zero. The conditions (1) and (2) together give *dynamic balancing.*

1. ***When the plane of the disturbing mass lies in between the planes of the two balancing masses***

Consider a disturbing mass m lying in a plane A to be balanced by two rotating masses m_1 and m_2 lying in two different planes L and M as shown in Fig. 21.2. Let r, r_1 and r_2 be the radii of rotation of the masses in planes A, L and M respectively.

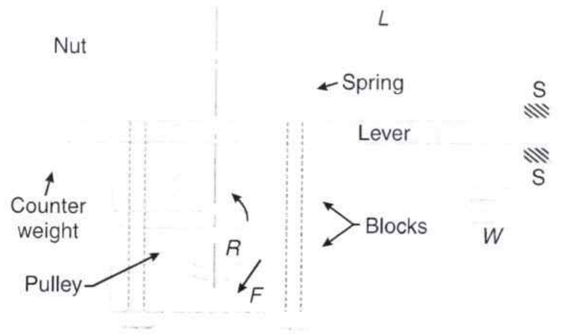


Fig. 19.31. Prony brake dynamometer.

When the brake is to be put in operation, the long end of the lever is loaded with suitable weights W and the nuts are tightened until the engine shaft runs at a constant speed and the lever is in horizontal position. Under these conditions, the moment due to the weight W must balance the moment of the frictional resistance between the blocks and the pulley.

Let W = Weight at the outer end of the lever in newtons,

L = Horizontal distance of the weight W from the centre of the pulley in metres,

F = Frictional resistance between the blocks and the pulley in newtons,

R = Radius of the pulley in metres, and N = Speed of the shaft in r.p.m.

We know that the moment of the frictional resistance or torque on the shaft,

$$T = W.L = F.R \text{ N-m}$$

Work done in one revolution = Torque × Angle turned in radians = $T \times 2\pi$ N-m

∴ Work done per minute = $T \times 2\pi N$ N-m

We know that brake power of the engine,

$$B.P. = \frac{\text{Work done per min.}}{60} = \frac{T \times 2\pi N}{60} = \frac{W.L \times 2\pi N}{60} \text{ watts}$$

19.16. Rope Brake Dynamometer

It is another form of absorption type dynamometer which is most commonly used for measuring the brake power of the engine. It consists of one, two or more ropes wound around the flywheel or rim of a pulley fixed rigidly to the shaft of an engine. The upper end of the ropes is attached to a spring balance while the lower end of the ropes is kept in position by applying a dead weight as shown in Fig. 19.32. In order to prevent the slipping of the rope over the flywheel, wooden blocks are placed at intervals around the circumference of the flywheel.

In the operation of the brake, the engine is made to run at a constant speed. The frictional torque, due to the rope, must be equal to the torque being transmitted by the engine.

Let W = Dead load in newtons,

S = Spring balance reading in newtons,

D = Diameter of the wheel in metres,

d = diameter of rope in metres, and

N = Speed of the engine shaft in r.p.m.

∴ Net load on the brake

$$= (W - S) \text{ N}$$

We know that distance moved in one revolution

$$= \pi(D + d) \text{ m}$$

∴ Work done per revolution

$$= (W - S)\,\pi(D + d) \text{ N-m}$$

and work done per minute

$$= (W - S)\,\pi(D + d)\,N \text{ N-m}$$

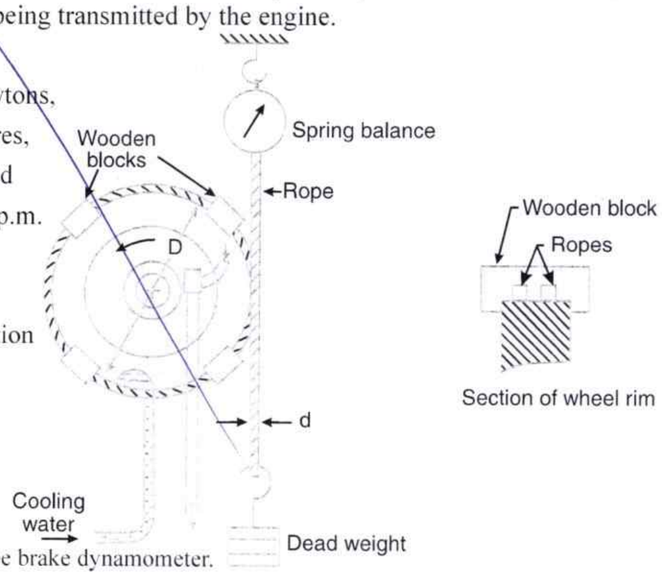


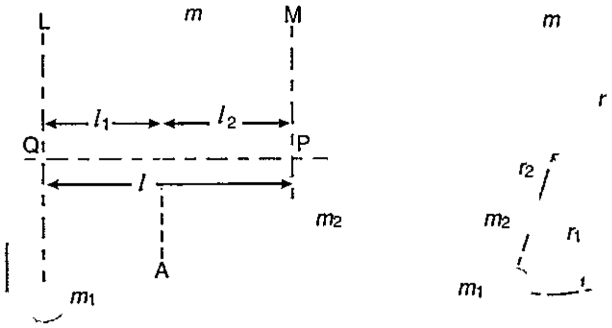
Fig. 19.32. Rope brake dynamometer.

∴ Brake power of the engine,

$$\text{B.P} = \frac{\text{Work done per min}}{60} = \frac{(W - S)\,\pi\,(D + d)N}{60} \text{ watts}$$

If the diameter of the rope (d) is neglected, then brake power of the engine,

$$\text{B.P.} = \frac{(W - S)\,\pi\, D\, N}{60} \text{ watts}$$



We know that the centrifugal force exerted by the mass m in the plane A,

$$F_C = m \cdot \omega^2 \cdot r$$

Similarly, the centrifugal force exerted by the mass m_1 in the plane L,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1$$

and, the centrifugal force exerted by the mass m_2 in the plane M,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2$$

Since the net force acting on the shaft must be equal to zero, therefore the centrifugal force on the disturbing mass must be equal to the sum of the centrifugal forces on the balancing masses, therefore

$$F_C \quad F_{C1} \quad F_{C2}$$

$$\therefore \qquad m \cdot r = m_1 \cdot r_1 + m_2 \cdot r_2 \qquad \ldots (i)$$

Now in order to find the magnitude of balancing force in the plane L (or the dynamic force at the bearing Q of a shaft), take moments about P which is the point of intersection of the plane M and the axis of rotation. Therefore

$$F_{C1} \times l = F_C \times l_2$$

$$\therefore \qquad m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2 \qquad \ldots (ii)$$

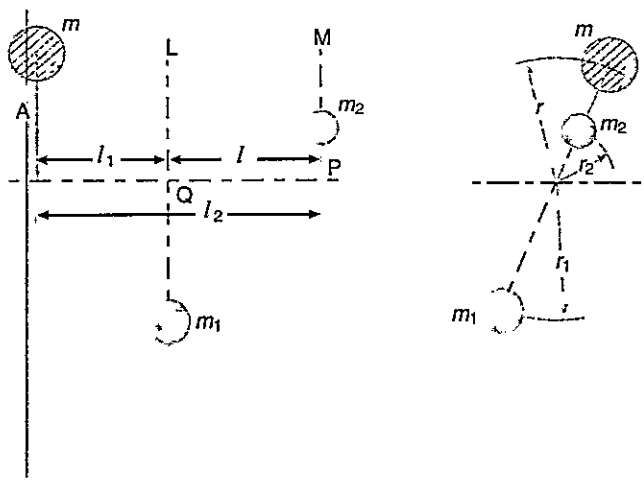
Similarly, in order to find the balancing force in plane M (or the dynamic force at the bearing P of a shaft), take moments about Q which is the point of intersection of the plane L and the axis of rotation. Therefore

$$F_{C2} \times l = F_C \times l_1$$

$$\therefore \qquad m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1 \qquad \ldots (iii)$$

•

2. ***When the plane of the disturbing mass lies on one end of the planes of the balancing masses***



In this case, the mass m lies in the plane A and the balancing masses lie in the planes L and M, the following conditions must be satisfied in order
to balance the system, *i.e.*

$$F_C + F_{C2} = F_{C1}$$

$$\therefore \quad m \cdot r + m_2 \cdot r_2 = m_1 \cdot r_1 \quad \ldots (iv)$$

Now, to find the balancing force in the plane L (or the dynamic force at the bearing Q of a shaft), take moments about P which is the point of intersection of the plane M and the axis of rotation. Therefore

$$F_{C1} \times l = F_C \times l_2$$

$$\therefore \quad m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2 \quad \ldots (v)$$

... [Same as equation (*ii*)]

Similarly, to find the balancing force in the plane M (or the dynamic force at the bearing P of a shaft), take moments about Q which is the point of intersection of the plane L and the axis of rotation. Therefore

$$F_{C2} \times l = F_C \times l_1$$

$$m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1$$

21.5. Balancing of Several Masses Rotating in the Same Plane

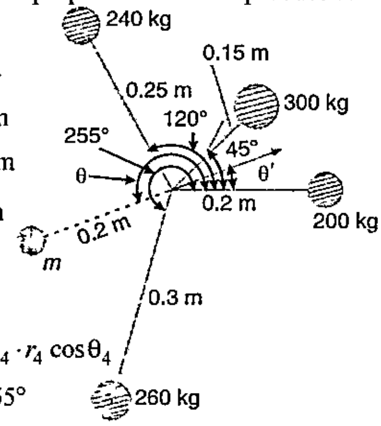
Example 1. *Four masses m_1, m_2, m_3 and m_4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m r espectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass r equired, if its radius of r otation is 0.2 m.*

Solution. Given : $m_1 = 200$ kg ; $m_2 = 300$ kg ; $m_3 = 240$ kg ; $m_4 = 260$ kg ; $r_1 = 0.2$ m ; $r_2 = 0.15$ m ; $r_3 = 0.25$ m ; $r_4 = 0.3$ m ; $\theta_1 = 0°$; $\theta_2 = 45°$; $\theta_3 = 45° + 75° = 120°$; $\theta_4 = 45° + 75° + 135° = 255°$; $r = 0.2$ m

Let m = Balancing mass, and

θ = The angle which the balancing mass makes with m_1.

Since the magnitude of centrifugal forces are proportional to the product of each mass and its radius, therefore



$$m_1 \cdot r_1 = 200 \times 0.2 = 40 \text{ kg-m}$$

$$m_2 \cdot r_2 = 300 \times 0.15 = 45 \text{ kg-m}$$

$$m_3 \cdot r_3 = 240 \times 0.25 = 60 \text{ kg-m}$$

$$m_4 \cdot r_4 = 260 \times 0.3 = 78 \text{ kg-m}$$

1. *Analytical method*

$$\Sigma H = m_1 \cdot r_1 \cos\theta_1 + m_2 \cdot r_2 \cos\theta_2 + m_3 \cdot r_3 \cos\theta_3 + m_4 \cdot r_4 \cos\theta_4$$

$$= 40 \cos 0° + 45 \cos 45° + 60 \cos 120° + 78 \cos 255°$$

$$= 40 + 31.8 - 30 - 20.2 = 21.6 \text{ kg-m}$$

Now resolving vertically,

$$\Sigma V = m_1 \cdot r_1 \sin\theta_1 + m_2 \cdot r_2 \sin\theta_2 + m_3 \cdot r_3 \sin\theta_3 + m_4 \cdot r_4 \sin\theta_4$$

$$= 40 \sin 0° + 45 \sin 45° + 60 \sin 120° + 78 \sin 255°$$

$$= 0 + 31.8 + 52 - 75.3 = 8.5 \text{ kg-m}$$

∴ Resultant, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(21.6)^2 + (8.5)^2} = 23.2$ kg-m

We know that

$$m \cdot r = R = 23.2 \quad \text{or} \quad m = 23.2 / r = 23.2 / 0.2 = 116 \text{ kg} \quad \textbf{Ans.}$$

and $\tan\theta' = \Sigma V / \Sigma H = 8.5 / 21.6 = 0.3935 \quad \text{or} \quad \theta' = 21.48°$

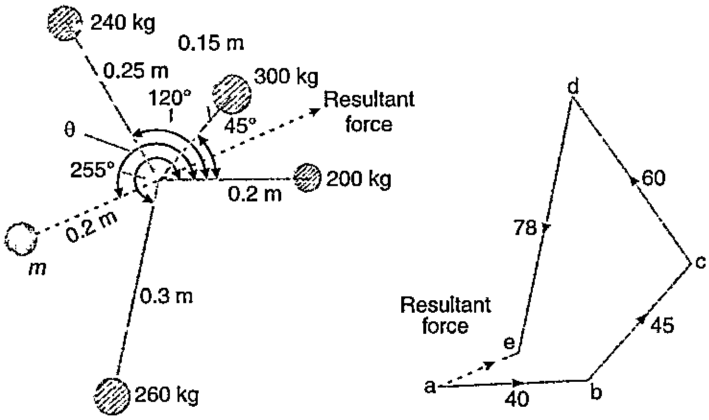
Since θ' is the angle of the resultant R from the horizontal mass of 200 kg, therefore the angle of the balancing mass from the horizontal mass of 200 kg

$$\theta = 180° + 21.48° = 201.48° \quad \textbf{Ans.}$$

2. *Graphical method*

1. First of all, draw the space diagram showing the positions of all the given masses as shown in Fig 21.6 (*a*).
2. Since the centrifugal force of each mass is proportional to the product of the mass and radius, therefore

$$m_1.r_1 = 200 \times 0.2 = 40 \text{ kg-m} \;\&\; m_2.r_2 = 300 \times 0.15 = 45 \text{ kg-m}$$



(*a*) Space diagram. (*b*) Vector diagram

Fig. 21.6

4. The balancing force is equal to the resultant force, but *opposite* in direction as shown in Fig. 21.6 (*a*). Since the balancing force is proportional to *m.r*, therefore

$$m \times 0.2 = \text{vector } ea = 23 \text{ kg-m} \quad \text{or} \quad m = 23/0.2 = \textbf{115 kg Ans.}$$

By measurement we also find that the angle of inclination of the balancing mass (m) from the horizontal mass of 200 kg

$$\theta = 201° \quad \textbf{Ans.}$$

21.6. Balancing of Several Masses Rotating in Different Planes

Example 2. *A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.*

Solution. Given : $m_A = 200$ kg ; $m_B = 300$ kg ; $m_C = 400$ kg ; $m_D = 200$ kg ; $r_A = 80$ mm $= 0.08$m ; $r_B = 70$ mm $= 0.07$ m ; $r_C = 60$ mm $= 0.06$ m ; $r_D = 80$ mm $= 0.08$ m ; $r_X = r_Y = 100$ mm $= 0.1$ m

Let m_X = Balancing mass placed in plane X, and

m_Y = Balancing mass placed in plane Y.

The position of planes and angular position of the masses (assuming the mass A as horizontal) are shown in Fig. 21.8 (a) and (b) respectively.

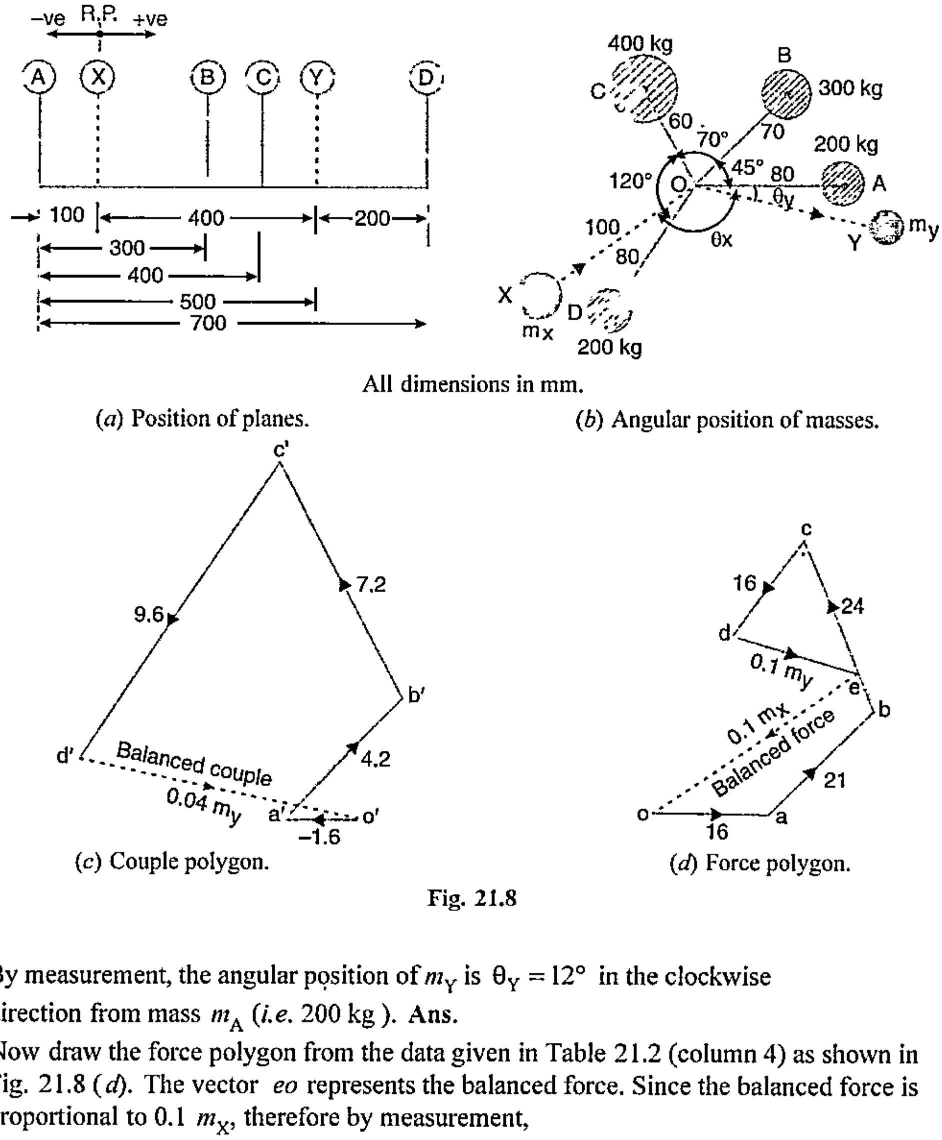
Assume the plane X as the reference plane (*R.P.*). The distances of the planes to the right of plane X are taken as + ve while the distances of the planes to the left of plane X are taken as – ve.

Table 21.2

Plane *(1)*	*Mass (m)* *kg* *(2)*	*Radius (r)* *m* *(3)*	*Cent.force* $\div \omega^2$ *(m.r) kg-m* *(4)*	*Distance from* *Plane x(l) m* *(5)*	*Couple* $\div \omega^2$ *(m.r.l) kg-m²* *(6)*
A	200	0.08	16	− 0.1	− 1.6
X(R.P.)	m_X	0.1	0.1 m_X	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_Y	0.1	0.1 m_Y	0.4	0.04 m_Y
D	200	0.08	16	0.6	9.6

1. First of all, draw the couple polygon from the data given in Table 21.2 (column 6) as shown in Fig. 21.8 (*c*) to some suitable scale. The vector $d'o'$ represents the balanced couple. Since the balanced couple is proportional to 0.04 m_Y, therefore by measurement,

$$0.04\, m_Y = \text{vector } d'o' = 7.3 \text{ kg-m}^2 \quad \text{or} \quad m_Y = 182.5 \text{ kg } \textbf{Ans.}$$



All dimensions in mm.

(*a*) Position of planes. (*b*) Angular position of masses.

(*c*) Couple polygon. (*d*) Force polygon.

Fig. 21.8

By measurement, the angular position of m_Y is $\theta_Y = 12°$ in the clockwise direction from mass m_A (*i.e.* 200 kg). **Ans.**

2. Now draw the force polygon from the data given in Table 21.2 (column 4) as shown in Fig. 21.8 (*d*). The vector *eo* represents the balanced force. Since the balanced force is proportional to 0.1 m_X, therefore by measurement,

$$0.1\, m_X = \text{vector } eo = 35.5 \text{ kg-m} \quad \text{or} \quad m_X = 355 \text{ kg } \textbf{Ans.}$$

By measurement, the angular position of m_X is $\theta_X = 145°$ in the clockwise direction from mass m_A (*i.e.* 200 kg). **Ans.**

Table 21.3

Example 3. *A, B, C and D ar e four masses carried by a r otating shaft at radii 100, 125, 200 and 150 mm r espectively. The planes in which the masses r evolve are spaced 600 mm apart and the mass of B, C and D ar e 10 kg, 5 kg, and 4 kg r espectively.*

Find the required mass A and the r elative angular settings of the four masses so that the shaft shall be in complete balance.

Solution. Given : $r_A = 100$ mm = 0.1 m ; $r_B = 125$ mm = 0.125 m ; $r_C = 200$ mm = 0.2 m ; $r_D = 150$ mm = 0.15 m ; $m_B = 10$ kg ; $m_C = 5$ kg ; $m_D = 4$ kg

The position of planes is shown in Fig. 21.10 (*a*). Assuming the plane of mass *A* as the reference plane (*R.P.*), the data may be tabulated as below :

Table 21.4

Plane (1)	*Mass (m)* kg (2)	*Radius (r)* m (3)	*Cent. Force* $\div \omega^2$ (m.r) kg-m (4)	*Distance from plane A (l)* m (5)	*Couple* $\div \omega^2$ (m.r.l) kg-m^2 (6)
A(R.P.)	m_A	0.1	0.1 m_A	0	0
B	10	0.125	1.25	0.6	0.75
C	5	0.2	1	1.2	1.2
D	4	0.15	0.6	1.8	1.08

Drawing the couple polygon from the data given in Table 21.4 (column 6). Assume the position of mass *B* in the horizontal direction. By measurement, we find that the angular setting of mass *C* from mass *B* in the anticlockwise direction, *i.e.*

$\angle BOC = 240°$ **Ans.**

and angular setting of mass *D* from mass *B* in the anticlockwise direction, *i.e* $\angle BOD = 100°$ **Ans.**

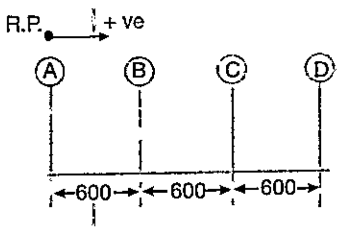
Draw the force polygon to some suitable scale, as shown in Fig. 21.10 (*d*).

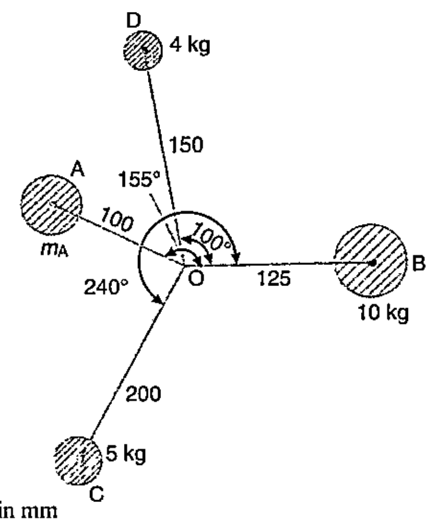
Since the closing side of the force polygon (vector *do*) is proportional to 0.1 m_A, therefore by measurement,

$0.1\, m_A = 0.7$ kg-m^2 or $m_A = 7$ kg **Ans.**

Now draw *OA* in Fig. 21.10 (*b*), parallel to vector *do*. By measurement, we find that the angular setting of mass *A* from mass *B* in the anticlockwise direction, *i.e.*

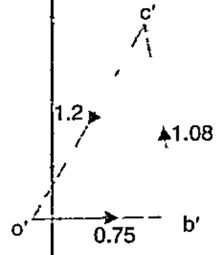
$\angle BOA = 155°$ **Ans.**

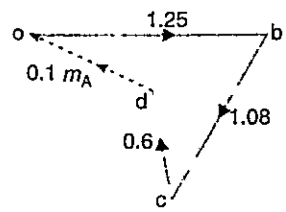




All dimensions in mm

(*a*) Position of planes. (*b*) Angular position of masses.





Explain about primary and secondary unbalanced forces of rotating masses ?

22.1. Introduction

BALANCING OF RECIPROCATING MASSES:

The resultant of all the forces acting on the body of the engine due to inertia forces only is known as *unbalanced force* or *shaking force.*

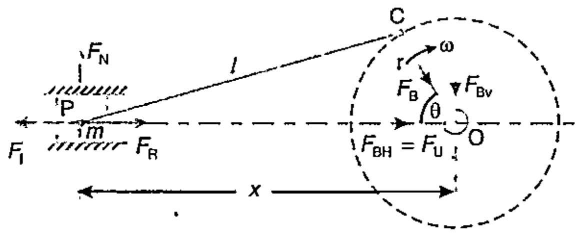


Fig. 22.1. Reciprocating engine mechanism.

Let F_R = Force required to accelerate the reciprocating parts,

F_I = Inertia force due to reciprocating parts,

F_N = Force on the sides of the cylinder walls or normal force acting on the cross-head guides, and

F_B = Force acting on the crankshaft bearing or main bearing.

22.2. Primary and Secondary Unbalanced Forces of Reciprocating Masses

Consider a reciprocating engine mechanism as shown in Fig. 22.1.

Let m = Mass of the reciprocating parts,

l = Length of the connecting rod PC,

r = Radius of the crank OC,

θ = Angle of inclination of the crank with the line of stroke PO,

ω = Angular speed of the crank,

n = Ratio of length of the connecting rod to the crank radius = l / r.

The acceleration of the reciprocating parts is approximately given by the expression,

$$a_R = \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$\therefore$ Inertia force due to reciprocating parts or force required to accelerate the reciprocating parts,

$$F_I = F_R = \text{Mass} \times \text{acceleration} = m \cdot \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

The horizontal component of the force exerted on the crank shaft bearing (*i.e.* F_{BH}) is equal and opposite to inertia force (F_I). This force is an unbalanced one and is denoted by F_U.

$\therefore$ Unbalanced force,

$$F_U = m \cdot \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right) = m.\omega^2 \cdot r\cos\theta + m \cdot \omega^2 \cdot r \times \frac{\cos 2\theta}{n} = F_P + F_S$$

The expression $(m \cdot \omega^2 \cdot r\cos\theta)$ is known as *primary unbalanced force* and $\left(m \cdot \omega^2 \cdot r \times \frac{\cos 2\theta}{n}\right)$ is called *secondary unbalanced force.*

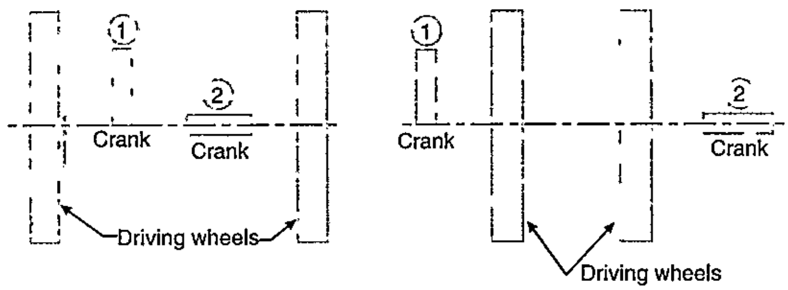
22.4. Partial Balancing of Locomotives

The locomotives, usually, have two cylinders with cranks placed at right angles to each other in order to have uniformity in turning moment diagram. The two cylinder locomotives may be classified as :

1. Inside cylinder locomotives ; and 2. Outside cylinder locomotives.

In the ***inside cylinder locomotives,*** the two cylinders are placed in between the planes of two driving wheels as shown in Fig. 22.3 (*a*) ; whereas in the ***outside cylinder locomotives,*** the two cylinders are placed outside the driving wheels, one on each side of the driving wheel, as shown in Fig. 22.3 (*b*). The locomotives may be

(*a*) Single or uncoupled locomotives ; and (*b*) Coupled locomotives.



(*a*) Inside cylinder locomotives. (*b*) Outside cylinder locomotives.

Fig. 22.3

22.5. Effect of Partial Balancing of Reciprocating Parts of Two Cylinder Locomotives

22.6. Variation of Tractive Force

The resultant unbalanced force due to the two cylinders, along the line of stroke, is known as *tractive force.*

∴ As per definition, the tractive force,

F_T = Resultant unbalanced force along the line of stroke

$$= (1-c)m.\omega^2.r\cos\theta + (1-c)m.\omega^2.r\cos(90°+\theta)$$

$$= (1-c)m.\omega^2.r(\cos\theta - \sin\theta)$$

Thus, the tractive force is maximum or minimum when θ = 135° or 315°.

∴ Maximum and minimum value of the tractive force or the variation in tractive force

$$= \pm(1-c)\,m.\omega^2.r(\cos 135° - \sin 135°) = \pm\sqrt{2}\,(1-c)\,m.\omega^2.r$$

22.7. Swaying Couple

The unbalanced forces along the line of stroke for the two cylinders constitute a couple about the centre line *YY* between the cylinders as shown in Fig. 22.5.

This couple has swaying effect about a vertical axis, and tends to sway the engine alternately in clockwise and anticlockwise directions. Hence the couple is known as ***swaying couple.***

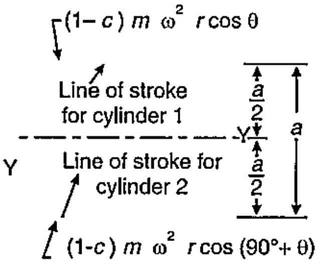
Let a = Distance between the centre lines of the two cylinders.

∴ Swaying couple

$$= (1-c)m.\omega^2.r\cos\theta \times \frac{a}{2}$$

$$-(1-c)m.\omega^2.r\cos(90^\circ+\theta)\frac{a}{2}$$

$$=(1-c)m.\omega^2.r\times\frac{a}{2}(\cos\theta+\sin\theta)$$



Thus, the swaying couple is maximum or minimum when $\theta = 45^\circ$ or 225°.

$\therefore$ Maximum and minimum value of the swaying couple

$$=\pm(1-c)m.\omega^2.r\times\frac{a}{2}(\cos 45^\circ+\sin 45^\circ)=\pm\frac{a}{\sqrt{2}}(1-c)m.\omega^2.r$$

22.8. Hammer Blow

The maximum magnitude of the unbalanced force along the perpendicular to the line of stroke is known as ***hammer blow***.

$\therefore$ Hammer blow $= B.\omega^2.b$

Example 4. *An inside cylinder locomotive has its cylinder centre lines 0.7 m apart and has a stroke of 0.6 m. The rotating masses per cylinder are equivalent to 150 kg at the crank pin, and the reciprocating masses per cylinder to 180 kg. The wheel centre lines are 1.5 m apart. The cranks are at right angles. The whole of the rotating and 2/3 of the reciprocating masses are to be balanced by masses placed at a radius of 0.6 m. Find the magnitude and direction of the balancing masses. Find the fluctuation in rail pressure under one wheel, variation of tractive effort and the magnitude of swaying couple at a crank speed of 300 r.p.m.*

Solution. Given : $a = 0.7$ m; $l_B = l_C = 0.6$ m or $r_B = r_C = 0.3$ m; $m_1 = 150$ kg; $m_2 = 180$ kg; $c = 2/3$; $r_A = r_D = 0.6$ m; $N = 300$ r.p.m. or $\omega = 2\pi\times 300/60 = 31.42$ rad/s

The equivalent mass of the rotating parts to be balanced per cylinder at the crank pin,

$$m = m_B = m_C = m_1 + c.m_2 = 150 + \frac{2}{3}\times 180 = 270 \text{ kg}$$

Magnitude and direction of the balancing masses

Let m_A and m_D = Magnitude of the balancing masses

θ_A and θ_D = Angular position of the balancing masses m_A and m_D from the first crank B.

The magnitude and direction of the balancing masses may be determined graphically as discussed below :

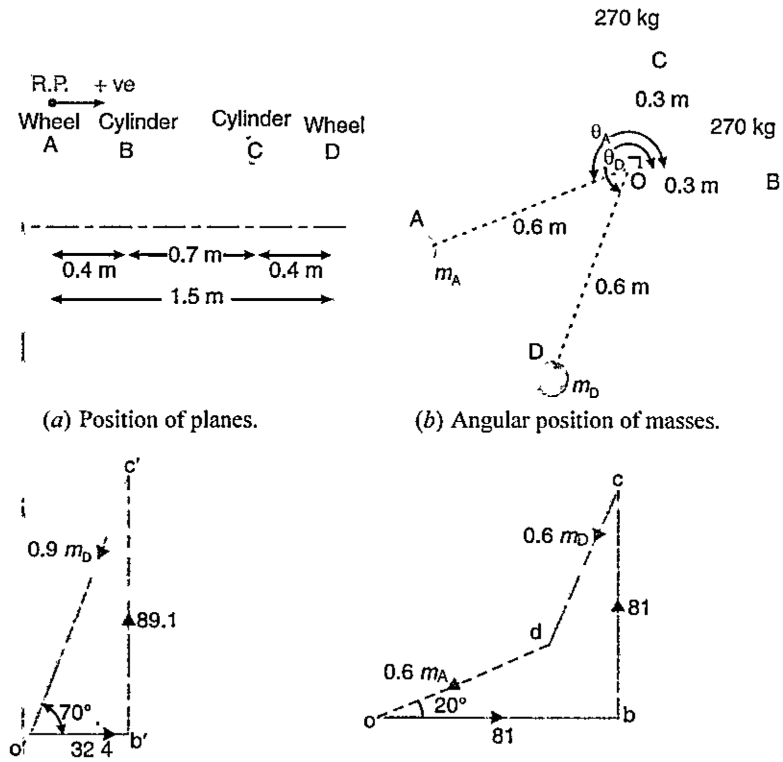
1. First of all, draw the space diagram to show the positions of the planes of the wheels and the cylinders, as shown in Fig. 22.7 (*a*). Since the cranks of the cylinders are at right angles, therefore assuming the position of crank of the cylinder B in the horizontal direction, draw OC and OB at right angles to each other as shown in Fig. 22.7 (*b*).
2. Tabulate the data as given in the following table. Assume the plane of wheel A as the reference plane.

Table 22.1

Plane (1)	*mass.* *(m) kg* (2)	*Radius* *(r)m* (3)	*Cent. force* $\div\ \omega^2$ *(m.r) kg-m* (4)	*Distance from plane A (l)m* (5)	*Couple* $\div\ \omega^2$ *(m.r.l) kg-m²* (6)
A (R.P.)	m_A	0.6	0.6 m_A	0	0
B	270	0.3	81	0.4	32.4
C	270	0.3	81	1.1	89.1
D	m_D	0.6	$0.6m_D$	1.5	0.9 m_D

3. Now, draw the couple polygon from the data given in Table 22.1 (column 6), to some suitable scale, as shown in Fig 22.7 (*c*). The closing side *c′o′* represents the balancing couple and it is proportional to 0.9 m_D. Therefore, by measurement,

$$0.9\ m_D = \text{vector } c'o' = 94.5 \text{ kg-m}^2 \quad\text{or}\quad m_D = 105 \text{ kg} \quad \textbf{Ans.}$$



(*a*) Position of planes. (*b*) Angular position of masses.

(*c*) Couple polygon. (*d*) Force polygon.

Fig. 22.7

4. To determine the angular position of the balancing mass *D*, draw *OD* in Fig. 22.7 (*b*) parallel to vector $c'o'$. By measurement,

$$\theta_D = 250° \textbf{ Ans.}$$

5. In order to find the balancing mass *A*, draw the force polygon from the data given in Table 22.1 (column 4), to some suitable scale, as shown in Fig. 22.7 (*d*), The vector *do* represents the balancing force and it is proportional to 0.6 m_A. Therefore by measurement,

$$0.6\ m_A = \text{vector } do = 63 \text{ kg-m or } m_A = 105 \text{ kg } \textbf{Ans.}$$

6. To determine the angular position of the balancing mass *A*, draw *OA* in Fig. 22.7 (*b*) parallel to vector *do*. By measurement,

$$\theta_A = 200° \textbf{ Ans.}$$

Fluctuation in rail pressure

We know that each balancing mass = 105 kg

Balancing mass for rotating masses,

$$D = \frac{m_1}{m} \times 105 = \frac{150}{270} \times 105 = 58.3 \text{ kg}$$

and balancing mass for reciprocating masses,

$$B = \frac{c.m_2}{m} \times 105 = \frac{2}{3} \times \frac{180}{270} \times 105 = 46.6 \text{ kg}$$

This balancing mass of 46.6 kg for reciprocating masses gives rise to the centrifugal force.

$\therefore$ Fluctuation in rail pressure or hammer blow

$$= B.\omega^2.b = 46.6\ (31.42)^2\ 0.6 = 27\ 602 \text{ N. } \textbf{Ans.} \qquad \ldots (\because\ b = r_A = r_D)$$

Variation of tractive ef fort

We know that maximum variation of tractive ef fort

$$= \pm\sqrt{2}(1-c)m_2.\omega^2.r = \pm\sqrt{2}\left(1-\frac{2}{3}\right)180(31.42)^2 0.3 \text{ N}$$

$= \pm 25\ 127\ \text{N}$ **Ans.** ... ($\because r = r_B = r_C$)

Swaying couple

We know that maximum swaying couple

$$= \frac{a(1-c)}{\sqrt{2}} \times m_2.\omega^2.r = \frac{0.7\left(1-\frac{2}{3}\right)}{\sqrt{2}} \times 180(31.42)^2 0.3 \text{ N-m}$$

$= 8797$ N-m **Ans.**

$$\omega = \frac{v}{D/2} = \frac{33.33}{1.8/2} = 37 \text{rad/s}$$

We know that hammer blow

$$= \pm B.\omega^2.b = 33(37)^2 0.675 = \pm 30.494 \text{ N} \quad \textbf{Ans.}$$

... ($\because B = m''_E$, and $b = r_B = r_E$)

22.10. Balancing of Primary Forces of Multi-cylinder In-line Engines

The multi-cylinder engines with the cylinder centre lines in the same plane and on the same side of the centre line of the crankshaft, are known as ***In-line engines.*** The following two conditions must be satisfied in order to give the primary balance of the reciprocating parts of a multi-cylinder engine :

1. The algebraic sum of the primary forces must be equal to zero. In other words, the primary force polygon must "close ; and
2. The algebraic sum of the couples about any point in the plane of the primary forces must be equal to zero. In other words, the primary couple polygon must close.

22.11. Balancing of Secondary Forces of Multi-cylinder In-line Engines

When the connecting rod is not too long (*i.e.* when the obliquity of the connecting rod is considered), then the secondary disturbing force due to the reciprocating mass arises.

The secondary force,

$$F_S = m.\omega^2.r \times \frac{\cos 2\theta}{n}$$

Example 5. *A four cylinder vertical engine has cranks 150 mm long. The planes of rotation of the first, second and fourth cranks ar e 400 mm, 200 mm and 200 mm r espectively from the third crank and their r eciprocating masses are 50 kg, 60 kg and 50 kg r espectively. Find the mass of the r eciprocating parts for the thir d cylinder and the r elative angular positions of the cranks in or der that the engine may be in complete primary balance.*

Solution. Given $r_1 = r_2 = r_3 = r_4 = 150$ mm $= 0.15$ m ; $m_1 = 50$ kg ; $m_2 = 60$ kg ; $m_4 = 50$ kg

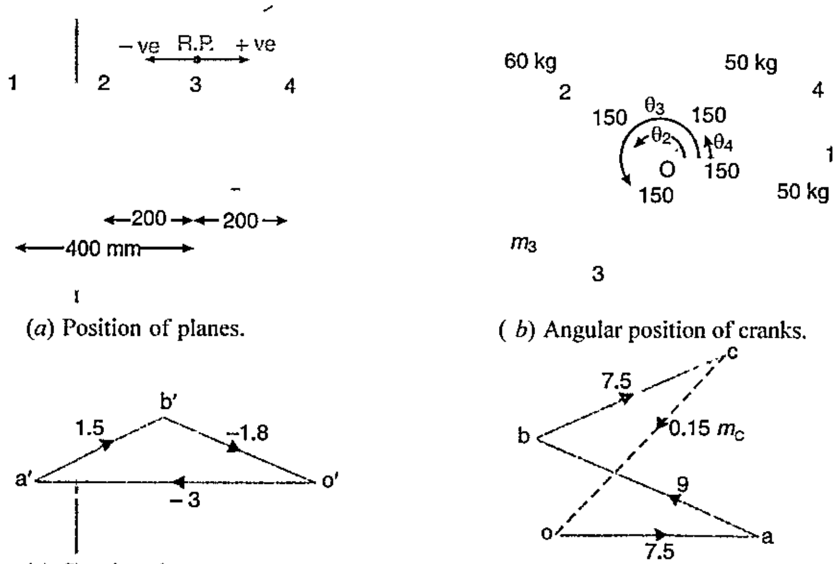
The position of planes is shown in Fig. 22.17 (*a*). Assuming the plane of third cylinder as the reference plane, the data may be tabulated as given in T able 22.8.

Table 22.8

Plane (1)	*Mass* (*m*) *kg* (2)	*Radius* (*r*) *m* (3)	*Cent. force* $\div \omega^2$ (*m.r*) *kg-m* (4)	*Distance from plane 3*(*l*) *m* (5)	*Couple* $\div \omega^2$ (*m.r.l*) *kg-m*^{2} (6)
1	50	0.15	7.5	– 0.4	– 3
2	60	0.15	9	– 0.2	– 1.8
3(*R.P.*)	m_3	0.15	$0.15m_3$	0	0
4	50	0.15	7.5	0.2	1.5

First of all, the angular position of cranks 2 and 4 are obtained by drawing the couple polygon from the data given in T able 22.8 (column 6). Assume the position of crank 1 in the horizontal direction as shown in Fig 22.17 (*b*), The couple polygon, as shown in Fig. 22.17 (*c*), is drawn as discussed below:

1. Draw vector $o'a'$ in the horizontal direction (*i.e.* parallel to $O1$) and equal to – 3 kg-m^2, to some suitable scale.
2. From point o' and a', draw vectors $o'b'$ and $a'b'$ equal to – 1.8 kg-m^2 and 1.5 kg-m^2 respectively. These vectors intersect at b'.



(a) Position of planes. (b) Angular position of cranks.

(c) Couple polygon. (d) Force polygon.

Fig. 22.17

3. Now in Fig. 22.17 (b), draw $O2$ parallel to vector $o'b'$ and $O4$ parallel to vector $a'b'$.

By measurement, we find that the angular position of crank 2 from crank 1 in the anticlockwise direction is

$$\theta_2 = 160° \textbf{ Ans.}$$

and the angular position of crank 4 from crank 1 in the anticlockwise direction is

$$\theta_4 = 26° \textbf{ Ans.}$$

In order to find the mass of the third cylinder (m_3) and its angular position, draw the force polygon, to some suitable scale, as shown in Fig. 22.17 (d), from the data given in T able 22.8 (column 4). Since the closing side of the force polygon (vector co) is proportional to 0.15 m_3, therefore by measurement,

$$0.15 m_3 = 9 \text{ kg-m} \quad \text{or} \quad m_3 = 60 \text{ kg} \textbf{ Ans.}$$

Now draw O3 in Fig 22.17 (b), parallel to vector co. By measurement, we find that theangular position of crank 3 from crank 1 in the anticlockwise direction is θ3 = 227° Ans.

22.13. Balancing of V-engines

Consider a symmetrical two cylinder *V*-engine as shown in Fig. 22.33, The common crank *OC* is driven by two connecting rods *PC* and *QC*. The lines of stroke *OP* and *OQ* are inclined to the vertical *OY*, at an angle α as shown in Fig 22.33.

Let m = Mass of reciprocating parts per cylinder ,

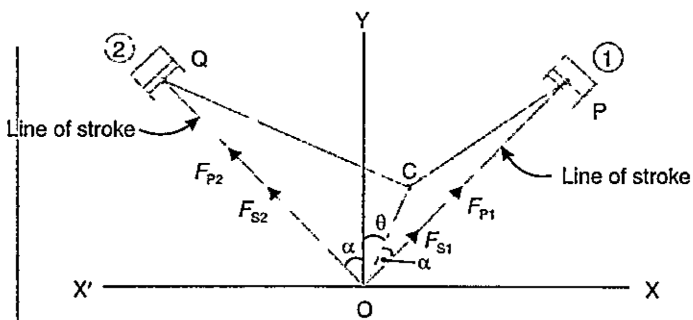
l = Length of connecting rod,

r = Radius of crank,

n = Ratio of length of connecting rod to crank radius = l / r

θ = Inclination of crank to the vertical at any instant,

ω = Angular velocity of crank.



We know that inertia force due to reciprocating parts of cylinder 1, along the line of stroke

$$= m.\omega^2.r\left[\cos(\alpha-\theta)+\frac{\cos 2(\alpha-\theta)}{n}\right]$$

and the inertia force due to reciprocating parts of cylinder 2, along the line of stroke

$$= m.\omega^2.r\left[\cos(\alpha-\theta)+\frac{\cos 2(\alpha+\theta)}{n}\right]$$

•

The balancing of V-engines is only considered for primary and secondary forces * as discussed below :

Considering primary forces

We know that primary force acting along the line of stroke of cylinder 1,

$$F_{P1} = m.\omega^2.r\cos(\alpha-\theta)$$

$\therefore$ Component of F_{P1} along the vertical line OY,

$$= F_{P1}\cos\alpha = m.\omega^2 r.\cos(\alpha-\theta)\cos\alpha \qquad \ldots (i)$$

and component of F_{P1} along the horizontal line OX

$$= F_{P1}\sin\alpha = m.\omega^2 r\cos(\alpha-\theta)\sin\alpha \qquad \ldots (ii)$$

Similarly, primary force acting along the line of stroke of cylinder 2,

$$F_{P2} = m.\omega^2.r\cos(\alpha+\theta)$$

$\therefore$ Component of F_{P2} along the vertical line OY

$$= F_{P2}\cos\alpha = m.\omega^2.r\cos(\alpha+\theta)\cos\alpha \qquad \ldots (iii)$$

and component of F_{P2} along the horizontal line OX'

$$= F_{P2}\sin\alpha = m.\omega^2.r\cos(\alpha+\theta)\sin\alpha \qquad \ldots (iv)$$

Total component of primary force along the vertical line OY

$$F_{PV} = (i) + (iii) = m.\omega^2.r\cos\alpha\,[\cos(\alpha-\theta)+\cos(\alpha+\theta)]$$

$$= m.\omega^2.r\cos\alpha\times 2\cos\alpha\cos\theta$$

$$\ldots [\because\ \cos(\alpha-\theta)+\cos(\alpha+\theta) = 2\cos\alpha\cos\theta]$$

$$= 2\,m.\omega^2.r\cos^2\alpha.\cos\theta$$

and total component of primary force along the horizontal line OX

$$F_{PH} = (ii) - (iv) = m.\omega^2.r\sin\alpha[\cos(\alpha-\theta)-\cos(\alpha+\theta)]$$

$$= m.\omega^2.r\sin\alpha\times 2\sin\alpha\sin\theta$$

$$\ldots [\because\ \cos(\alpha-\theta)-\cos(\alpha+\theta) = 2\sin\alpha\sin\theta]$$

$$= 2m.\omega^2.r\sin^2\alpha.\sin\theta$$

$\therefore$ Resultant primary force,

$$F_P = \sqrt{(F_{PV})^2+(F_{PH})^2}$$

$$= 2m.\omega^2.r\sqrt{(\cos^2\alpha.\cos\theta)^2+(\sin^2\alpha.\sin\theta)^2} \qquad \ldots (v)$$

Notes : The following results, derived from equation (v), depending upon the value of α may be noted :

1. When $2\alpha = 60°$ or $\alpha = 30°$,

$$F_P = 2m.\omega^2.r\sqrt{(\cos^2 30°\cos\theta)^2+(\sin^2 30°\sin\theta)^2}$$

$$2m.\omega^2.r\sqrt{\left(\frac{3}{4}\cos\theta\right)^2+\left(\frac{1}{4}\sin\theta\right)^2} = \frac{m}{2}\times\omega^2.r\sqrt{9\cos^2\theta+\sin^2\theta} \quad \text{...(vi)}$$

2. When $2\alpha = 90°$ or $\alpha = 45°$

$$F_P = 2m.\omega^2.r\sqrt{(\cos^2 45°\cos\theta)^2+(\sin^2 45°\sin\theta)^2}$$

$$= 2m.\omega^2.r\sqrt{\left(\frac{1}{2}\cos\theta\right)^2+\left(\frac{1}{2}\sin\theta\right)^2} = m.\omega^2.r \quad \text{... (vii)}$$

3. When $2\alpha = 120°$ or $\alpha = 60°$.

$$F_P = 2m.\omega^2.r\sqrt{(\cos^2 60°\cos\theta)^2+(\sin^2 60°\sin\theta)^2}$$

$$= 2m.\omega^2.r\sqrt{\left(\frac{1}{4}\cos\theta\right)^2+\left(\frac{3}{4}\sin\theta\right)^2} = \frac{m}{2}\times\omega^2.r\sqrt{\cos^2\theta+9\sin^2\theta} \quad \text{... (viii)}$$

Considering secondary forces

We know that secondary force acting along the line of stroke of cylinder 1,

$$F_{S1} = m.\omega^2.r\times\frac{\cos 2(\alpha-\theta)}{n}$$

$\therefore$ Component of F_{S1} along the vertical line OY

$$= F_{S1}\cos\alpha = m.\omega^2.r\times\frac{\cos 2(\alpha-\theta)}{n}\times\cos\alpha \quad \text{... (ix)}$$

and component of F_{S1} along the horizontal line OX

$$= F_{S1}\sin\alpha = m.\omega^2.r\times\frac{\cos 2(\alpha-\theta)}{n}\times\sin\alpha \quad \text{... (x)}$$

Similarly, secondary force acting along the line of stroke of cylinder 2,

$$F_{S2} = m.\omega^2 r\times\frac{\cos 2(\alpha+\theta)}{n}$$

$\therefore$ Component of F_{S2} along the vertical line OY

$$= F_{S2}\cos\alpha = m.\omega^2.r\times\frac{\cos 2(\alpha+\theta)}{n}\times\cos\alpha \quad \text{... (xi)}$$

and component of F_{S2} along the horizontal line OX'

$$= F_{S2}\sin\alpha = m.\omega^2.r\times\frac{\cos 2(\alpha+\theta)}{n}\times\sin\alpha \quad \text{... (xii)}$$

Total component of secondary force along the vertical line OY,

$$F_{SV} = (ix) + (xi) = \frac{m}{n}\times\omega^2.r\cos\alpha\,[\cos 2(\alpha-\theta)+\cos 2(\alpha+\theta)]$$

$$= \frac{m}{n}\times\omega^2.r\cos\alpha\times 2\cos 2\alpha\cos 2\theta = \frac{2m}{n}\times\omega^2.r\cos\alpha.\cos 2\alpha\cos 2\theta$$

and total component of secondary force along the horizontal line OX,

$$F_{SH} = (x) - (xii) = \frac{m}{n}\times\omega^2.r\sin\alpha\,[\cos 2(\alpha-\theta)-\cos 2(\alpha+\theta)]$$

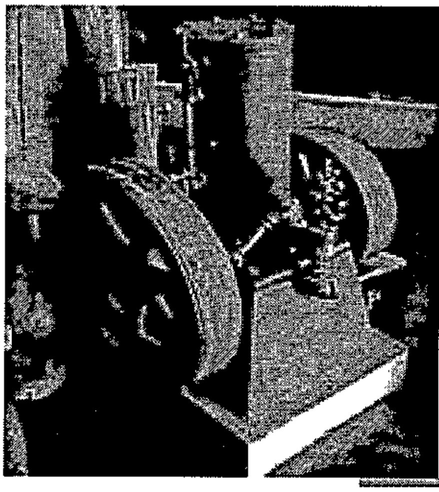
$$= \frac{m}{n}\times\omega^2.r\sin\alpha\times 2\sin 2\alpha.\sin 2\theta$$

$$= \frac{2m}{n}\times\omega^2.r\sin\alpha.\sin 2\alpha.\sin 2\theta$$

$\therefore$ Resultant secondary force,

$$F_S = \sqrt{(F_{SV})^2+(F_{SH})^2}$$

$$= \frac{2m}{n}\times\omega^2.r\sqrt{(\cos\alpha.\cos 2\alpha.\cos 2\theta)^2+(\sin\alpha.\sin 2\alpha.\sin 2\theta)^2} \quad \text{... (xiii)}$$



22

Balancing of Reciprocating Masses

Features

22.1. Introduction

We have discussed in Chapter 15 (Art. 15.10), the various forces acting on the reciprocating parts of an engine. The resultant of all the forces acting on the body of the engine due to inertia forces only is known as ***unbalanced force*** or ***shaking force***. Thus if the resultant of all the forces due to inertia effects is zero, then there will be no unbalanced force, but even then an unbalanced couple or shaking couple will be present.

Consider a horizontal reciprocating engine mechanism as shown in Fig. 22.1.

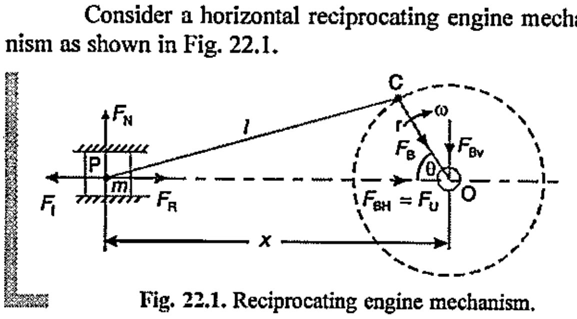


Fig. 22.1. Reciprocating engine mechanism.

Let F_R = Force required to accelerate the reciprocating parts,

F_I = Inertia force due to reciprocating parts,

F_N = Force on the sides of the cylinder walls or normal force acting on the cross-head guides, and

F_B = Force acting on the crankshaft bearing or main bearing.

Since F_R and F_I are equal in magnitude but opposite in direction, therefore they balance each other. The horizontal component of F_B (*i.e.* F_{BH}) acting along the line of reciprocation is also equal and opposite to F_I. This force $F_{BH} = F_U$ is an unbalanced force or shaking force and required to be properly balanced.

The force on the sides of the cylinder walls (F_N) and the vertical component of F_B (*i.e.* F_{BV}) are equal and opposite and thus form a shaking couple of magnitude $F_N \times x$ or $F_{BV} \times x$.

From above we see that the effect of the reciprocating parts is to produce a shaking force and a shaking couple. Since the shaking force and a shaking couple vary in magnitude and direction during the engine cycle, therefore they cause very objectionable vibrations.

Thus the purpose of balancing the reciprocating masses is to eliminate the shaking force and a shaking couple. In most of the mechanisms, we can reduce the shaking force and a shaking couple by adding appropriate balancing mass, but it is usually not practical to eliminate them completely. In other words, the reciprocating masses are only partially balanced.

Note : The masses rotating with the crankshaft are normally balanced and they do not transmit any unbalanced or shaking force on the body of the engine.

22.2. Primary and Secondary Unbalanced Forces of Reciprocating Masses

Consider a reciprocating engine mechanism as shown in Fig. 22.1.

Let
m = Mass of the reciprocating parts,
l = Length of the connecting rod PC,
r = Radius of the crank OC,
θ = Angle of inclination of the crank with the line of stroke PO,
ω = Angular speed of the crank,
n = Ratio of length of the connecting rod to the crank radius = l / r.

We have already discussed in Art. 15.8 that the acceleration of the reciprocating parts is approximately given by the expression,

$$a_R = \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$\therefore$ Inertia force due to reciprocating parts or force required to accelerate the reciprocating parts,

$$F_I = F_R = \text{Mass} \times \text{acceleration} = m \cdot \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

We have discussed in the previous article that the horizontal component of the force exerted on the crank shaft bearing (*i.e.* F_{BH}) is equal and opposite to inertia force (F_I). This force is an unbalanced one and is denoted by F_U.

$\therefore$ Unbalanced force,

$$F_U = m \cdot \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right) = m.\omega^2 \cdot r\cos\theta + m \cdot \omega^2 \cdot r \times \frac{\cos 2\theta}{n} = F_P + F_S$$

The expression $(m \cdot \omega^2 \cdot r\cos\theta)$ is known as ***primary unbalanced force*** and $\left(m \cdot \omega^2 \cdot r \times \frac{\cos 2\theta}{n}\right)$ is called ***secondary unbalanced force.***

∴ Primary unbalanced force, $F_P = m \cdot \omega^2 \cdot r\cos\theta$

and secondary unbalanced force, $$F_S = m \cdot \omega^2 \cdot r \times \frac{\cos 2\theta}{n}$$

Notes: **1.** The primary unbalanced force is maximum, when θ = 0° or 180°. Thus, the primary force is maximum twice in one revolution of the crank. The maximum primary unbalanced force is given by

$$F_{P(max)} = m \cdot \omega^2 \cdot r$$

2. The secondary unbalanced force is maximum, when θ = 0°, 90°,180° and 360°. Thus, the secondary force is maximum four times in one revolution of the crank. The maximum secondary unbalanced force is given by

$$F_{S(max)} = m \cdot \omega^2 \times \frac{r}{n}$$

3. From above we see that secondary unbalanced force is $1/n$ times the maximum primary unbalanced force.

4. In case of moderate speeds, the secondary unbalanced force is so small that it may be neglected as compared to primary unbalanced force.

5. The unbalanced force due to reciprocating masses varies in magnitude but constant in direction while due to the revolving masses, the unbalanced force is constant in magnitude but varies in direction.

22.3. Partial Balancing of Unbalanced Primary Force in a Reciprocating Engine

The primary unbalanced force $(m \cdot \omega^2 \cdot r\cos\theta)$ may be considered as the component of the centrifugal force produced by a rotating mass m placed at the crank radius r, as shown in Fig. 22.2.

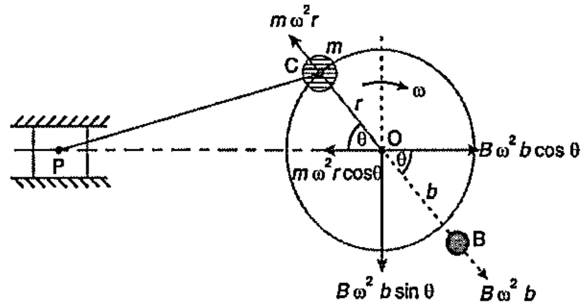


Fig. 22.2. Partial balancing of unbalanced primary force in a reciprocating engine.

The primary force acts from O to P along the line of stroke. Hence, balancing of primary force is considered as equivalent to the balancing of mass m rotating at the crank radius r. This is balanced by having a mass B at a radius b, placed diametrically opposite to the crank pin C.

We know that centrifugal force due to mass B,

$$= B \cdot \omega^2 \cdot b$$

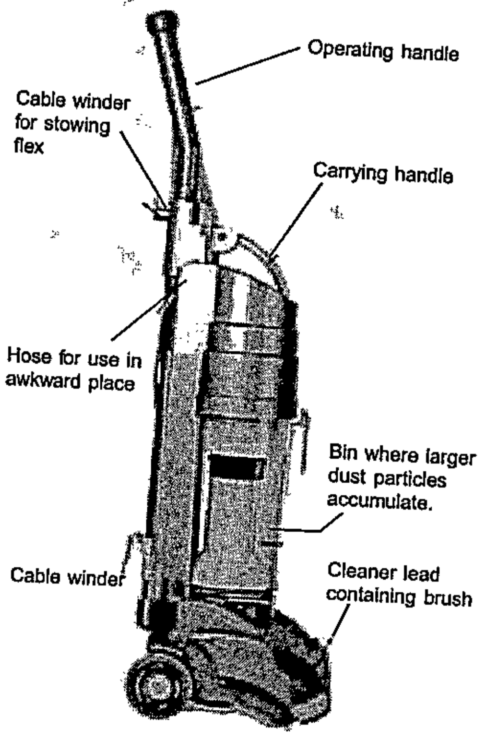
and horizontal component of this force acting in opposite direction of primary force

$$= B \cdot \omega^2 \cdot b\cos\theta$$

The primary force is balanced, if

$$B \cdot \omega^2 \cdot b\cos\theta = m \cdot \omega^2 \cdot r\cos\theta \quad \text{or} \quad B.b = m.r$$

A little consideration will show, that the primary force is completely balanced if $B.b = m.r$, but the centrifugal force produced due to the revolving mass B, has also a vertical component (perpendicular to the line of stroke) of magnitude $B \cdot \omega^2 \cdot b \sin\theta$. This force remains unbalanced. The maximum value of this force is equal to $B \cdot \omega^2 \cdot b$ when θ is 90° and 270°, which is same as the maximum value of the primary force $m \cdot \omega^2 \cdot r$.



Cyclone cleaner.

From the above discussion, we see that in the first case, the primary unbalanced force acts along the line of stroke whereas in the second case, the unbalanced force acts along the perpendicular to the line of stroke. The maximum value of the force remains same in both the cases. It is thus obvious, that the effect of the above method of balancing is to change the direction of the maximum unbalanced force from the line of stroke to the perpendicular of line of stroke. As a compromise let a fraction 'c' of the reciprocating masses is balanced, such that

$$c.m.r = B.b$$

∴ Unbalanced force along the line of stroke

$$= m \cdot \omega^2 \cdot r \cos\theta - B \cdot \omega^2 \cdot b \cos\theta$$

$$= m \cdot \omega^2 \cdot r \cos\theta - c \cdot m \cdot \omega^2 \cdot r \cos\theta \qquad \text{... } (\because B.b = c.m.r)$$

$$= (1-c) m \cdot \omega^2 \cdot r \cos\theta$$

and unbalanced force along the perpendicular to the line of stroke

$$= B \cdot \omega^2 \cdot b \sin\theta = c \cdot m \cdot \omega^2 \cdot r \sin\theta$$

∴ Resultant unbalanced force at any instant

$$= \sqrt{\left[(1-c) m \cdot \omega^2 \cdot r \cos\theta\right]^2 + \left[c \cdot m \cdot \omega^2 \cdot r \sin\theta\right]^2}$$

$$= m \cdot \omega^2 \cdot r \sqrt{(1-c)^2 \cos^2\theta + c^2 \sin^2\theta}$$

Note : If the balancing mass is required to balance the revolving masses as well as reciprocating masses, then

$$B.b = m_1 \cdot r + c \cdot m \cdot r = (m_1 + c \cdot m) r$$

where m_1 = Magnitude of the revolving masses, and

m = magnitude of the reciprocating masses.

Example 22.1. *A single cylinder reciprocating engine has speed 240 r.p.m., stroke 300 mm, mass of reciprocating parts 50 kg, mass of revolving parts at 150 mm radius 37 kg. If two-third of the reciprocating parts and all the revolving parts are to be balanced, find : 1. The balance mass required at a radius of 400 mm, and 2. The residual unbalanced force when the crank has rotated 60° from top dead centre.*

Solution. Given : $N = 240$ r.p.m. or $\omega = 2\pi \times 240/60 = 25.14$ rad/s ; Stroke = 300 mm = 0.3 m; $m = 50$ kg ; $m_1 = 37$ kg ; $r = 150$ mm = 0.15 m ; $c = 2/3$

1. ***Balance mass required***

Let B = Balance mass required, and

b = Radius of rotation of the balance mass = 400 mm = 0.4 m ... (Given)

We know that

$$B.b = (m_1 + c.m)\, r$$

$$B \times 0.4 = \left(37 + \frac{2}{3} \times 50\right) 0.15 = 10.55 \quad \text{or} \quad B = 26.38 \text{ kg } \textbf{Ans.}$$

2. ***Residual unbalanced force***

Let θ = Crank angle from top dead centre = 60° ... (Given)

We know that residual unbalanced force

$$= m \cdot \omega^2 \cdot r \sqrt{(1-c)^2 \cos^2\theta + c^2 \sin^2\theta}$$

$$= 50(25.14)^2\, 0.15 \sqrt{\left(1-\frac{2}{3}\right)^2 \cos^2 60° + \left(\frac{2}{3}\right)^2 \sin^2 60°} \text{ N}$$

$$= 4740 \times 0.601 = 2849 \text{ N } \textbf{Ans.}$$

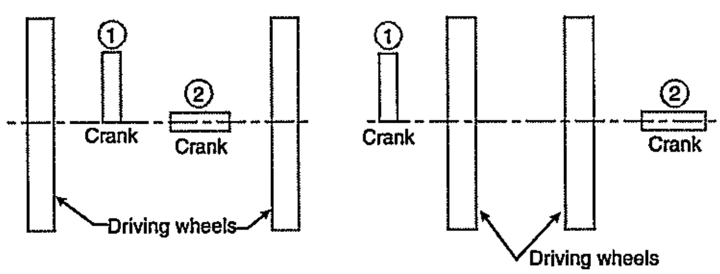
22.4. Partial Balancing of Locomotives

The locomotives, usually, have two cylinders with cranks placed at right angles to each other in order to have uniformity in turning moment diagram. The two cylinder locomotives may be classified as :

1. Inside cylinder locomotives ; and **2.** Outside cylinder locomotives.

In the ***inside cylinder locomotives***, the two cylinders are placed in between the planes of two driving wheels as shown in Fig. 22.3 (*a*) ; whereas in the ***outside cylinder locomotives***, the two cylinders are placed outside the driving wheels, one on each side of the driving wheel, as shown in Fig. 22.3 (*b*). The locomotives may be

(*a*) Single or uncoupled locomotives ; and (*b*) Coupled locomotives.



(*a*) Inside cylinder locomotives. (*b*) Outside cylinder locomotives.

Fig. 22.3

A ***single*** or ***uncoupled locomotive*** is one, in which the effort is transmitted to one pair of the wheels only ; whereas in ***coupled locomotives***, the driving wheels are connected to the leading and trailing wheel by an outside coupling rod.

22.5. Effect of Partial Balancing of Reciprocating Parts of Two Cylinder Locomotives

We have discussed in the previous article that the reciprocating parts are only partially balanced. Due to this partial balancing of the reciprocating parts, there is an unbalanced primary force along the line of stroke and also an unbalanced primary force perpendicular to the line of stroke. The effect of an unbalanced primary force along the line of stroke is to produce;

1. Variation in tractive force along the line of stroke ; and **2.** Swaying couple.

The effect of an unbalanced primary force perpendicular to the line of stroke is to produce variation in pressure on the rails, which results in hammering action on the rails. The maximum magnitude of the unbalanced force along the perpendicular to the line of stroke is known as a ***hammer blow***. We shall now discuss the effects of an unbalanced primary force in the following articles.

22.6. Variation of Tractive Force

The resultant unbalanced force due to the two cylinders, along the line of stroke, is known as ***tractive force***. Let the crank for the first cylinder be inclined at an angle θ with the line of stroke, as shown in Fig. 22.4. Since the crank for the second cylinder is at right angle to the first crank, therefore the angle of inclination for the second crank will be $(90° + \theta)$.

Let m = Mass of the reciprocating parts per cylinder, and

c = Fraction of the reciprocating parts to be balanced.

We know that unbalanced force along the line of stroke for cylinder 1

$$= (1-c)m.\omega^2.r\cos\theta$$

Similarly, unbalanced force along the line of stroke for cylinder 2,

$$= (1-c)m.\omega^2 \cdot r\cos(90°+\theta)$$

∴ As per definition, the tractive force,

F_T = Resultant unbalanced force along the line of stroke

$$= (1-c)m.\omega^2.r\cos\theta + (1-c)m.\omega^2.r\cos(90°+\theta)$$

$$= (1-c)m.\omega^2.r(\cos\theta - \sin\theta)$$

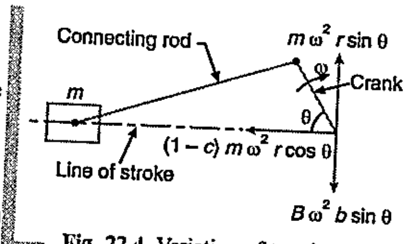


Fig. 22.4. Variation of tractive force.

The tractive force is maximum or minimum when $(\cos\theta - \sin\theta)$ is maximum or minimum. For $(\cos\theta - \sin\theta)$ to be maximum or minimum,

$$\frac{d}{d\theta}(\cos\theta - \sin\theta) = 0 \quad \text{or} \quad -\sin\theta - \cos\theta = 0 \quad \text{or} \quad -\sin\theta = \cos\theta$$

$$\therefore \quad \tan\theta = -1 \quad \text{or} \quad \theta = 135° \quad \text{or} \quad 315°$$

Thus, the tractive force is maximum or minimum when $\theta = 135°$ or $315°$.

∴ Maximum and minimum value of the tractive force or the variation in tractive force

$$= \pm(1-c)\,m.\omega^2.r(\cos 135° - \sin 135°) = \pm\sqrt{2}\,(1-c)\,m.\omega^2.r$$

22.7. Swaying Couple

The unbalanced forces along the line of stroke for the two cylinders constitute a couple about the centre line *YY* between the cylinders as shown in Fig. 22.5.

This couple has swaying effect about a vertical axis, and tends to sway the engine alternately in clockwise and anticlockwise directions. Hence the couple is known as ***swaying couple.***

Let a = Distance between the centre lines of the two cylinders.

$\therefore$ Swaying couple

$$= (1-c)m.\omega^2.r\cos\theta\times\frac{a}{2} - (1-c)m.\omega^2.r\cos(90°+\theta)\frac{a}{2}$$

$$= (1-c)m.\omega^2.r\times\frac{a}{2}(\cos\theta+\sin\theta)$$

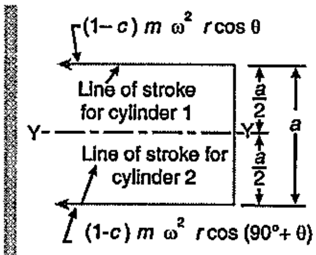


Fig. 22.5. Swaying couple.

The swaying couple is maximum or minimum when $(\cos\theta+\sin\theta)$ is maximum or minimum. For $(\cos\theta+\sin\theta)$ to be maximum or minimum,

$$\frac{d}{d\theta}(\cos\theta+\sin\theta)=0 \quad \text{or} \quad -\sin\theta+\cos\theta=0 \quad \text{or} \quad -\sin\theta=-\cos\theta$$

$$\therefore \quad \tan\theta=1 \quad \text{or} \quad \theta=45° \quad \text{or} \quad 225°$$

Thus, the swaying couple is maximum or minimum when $\theta = 45°$ or $225°$.

$\therefore$ Maximum and minimum value of the swaying couple

$$= \pm(1-c)m.\omega^2.r\times\frac{a}{2}(\cos 45°+\sin 45°) = \pm\frac{a}{\sqrt{2}}(1-c)m.\omega^2.r$$

Note : In order to reduce the magnitude of the swaying couple, revolving balancing masses are introduced. But, as discussed in the previous article, the revolving balancing masses cause unbalanced forces to act at right angles to the line of stroke. These forces vary the downward pressure of the wheels on the rails and cause oscillation of the locomotive in a vertical plane about a horizontal axis. Since a swaying couple is more harmful than an oscillating couple, therefore a value of 'c' from 2/3 to 3/4, in two-cylinder locomotives with two pairs of coupled wheels, is usually used. But in large four cylinder locomotives with three or more pairs of coupled wheels, the value of 'c' is taken as 2/5.

22.8. Hammer Blow

We have already discussed that the maximum magnitude of the unbalanced force along the perpendicular to the line of stroke is known as ***hammer blow.***

We know that the unbalanced force along the perpendicular to the line of stroke due to the balancing mass B, at a radius b, in order to balance reciprocating parts only is $B.\omega^2.b\sin\theta$. This force will be maximum when $\sin\theta$ is unity, *i.e.* when $\theta = 90°$ or $270°$.

$\therefore$ Hammer blow $= B.\omega^2.b$ (Substituiting $\sin\theta = 1$)

The effect of hammer blow is to cause the variation in pressure between the wheel and the rail. This variation is shown in Fig. 22.6, for one revolution of the wheel.

Let P be the downward pressure on the rails (or static wheel load).

∴ Net pressure between the wheel and the rail

$$= P \pm B.\omega^2.b$$

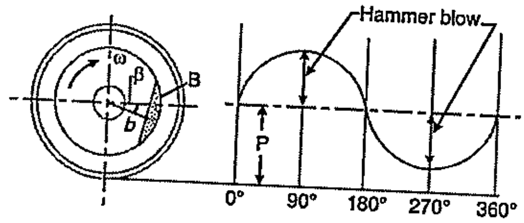


Fig. 22.6. Hammer blow.

If $(P - B.\omega^2.b)$ is ***negative***, then the wheel will be lifted from the rails. Therefore the limiting condition in order that the wheel does not lift from the rails is given by

$$P = B.\omega^2.b$$

and the permissible value of the angular speed,

$$\omega = \sqrt{\frac{P}{B.b}}$$

Example 22.2. *An inside cylinder locomotive has its cylinder centre lines 0.7 m apart and has a stroke of 0.6 m. The rotating masses per cylinder are equivalent to 150 kg at the crank pin, and the reciprocating masses per cylinder to 180 kg. The wheel centre lines are 1.5 m apart. The cranks are at right angles.*

The whole of the rotating and 2/3 of the reciprocating masses are to be balanced by masses placed at a radius of 0.6 m. Find the magnitude and direction of the balancing masses.

Find the fluctuation in rail pressure under one wheel, variation of tractive effort and the magnitude of swaying couple at a crank speed of 300 r.p.m.

Solution. Given : $a = 0.7$ m; $l_B = l_C = 0.6$ m or $r_B = r_C = 0.3$ m; $m_1 = 150$ kg; $m_2 = 180$ kg; $c = 2/3$; $r_A = r_D = 0.6$ m; $N = 300$ r.p.m. or $\omega = 2\pi \times 300/60 = 31.42$ rad/s

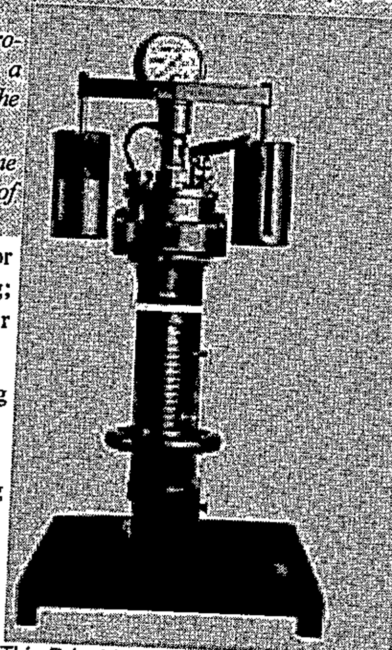
We know that the equivalent mass of the rotating parts to be balanced per cylinder at the crank pin,

$$m = m_B = m_C = m_1 + c.m_2 = 150 + \frac{2}{3} \times 180 = 270 \text{ kg}$$

Magnitude and direction of the balancing masses

Let m_A and m_D = Magnitude of the balancing masses

θ_A and θ_D = Angular position of the balancing masses m_A and m_D from the first crank B.



This Brinel hardness testing machine is used to test the hardness of the metal.

Note : This picture is given as additional information and is not a direct example of the current chapter.

The magnitude and direction of the balancing masses may be determined graphically as discussed below :

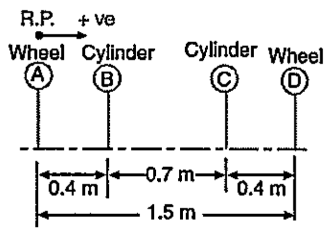
1. First of all, draw the space diagram to show the positions of the planes of the wheels and the cylinders, as shown in Fig. 22.7 (*a*). Since the cranks of the cylinders are at right angles, therefore assuming the position of crank of the cylinder *B* in the horizontal direction, draw *OC* and *OB* at right angles to each other as shown in Fig. 22.7 (*b*).
2. Tabulate the data as given in the following table. Assume the plane of wheel *A* as the reference plane.

Table 22.1

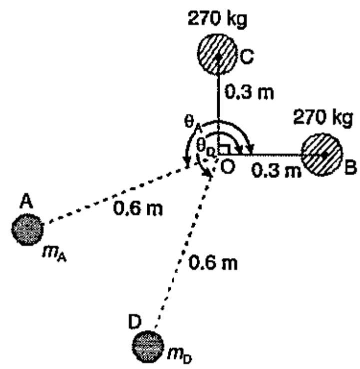
Plane (1)	mass. (m) kg (2)	Radius (r) m (3)	Cent. force ÷ ω^2 (m.r) kg-m (4)	Distance from plane A (l) m (5)	Couple ÷ ω^2 (m.r.l) kg-m² (6)
A (R.P.)	m_A	0.6	$0.6\ m_A$	0	0
B	270	0.3	81	0.4	32.4
C	270	0.3	81	1.1	89.1
D	m_D	0.6	$0.6 m_D$	1.5	$0.9\ m_D$

3. Now, draw the couple polygon from the data given in Table 22.1 (column 6), to some suitable scale, as shown in Fig 22.7 (*c*). The closing side $c'o'$ represents the balancing couple and it is proportional to $0.9\ m_D$. Therefore, by measurement,

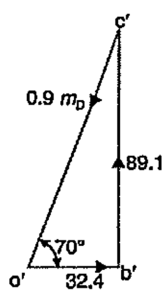
$$0.9\ m_D = \text{vector } c'o' = 94.5 \text{ kg-m}^2 \quad \text{or} \quad m_D = 105 \text{ kg} \quad \textbf{Ans.}$$



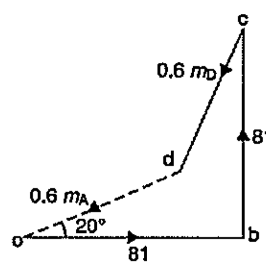
(*a*) Position of planes.



(*b*) Angular position of masses.



(*c*) Couple polygon.



(*d*) Force polygon.

Fig. 22.7

4. To determine the angular position of the balancing mass *D*, draw *OD* in Fig. 22.7 (*b*) parallel to vector $c'o'$. By measurement,

$$\theta_D = 250° \textbf{ Ans.}$$

5. In order to find the balancing mass *A*, draw the force polygon from the data given in Table 22.1 (column 4), to some suitable scale, as shown in Fig. 22.7 (*d*), The vector *do* represents the balancing force and it is proportional to 0.6 m_A. Therefore by measurement,

$$0.6\, m_A = \text{vector } do = 63 \text{ kg-m or } m_A = 105 \text{ kg } \textbf{Ans.}$$

6. To determine the angular position of the balancing mass *A*, draw *OA* in Fig. 22.7 (*b*) parallel to vector *do*. By measurement,

$$\theta_A = 200° \textbf{ Ans.}$$

Fluctuation in rail pressure

We know that each balancing mass

$= 105$ kg

∴ Balancing mass for rotating masses,

$$D = \frac{m_1}{m} \times 105 = \frac{150}{270} \times 105 = 58.3 \text{ kg}$$

and balancing mass for reciprocating masses,

$$B = \frac{c.m_2}{m} \times 105 = \frac{2}{3} \times \frac{180}{270} \times 105 = 46.6 \text{ kg}$$

This balancing mass of 46.6 kg for reciprocating masses gives rise to the centrifugal force.

∴ Fluctuation in rail pressure or hammer blow

$$= B.\omega^2.b = 46.6\,(31.42)^2\, 0.6 = 27\,602 \text{ N. } \textbf{Ans.} \qquad \ldots (\because\ b = r_A = r_D)$$

Variation of tractive effort

We know that maximum variation of tractive effort

$$= \pm\sqrt{2}(1-c)m_2.\omega^2.r = \pm\sqrt{2}\left(1-\frac{2}{3}\right)180(31.42)^2 0.3 \text{ N}$$

$$= \pm\ 25\ 127 \text{ N } \textbf{Ans.} \qquad \ldots (\because\ r = r_B = r_C)$$

Swaying couple

We know that maximum swaying couple

$$= \frac{a(1-c)}{\sqrt{2}} \times m_2.\omega^2.r = \frac{0.7\left(1-\frac{2}{3}\right)}{\sqrt{2}} \times 180(31.42)^2 0.3 \text{ N-m}$$

$$= 8797 \text{ N-m } \textbf{Ans.}$$

Example 22.3 *The three cranks of a three cylinder locomotive are all on the same axle and are set at 120°. The pitch of the cylinders is 1 metre and the stroke of each piston is 0.6 m. The reciprocating masses are 300 kg for inside cylinder and 260 kg for each outside cylinder and the planes of rotation of the balance masses are 0.8 m from the inside crank.*

If 40% of the reciprocating parts are to be balanced, find :

1. the magnitude and the position of the balancing masses required at a radius of 0.6 m ; and

2. the hammer blow per wheel when the axle makes 6 r.p.s.

Solution. Given : $\angle AOB = \angle BOC = \angle COA = 120°$; $l_A = l_B = l_C = 0.6$ m or $r_A = r_B = r_C = 0.3$ m ; $m_I = 300$ kg ; $m_O = 260$ kg ; $c = 40\% = 0.4$; $b_1 = b_2 = 0.6$ m ; $N = 6$ r.p.s. $= 6 \times 2\pi = 37.7$ rad/s

Since 40% of the reciprocating masses are to be balanced, therefore mass of the reciprocating parts to be balanced for each outside cylinder,

$$m_A = m_C = c \times m_O = 0.4 \times 260 = 104 \text{ kg}$$

and mass of the reciprocating parts to be balanced for inside cylinder,

$$m_B = c \times m_1 = 0.4 \times 300 = 120 \text{ kg}$$

1. ***Magnitude and position of the balancing masses***

Let B_1 and B_2 = Magnitude of the balancing masses in kg,

θ_1 and θ_2 = Angular position of the balancing masses B_1 and B_2 from crank A.

The magnitude and position of the balancing masses may be determined graphically as discussed below :

1. First of all, draw the position of planes and cranks as shown in Fig. 22.8 (*a*) and (*b*) respectively. The position of crank A is assumed in the horizontal direction.
2. Tabulate the data as given in the following table. Assume the plane of balancing mass B_1 (*i.e.* plane 1) as the reference plane.

Table 22.2

Plane (1)	*Mass* (*m*) *kg* (2)	*Radius* (*r*) *m* (3)	*Cent. force* $\div \omega^2$ (*m.r*) *kg-m* (4)	*Distance from plane 1* (*l*) *m* (5)	*Couple* $\div \omega^2$ (*m.r.l.*) *kg-m*^{2} (6)
A	104	0.3	31.2	− 0.2	− 6.24
1 (*R.P.*)	B_1	0.6	0.6 B_1	0	0
B	120	0.3	36	0.8	28.8
2	B_2	0.6	0.6 B_2	1.6	0.96 B_2
C	104	0.3	31.2	1.8	56.16

3. Now draw the couple polygon with the data given in Table 22.2 (column 6), to some suitable scale, as shown in Fig. 22.8 (*c*). The closing side $c'o'$ represents the balancing couple and it is proportional to 0.96 B_2. Therefore, by measurement,

$$0.96\, B_2 = \text{vector } c'o' = 55.2 \text{ kg-m}^2 \text{ or } B_2 = 57.5 \text{ kg } \textbf{Ans.}$$

4. To determine the angular position of the balancing mass B_2, draw OB_2 parallel to vector $c'o'$ as shown in Fig. 22.8 (*b*). By measurement,

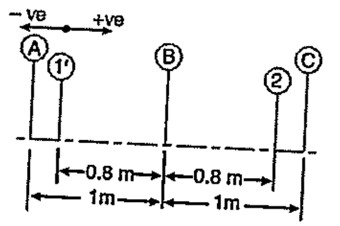
$$\theta_2 = 24° \textbf{ Ans.}$$

5. In order to find the balance mass B_1, draw the force polygon with the data given in Table 22.2 (column 4), to some suitable scale, as shown in Fig. 22.8 (*d*). The closing side *co* represents the balancing force and it is proportional to 0.6 B_1. Therefore, by measurement,

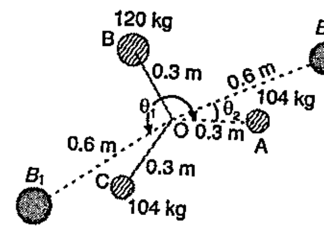
$$0.6\, B_1 = \text{vector } co = 34.5 \text{ kg-m} \quad \text{or} \quad B_1 = 57.5 \text{ kg } \textbf{Ans.}$$

6. To determine the angular position of the balancing mass B_1, draw OB_1 parallel to vector *co*, as shown in Fig. 22.8 (*b*). By measurement,

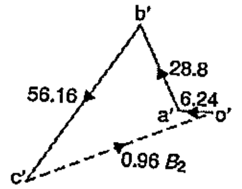
$$\theta_1 = 215° \textbf{ Ans.}$$



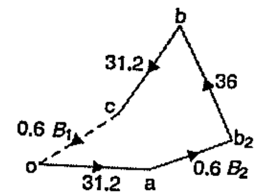
(*a*) Position of planes.



(*b*) Position of cranks.



(*c*) Couple polygon.



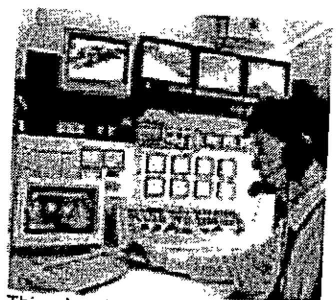
(*d*) Force polygon.

Fig. 22.8

2. *Hammer blow per wheel*

We know that hammer blow per wheel

$$= B_1.\omega^2.b_1 = 57.5\ (37.7)^2\ 20.6 = 49\ 035 \text{ N \textbf{Ans.}}$$



This chamber is used to test the acoustics of a vehicle so that the noise it produces can be reduced. The panels in the walls and ceiling of the room absorb the sound which is monitored (above)

Note : This picture is given as additional information and is not a direct example of the current chapter.

Example 22.4. *The following data refer to two cylinder locomotive with cranks at 90° :*

Reciprocating mass per cylinder = 300 kg ; Crank radius = 0.3 m ; Driving wheel diameter = 1.8 m ; Distance between cylinder centre lines = 0.65 m ; Distance between the driving wheel central planes = 1.55 m.

Determine : 1. the fraction of the reciprocating masses to be balanced, if the hammer blow is not to exceed 46 kN at 96.5 km. p.h. ; 2. the variation in tractive effort ; and 3. the maximum swaying couple.

Solution. Given : $m = 300$ kg ; $r = 0.3$ m ; $D = 1.8$ m or $R = 0.9$ m ; $a = 0.65$ m ; Hammer blow = 46 kN = 46×10^3 N ; $v = 96.5$ km/h = 26.8 m/s

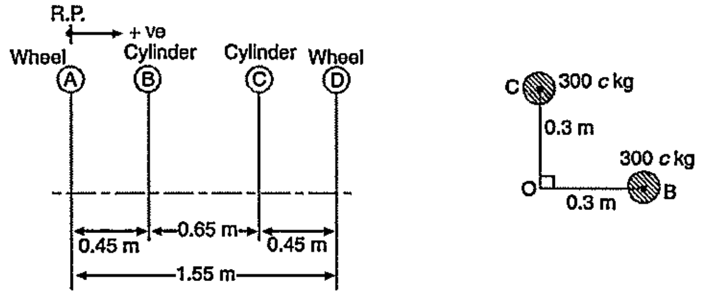
1. ***Fraction of the reciprocating masses to be balanced***

Let c = Fraction of the reciprocating masses to be balanced, and

B = Magnitude of balancing mass placed at each of the driving wheels at radius b.

We know that the mass of the reciprocating parts to be balanced

$$= c.m = 300c \text{ kg}$$



(*a*) Position of planes. (*b*) Position of cranks.

Fig. 22.9

The position of planes of the wheels and cylinders is shown in Fig. 22.9 (*a*), and the position of cranks is shown in Fig 22.9 (*b*). Assuming the plane of wheel *A* as the reference plane, the data may be tabulated as below :

Table 22.3

Plane (1)	*Mass* (*m*) *kg* (2)	*Radius* (*r*) *m* (3)	*Cent. force* ÷ ω^2 (*m.r*) *kg-m* (4)	*Distance from plane A* (*l*)*m* (5)	*Couple* ÷ ω^2 (*m.r.l.*) *kg-m*^{2} (6)
A (*R.P.*)	*B*	*b*	*B.b*	0	0
B	300 *c*	0.3	90 *c*	0.45	40.5 *c*
C	300 *c*	0.3	90 *c*	1.1	99 *c*
D	*B*	*b*	*B.b*	1.55	1.55 *B.b*

Now the couple polygon, to some suitable scale, may be drawn with the data given in Table 22.3 (column 6), as shown in Fig. 22.10. The closing side of the polygon (vector $c'o'$) represents the balancing couple and is proportional to 1.55 *B.b*.

From the couple polygon,

$$1.55\, B.b = \sqrt{(40.5c)^2 + (99c)^2} = 107c$$

$$\therefore \quad B.b = 107\, c\, /\, 1.55 = 69\, c$$

We know that angular speed,

$$\omega = v/R = 26.8/0.9 = 29.8 \text{ rad/s}$$

$\therefore$ Hammer blow,

$$46 \times 10^3 = B.\, \omega^2 .b$$

$$= 69\, c\, (29.8)^2 = 61\,275\, c$$

$$\therefore \quad c = 46 \times 10^3/61\,275 = 0.751 \text{ Ans.}$$

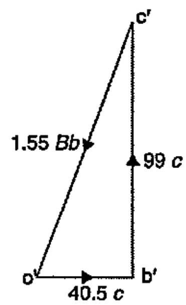


Fig. 22.10

2. Variation in tractive effort

We know that variation in tractive effort

$$= \pm\sqrt{2}(1-c)m.\omega^2.r = \pm\sqrt{2}(1-0.751)\,300(29.8)^2 0.3$$
$$= 28\ 140 \text{ N} = 28.14 \text{ kN } \textbf{Ans.}$$

Maximum swaying couple

We know the maximum swaying couple

$$= \frac{a(1-c)}{\sqrt{2}} \times m.\omega^2.r = \frac{0.65(1-0.751)}{\sqrt{2}} \times 300(29.8)^2\, 0.3 = 9148 \text{ N-m}$$
$$= 9.148 \text{ kN-m } \textbf{Ans.}$$

Example 22.5. *The following data apply to an outside cylinder uncoupled locomotive :*

Mass of rotating parts per cylinder = 360 kg ; Mass of reciprocating parts per cylinder = 300 kg ; Angle between cranks = 90° ; Crank radius = 0.3 m ; Cylinder centres = 1.75 m ; Radius of balance masses = 0.75 m ; Wheel centres = 1.45 m.

If whole of the rotating and two-thirds of reciprocating parts are to be balanced in planes of the driving wheels, find :

1. Magnitude and angular positions of balance masses,

2. Speed in kilometres per hour at which the wheel will lift off the rails when the load on each driving wheel is 30 kN and the diameter of tread of driving wheels is 1.8 m, and

3. Swaying couple at speed arrived at in (2) above.

Solution : Given : $m_1 = 360$ kg ; $m_2 = 300$ kg ; $\angle AOD = 90°$; $r_A = r_D = 0.3$ m ; $a = 1.75$ m ; $r_B = r_C = 0.75$ m ; $c = 2/3$.

We know that the equivalent mass of the rotating parts to be balanced per cylinder,

$$m = m_A = m_D = m_1 + c.m_2 = 360 + \frac{2}{3} \times 300 = 560 \text{ kg}$$

1. ***Magnitude and angular position of balance masses***

Let m_B and m_C = Magnitude of the balance masses, and

θ_B and θ_C = angular position of the balance masses m_B and m_C from the crank A.

The magnitude and direction of the balance masses may be determined, graphically, as discussed below :

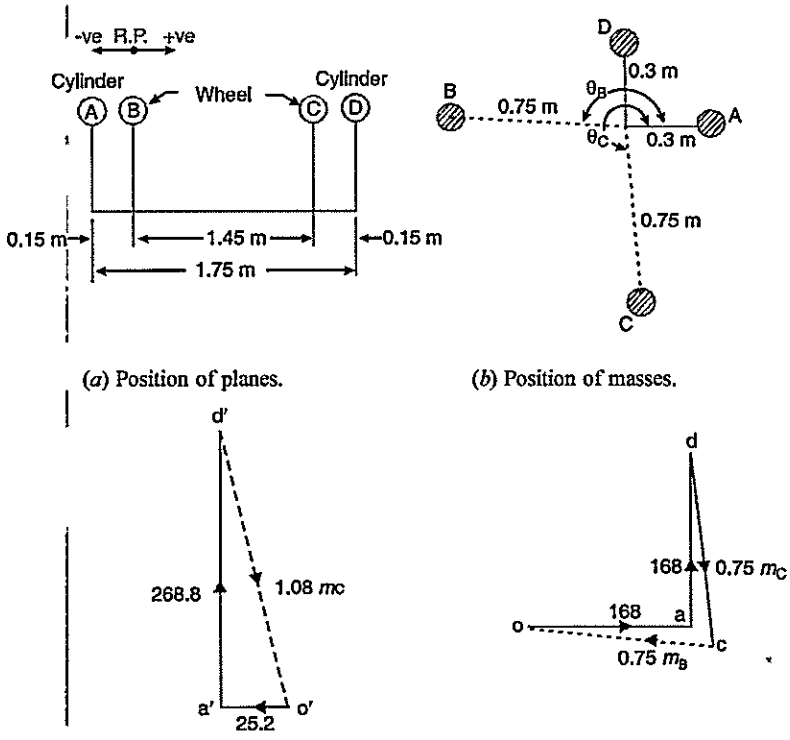
1. First of all, draw the positions of the planes of the wheels and the cylinders as shown in Fig. 22.11 (*a*). Since the cranks of the two cylinders are at right angles, therefore assuming the position of the cylinder A in the horizontal direction, draw OA and OD at right angles to each other as shown in Fig. 22.11 (*b*).
2. Assuming the plane of wheel B as the reference plane, the data may be tabulated as below:

Table 22.4

Plane (1)	*Mass* (m) kg (2)	*Radius* (r) m (3)	*Cent. force* ÷ ω^2 (m.r) kg-m (4)	*Distance from plane B(l) m* (5)	*Couple* ÷ ω^2 (m.r.l) kg-m² (6)
A	560	0.3	168	– 0.15	– 25.2
B (R.P.)	m_B	0.75	0.75 m_B	0	0
C	m_C	0.75	0.75 m_C	1.45	1.08 m_C
D	560	0.3	168	1.6	268.8

3. Now draw the couple polygon with the data given in Table 22.4 column (6), to some suitable scale as shown in Fig. 22.11(*c*). The closing side *d′o′* represents the balancing couple and it is proportional to 1.08 m_C. Therefore, by measurement,

$$1.08\ m_C = 269.6 \text{ kg-m}^2 \quad \text{or} \quad m_C = 249 \text{ kg} \quad \textbf{Ans.}$$



(*a*) Position of planes. (*b*) Position of masses.

(*c*) Couple polygon. (*d*) Force polygon.

Fig. 22.11

4. To determine the angular position of the balancing mass *C*, draw *OC* parallel to vector *d′o′* as shown in Fig. 22.11 (*b*). By measurement,

$$\theta_C = 275° \textbf{ Ans.}$$

5. In order to find the balancing mass *B*, draw the force polygon with the data given in Table 22.4 column (4), to some suitable scale, as shown in Fig. 22.11 (*d*). The vector *co* represents

the balancing force and it is proportional to 0.75 m_B. Therefore, by measurement,

$$0.75\ m_B = 186.75 \text{ kg-m} \quad \text{or} \quad m_B = 249 \text{ kg } \textbf{Ans.}$$

6. To determine the angular position of the balancing mass *B*, draw *OB* parallel to vector *oc* as shown Fig. 22.11 (*b*). By measurement,

$$\theta_B = 174.5° \textbf{ Ans.}$$

2. *Speed at which the wheel will lift off the rails*

Given : $P = 30 \text{ kN} = 30 \times 10^3 \text{ N}$; $D = 1.8$ m

Let ω = Angular speed at which the wheels will lift off the rails in rad/s, and

v = Corresponding linear speed in km/h.

We know that each balancing mass,

$$m_B = m_C = 249 \text{ kg}$$

∴ Balancing mass for reciprocating parts,

$$B = \frac{c.m_2}{m} \times 249 = \frac{2}{3} \times \frac{300}{560} \times 249 = 89 \text{ kg}$$

We know that $\omega = \sqrt{\dfrac{P}{B.b}} = \sqrt{\dfrac{30 \times 10^3}{89 \times 0.75}} = 21.2$ rad/s ...(∵ $b = r_B = r_C$)

and $v = \omega \times D/2 = 21.2 \times 1.8/2 = 19.08$ m/s

$= 19.08 \times 3600/1000 = 68.7$ km/h **Ans.**

3. *Swaying couple at speed* ω = *21.1 rad/s*

We know that the swaying couple

$$= \frac{a(1-c)}{\sqrt{2}} \times m_2 . \omega^2 . r = \frac{1.75\left[1 - \frac{2}{3}\right]}{\sqrt{2}} \times 300 (21.2)^2 0.3 \text{ N-m}$$

$= 16\ 687$ N-m $= 16.687$ kN-m **Ans.**

22.9. Balancing of Coupled Locomotives

The uncoupled locomotives as discussed in the previous article, are obsolete now-a-days. In a coupled locomotive, the driving wheels are connected to the leading and trailing wheels by an outside coupling rod. By such an arrangement, a greater portion of the engine mass is utilised by tractive purposes. In coupled locomotives, the coupling rod cranks are placed diametrically opposite to the adjacent main cranks (*i.e.* driving cranks). The coupling rods together with cranks and pins may be treated as rotating masses

A dynamo converts mechanical energy into electrical energy.

Note : This picture is given as additional information and is not a direct example of the current chapter.

and completely balanced by masses in the respective wheels. Thus in a coupled engine, the rotating and reciprocating masses must be treated separately and the balanced masses for the two systems are suitably combined in the wheel.

It may be noted that the variation of pressure between the wheel and the rail (*i.e.* hammer blow) may be reduced by equal distribution of balanced mass (B) between the driving, leading and trailing wheels respectively.

Example 22.6. *The following particulars relate to a two-cylinder locomotive with two coupled wheels on each side :*

Stroke	*= 650 mm*
Mass of reciprocating parts per cylinder	*= 240 kg*
Mass of revolving parts per cylinder	*= 200 kg*
Mass of each coupling rod	*= 250 kg*
Radius of centre of coupling rod pin	*= 250 mm*
Distances between cylinders	*= 0.6 m*
Distance between wheels	*= 1.5 m*
Distance between coupling rods	*= 1.8 m*

The main cranks are at right angles and the coupling rod pins are at 180° to their respective main cranks. The balance masses are to be placed in the wheels at a mean radius of 675 mm in order to balance whole of the revolving and 3/4th of the reciprocating masses. The balance mass for the reciprocating masses is to be divided equally between the driving wheels and the coupled wheels. Find : 1. The magnitudes and angular positions of the masses required for the driving and trailing wheels, and 2. The hammer blow at 120 km/h, if the wheels are 1.8 metre diameter.

Solution. Given : $L_C = L_D = 650$ mm or $r_C = r_D = 325$ mm $= 0.325$ m ; $m_1 = 240$ kg ; $m_2 = 200$ kg ; $m_3 = 250$ kg ; $r_A = r_F = 250$ mm $= 0.25$ m ; $CD = 0.6$ m ; $BE = 1.5$ m ; $AF = 1.8$ m ; $r_B = r_E = 675$ mm $= 0.675$ m ; $c = 3/4$

The position of planes for the driving wheels B and E, cylinders C and D, and coupling rods A and F, are shown in Fig. 22.12 (*a*).

The angular position of cranks C and D and coupling pins A and F are shown in Fig. 22.12(*b*).

We know that mass of the reciprocating parts per cylinder to be balanced

$$= c.m_1 = \frac{3}{4} \times 240 = 180 \text{ kg}$$

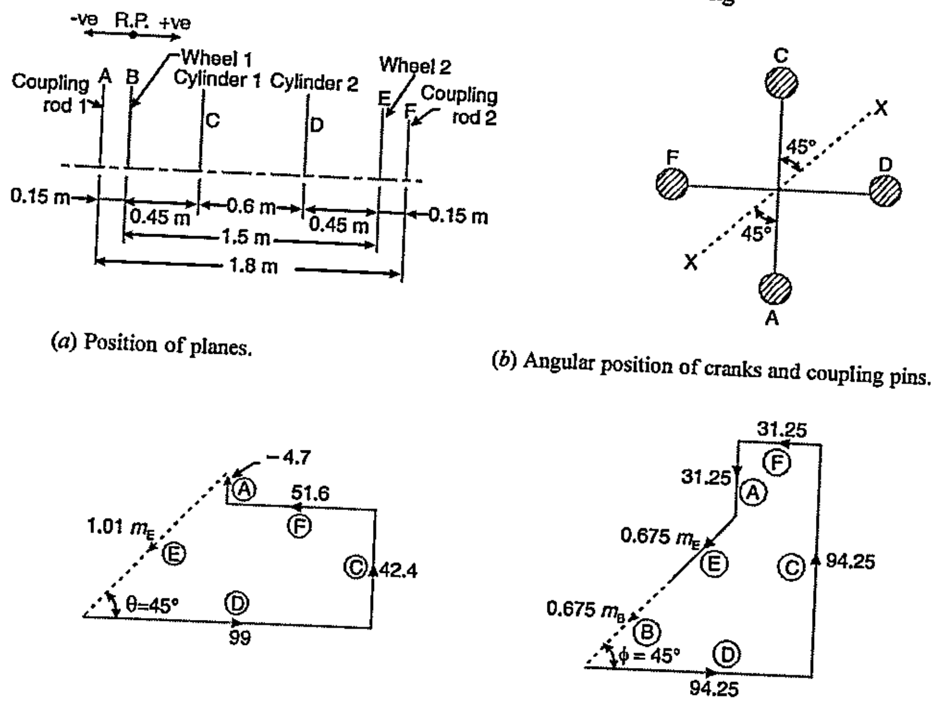
Since the reciprocating masses are to be divided equally between the driving wheels and trailing wheels, therefore 90 kg is taken for driving wheels and 90 kg for trailing wheels. Now for each driving wheel, the following masses are to be balanced :

1. Half of the mass of coupling rod *i.e.* $\frac{1}{2} \times 250 = 125 \text{ kg}$. In other words, the masses at the coupling rods A and F to be balanced for each driving wheel are

$$m_A = m_F = 125 \text{ kg}$$

2. Whole of the revolving mass *i.e.* 200 kg and the mass of the reciprocating parts *i.e.* 90 kg. In other words, total mass at the cylinders *C* and *D* to be balanced for each driving wheel are

$$m_C = m_D = 200 + 90 = 290 \text{ kg}$$



(*a*) Position of planes.

(*b*) Angular position of cranks and coupling pins.

(*c*) Couple polygon : Driving wheel *E*.

(*d*) Force polygon : Driving wheel *B*.

Fig. 22.12

Balanced masses in the driving wheels

Let m_B and m_E be the balance masses placed in the driving wheels *B* and *E* respectively. Taking the plane of *B* as reference plane, the data may be tabulated as below :

Table 22.5. (For driving wheels)

Plane (1)	*Mass* (m) kg (2)	*Radius* (r) m (3)	*Cent. force* $\div \omega^2$ (m.r) kg-m (4)	*Distance from Plane B(l) m* (5)	*Couple* $\div \omega^2$ (m.r.l) kg-m^2 (6)
A	125	0.25	31.25	− 0.15	− 4.7
B (R.P.)	m_B	0.675	0.675 m_B	0	0
C	290	0.325	94.25	0.45	42.4
D	290	0.325	94.25	1.05	99
E	m_E	0.675	0.675 m_E	1.5	1.01 m_E
F	125	0.25	31.25	1.65	51.6

In order to find the balance mass m_E in the driving wheel *E*, draw a couple polygon from the data given in Table 22.5 (column 6), to some suitable scale as shown in Fig 22.12 (*c*). The closing side of polygon as shown dotted is proportional to 1.01 m_E, Therefore by measurement, we find that

$$1.01\ m_E = 67.4 \text{ kg-m}^2 \text{ or } m_E = 66.7 \text{ kg } \textbf{Ans.}$$

and $\theta = 45°$ **Ans.**

Now draw the force polygon from the data given in Table 22.5 (column 4), to some suitable scale, as shown in Fig. 22.12 (*d*). The closing side of the polygon as shown dotted is proportional to 0.675 m_B. Therefore by measurement, we find that

$$0.675\ m_B = 45 \text{ kg-m} \quad \text{or} \quad m_B = 66.7 \text{ kg } \textbf{Ans.}$$

and $\phi = 45°$ **Ans.**

Balance masses in the trailing wheels

For each trailing wheel, the following masses are to be balanced :

1. Half of the mass of the coupling rod *i.e.* 125 kg. In other words, the masses at the coupling rods *A* and *F* to be balanced for each trailing wheel are

$$m_A = m_F = 125 \text{ kg}$$

2. Mass of the reciprocating parts *i.e.* 90 kg. In other words, the mass at the cylinders *C* and *D* to be balanced for each trailing wheel are

$$m_C = m_D = 90 \text{ kg}$$

Let m'_B and m'_E be the balanced masses placed in the trailing wheels. Taking the plane of *B* as the reference plane, the data may be tabulated as below :

Table 22.6. (For trailing wheels)

Plane (1)	*Mass (m) kg* (2)	*Radius (r) m* (3)	*Cent. force ÷ ω^2 (m.r) kg-m* (4)	*Distance from plane B (l) m* (5)	*Couple ÷ ω^2 (m.r.l) kg-m^2* (6)
A	125	0.25	31.25	– 0.15	– 4.7
B (R.P.)	m'_B	0.675	0.675 m'_B	0	0
C	90	0.325	29.25	0.45	13.2
D	90	0.325	29.25	1.05	30.7
E	m'_E	0.675	0.675 m'_E	1.5	1.01 m'_E
F	125	0.25	31.25	1.65	51.6

In order to find the balance mass m'_E in the trailing wheel *E*, draw a couple polygon from the data given in Table 22.6 (column 6), to some suitable scale, as shown in Fig. 22.13 (*a*). The closing side of the polygon as shown dotted is proportional to 1.01 m'_E. Therefore by measurement, we find that

$$1.01\ m'_E = 27.5 \text{ m}^2 \quad \text{or} \quad m'_E = 27.5 \text{ kg } \textbf{Ans.}$$

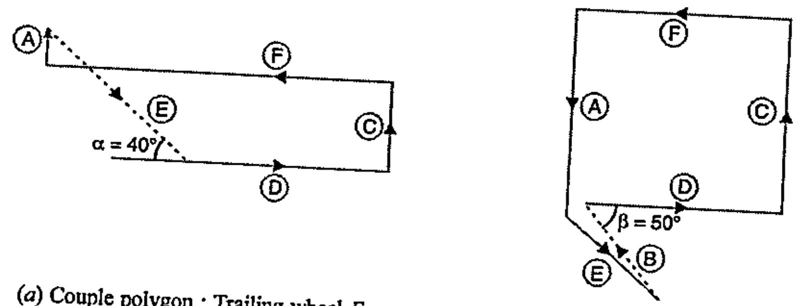
and $\alpha = 40°$ **Ans.**

Now draw the force polygon from the data given in Table 22.6 (column 4), to some suitable scale, as shown in Fig. 22.13 (*b*). The closing side of the polygon as shown dotted is proportional to 0.675 m'_B. Therefore by measurement, we find that

$$0.675\ m'_B = 18.35 \text{ kg-m} \quad \text{or} \quad m'_B = 27.2 \text{ kg } \textbf{Ans.}$$

and $\beta = 50°$ **Ans.**

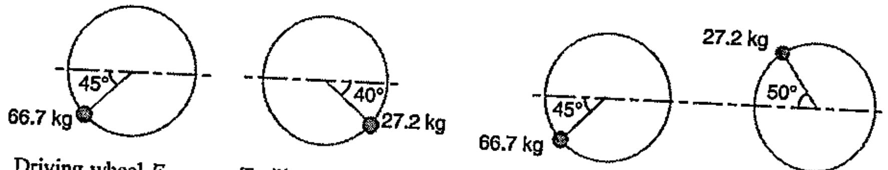
Fig. 22.14 shows the balance masses in the four wheels and it will be seen that the balance masses for the driving wheels are symmetrical about the axis *X-X* [Fig. 22.12 (*b*)]. Similarly the balance masses for the trailing wheels are symmetrical about the axis *X-X*.



(*a*) Couple polygon : Trailing wheel *E*.

(*b*) Force polygon : Trailing wheel *B*.

Fig. 22.13



Driving wheel *E*. (*a*) Trailing wheel *E*. (*b*) Driving wheel *B*. (*c*) Trailing wheel *B*. (*d*)

Fig. 22.14

Hammer blow

In order to find the hammer blow, we must find the balance mass required for reciprocating masses only. For this, the data may be tabulated as below. Let m''_B and m''_E be the balanced masses required for the reciprocating masses.

Table 22.7. (For hammer blow)

Plane (*1*)	*Mass* (*m*) *kg* (*2*)	*Radius* (*r*) *m* (*3*)	*Cent. force* ÷ ω^2 (*m.r*) *kg-m* (*4*)	*Distance from Plane B*(*l*) *m* (*5*)	*Couple* ÷ ω^2 (*m.r.l*) *kg-m*^{2} (*6*)
B(*R.P.*)	m''_B	0.675	0675 m''_B	0	0
C	90	0.325	29.25	0.45	13.2
D	90	0.325	29.25	1.05	30.7
E	m''_E	0.675	0.675 m''_E	1.5	1.01 m''_E

Now the couple polygon and the force polygon may be drawn, but due to symmetry we shall only draw the couple polygon from the data given in Table 22.7 (column 6), to some suitable scale as shown in Fig 22.15.

From Fig. 22.15,

$$1.01\, m''_E = \sqrt{(30.7)^2 + (13.2)^2} = 33.4$$

$$\therefore \quad m''_E = 33 \text{ kg}$$

We know that linear speed of the wheel,

$$v = 120 \text{ km/h} = 33.33 \text{ m/s}$$

and diameter of the wheel, $D = 1.8$ m

$\therefore$ Angular speed of the wheel

$$\omega = \frac{v}{D/2} = \frac{33.33}{1.8/2} = 37 \text{rad/s}$$

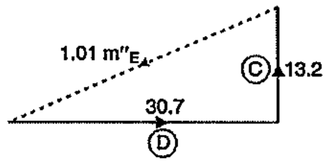


Fig. 22.15

We know that hammer blow

$$= \pm B.\omega^2.b = 33(37)^2 0.675 = \pm 30.494 \text{ N } \textbf{Ans.}$$

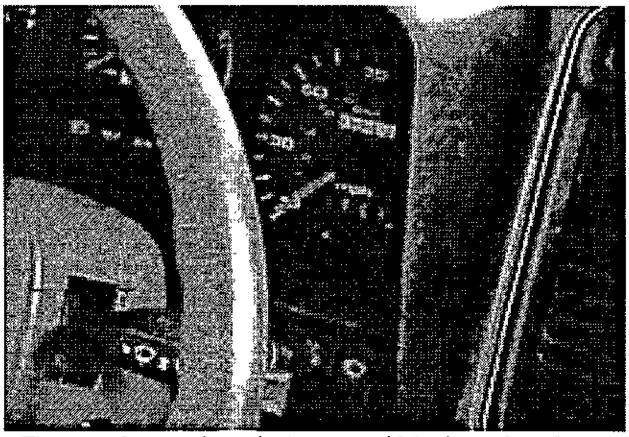
$$\ldots (\because B = m''_E, \text{ and } b = r_B = r_E)$$

22.10. Balancing of Primary Forces of Multi-cylinder In-line Engines

The multi-cylinder engines with the cylinder centre lines in the same plane and on the same side of the centre line of the crankshaft, are known as ***In-line engines.*** The following two conditions must be satisfied in order to give the primary balance of the reciprocating parts of a multi-cylinder engine :

1. The algebraic sum of the primary forces must be equal to zero. In other words, the primary force polygon must *close ; and
2. The algebraic sum of the couples about any point in the plane of the primary forces must be equal to zero. In other words, the primary couple polygon must close.

We have already discussed, that the primary unbalanced force due to the reciprocating masses is equal to the component, parallel to the line of stroke, of the centrifugal force produced by the equal mass placed at the crankpin and revolving with it. Therefore, in order to give the ***primary balance of the reciprocating parts of a multi-cylinder engine, it is convenient to imagine the reciprocating masses to be transferred to their respective crankpins and to treat the problem as one of revolving masses.***


The speedometer is an instrument which shows how fast a car is moving. It works with a magnet that spins around as the car moves.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Notes : 1. For a two cylinder engine with cranks at 180°, condition (1) may be satisfied, but this will result in an unbalanced couple. Thus the above method of primary balancing cannot be applied in this case.

2. For a three cylinder engine with cranks at 120° and if the reciprocating masses per cylinder are same, then condition (1) will be satisfied because the forces may be represented by the sides of an equilateral triangle. However, by taking a reference plane through one of the cylinder centre lines, two couples with non-parallel axes will remain and these cannot vanish vectorially. Hence the above method of balancing fails in this case also.

* The closing side of the primary force polygon gives the maximum unbalanced primary force and the closing side of the primary couple polygon gives the maximum unblanced primary couple.

3. For a four cylinder engine, similar reasoning will show that complete primary balance is possible and it follows that

'For a multi-cylinder engine, the primary forces may be completely balanced by suitably arranging the crank angles, provided that the number of cranks are not less than four'.

22.11. Balancing of Secondary Forces of Multi-cylinder In-line Engines

When the connecting rod is not too long (*i.e.* when the obliquity of the connecting rod is considered), then the secondary disturbing force due to the reciprocating mass arises.

We have discussed in Art. 22.2, that the secondary force,

$$F_S = m.\omega^2.r \times \frac{\cos 2\theta}{n}$$

This expression may be written as

$$F_S = m.(2\omega)^2 \times \frac{r}{4n} \times \cos 2\theta$$

As in case of primary forces, the secondary forces may be considered to be equivalent to the component, parallel to the line of stroke, of the centrifugal force produced by an equal mass placed at the imaginary crank of length $r / 4n$ and revolving at twice the speed of the actual crank (*i.e.* 2ω) as shown in Fig. 22.16.

Thus, in multi-cylinder in-line engines, each imaginary secondary crank with a mass attached to the crankpin is inclined to the line of stroke at twice the angle of the actual crank. The values of the secondary forces and couples may be obtained by considering the revolving mass. This is done in the similar way as discussed for primary forces. The following two conditions must be satisfied in order to give a complete secondary balance of an engine :

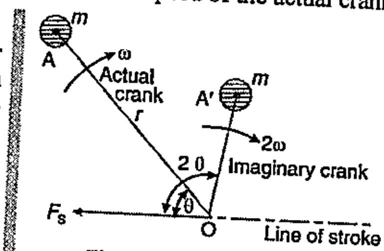


Fig. 22.16. Secondary force.

1. The algebraic sum of the secondary forces must be equal to zero. In other words, the secondary force polygon must close, and
2. The algebraic sum of the couples about any point in the plane of the secondary forces must be equal to zero. In other words, the secondary couple polygon must close.

Note : The closing side of the secondary force polygon gives the maximum unbalanced secondary force and the closing side of the secondary couple polygon gives the maximum unbalanced secondary couple.

Example 22.7. *A four cylinder vertical engine has cranks 150 mm long. The planes of rotation of the first, second and fourth cranks are 400 mm, 200 mm and 200 mm respectively from the third crank and their reciprocating masses are 50 kg, 60 kg and 50 kg respectively. Find the mass of the reciprocating parts for the third cylinder and the relative angular positions of the cranks in order that the engine may be in complete primary balance.*

Solution. Given $r_1 = r_2 = r_3 = r_4 = 150$ mm $= 0.15$ m ; $m_1 = 50$ kg ; $m_2 = 60$ kg ; $m_4 = 50$ kg

We have discussed in Art. 22.10 that in order to give the primary balance of the reciprocating parts of a multi-cylinder engine, the problem may be treated as that of revolving masses with the reciprocating masses transferred to their respective crank pins.

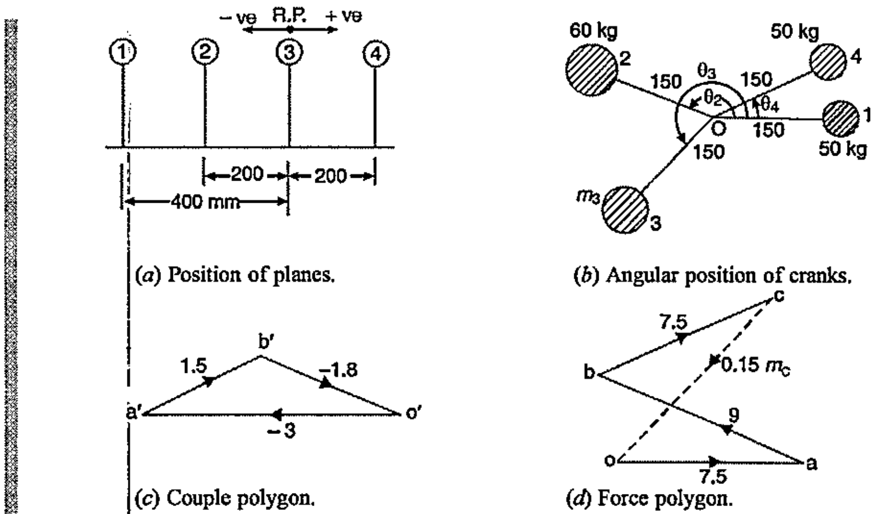
The position of planes is shown in Fig. 22.17 (*a*). Assuming the plane of third cylinder as the reference plane, the data may be tabulated as given in Table 22.8.

Table 22.8

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. force ÷ ω^2 (m.r) kg-m (4)	Distance from plane 3(l) m (5)	Couple ÷ ω^2 (m.r.l) kg-m^2 (6)
1	50	0.15	7.5	− 0.4	− 3
2	60	0.15	9	− 0.2	− 1.8
3(R.P.)	m_3	0.15	$0.15m_3$	0	0
4	50	0.15	7.5	0.2	1.5

First of all, the angular position of cranks 2 and 4 are obtained by drawing the couple polygon from the data given in Table 22.8 (column 6). Assume the position of crank 1 in the horizontal direction as shown in Fig 22.17 (*b*), The couple polygon, as shown in Fig. 22.17 (*c*), is drawn as discussed below:

1. Draw vector $o'a'$ in the horizontal direction (*i.e.* parallel to $O1$) and equal to − 3 kg-m^2, to some suitable scale.
2. From point o' and a', draw vectors $o'b'$ and $a'b'$ equal to − 1.8 kg-m^2 and 1.5 kg-m^2 respectively. These vectors intersect at b'.



(*a*) Position of planes.

(*b*) Angular position of cranks.

(*c*) Couple polygon.

(*d*) Force polygon.

Fig. 22.17

3. Now in Fig. 22.17 (*b*), draw $O2$ parallel to vector $o'b'$ and $O4$ parallel to vector $a'b'$.

By measurement, we find that the angular position of crank 2 from crank 1 in the anticlockwise direction is

$$\theta_2 = 160° \textbf{ Ans.}$$

and the angular position of crank 4 from crank 1 in the anticlockwise direction is

$$\theta_4 = 26° \textbf{ Ans.}$$

In order to find the mass of the third cylinder (m_3) and its angular position, draw the force polygon, to some suitable scale, as shown in Fig. 22.17 (*d*), from the data given in Table 22.8 (column 4). Since the closing side of the force polygon (vector *co*) is proportional to 0.15 m_3, therefore by measurement,

$$0.15m_3 = 9 \text{ kg-m} \quad \text{or} \quad m_3 = 60 \text{ kg } \textbf{Ans.}$$

Now draw $O3$ in Fig 22.17 (*b*), parallel to vector *co*. By measurement, we find that the angular position of crank 3 from crank 1 in the anticlockwise direction is

$$\theta_3 = 227° \textbf{ Ans.}$$

Example 22.8. *A four crank engine has the two outer cranks set at 120° to each other, and their reciprocating masses are each 400 kg. The distance between the planes of rotation of adjacent cranks are 450 mm, 750 mm and 600 mm. If the engine is to be in complete primary balance, find the reciprocating mass and the relative angular position for each of the inner cranks.*

If the length of each crank is 300 mm, the length of each connecting rod is 1.2 m and the speed of rotation is 240 r.p.m., what is the maximum secondary unbalanced force ?

Solution. Given : $m_1 = m_4 = 400$ kg ; $r = 300$ mm $= 0.3$ m ; $l = 1.2$ m ; $N = 240$ r.p.m. or $\omega = 2\pi \times 240/60 = 25.14$ rad/s

Reciprocating mass and the relative angular position for each of the inner cranks

Let m_2 and m_3 = Reciprocating mass for the inner cranks 2 and 3 respectively, and

θ_2 and θ_3 = Angular positions of the cranks 2 and 3 with respect to crank 1 respectively.

The position of the planes of rotation of the cranks and their angular setting are shown in Fig. 22.18 (*a*) and (*b*) respectively. Taking the plane of crank 2 as the reference plane, the data may be tabulated as below :

Table 22.9

Plane (1)	*Mass* (*m*) kg (2)	*Radius* (*r*) m (3)	*Cent. force* ÷ ω^2 (*m.r*) kg-m (4)	*Distance from plane* (2) (*l*) m (5)	*Couple* ÷ ω^2 (*m.r.l.*) kg-m² (6)
1	400	0.3	120	– 0.45	– 54
2(*R.P.*)	m_2	0.3	0.3 m_2	0	0
3	m_3	0.3	0.3 m_3	0.75	0.225 m_3
4	400	0.3	120	1.35	162

Since the engine is to be in complete primary balance, therefore the primary couple polygon and the primary force polygon must close. First of all, the primary couple polygon, as shown in Fig. 22.18 (*c*), is drawn to some suitable scale from the data given in Table 22.9 (column 6), in order to find the reciprocating mass for crank 3. Now by measurement, we find that

$$0.225\, m_3 = 196 \text{kg-m}^2 \quad \text{or} \quad m_3 = 871 \text{ kg } \textbf{Ans.}$$

and its angular position with respect to crank 1 in the anticlockwise direction,

$$\theta_3 = 326° \textbf{ Ans.}$$

Now in order to find the reciprocating mass for crank 2, draw the primary force polygon, as shown in Fig. 22.18 (*d*), to some suitable scale from the data given in Table 22.9 (column 4). Now by measurement, we find that

$$0.3\, m_2 = 284 \text{ kg-m} \quad \text{or} \quad m_2 = 947 \text{ kg } \textbf{Ans.}$$

and its angular position with respect to crank 1 in the anticlockwise direction,

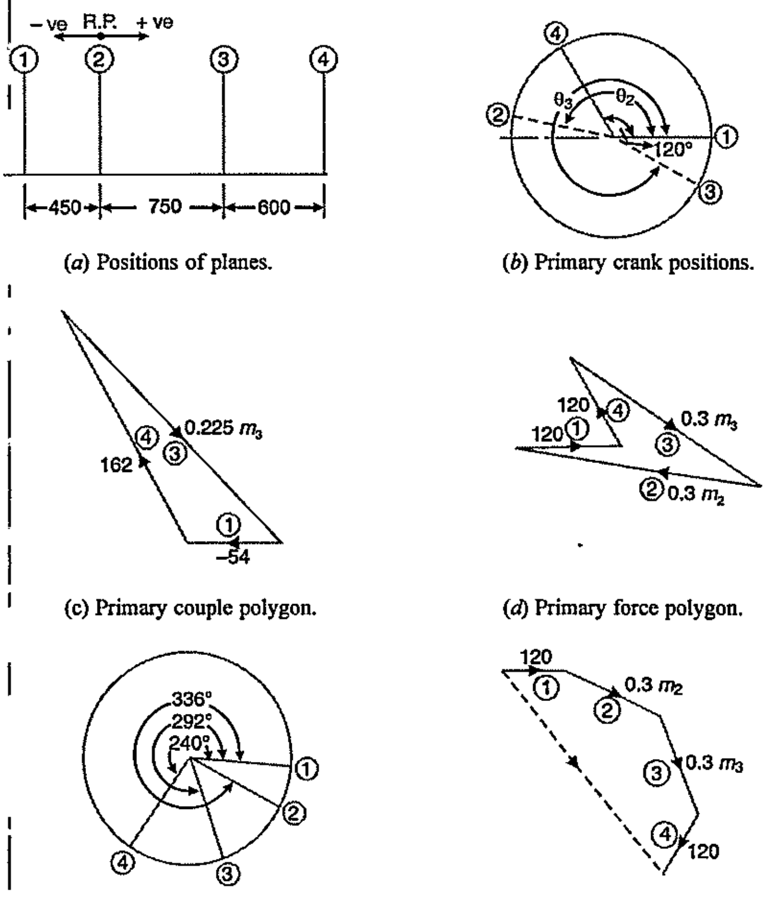
$$\theta_2 = 168° \textbf{ Ans.}$$

Maximum secondary unbalanced force

The secondary crank positions obtained by rotating the primary cranks at twice the angle, is shown in Fig. 22.18 (*e*). Now draw the secondary force polygon, as shown in Fig. 22.18 (*f*), to some suitable scale, from the data given in Table 22.9 (column 4). The closing side of the polygon shown dotted in Fig. 22.18 (*f*) represents the maximum secondary unbalanced force. By measurement, we find that the maximum secondary unbalanced force is proportional to 582 kg-m.

∴ Maximum secondary unbalanced force

$$= 582 \times \frac{\omega^2}{n} = \frac{582(25.14)^2}{1.2/0.3} = 91\,960\text{N} = 91.96 \text{ kN } \textbf{Ans.} \quad \ldots (\because n = l/r)$$



(*a*) Positions of planes. (*b*) Primary crank positions.

(c) Primary couple polygon. (*d*) Primary force polygon.

(*e*) Secondary crank positions. (*f*) Secondary force polygon.

Fig. 22.18

Example 22.9. *The cranks and connecting rods of a 4-cylinder in-line engine running at 1800 r.p.m. are 60 mm and 240 mm each respectively and the cylinders are spaced 150 mm apart. If the cylinders are numbered 1 to 4 in sequence from one end, the cranks appear at intervals of 90° in an end view in the order 1-4-2-3. The reciprocating mass corresponding to each cylinder is 1.5 kg.*

Determine : 1. Unbalanced primary and secondary forces, if any, and 2. Unbalanced primary and secondary couples with reference to central plane of the engine.

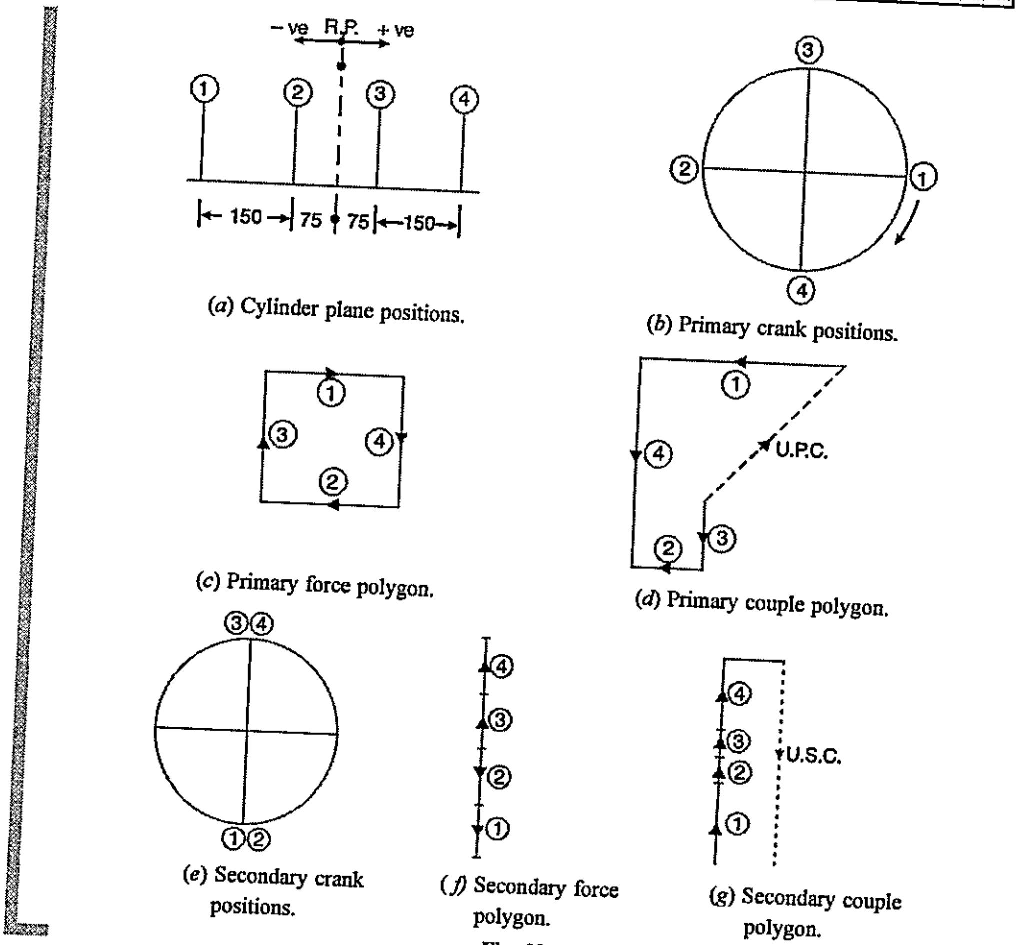
Solution. Given : N = 1800 r.p.m. or $\omega = 2\pi \times 1800/60 = 188.52$ rad/s ; r = 60 mm = 0.6 m ; l = 240 mm = 0.24 m ; m = 1.5 kg

1. ***Unbalanced primary and secondary forces***

The position of the cylinder planes and cranks is shown in Fig.22.19 (*a*) and (*b*) respectively. With reference to central plane of the engine, the data may be tabulated as below :

Table 22.10

Plane (*1*)	*Mass* (*m*) *kg* (*2*)	*Radius* (*r*) *m* (*3*)	*Cent. force* ÷ ω^2 (*m.r*) *kg-m* (*4*)	*Distance from ref. plane 3* (*l*) *m* (*5*)	*Couple* ÷ ω^2 (*m.r.l.*) *kg-m*^{2} (*6*)
1	1.5	0.6	0.9	− 0.225	− 0.2025
2	1.5	0.6	0.9	− 0.075	− 0.0675
3	1.5	0.6	0.9	+ 0.075	+ 0.0675
4	1.5	0.6	0.9	+ 0.225	+ 0.2025



(*a*) Cylinder plane positions.

(*b*) Primary crank positions.

(*c*) Primary force polygon.

(*d*) Primary couple polygon.

(*e*) Secondary crank positions.

(*f*) Secondary force polygon.

(*g*) Secondary couple polygon.

Fig. 22.19

The primary force polygon from the data given in Table 22.10 (column 4) is drawn as shown in Fig. 22.19 (*c*). Since the primary force polygon is a closed figure, therefore there are no unbalanced primary forces. **Ans.**

The secondary crank positions, taking crank 3 as the reference crank, is shown in Fig. 22.19 (*e*). From the secondary force polygon as shown in Fig. 22.19 (*f*), we see that it is a closed figure. Therefore there are no unbalanced secondary forces. **Ans.**

2. *Unbalanced primary and secondary couples*

The primary couple polygon from the data given in Table 22.10 (column 6) is drawn as shown in Fig. 22.19 (*d*). The closing side of the polygon, shown dotted in the figure, represents unbalanced primary couple. By measurement, we find the unbalanced primary couple is proportional to 0.19 kg-m^2.

∴ Unbalanced primary couple,

$$U.P.C = 0.19 \times \omega^2 = 0.19\,(188.52)^2 = 6752 \text{ N-m } \textbf{Ans.}$$

The secondary couple polygon is shown in Fig. 22.1 (*g*). The unbalanced secondary couple is shown by dotted line. By measurement, we find that unbalanced secondary couple is proportional to 0.54 kg-m^2.

∴ Unbalanced secondary couple,

$$U.S.C. = 0.54 \times \frac{\omega^2}{n} = 0.54 \times \frac{(188.52)^2}{0.24/0.6} = 4798 \text{ N-m } \textbf{Ans.} \quad \ldots (\because n = l/r)$$

Example 22.10. *Fig. 22.20 shows the arrangement of the cranks in a four crank symmetrical engine in which the masses of the reciprocating parts at cranks 1 and 4 are each equal to m_1 and at cranks 2 and 3 are each equal to m_2.*

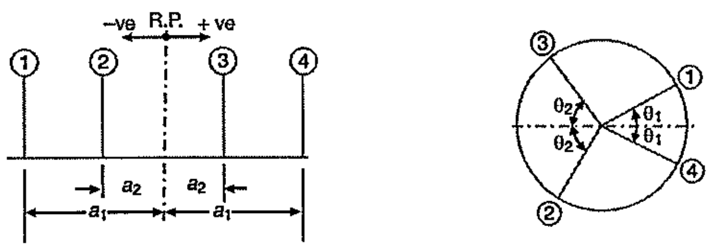


Fig. 22.20

Show that the arrangement is balanced for primary forces and couples and for secondary forces provided that

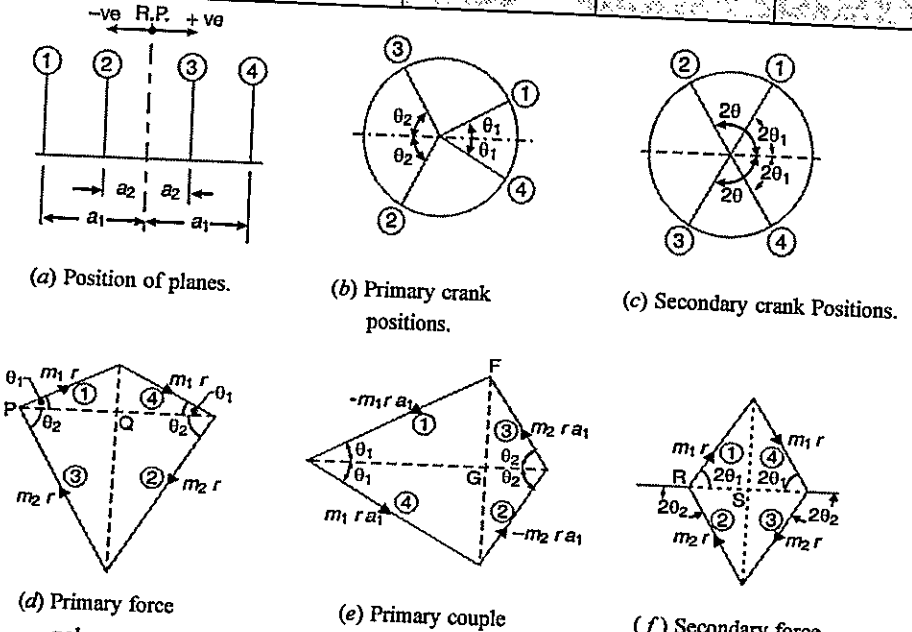
$$\frac{m_1}{m_2} = \frac{\cos\theta_2}{\cos\theta_1}\,;\quad \frac{a_1}{a_2} = \frac{\tan\theta_2}{\tan\theta_1},\quad \textit{and}\quad \cos\theta_1 \cdot \cos\theta_2 = \frac{1}{2}.$$

Solution. Given : Mass of reciprocating parts at cranks 1 and 4 = m_1 ; Mass of the reciprocating parts at cranks 2 and 3 = m_2

The position of planes and primary and secondary crank positions are shown in Fig. 22.21 (*a*), (*b*) and (*c*) respectively. Assuming the reference plane midway between the planes of rotation of cranks 2 and 3, the data may be tabulated as below :

Table 22.11

Plane (1)	*Mass (m)* (2)	*Radius (r)* (3)	*Cent. force* $\div \omega^2$ *(m.r)* (4)	*Distance from ref. plane (l)* (5)	*Couple* $\div \omega^2$ *(m.r.l)* (6)
1	m_1	r	$m_1.r$	$-a_1$	$-m_1.r.a_1$
2	m_2	r	$m_2.r$	$-a_2$	$-m_2.r.a_2$
3	m_2	r	$m_2.r$	$+a_2$	$+m_2.r.a_2$
4	m_1	r	$m_1.r$	$+a_1$	$+m_1.r.a_1$



(*a*) Position of planes. (*b*) Primary crank positions. (*c*) Secondary crank Positions.

(*d*) Primary force polygon. (*e*) Primary couple polygon. (*f*) Secondary force polygon.

Fig. 22.21

In order to balance the arrangement for primary forces and couples, the primary force and couple polygons must close. Fig. 22.21 (*d*) and (*e*) show the primary force and couple polygons, which are closed figures. From Fig. 22.21 (*d*),

$$PQ = m_1.r\cos\theta_1 = m_2.r\cos\theta_2 \quad \text{or} \quad \frac{m_1}{m_2} = \frac{\cos\theta_2}{\cos\theta_1} \quad \textbf{Ans.}$$

From Fig. 22.21 (*e*),

$$FG = m_1.r.a_1\sin\theta_1 = m_2.r.a_2\sin\theta_2$$

or

$$m_1.a_1\sin\theta_1 = m_2.a_2\sin\theta_2$$

$$\frac{m_1}{m_2}\times\frac{a_1}{a_2} = \frac{\sin\theta_2}{\sin\theta_1} \quad \text{or} \quad \frac{\cos\theta_2}{\cos\theta_1}\times\frac{a_1}{a_2} = \frac{\sin\theta_2}{\sin\theta_1} \quad \ldots\left(\because \frac{m_1}{m_2} = \frac{\cos\theta_2}{\cos\theta_1}\right)$$

$$\therefore \quad \frac{a_1}{a_2} = \frac{\sin\theta_2}{\sin\theta_1} \times \frac{\cos\theta_1}{\cos\theta_2} = \frac{\tan\theta_2}{\tan\theta_1} \text{ Ans.}$$

In order to balance the arrangement for secondary forces, the secondary force polygon must close. The position of the secondary cranks is shown in Fig. 22.21 (*c*) and the secondary force polygon is shown in Fig. 22.21 (*f*).

Now from Fig. 22.21 (*f*),

$$RS = m_1.r\cos 2\theta_1 = m_2.r\cos(180° - 2\theta_2)$$

or
$$m_1.\cos 2\theta_2 = -m_2.\cos 2\theta_2$$

$$\therefore \quad \frac{m_1}{m_2} = \frac{-\cos 2\theta_2}{\cos 2\theta_1} = \frac{-(2\cos^2\theta_2 - 1)}{2\cos^2\theta_1 - 1} \quad \ldots (\because \cos 2\theta = 2\cos^2\theta - 1)$$

$$\frac{\cos\theta_2}{\cos\theta_1} = \frac{(1 - 2\cos^2\theta_2)}{2\cos^2\theta_1 - 1} \quad \ldots \left[\because \frac{m_1}{m_2} = \frac{\cos\theta_2}{\cos\theta_1}\right]$$

$$2\cos^2\theta_1.\cos\theta_2 - \cos\theta_2 = \cos\theta_1 - 2\cos^2\theta_2.\cos\theta_1$$

$$2\cos\theta_1.\cos\theta_2(\cos\theta_1 + \cos\theta_2) = \cos\theta_1 + \cos\theta_2$$

$$2\cos\theta_1.\cos\theta_2 = 1 \quad \text{or} \quad \cos\theta_1.\cos\theta_2 = \frac{1}{2} \text{ Ans.}$$

Example 22.11. *A four cylinder engine has cranks arranged symmetrically along the shaft as shown in Fig. 22.22. The distance between the outer cranks A and D is 5.4 metres and that between the inner cranks B and C is 2.4 metres. The mass of the reciprocating parts belonging to each of the outer cylinders is 2 tonnes, and that belonging to each of the inner cylinders is m tonnes.*

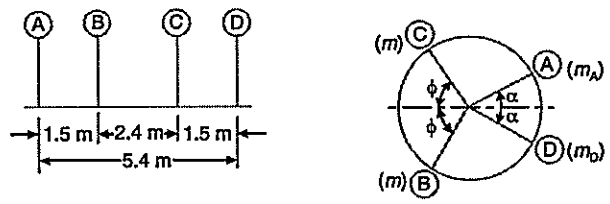


Fig. 22.22

If the primary and secondary forces are to be balanced and also the primary couples, determine the crank angle positions and the mass of the reciprocating parts (m) corresponding to the inner cylinders.

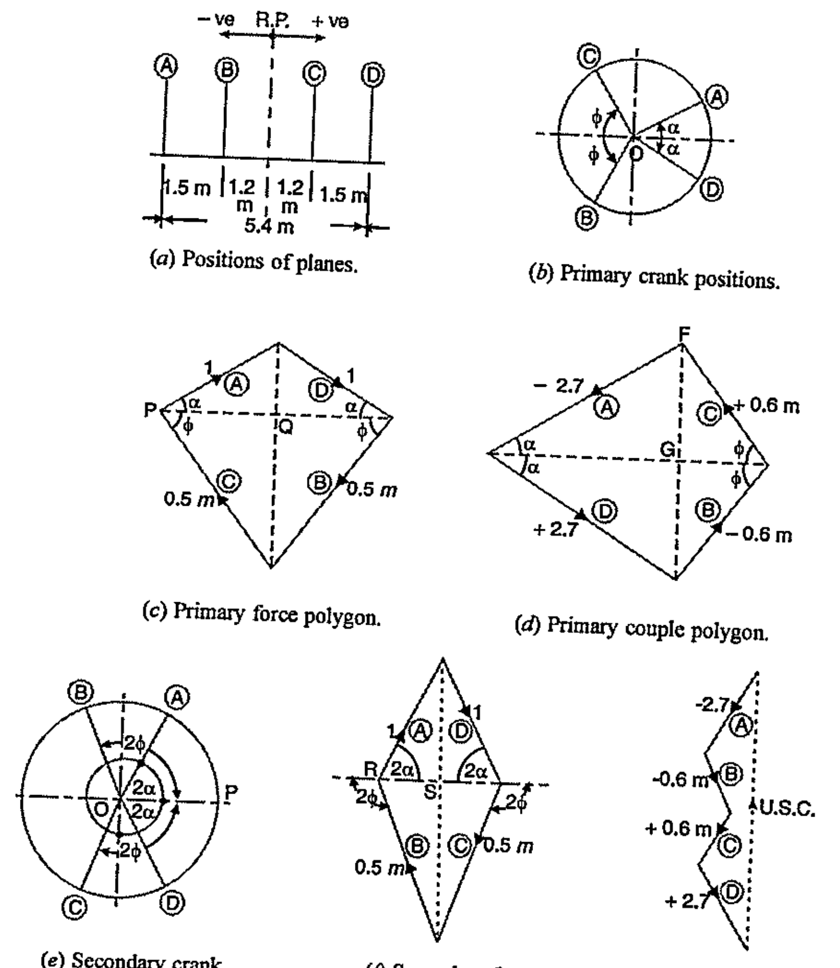
Find also the maximum value of the unbalanced secondary couple, if the stroke is 1 metre, the connecting rod length 2 metres, and the speed of the engine is 110 r.p.m.

Solution. Given : $AD = 5.4$ m ; $BC = 2.4$ m ; $m_A = m_D = 2$ t ; $L = 1$ m or $r = L/2 = 0.5$ m ; $l = 2$ m ; $N = 110$ r.p.m. or $\omega = 2\pi \times 110/60 = 11.52$ rad/s

Fig. 22.23 (*a*) shows the position of planes and Fig. 22.23 (*b*) shows the end view of the cranks with primary crank angles α and ϕ which are to be determined. Assuming the reference plane mid-way between the planes of rotation of cranks A and D, the data may be tabulated as below :

Table 22.12

Plane (1)	Mass (m) t (2)	Radius (r) m (3)	Cent. force ÷ ω^2 ($m.r$) t-m (3)	Distance from ref. plane (l) m (4)	Couple ÷ ω^2 ($m.r.l$) t-m^2 (5)
A	2	0.5	1	− 2.7	− 2.7
B	m	0.5	0.5 m	− 1.2	− 0.6 m
C	m	0.5	0.5 m	+ 1.2	+ 0.6 m
D	2	0.5	1	+ 2.7	+ 2.7



(a) Positions of planes. (b) Primary crank positions. (c) Primary force polygon. (d) Primary couple polygon. (e) Secondary crank positions. (f) Secondary force polygon. (g) Secondary couple polygon.

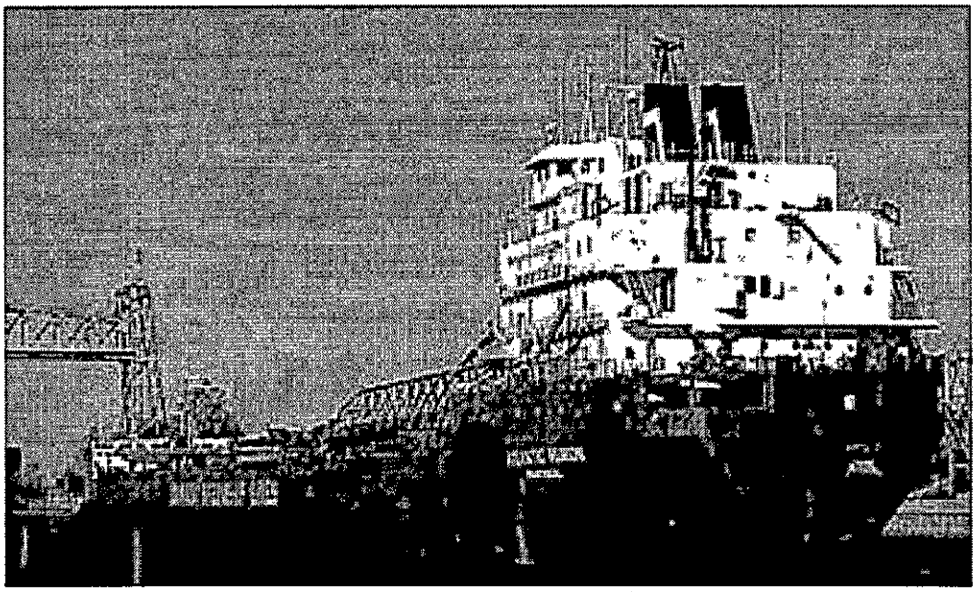
Fig. 22.23

Since the primary forces and couples are to be balanced, therefore the primary force and couple polygons, drawn from the data given in Table 22.12 column (4) and (6) respectively, as shown in Fig. 22.23 (*c*) and (*d*), must close.

From Fig. 22.23 (*c*),

$$PQ = 1\cos\alpha = 0.5\, m\cos\phi$$

$$\therefore \quad \cos\phi = \frac{1\cos\alpha}{0.5\, m} = \frac{2\cos\alpha}{m} \quad \ldots (i)$$



A Steam-powered ship.

From Fig. 22.23 (*d*),

$$FG = 2.7\sin\alpha = 0.6 m\sin\phi$$

$$\therefore \quad \sin\alpha = \frac{0.6m\,\sin\phi}{2.7} = \frac{m\sin\phi}{4.5} \quad \ldots (ii)$$

Now draw the secondary crank positions as shown in Fig. 22.23 (*e*). Let *OP* be the reference line. The secondary crank angles are given below :

OP to $OA = 2\alpha$

OP to $OC = 2\,(180° - \phi) = 360° - 2\phi$

OP to $OB = 2\,(180° + \phi) = 360° + 2\phi$

OP to $OD = 2\,(360° - \alpha) = 720° - 2\alpha$

Since the secondary forces are to be balanced, therefore the secondary force polygon, as shown in Fig. 22.23 (*f*), must close. Now from Fig. 22.23 (*f*),

$$RS = 1\,\cos\,2\alpha = 0.5\,m\cos\,(180° - 2\phi)$$

or

$$\frac{1}{0.5m} = \frac{-\cos 2\phi}{\cos 2\alpha} = \frac{-(2\cos^2\theta - 1)}{2\cos^2\alpha - 1} \quad \ldots (\because \cos 2\theta = 2\cos^2\theta - 1)$$

$$2\cos^2\alpha - 1 = 0.5m(1 - 2\cos^2\phi) = 0.5m\left[1 - 2\left(\frac{2\cos\alpha}{m}\right)^2\right] \quad \text{. . . [From equation } (i)]$$

$$= 0.5m\left[1 - \frac{8\cos^2\alpha}{m^2}\right] = 0.5m - \frac{4\cos^2\alpha}{m}$$

$$2\cos^2\alpha + \frac{4\cos^2\alpha}{m} = 1 + 0.5m \quad \text{or} \quad \cos^2\alpha\left(\frac{2m+4}{m}\right) = 1 + 0.5m$$

$$\therefore \quad \cos^2\alpha = (1 + 0.5m) \times \frac{m}{2m+4} = \frac{m}{4} \quad \text{. . . } (iii)$$

Now from equation *(ii)*

$$\sin^2\alpha = \left(\frac{m\sin\phi}{4.5}\right)^2$$

or

$$1 - \cos^2\alpha = \frac{m^2\sin^2\phi}{20.25} = \frac{m^2}{20.25}(1 - \cos^2\phi) = \frac{m^2}{20.25}\left[1 - \left(\frac{2\cos\alpha}{m}\right)^2\right]$$

. . . [From equations *(i)*]

$$1 - \frac{m}{4} = \frac{m^2}{20.25}\left(1 - \frac{4}{m^2} \times \frac{m}{4}\right) = \frac{m^2}{20.25}\left(1 - \frac{1}{m}\right) = \frac{m^2}{20.25} - \frac{m}{20.25}$$

. . . [From equation *(iii)*]

or
$$\frac{m^2}{20.25} - \frac{m}{20.25} + \frac{m}{4} - 1 = 0 \quad \text{or} \quad m^2 + 4.0625m - 20.25 = 0$$

$$\therefore \quad m = \frac{-4.0625 \pm \sqrt{(4.0625)^2 + 4 \times 20.25}}{2} = 2.9 \text{ t}$$

We know that $\cos^2\alpha = \dfrac{m}{4} = \dfrac{2.9}{4} = 0.725$

$\therefore \quad \cos\alpha = 0.851$ or $\alpha = 31.6°$ **Ans.**

Also $\quad \cos\phi = \dfrac{2\cos\alpha}{m} = \dfrac{2 \times 0.851}{2.9} = 0.5869 \quad$ or $\quad \phi = 54.06°$ **Ans.**

Maximum unbalanced secondary couple

The secondary couple polygon is shown in Fig. 22.23 (*g*). The maximum unbalanced secondary couple is shown by a dotted line. By measurement, we find that the maximum unbalanced secondary couple is proportional to 8 t-m^2.

∴ Maximum unbalanced secondary couple,

$$U.S.C = 8 \times \frac{\omega^2}{n} = 8 \times \frac{(11.52)^2}{2/0.5} = 265.4 \text{ kN-m } \textbf{Ans.} \quad \text{. . . } (\because n = l/r)$$

Example 22.12. *A five cylinder in-line engine running at 750 r.p.m. has successive cranks 144° apart, the distance between the cylinder centre lines being 375 mm. The piston stroke is 225 mm and the ratio of the connecting rod to the crank is 4. Examine the engine for balance of primary and secondary forces and couples. Find the maximum values of these and the position of the central crank at which these maximum values occur. The reciprocating mass for each cylinder is 15 kg.*

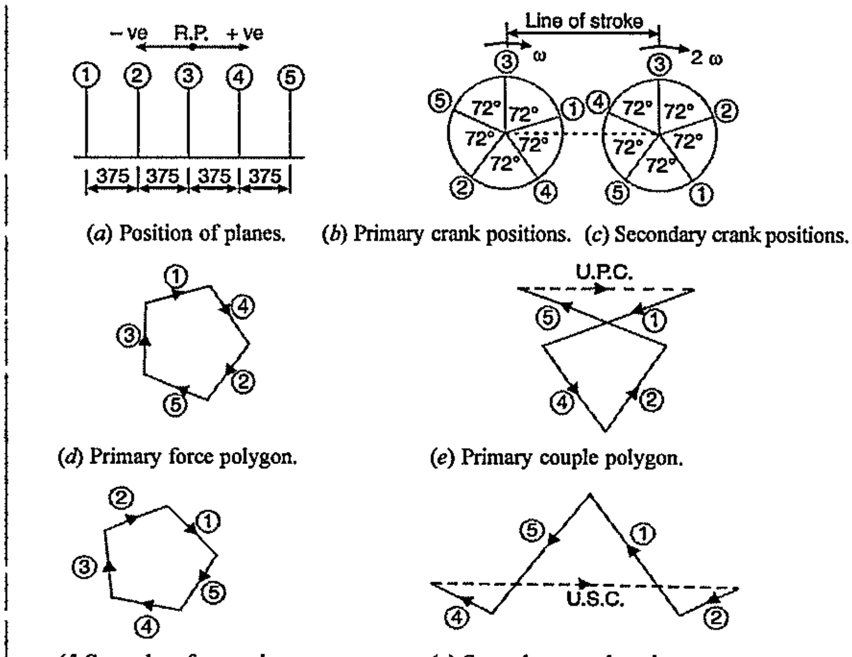
Solution. Given : $N = 750$ r.p.m. or $\omega = 2\pi \times 750/60 = 78.55$ rad/s ; $L = 225$ mm = 0.225 m or $r = 0.1125$ m ; $n = l/r = 4$; $m = 15$ kg

Assuming the engine to be a vertical engine, the positions of the cylinders and the cranks are shown in Fig. 22.24 (*a*), (*b*) and (*c*). The plane 3 may be taken as the reference plane and the crank 3 as the reference crank. The data may be tabulated as given in the following table.

Table 22.13

Plane (1)	*Mass* (*m*) *kg* (2)	*Radius* (*r*) *m* (3)	*Cent. force* ÷ ω^2 (*m.r*) *kg-m* (4)	*Distance from ref. Plane 3* (*l*) *m* (5)	*Couple* ÷ ω^2 (*m.r.l*) *kg-m*^{2} (6)
1	15	0.1125	1.6875	− 0.75	− 1.265
2	15	0.1125	1.6875	− 0.375	− 0.6328
3(*R.P.*)	15	0.1125	1.6875	0	0
4	15	0.1125	1.6875	+ 0.375	+ 0.6328
5	15	0.1125	1.6875	+ 0.75	+ 1.265

Now, draw the force and couple polygons for primary and secondary cranks as shown in Fig. 22.24 (*d*), (*e*), (*f*), and (*g*). Since the primary and secondary force polygons are close, therefore the engine is balanced for primary and secondary forces. **Ans.**



(*a*) Position of planes. (*b*) Primary crank positions. (*c*) Secondary crank positions.

(*d*) Primary force polygon. (*e*) Primary couple polygon.

(*f*) Secondary force polygon. (*g*) Secondary couple polygon.

Fig. 22.24

Maximum unbalanced primary couple

We know that the closing side of the primary couple polygon [shown dotted in Fig. 22.24 (*e*)] gives the maximum unbalanced primary couple. By measurement, we find that maximum unbalanced primary couple is proportional to 1.62 kg-m^2.

∴ Maximum unbalanced primary couple,

$$U.P.C. = 1.62 \times \omega^2 = 1.62\,(78.55)^2 = 9996 \text{ N-m } \textbf{Ans.}$$

We see from Fig. 22.24 (*e*) [shown by dotted line] that the maximum unbalanced primary couple occurs when crank 3 is at 90° from the line of stroke.

Maximum unbalanced secondary couple

We know that the closing side of the secondary couple polygon [shown dotted in Fig. 22.24 (*g*)] gives the maximum unbalanced secondary couple. By measurement, we find that maximum unbalanced secondary couple is proportional to 2.7 kg-m^2.

∴ Maximum unbalanced secondary couple.

$$U.S.C = 2.7 \times \frac{\omega^2}{n} = 2.7 \times \frac{(78.55)^2}{4} = 4165 \text{ N-m } \textbf{Ans.}$$

We see from Fig. 22.24 (*g*) that if the vector representing the unbalanced secondary couple (shown by dotted line) is rotated through 90°, it will coincide with the line of stroke. Hence the original crank will be rotated through 45°. Therefore, the maximum unbalanced secondary couple occurs when crank 3 is at 45° and at successive intervals of 90° (*i.e.* 135°, 225° and 315°) from the line of stroke.

Example 22.13. *The firing order in a 6 cylinder vertical four stroke in-line engine is 1-4-2-6-3-5. The piston stroke is 100 mm and the length of each connecting rod is 200 mm. The pitch distances between the cylinder centre lines are 100 mm, 100 mm, 150 mm, 100 mm, and 100 mm respectively. The reciprocating mass per cylinder is 1 kg and the engine runs at 3000 r.p.m.*

Determine the out-of-balance primary and secondary forces and couples on this engine, taking a plane midway between the cylinder 3 and 4 as the reference plane.

Solution. Given : $L = 100$ mm or $r = L/2 = 50$ mm $= 0.05$ m ; $l = 200$ mm ; $m = 1$ kg ; $N = 3000$ r.p.m.

The position of the cylinders and the cranks are shown in Fig. 22.25 (*a*), (*b*) and (*c*). With the reference plane midway between the cylinders 3 and 4, the data may be tabulated as given in the following table :

Table 22.14

Plane (1)	*Mass* (*m*) kg (2)	*Radius* (*r*) m (3)	*Cent. force* ÷ ω^2 (*m.r*) kg-m (4)	*Distance from plane 3* (*l*) m (5)	*Couple* ÷ ω^2 (*m.r.l*) kg-m^2 (6)
1	1	0.05	0.05	− 0.275	− 0.01375
2	1	0.05	0.05	− 0.175	− 0.00875
3	1	0.05	0.05	− 0.075	− 0.00375
4	1	0.05	0.05	+ 0.075	+ 0.00375
5	1	0.05	0.05	+ 0.175	+ 0.00875
6	1	0.05	0.05	+ 0.275	+ 0.01375

Now, draw the force and couple polygons for the primary and secondary cranks as shown in Fig. 22.25 (*d*), (*e*), (*f*) and (*g*).

UNIT-IV BALANCING OF ROTATING MASSES

The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass, is called *balancing of rotating masses*.

21.3. Balancing of a Single Rotating Mass By a Single Mass Rotating in the Same Plane

Consider a disturbing mass m_1 attached to a shaft rotating at ω rad/s as shown in Fig. 21.1. Let r_1 be the radius of rotation of the mass m_1.

We know that the centrifugal force exerted by the mass m_1 on the shaft,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1 \qquad \ldots (i)$$

This centrifugal force acts radially outwards and thus produces bending moment on the shaft.

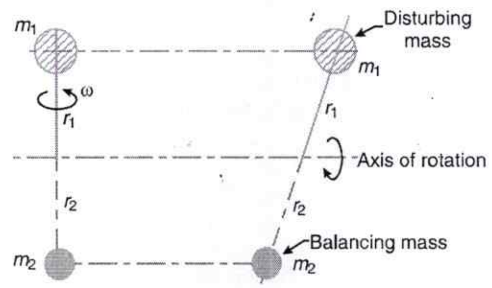


Fig. 21.1. Balancing of a single rotating mass by a single mass rotating in the same plane.

Let r_2 = Radius of rotation of the balancing mass m_2.

$\therefore$ Centrifugal force due to mass m_2,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2 \qquad \ldots (ii)$$

Equating equations (*i*) and (*ii*),

$$m_1 . \omega^2 \cdot r_1 = m_2 \cdot \omega^2 \cdot r_2 \quad \text{or} \quad m_1 \cdot r_1 = m_2 \cdot r_2$$

Discuss how a single revolving mass is balanced by two masses revolving in different planes ?

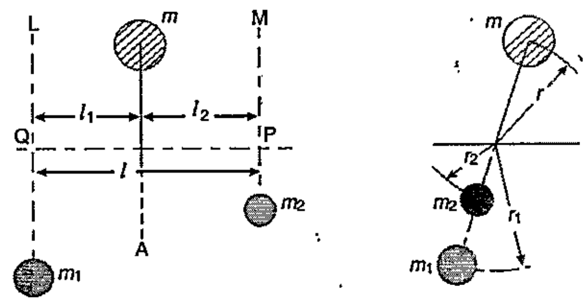
21.4. Balancing of a Single Rotating Mass By Two Masses Rotating in Different Planes

Two balancing masses are placed in two different planes, parallel to the plane of rotation of the disturbing mass, in such a way that they satisfy the following two conditions of equilibrium.

1. The net dynamic force acting on the shaft is equal to zero. This requires that the line of action of three centrifugal forces must be the same. In other words, the centre of the masses of the system must lie on the axis of rotation. This is the condition for *Static balancing*.
2. The net couple due to the dynamic forces acting on the shaft is equal to zero. In other words, the algebraic sum of the moments about any point in the plane must be zero. The conditions (1) and (2) together give *dynamic balancing*.

1. *When the plane of the disturbing mass lies in between the planes of the two balancing masses*

Consider a disturbing mass m lying in a plane A to be balanced by two rotating masses m_1 and m_2 lying in two different planes L and M as shown in Fig. 21.2. Let r, r_1 and r_2 be the radii of rotation of the masses in planes A, L and M respectively.



We know that the centrifugal force exerted by the mass m in the plane A,

$$F_C = m \cdot \omega^2 \cdot r$$

Similarly, the centrifugal force exerted by the mass m_1 in the plane L,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1$$

and, the centrifugal force exerted by the mass m_2 in the plane M,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2$$

Since the net force acting on the shaft must be equal to zero, therefore the centrifugal force on the disturbing mass must be equal to the sum of the centrifugal forces on the balancing masses, therefore

$$F_C \quad F_{C1} \quad F_{C2}$$

$$\therefore \quad m \cdot r = m_1 \cdot r_1 + m_2 \cdot r_2 \qquad \ldots (i)$$

Now in order to find the magnitude of balancing force in the plane L (or the dynamic force at the bearing Q of a shaft), take moments about P which is the point of intersection of the plane M and the axis of rotation. Therefore

$$F_{C1} \times l = F_C \times l_2$$

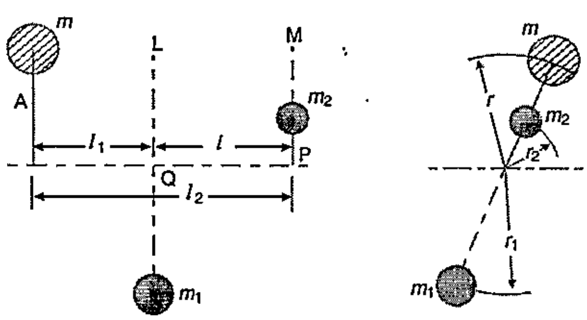
$$\therefore \quad m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2 \qquad \ldots (ii)$$

Similarly, in order to find the balancing force in plane M (or the dynamic force at the bearing P of a shaft), take moments about Q which is the point of intersection of the plane L and the axis of rotation. Therefore

$$F_{C2} \times l = F_C \times l_1$$

$$\therefore \quad m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1 \qquad \ldots (iii)$$

2. ***When the plane of the disturbing mass lies on one end of the planes of the balancing masses***



In this case, the mass m lies in the plane A and the balancing masses lie in the planes L and M, the following conditions must be satisfied in order to balance the system, *i.e.*

$$F_C + F_{C2} = F_{C1}$$

$$\therefore \quad m \cdot r + m_2 \cdot r_2 = m_1 \cdot r_1 \qquad \dots (iv)$$

Now, to find the balancing force in the plane L (or the dynamic force at the bearing Q of a shaft), take moments about P which is the point of intersection of the plane M and the axis of rotation. Therefore

$$F_{C1} \times l = F_C \times l_2$$

$$\therefore \quad m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2 \qquad \dots (v)$$

... [Same as equation (*ii*)]

Similarly, to find the balancing force in the plane M (or the dynamic force at the bearing P of a shaft), take moments about Q which is the point of intersection of the plane L and the axis of rotation. Therefore

$$F_{C2} \times l = F_C \times l_1$$

$$m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1$$

21.5. Balancing of Several Masses Rotating in the Same Plane

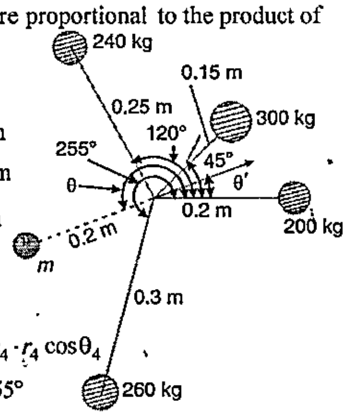
Example 1. *Four masses m_1, m_2, m_3 and m_4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required, if its radius of rotation is 0.2 m.*

Solution. Given : $m_1 = 200$ kg ; $m_2 = 300$ kg ; $m_3 = 240$ kg ; $m_4 = 260$ kg ; $r_1 = 0.2$ m ; $r_2 = 0.15$ m ; $r_3 = 0.25$ m ; $r_4 = 0.3$ m ; $\theta_1 = 0°$; $\theta_2 = 45°$; $\theta_3 = 45° + 75° = 120°$; $\theta_4 = 45° + 75° + 135° = 255°$; $r = 0.2$ m

Let m = Balancing mass, and

θ = The angle which the balancing mass makes with m_1.

Since the magnitude of centrifugal forces are proportional to the product of each mass and its radius, therefore



$$m_1 \cdot r_1 = 200 \times 0.2 = 40 \text{ kg-m}$$

$$m_2 \cdot r_2 = 300 \times 0.15 = 45 \text{ kg-m}$$

$$m_3 \cdot r_3 = 240 \times 0.25 = 60 \text{ kg-m}$$

$$m_4 \cdot r_4 = 260 \times 0.3 = 78 \text{ kg-m}$$

1. ***Analytical method***

$$\Sigma H = m_1 \cdot r_1 \cos\theta_1 + m_2 \cdot r_2 \cos\theta_2 + m_3 \cdot r_3 \cos\theta_3 + m_4 \cdot r_4 \cos\theta_4$$

$$= 40 \cos 0° + 45 \cos 45° + 60 \cos 120° + 78 \cos 255°$$

$$= 40 + 31.8 - 30 - 20.2 = 21.6 \text{ kg-m}$$

Now resolving vertically,

$$\Sigma V = m_1 \cdot r_1 \sin\theta_1 + m_2 \cdot r_2 \sin\theta_2 + m_3 \cdot r_3 \sin\theta_3 + m_4 \cdot r_4 \sin\theta_4$$

$$= 40 \sin 0° + 45 \sin 45° + 60 \sin 120° + 78 \sin 255°$$

$$= 0 + 31.8 + 52 - 75.3 = 8.5 \text{ kg-m}$$

.Resultant, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(21.6)^2 + (8.5)^2} = 23.2$ kg-m

We know that

$$m \cdot r = R = 23.2 \quad \text{or} \quad m = 23.2/r = 23.2/0.2 = 116 \text{ kg} \quad \textbf{Ans.}$$

and $$\tan\theta' = \Sigma V/\Sigma H = 8.5/21.6 = 0.3935 \quad \text{or} \quad \theta' = 21.48°$$

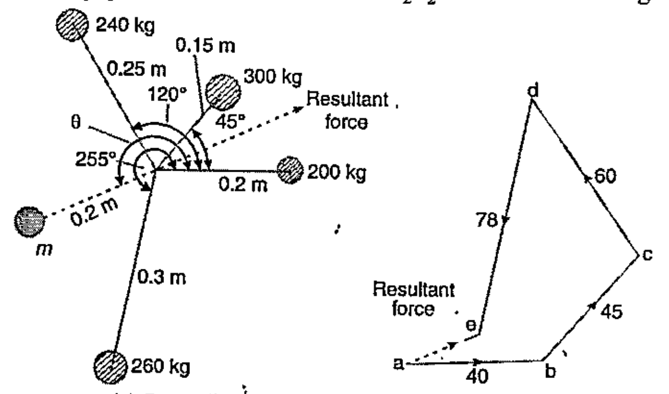
Since θ' is the angle of the resultant R from the horizontal mass of 200 kg, therefore the angle of the balancing mass from the horizontal mass of 200 kg

$$\theta = 180° + 21.48° = 201.48° \quad \textbf{Ans.}$$

2. *Graphical method*

1. First of all, draw the space diagram showing the positions of all the given masses as shown in Fig 21.6 (a).
2. Since the centrifugal force of each mass is proportional to the product of the mass and radius, therefore

$$m_1.r_1 = 200 \times 0.2 = 40 \text{ kg-m} \ \& \ m_2.r_2 = 300 \times 0.15 = 45 \text{ kg-m}$$



(a) Space diagram. (b) Vector diagram

Fig. 21.6

4. The balancing force is equal to the resultant force, but *opposite* in direction as shown in Fig. 21.6 (a). Since the balancing force is proportional to $m.r$, therefore

$$m \times 0.2 = \text{vector } ea = 23 \text{ kg-m} \quad \text{or} \quad m = 23/0.2 = 115 \text{ kg} \quad \textbf{Ans.}$$

By measurement we also find that the angle of inclination of the balancing mass (m) from the horizontal mass of 200 kg

$$\theta = 201° \quad \textbf{Ans.}$$

21.6. Balancing of Several Masses Rotating in Different Planes

Example 2. *A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses ar e to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses r evolve at a radius of 100 mm, find their magnitudes and angular positions.*

Solution. Given : $m_A = 200$ kg ; $m_B = 300$ kg ; $m_C = 400$ kg ; $m_D = 200$ kg ; $r_A = 80$ mm $= 0.08$m ; $r_B = 70$ mm $= 0.07$ m ; $r_C = 60$ mm $= 0.06$ m ; $r_D = 80$ mm $= 0.08$ m ; $r_X = r_Y = 100$ mm $= 0.1$ m

Let m_X = Balancing mass placed in plane X, and

m_Y = Balancing mass placed in plane Y.

The position of planes and angular position of the masses (assuming the mass A as horizontal) are shown in Fig. 21.8 (a) and (b) respectively.

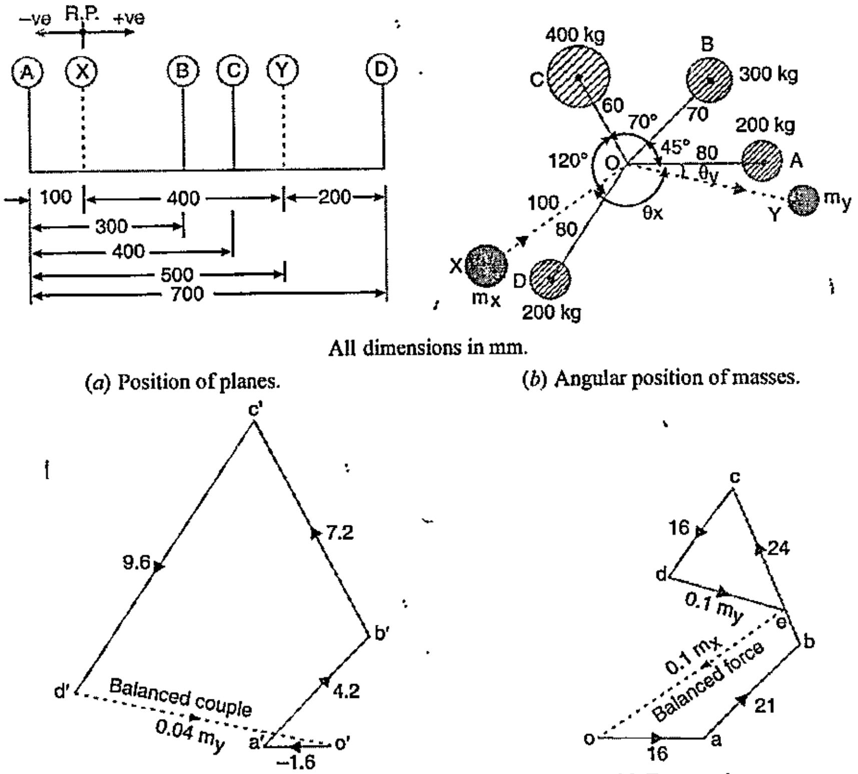
Assume the plane X as the reference plane ($R.P.$). The distances of the planes to the right of plane X are taken as + ve while the distances of the planes to the left of plane X are taken as – ve.

Table 21.2

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent.force ÷ ω^2 (m.r) kg-m (4)	Distance from Plane x(l) m (5)	Couple ÷ ω^2 (m.r.l) kg-m^2 (6)
A	200	0.08	16	– 0.1	– 1.6
X(R.P.)	m_X	0.1	0.1 m_X	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_Y	0.1	0.1 m_Y	0.4	0.04 m_Y
D	200	0.08	16	0.6	9.6

1. First of all, draw the couple polygon from the data given in Table 21.2 (column 6) as shown in Fig. 21.8 (*c*) to some suitable scale. The vector $d'o'$ represents the balanced couple. Since the balanced couple is proportional to 0.04 m_Y, therefore by measurement,

$$0.04\,m_Y = \text{vector } d'\,o' = 7.3 \text{ kg-m}^2 \quad \text{or} \quad m_Y = 182.5 \text{ kg } \textbf{Ans.}$$



All dimensions in mm.

(*a*) Position of planes. (*b*) Angular position of masses.

(*c*) Couple polygon. (*d*) Force polygon.

Fig. 21.8

By measurement, the angular position of m_Y is $\theta_Y = 12°$ in the clockwise direction from mass m_A (*i.e.* 200 kg). **Ans.**

2. Now draw the force polygon from the data given in Table 21.2 (column 4) as shown in Fig. 21.8 (*d*). The vector *eo* represents the balanced force. Since the balanced force is proportional to 0.1 m_X, therefore by measurement,

$$0.1\,m_X = \text{vector } eo = 35.5 \text{ kg-m} \quad \text{or} \quad m_X = 355 \text{ kg } \textbf{Ans.}$$

By measurement, the angular position of m_X is $\theta_X = 145°$ in the clockwise direction from mass m_A (*i.e.* 200 kg). **Ans.**

Table 21.3

Example 3. *A, B, C and D are four masses carried by a rotating shaft at radii 100, 125, 200 and 150 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the mass of B, C and D are 10 kg, 5 kg, and 4 kg respectively.*

Find the required mass A and the relative angular settings of the four masses so that the shaft shall be in complete balance.

Solution. Given : $r_A = 100$ mm $= 0.1$ m ; $r_B = 125$ mm $= 0.125$ m ; $r_C = 200$ mm $= 0.2$ m ; $r_D = 150$ mm $= 0.15$ m ; $m_B = 10$ kg ; $m_C = 5$ kg ; $m_D = 4$ kg

The position of planes is shown in Fig. 21.10 (*a*). Assuming the plane of mass *A* as the reference plane (*R.P.*), the data may be tabulated as below :

Table 21.4

Plane (1)	*Mass (m) kg* (2)	*Radius (r) m* (3)	*Cent. Force* $\div \omega^2$ *(m.r) kg-m* (4)	*Distance from plane A (l) m* (5)	*Couple* $\div \omega^2$ *(m.r.l) kg-m²* (6)
A(R.P.)	m_A	0.1	0.1 m_A	0	0
B	10	0.125	1.25	0.6	0.75
C	5	0.2	1	1.2	1.2
D	4	0.15	0.6	1.8	1.08

Drawing the couple polygon from the data given in Table 21.4 (column 6). Assume the position of mass *B* in the horizontal direction. By measurement, we find that the angular setting of mass *C* from mass *B* in the anticlockwise direction, *i.e.*

$\angle BOC = 240°$ **Ans.**

and angular setting of mass *D* from mass *B* in the anticlockwise direction, *i.e* $\angle BOD = 100°$ **Ans.**

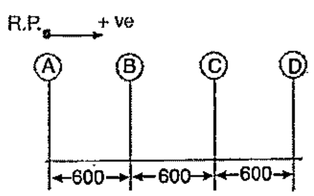
Draw the force polygon to some suitable scale, as shown in Fig. 21.10 (*d*).

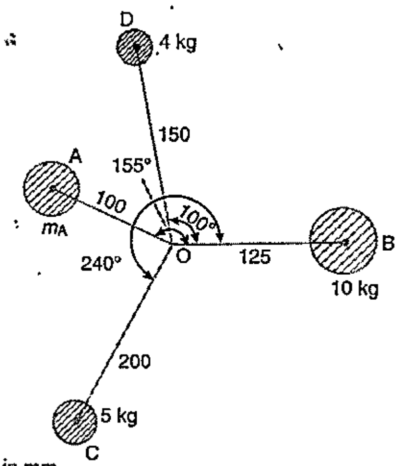
Since the closing side of the force polygon (vector *do*) is proportional to 0.1 m_A, therefore by measurement,

$0.1\ m_A = 0.7$ kg-m² or $m_A = 7$ kg **Ans.**

Now draw *OA* in Fig. 21.10 (*b*), parallel to vector *do*. By measurement, we find that the angular setting of mass *A* from mass *B* in the anticlockwise direction, *i.e.*

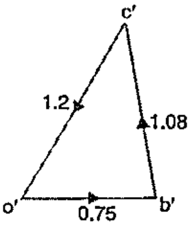
$\angle BOA = 155°$ **Ans.**

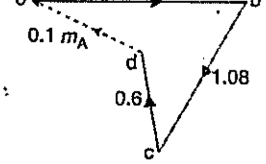




All dimensions in mm

(*a*) Position of planes. (*b*) Angular position of masses.





Explain about primary and secondary unbalanced forces of rotating masses ?

22.1. Introduction

BALANCING OF RECIPROCATING MASSES:

The resultant of all the forces acting on the body of the engine due to inertia forces only is known as ***unbalanced force*** or ***shaking force.***

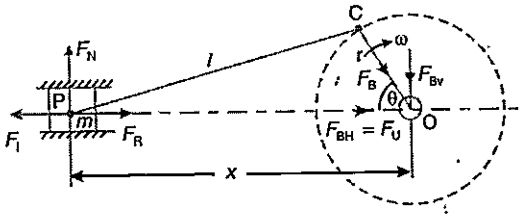


Fig. 22.1. Reciprocating engine mechanism.

Let F_R = Force required to accelerate the reciprocating parts,

F_I = Inertia force due to reciprocating parts,

F_N = Force on the sides of the cylinder walls or normal force acting on the cross-head guides, and

F_B = Force acting on the crankshaft bearing or main bearing.

22.2. Primary and Secondary Unbalanced Forces of Reciprocating Masses

Consider a reciprocating engine mechanism as shown in Fig. 22.1.

Let m = Mass of the reciprocating parts,

l = Length of the connecting rod PC,

r = Radius of the crank OC,

θ = Angle of inclination of the crank with the line of stroke PO,

ω = Angular speed of the crank,

n = Ratio of length of the connecting rod to the crank radius = l / r.

The acceleration of the reciprocating parts is approximately given by the expression,

$$a_R = \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$\therefore$ Inertia force due to reciprocating parts or force required to accelerate the reciprocating parts,

$$F_I = F_R = \text{Mass} \times \text{acceleration} = m \cdot \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

The horizontal component of the force exerted on the crank shaft bearing (*i.e.* F_{BH}) is equal and opposite to inertia force (F_I). This force is an unbalanced one and is denoted by F_U.

$\therefore$ Unbalanced force,

$$F_U = m \cdot \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right) = m.\omega^2 \cdot r\cos\theta + m \cdot \omega^2 \cdot r \times \frac{\cos 2\theta}{n} = F_P + F_S$$

The expression $(m \cdot \omega^2 \cdot r\cos\theta)$ is known as ***primary unbalanced force*** and $\left(m \cdot \omega^2 \cdot r \times \frac{\cos 2\theta}{n}\right)$ is called ***secondary unbalanced force.***

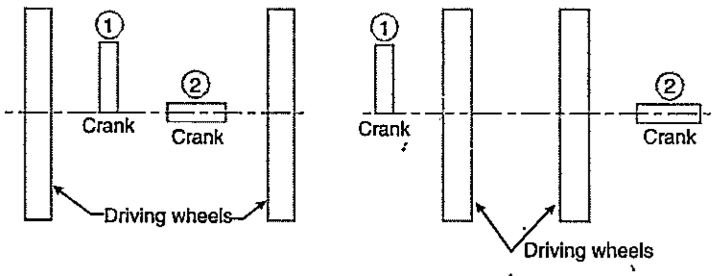
22.4. Partial Balancing of Locomotives

The locomotives, usually, have two cylinders with cranks placed at right angles to each other in order to have uniformity in turning moment diagram. The two cylinder locomotives may be classified as :

1. Inside cylinder locomotives ; and 2. Outside cylinder locomotives.

In the ***inside cylinder locomotives***, the two cylinders are placed in between the planes of two driving wheels as shown in Fig. 22.3 (*a*) ; whereas in the ***outside cylinder locomotives***, the two cylinders are placed outside the driving wheels, one on each side of the driving wheel, as shown in Fig. 22.3 (*b*). The locomotives may be

(*a*) Single or uncoupled locomotives ; and (*b*) Coupled locomotives.



(*a*) Inside cylinder locomotives. (*b*) Outside cylinder locomotives.

Fig. 22.3

22.5. Effect of Partial Balancing of Reciprocating Parts of Two Cylinder Locomotives

22.6. Variation of Tractive Force

The resultant unbalanced force due to the two cylinders, along the line of stroke, is known as ***tractive force.***

$\therefore$ As per definition, the tractive force,

$$F_T = \text{Resultant unbalanced force along the line of stroke}$$

$$= (1-c)m.\omega^2.r\cos\theta + (1-c)m.\omega^2.r\cos(90^\circ+\theta)$$

$$= (1-c)m.\omega^2.r(\cos\theta - \sin\theta)$$

Thus, the tractive force is maximum or minimum when $\theta = 135^\circ$ or 315°.

$\therefore$ Maximum and minimum value of the tractive force or the variation in tractive force

$$= \pm(1-c)\,m.\omega^2.r(\cos 135^\circ - \sin 135^\circ) = \pm\sqrt{2}\,(1-c)\,m.\omega^2.r$$

22.7. Swaying Couple

The unbalanced forces along the line of stroke for the two cylinders constitute a couple about the centre line YY between the cylinders as shown in Fig. 22.5.

This couple has swaying effect about a vertical axis, and tends to sway the engine alternately in clockwise and anticlockwise directions. Hence the couple is known as ***swaying couple.***

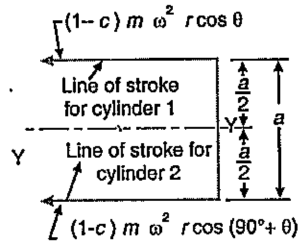
Let a = Distance between the centre lines of the two cylinders.

$\therefore$ Swaying couple

$$= (1-c)m.\omega^2.r\cos\theta \times \frac{a}{2}$$

$$-(1-c)m.\omega^2.r\cos(90°+\theta)\frac{a}{2}$$

$$= (1-c)m.\omega^2.r\times\frac{a}{2}(\cos\theta+\sin\theta)$$



Thus, the swaying couple is maximum or minimum when $\theta = 45°$ or $225°$.

$\therefore$ Maximum and minimum value of the swaying couple

$$= \pm(1-c)m.\omega^2.r\times\frac{a}{2}(\cos 45°+\sin 45°) = \pm\frac{a}{\sqrt{2}}(1-c)m.\omega^2.r$$

22.8. Hammer Blow

The maximum magnitude of the unbalanced force along the perpendicular to the line of stroke is known as *hammer blow.*

$\therefore$ Hammer blow $= B.\omega^2.b$

Example 4. *An inside cylinder locomotive has its cylinder centre lines 0.7 m apart and has a stroke of 0.6 m. The r otating masses per cylinder are equivalent to 150 kg at the crank pin, and the reciprocating masses per cylinder to 180 kg. The wheel centr e lines are 1.5 m apart. The cranks are at right angles. The whole of the r otating and 2/3 of the r eciprocating masses are to be balanced by masses placed at a radius of 0.6 m. Find the magnitude and dir ection of the balancing masses. Find the fluctuation in rail pr essure under one wheel, variation of tractive effort and the magnitude of swaying couple at a crank speed of 300 r .p.m.*

Solution. Given : $a = 0.7$ m; $l_B = l_C = 0.6$ m or $r_B = r_C = 0.3$ m; $m_1 = 150$ kg; $m_2 = 180$ kg; $c = 2/3$; $r_A = r_D = 0.6$ m; $N = 300$ r .p.m. or $\omega = 2\pi\times300/60 = 31.42$ rad/s

The equivalent mass of the rotating parts to be balanced per cylinder at the crank pin,

$$m = m_B = m_C = m_1 + c.m_2 = 150 + \frac{2}{3}\times 180 = 270 \text{ kg}$$

Magnitude and direction of the balancing masses

Let m_A and m_D = Magnitude of the balancing masses

θ_A and θ_D = Angular position of the balancing masses m_A and m_D from the first crank B.

The magnitude and direction of the balancing masses may be determined graphically as discussed below :

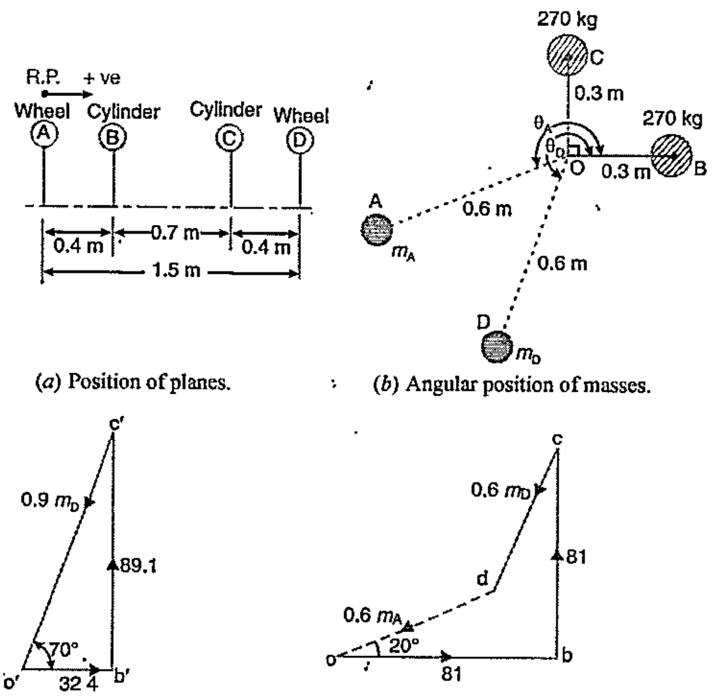
1. First of all, draw the space diagram to show the positions of the planes of the wheels and the cylinders, as shown in Fig. 22.7 (*a*). Since the cranks of the cylinders are at right angles, therefore assuming the position of crank of the cylinder B in the horizontal direction, draw OC and OB at right angles to each other as shown in Fig. 22.7 (*b*).
2. Tabulate the data as given in the following table. Assume the plane of wheel A as the reference plane.

Table 22.1

Plane (1)	*mass. (m) kg (2)*	*Radius (r)m (3)*	*Cent. force ÷ ω² (m.r) kg-m (4)*	*Distance from plane A (l)m (5)*	*Couple ÷ ω² (m.r.l) kg-m² (6)*
A (R.P.)	m_A	0.6	0.6 m_A	0	0
B	270	0.3	81	0.4	32.4
C	270	0.3	81	1.1	89.1
D	m_D	0.6	0.6m_D	1.5	0.9 m_D

3. Now, draw the couple polygon from the data given in T able 22.1 (column 6), to some suitable scale, as shown in Fig 22.7 (*c*). The closing side $c'o'$ represents the balancing couple and it is proportional to 0.9 m_D. Therefore, by measurement,

$$0.9\ m_D = \text{vector } c'o' = 94.5 \text{ kg-m}^2 \quad\text{or}\quad m_D = 105 \text{ kg} \quad \textbf{Ans.}$$



(*a*) Position of planes. (*b*) Angular position of masses.

(*c*) Couple polygon. (*d*) Force polygon.

Fig. 22.7

4. To determine the angular position of the balancing mass *D*, draw *QD* in Fig. 22.7 (*b*) parallel to vector $c'o'$. By measurement,

$$\theta_D = 250° \textbf{ Ans.}$$

5. In order to find the balancing mass *A*, draw the force polygon from the data given in Table 22.1 (column 4), to some suitable scale, as shown in Fig. 22.7 (*d*), The vector *do* represents the balancing force and it is proportional to 0.6 m_A. Therefore by measurement,

$$0.6\, m_A = \text{vector } do = 63 \text{ kg-m or } m_A = 105 \text{ kg } \textbf{Ans.}$$

6. To determine the angular position of the balancing mass *A*, draw *OA* in Fig. 22.7 (*b*) parallel to vector *do*. By measurement,

$$\theta_A = 200° \textbf{ Ans.}$$

Fluctuation in rail pressure

We know that each balancing mass = 105 kg

Balancing mass for rotating masses,

$$D = \frac{m_1}{m} \times 105 = \frac{150}{270} \times 105 = 58.3 \text{ kg}$$

and balancing mass for reciprocating masses,

$$B = \frac{c.m_2}{m} \times 105 = \frac{2}{3} \times \frac{180}{270} \times 105 = 46.6 \text{ kg}$$

This balancing mass of 46.6 kg for reciprocating masses gives rise to the centrifugal force.

∴ Fluctuation in rail pressure or hammer blow

$$= B.\omega^2.b = 46.6\,(31.42)^2\, 0.6 = 27\,602 \text{ N. } \textbf{Ans.} \qquad \ldots (\because\ b = r_A = r_D)$$

Variation of tractive effort

We know that maximum variation of tractive effort

$$= \pm\sqrt{2}(1-c)m_2.\omega^2.r = \pm\sqrt{2}\left(1-\frac{2}{3}\right)180(31.42)^2 0.3 \text{ N}$$

$= \pm 25\ 127$ N **Ans.** ... ($\because r = r_B = r_C$)

Swaying couple

We know that maximum swaying couple

$$= \frac{a(1-c)}{\sqrt{2}} \times m_2.\omega^2.r = \frac{0.7\left(1-\frac{2}{3}\right)}{\sqrt{2}} \times 180(31.42)^2 0.3 \text{ N-m}$$

$= 8797$ N-m **Ans.**

$$\omega = \frac{v}{D/2} = \frac{33.33}{1.8/2} = 37 \text{rad/s}$$

We know that hammer blow

$$= \pm B.\omega^2.b = 33(37)^2 0.675 = \pm 30.494 \text{ N} \textbf{ Ans.}$$

... ($\because B = m_E''$, and $b = r_B = r_E$)

22.10. Balancing of Primary Forces of Multi-cylinder In-line Engines

The multi-cylinder engines with the cylinder centre lines in the same plane and on the same side of the centre line of the crankshaft, are known as ***In-line engines***. The following two conditions must be satisfied in order to give the primary balance of the reciprocating parts of a multi-cylinder engine :

1. The algebraic sum of the primary forces must be equal to zero. In other words, the primary force polygon must *close ; and
2. The algebraic sum of the couples about any point in the plane of the primary forces must be equal to zero. In other words, the primary couple polygon must close.

22.11. Balancing of Secondary Forces of Multi-cylinder In-line Engines

When the connecting rod is not too long (*i.e.* when the obliquity of the connecting rod is considered), then the secondary disturbing force due to the reciprocating mass arises.

The secondary force,

$$F_S = m.\omega^2.r \times \frac{\cos 2\theta}{n}$$

Example 5. *A four cylinder vertical engine has cranks 150 mm long. The planes of rotation of the first, second and fourth cranks are 400 mm, 200 mm and 200 mm respectively from the third crank and their reciprocating masses are 50 kg, 60 kg and 50 kg respectively. Find the mass of the reciprocating parts for the third cylinder and the relative angular positions of the cranks in order that the engine may be in complete primary balance.*

Solution. Given $r_1 = r_2 = r_3 = r_4 = 150$ mm $= 0.15$ m ; $m_1 = 50$ kg ; $m_2 = 60$ kg ; $m_4 = 50$ kg

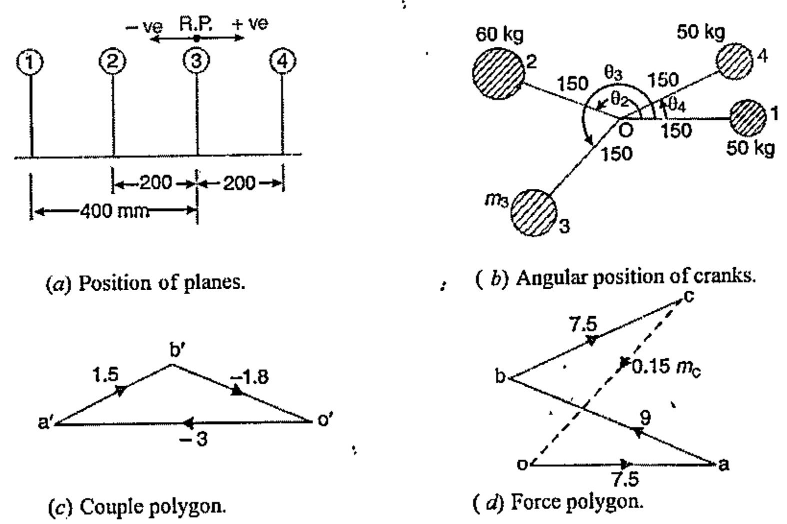
The position of planes is shown in Fig. 22.17 (*a*). Assuming the plane of third cylinder as the reference plane, the data may be tabulated as given in Table 22.8.

Table 22.8

Plane (*1*)	*Mass* (*m*) *kg* (*2*)	*Radius* (*r*) *m* (*3*)	*Cent. force* $\div \omega^2$ (*m.r*) *kg-m* (*4*)	*Distance from plane 3* (*l*) *m* (*5*)	*Couple* $\div \omega^2$ (*m.r.l*) *kg-m*^{2} (*6*)
1	50	0.15	7.5	– 0.4	– 3
2	60	0.15	9	– 0.2	– 1.8
3(*R.P.*)	m_3	0.15	$0.15m_3$	0	0
4	50	0.15	7.5	0.2	1.5

First of all, the angular position of cranks 2 and 4 are obtained by drawing the couple polygon from the data given in Table 22.8 (column 6). Assume the position of crank 1 in the horizontal direction as shown in Fig 22.17 (*b*), The couple polygon, as shown in Fig. 22.17 (*c*), is drawn as discussed below:

1. Draw vector $o'a'$ in the horizontal direction (*i.e.* parallel to $O1$) and equal to -3 kg-m^2, to some suitable scale.
2. From point o' and a', draw vectors $o'b'$ and $a'b'$ equal to -1.8 kg-m^2 and 1.5 kg-m^2 respectively. These vectors intersect at b'.



(*a*) Position of planes. (*b*) Angular position of cranks.

(*c*) Couple polygon. (*d*) Force polygon.

Fig. 22.17

3. Now in Fig. 22.17 (*b*), draw *O*2 parallel to vector $o'b'$ and *O*4 parallel to vector $a'b'$.

By measurement, we find that the angular position of crank 2 from crank 1 in the anticlockwise direction is

$$\theta_2 = 160° \text{ Ans.}$$

and the angular position of crank 4 from crank 1 in the anticlockwise direction is

$$\theta_4 = 26° \text{ Ans.}$$

In order to find the mass of the third cylinder (m_3) and its angular position, draw the force polygon, to some suitable scale, as shown in Fig. 22.17 (*d*), from the data given in Table 22.8 (column 4). Since the closing side of the force polygon (vector *co*) is proportional to $0.15\ m_3$, therefore by measurement,

$$0.15 m_3 = 9 \text{ kg-m} \quad \text{or} \quad m_3 = 60 \text{ kg Ans.}$$

Now draw O3 in Fig 22.17 (b), parallel to vector co. By measurement, we find that theangular position of crank 3 from crank 1 in the anticlockwise direction is θ3 = 227° Ans.

22.13. Balancing of V-engines

Consider a symmetrical two cylinder *V*-engine as shown in Fig. 22.33, The common crank *OC* is driven by two connecting rods *PC* and *QC*. The lines of stroke *OP* and *OQ* are inclined to the vertical *OY*, at an angle α as shown in Fig 22.33.

Let m = Mass of reciprocating parts per cylinder ,

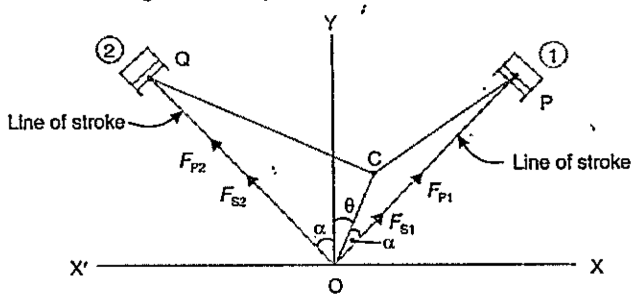
l = Length of connecting rod,

r = Radius of crank,

n = Ratio of length of connecting rod to crank radius = l / r

θ = Inclination of crank to the vertical at any instant,

ω = Angular velocity of crank.



FRICTION

Def :- When a block moves or tends to move tangentially w.r.t the surface on which it rests the inter locking property of the projecting particles opposes the motion. This opposing force is called force of friction.

Types of friction :

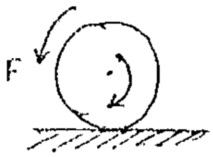
1. Static friction :

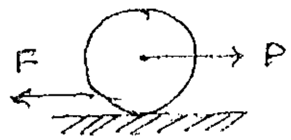
It is the friction experienced by a body when it is at rest.

2. Dynamic / Kinetic friction :

It is the friction experienced by a body when it is in motion.

(i) Rolling friction. (ii) Sliding friction.





Limiting friction :

The maximum value of the frictional force at which the body tends to produce motion over the surface of other body it is known as Limiting friction.

NOTE :

1. The static & Dynamic values are always in b/w 0 - Limiting friction (max.)
2. The force of friction is always independent of the area of contact b/w the two surfaces.
3. The force of friction depends on the roughness of the surface.
4. The force of friction always acts tangential to the contact surface and in a direction opp. to that in which the body

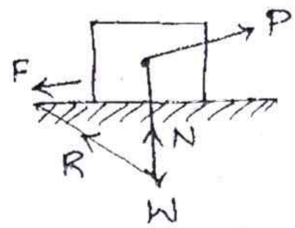
Normal Reaction :

The reaction offered by a contact surface against its body weight and act perpendicular to the plane is called Normal reaction.

Coefficient of friction :

The ratio of limiting friction to the ~~fr~~ normal reaction is constant and is called coeff. of friction.

i.e., $\boxed{\mu = F/N} = \tan\phi$



Here ϕ is called Angle of friction.

⇒ The angle of the inclined plane at which the body just begins to slide down the plane is called Angle of friction.

Application of Friction.

1. Ladder friction.
2. Wedge friction
3. Screw friction.

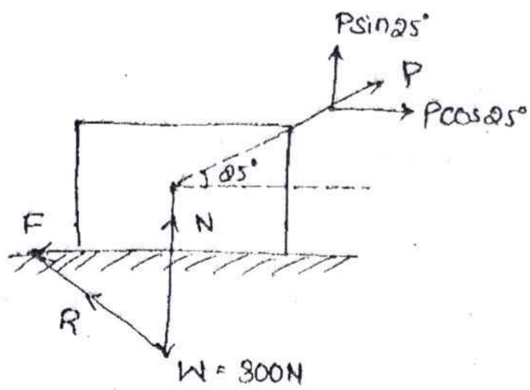
Problems.

1. A body of weight 300N is lying on a rough horizontal plane having a coeff. of friction as 0.3. Find the diameter, magnitude of the force which can move the body acting at an angle of 25° with the horizontal.

Sol: Given, $W = 300N$

Coeff. of friction $\mu = 0.3 = F/N = \tan\phi$

$\phi = 25°$



Let P be the magnitude of force which can move the body.

$$\mu = \frac{F}{N}$$

$$0.3 = \frac{F}{N}$$

$$\boxed{N = F/0.3}$$

Sum of all horizontal force :

$$\Sigma P_x = 0 \Rightarrow P\cos 25^\circ - F = 0$$

$$\Rightarrow F = P\cos 25^\circ \longrightarrow (1)$$

Sum of all vertical forces :

$$\Sigma P_y = 0 \Rightarrow P\sin 25^\circ - 300 + N = 0$$

$$\Rightarrow P\sin 25^\circ - 300 + \frac{F}{0.3} = 0$$

$$\Rightarrow P\sin 25^\circ - 300 + \frac{P\cos 25^\circ}{0.3} = 0$$

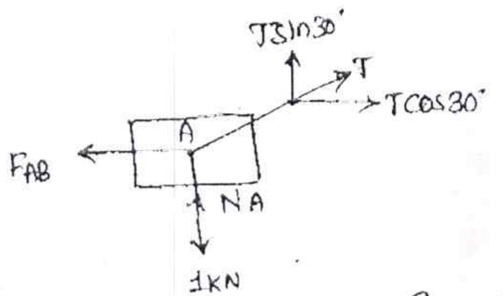
$$\Rightarrow P(\sin 25^\circ \times 0.3 + \cos 25^\circ) - 0.3 \times 300 = 0$$

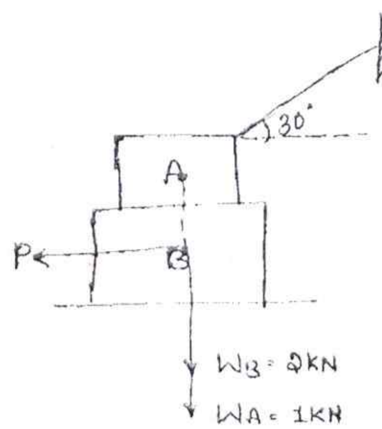
$$\Rightarrow P = \frac{0.3 \times 300}{\sin 25^\circ \times 0.3 + \cos 25^\circ}$$

$$\Rightarrow \boxed{P = 87.11 \text{ N}}$$

2. Two blocks A and B of weights 1KN and 2KN are in equilibrium positions as shown in fig. If the coeff. of friction b/w the two blocks as well as the block B and floor is 0.3. Find the force 'P' required to move the block 'P'.

Sol:- FBD of block A :





Given, $\mu = \frac{F}{N}$

Sum of horizontal forces :

$\Sigma P_x = 0 \Rightarrow T\cos 30° - F_{AB} = 0$

$\Rightarrow F_{AB} = T\cos 30°$

$\{\because F_{AB} = 0.295 \times \cos 30°$

$\boxed{F_{AB} = 0.255}$

Sum of vertical forces :

$\Sigma P_y = 0 \Rightarrow T\sin 30° + N_A - 1 = 0$

$\Rightarrow T\sin 30° + \frac{F_{AB}}{0.3} - 1 = 0$

$\Rightarrow T\sin 30° + \frac{T\cos 30°}{0.3} - 1 = 0$

$\Rightarrow T(0.3 \times \sin 30° + \cos 30°) - 1 \times 0.3 = 0$

$\Rightarrow T = \frac{0.3}{0.3 \times \sin 30° + \cos 30°}$

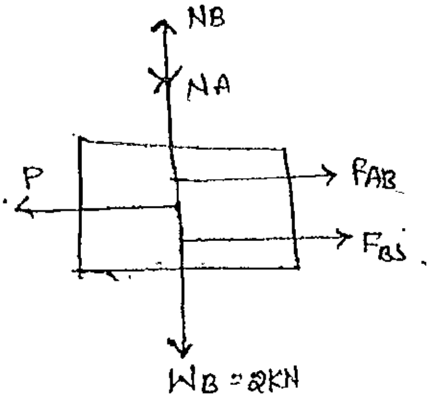
$\Rightarrow \boxed{T = 0.2952\,kN}$

$\Rightarrow \frac{F_{AB}}{N_A} = 0.3$

$N_A = \frac{0.255}{0.3}$

$\boxed{N_A = 0.852}$

FBD of block B :



Sum of horizontal forces :

$\Sigma P_x = 0 \Rightarrow F_{AB} + F_{BS} - P = 0 \longrightarrow (1)$

Sum of vertical forces :

$\Sigma P_y = 0 \Rightarrow N_B - N_A - 2 = 0$

$\Rightarrow N_B = 0.852 + 2$

$\Rightarrow \boxed{N_B = 2.85}$

$\mu_{BS} = \frac{F_{BS}}{N_B}$

$F_{BS} = 2.85 \times 0.3$

$\boxed{F_{BS} = 0.8556}$

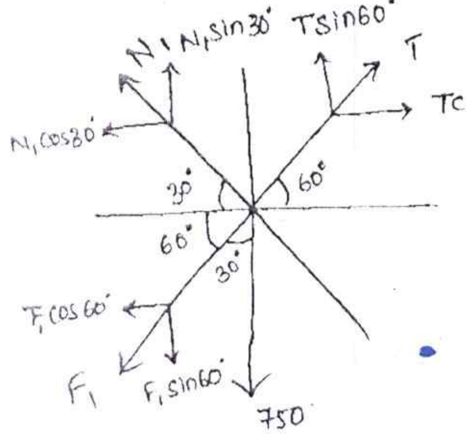
$(1) \Rightarrow 0.255 + 0.8556 = P$

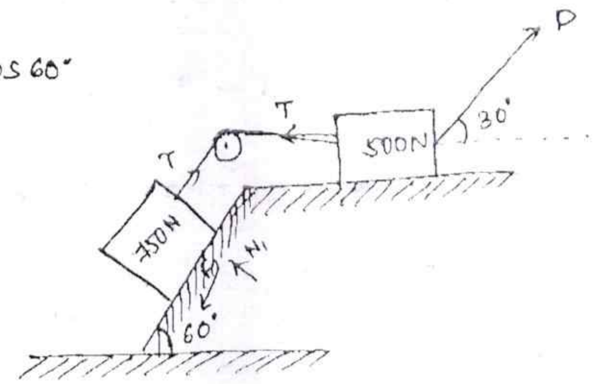
$\boxed{P = 1.1106\,KN}$

2/01/14

4. What is the value of 'P' in the system as shown in fig. to cause the motion of 500N block to the right side. Assume the pulley shown is smooth and the coeff. of friction b/w the other contact surfaces is 0.2.

Sol.





Given,

$\mu = 0.2$

$\frac{F}{N} = 0.2$

$F_1 = 0.2 N_1$

Sum of horizontal forces $\Sigma P_x = 0$

$T \cos 60° - N_1 \cos 30° - F_1 \cos 60° = 0 \rightarrow (1)$

Sum of vertical forces $\Sigma P_y = 0$

$T \sin 60° + N_1 \sin 30° - F_1 \sin 60° - 750 = 0 \rightarrow (2)$

$T \sin 60° + N_1 \sin 30° - 0.2 N_1 \sin 60° - 750 = 0$

$T \sin 60° - 750 + N_1 (\sin 30° - 0.2 \sin 60°) = 0 \rightarrow (3)$

$(1) \Rightarrow \quad T \cos 60° - N_1 \cos 30° - N_1 0.2 \cos 60° = 0$

$T \cos 60° - N_1 (\cos 30° + 0.2 \cos 60°) = 0$

$T \cos 60° = N_1 (\cos 30° + 0.2 \cos 60°)$

$$T = \frac{N_1 (\cos 30° + 0.2 \cos 60°)}{\cos 60°}$$

$③ \Rightarrow \quad 1.93 N_1 \sin 60° + N_1(\sin 30° - 0.2\sin 60°) - 750 = 0$

$N_1(1.93\sin 60° + \sin 30° - 0.2\sin 60°) = 750$

$\boxed{N_1 = 375.3}$

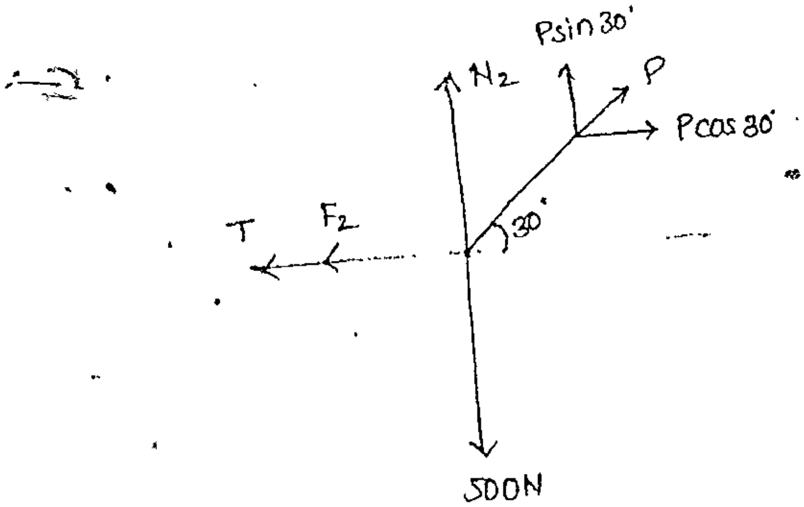
$④ \Rightarrow \quad T = 1.93 N_1$

$T = 1.93 \times 375.3$

$\boxed{T = 724.39}$

$F = 0.2 \times 375.3$

$\boxed{F_1 = 75.06}$



$F_2 = 0.2 N_2$

Sum of horizontal forces $\Sigma P_x = 0$

$P\cos 30° - T - F_2 = 0$

$P\cos 30° - 724.39 - F_2 = 0 \longrightarrow ①$

Sum of vertical forces $\Sigma P_y = 0$

$N_2 + P\sin 30° - 500 = 0 \longrightarrow ②$

$① \Rightarrow \quad P\cos 30° - 724.39 - 0.2 N_2 = 0$

$\dfrac{P\cos 30° - 724.39}{0.2} = N_2$

$② \Rightarrow \quad \dfrac{P\cos 30° - 724.39}{0.2} + P\sin 30° - 500 = 0$

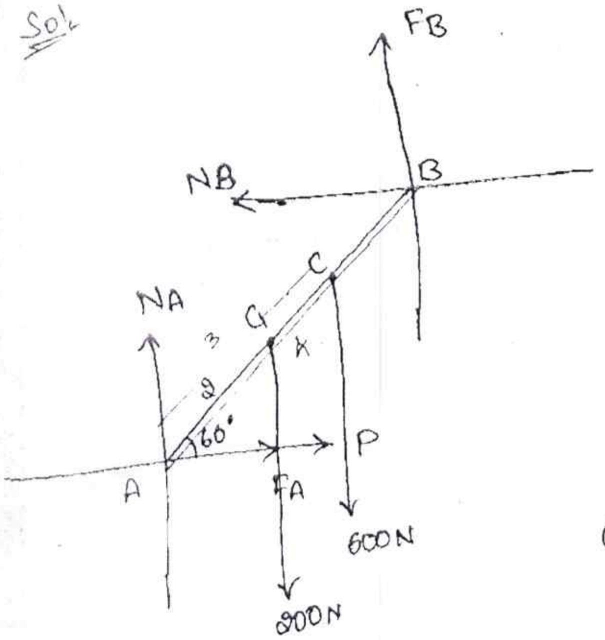
$P(\cos 30° + 0.2\sin 30°) - 724.39 - 0.2 \times 500 = 0$

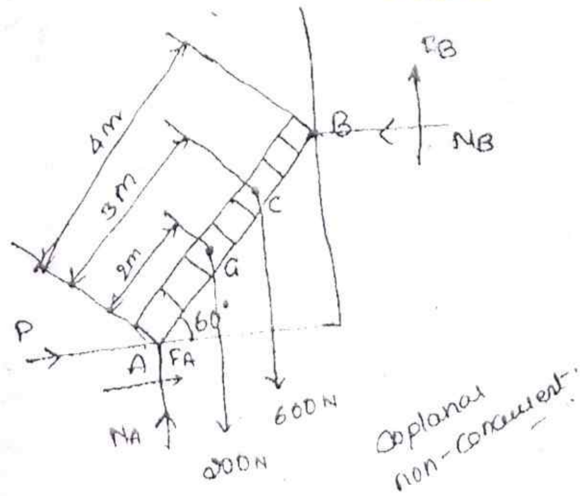
$P = \dfrac{724.39 + 0.2 \times 500}{}$

5. A ladder of length 4m weighing 200N is placed against a vertical wall as shown in the figure the coeff. of friction b/w the wall and the ladder is 0.2 and that b/w the floor and ladder is 0.3. In addition to self weight the ladder has to support a man weighing 600N at a distance of 3m from the 'A' total of the min horizontal force to be applied at 'A' to prevent slipping.

10M

Sol:





coplanar non-concurrent.

Given, $\mu_B = 0.2$ $\qquad$ $\mu_A = 0.3$

$$\frac{F_B}{N_B} = 0.2 \qquad \frac{F_A}{N_A} = 0.3$$

$$F_B = 0.2 N_B \qquad F_A = 0.3 N_A$$

$\Sigma P_x = 0 \Rightarrow$ ~~NA + FB~~

$\Sigma P_y = 0$

$$\Sigma M_A = 0 \Rightarrow \cancel{F_A} - 200 \times (2\cos 60°) - 600(3 \times \cos 60°) + F_B(4\cos 60°) + N_B(4\sin 60°) = 0$$

$$\Rightarrow -200 \times (2\cos 60°) - 600(3 \times \cos 60°) + 0.2 N_B(4\cos 60°) + N_B(4\sin 60°) = 0$$

$$\Rightarrow N_B = \frac{200 \times 2\cos 60 + 600(3\cos 60)}{0.2(4\cos 60) + 4\sin 60}$$

$\therefore F_B = 0.2 N_B$

$\boxed{F_B = 56.93\ N}$

$\Sigma P_y = 0 \Rightarrow F_B + N_A - 600 - 200 = 0$

$N_A = 600 + 200 - 56.93$

$\boxed{N_A = 743.07 N}$ $F_A = 0.3 \times 743.07$

$\boxed{F_A = 222.92 N}$

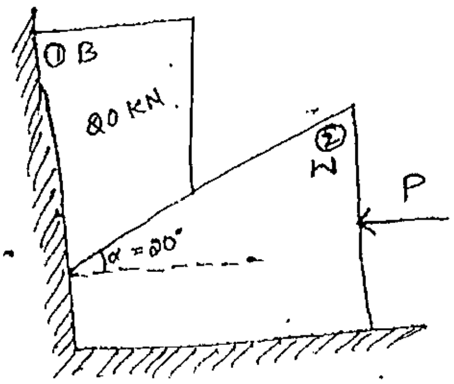
$\Sigma P_x = 0 \Rightarrow N_B - F_A - P = 0$

$P = 284.68 - 222.92$

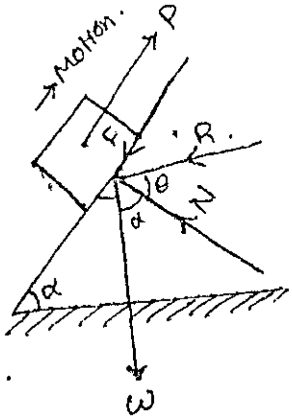
$\boxed{P = 61.75 N}$

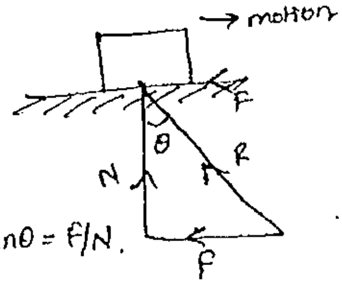
03/01/14.

Q. Determine the min force required to move the wedge. as shown in the figure and the angle of contact b/w the all the contanct surfaces is 15°.



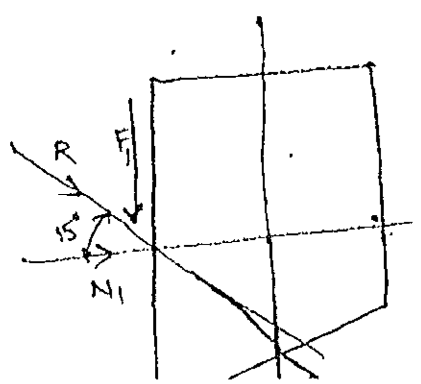
Sol. Given $\tan\theta = 15°$



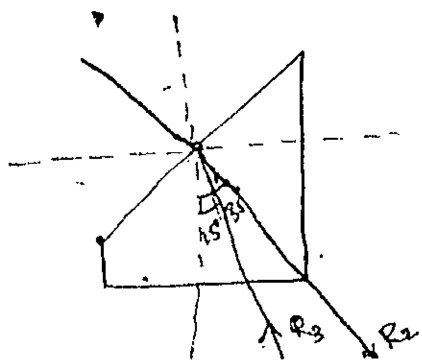


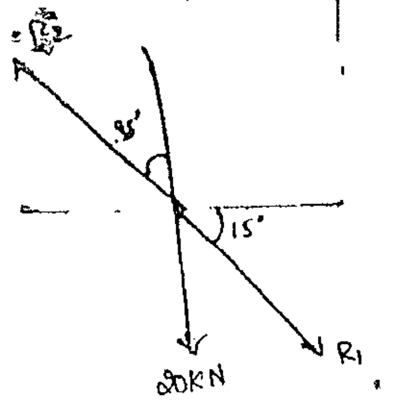
$\tan\theta = F/N$.

FBD of ① B.



FBD of ② W

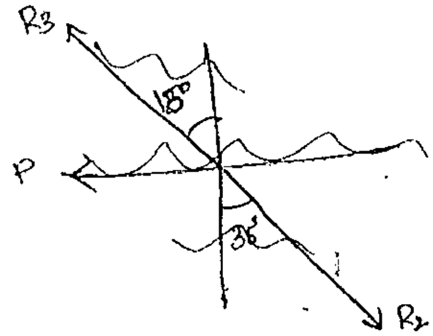




$$\frac{R_2}{\sin 75} = \frac{20}{\sin 140} = \frac{R_1}{\sin 145}$$

$R_1 = 17.85\,KN$

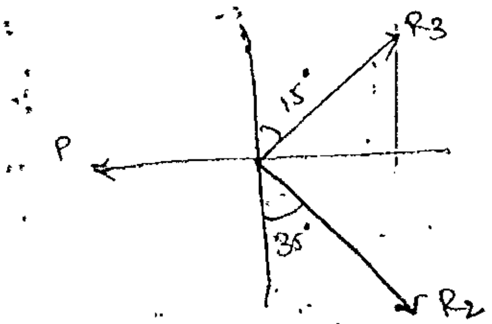
$R_2 = 30.05\,KN$



$$\cancel{\frac{P}{\sin 160} = \frac{R_2}{\sin 75} = \frac{R_3}{\sin 125}}$$

$\cancel{P = 10.64}$

$\cancel{R_3 = 25.48}$



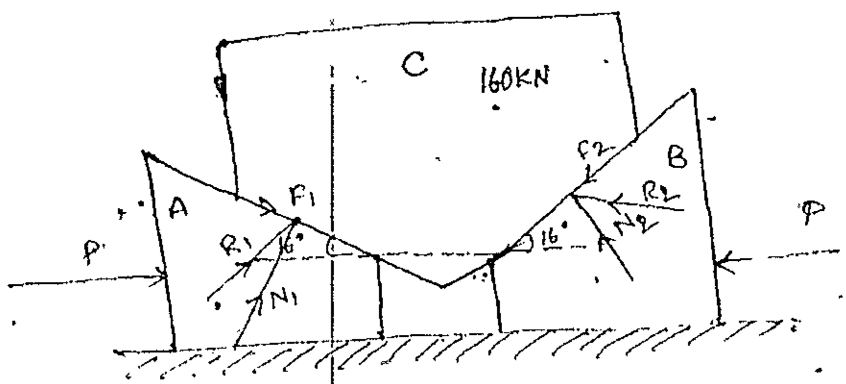
$$\frac{P}{\sin 130} = \frac{R_2}{\sin 105} = \frac{R_3}{\sin 125}.$$

$\boxed{P = 23.83}\,KN$.

$R_3 = 25.48$.

04/01/14

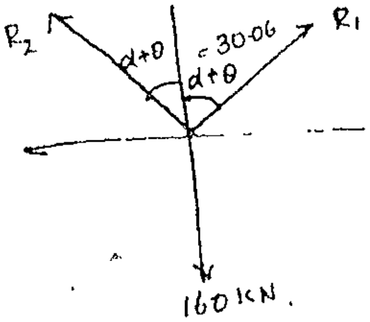
The block 'c' weighing 160 KN is to be raised by means driving wedges A and B as shown in the figure. Find the value of P for impending motion of the block upwards if the coeff. of friction is 0.25 for all contact surfaces and neglect self weight of wedges.



$\mu = 0.25 = \tan\theta$

$\theta = 14.036$

FBD of 'C'

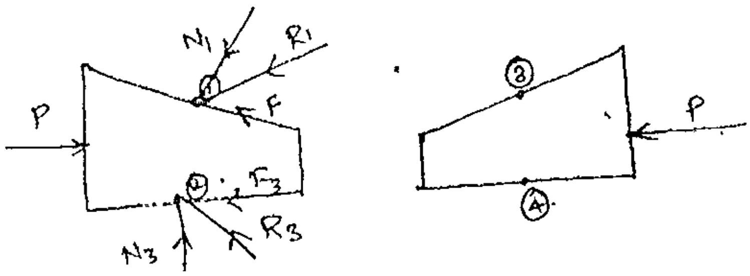


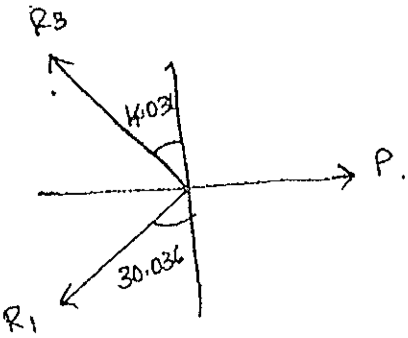
$$\frac{R_1}{\sin 149.96} = \frac{160}{\sin(60.072)} = \frac{R_2}{\sin 149.96}$$

$R_1 = 92.42$ KN. $\quad [\because R_1 = R_2]$

$R_2 = 92.42$ KN

30.036





$$\frac{P}{\sin 135.92} = \frac{R_3}{\sin 120} = \frac{R_1}{\sin 104.03}$$

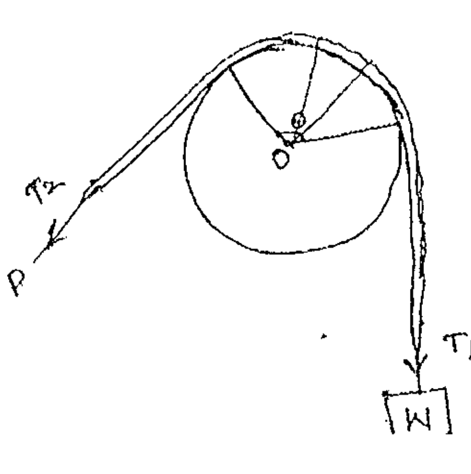
$P = 66.25$ KN.

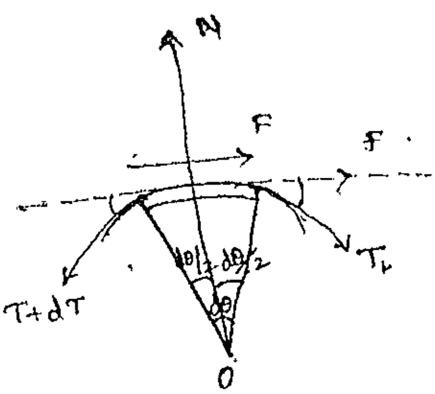
04/01/14

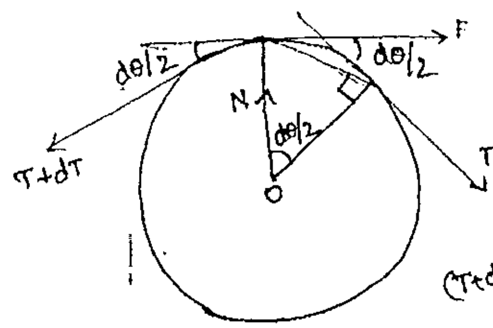
Rope Friction :

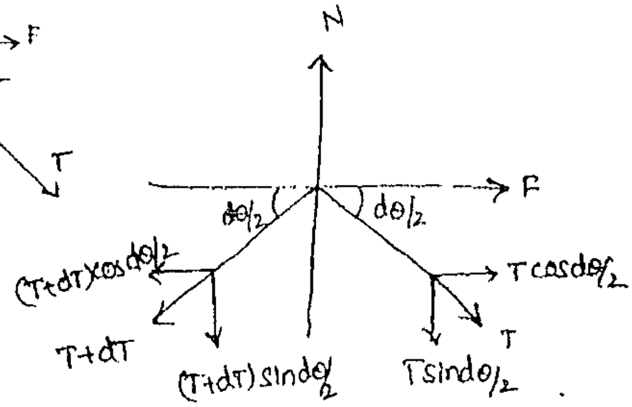
The transmission of power by means of rope & belts is possible becoz of friction.

- Relationship b/w force on tight side and slack side.









$\Sigma P_y = N - (T+dT)\sin d\theta/_2 - T\sin d\theta/_2 = 0.$

$\Rightarrow N - T\sin d\theta/_2 - dT\sin d\theta/_2 - T\sin d\theta/_2 = 0$

$N - 2T\sin d\theta/_2 - dT\sin d\theta/_2 = 0$

$N - \cancel{2}T.\, d\theta/_{\cancel{2}} - dT.\,\frac{d\theta}{2} = 0$ ($\because$ $\sin d\theta/_2$ is very small $\sin d\theta/_2 = d\theta/_2$)

$N - T.d\theta - dT.\frac{d\theta}{2} = 0.$

$N = Td\theta + dT.\frac{d\theta}{2}$

$N = Td\theta\left(1+\frac{d}{2}\right) \longrightarrow ①$

$\Sigma P_x = F - (T+dT)\cos d\theta/_2 + T\cos d\theta/_2 = 0$

$\Rightarrow F - T\cos d\theta/_2 - dT\cos d\theta/_2 + T\cos d\theta/_2 = 0$

$F - dT.\cos d\theta/_2 = 0$

$F = dT\cos d\theta/_2$ ($\because$ $\cos d\theta/_2$ is very small $\cos d\theta/_2 = 1$).

$F = dT \longrightarrow ②$

we have, $\mu = \frac{F}{N}$

$F = \mu N \longrightarrow ③$

$② \Rightarrow F = dT$

$\mu N = dT$

$\mu\left(1+\frac{d}{2}\right).Td\theta = dT$ ($\because$ from ①)

$\mu\, dT.\cancel{d\theta}^{0}$ dT

P

1.

Sol

$$\int_0^\theta \mu \cdot d\theta = \int_{T_1}^{T_2} \frac{dT}{T}$$

$$\mu(\theta)_0^\theta = \log(T_2/T_1)$$

$$\mu\theta = \log(T_2/T_1)$$

$$\log(T_2/T_1) = \mu\theta$$

$$\boxed{T_2/T_1 = e^{\mu\theta}}$$

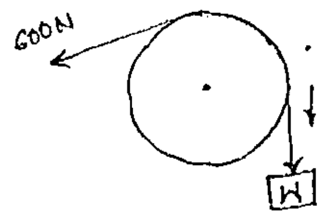
Problems

1. A rope making 1¼ terms around a stationary horizontal drum is used support a weight (W) if the coeff. of friction is 0.3. What range of weight can be supported by exerting at 600N force at the other end of the rope.

Sol:- Given,

$\mu = 0.3$

$1\frac{1}{4} = \frac{5}{4} \times 2\pi = 2.5\pi$



Case ①: Motion impending, weight to be down wards.

$T_1 = 600N \quad T_2 = W$

$$\frac{T_2}{T_1} = e^{\mu\theta}$$

$$\frac{W}{600} = e^{0.3 \times 2.5\pi}$$

$$W = 600 \times e^{0.3 \times 2.5\pi}$$

$$W = 6330.48\ N$$

Case ②: Motion of weight to be up wards.

$T_1 = \omega \ ; \ T_2 = 600$

$$\frac{T_2}{T_1} = e^{\mu\theta}$$

$con \quad \mu\theta \rightarrow \omega = \frac{600}{\ldots 5\pi}$

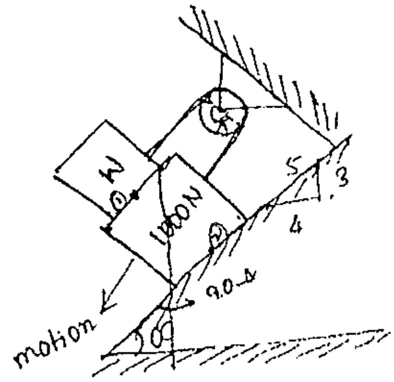
The coeff. of friction b/w the rope and fixed drum is 0.2 and b/w the other surfaces of contact is 0.3. Determine the min weight (W) to prevent downward motion of the 1000N block as shown in the figure.

Given:

$W = ?$

$\mu_d = 0.2$

$\mu_c = 0.3$

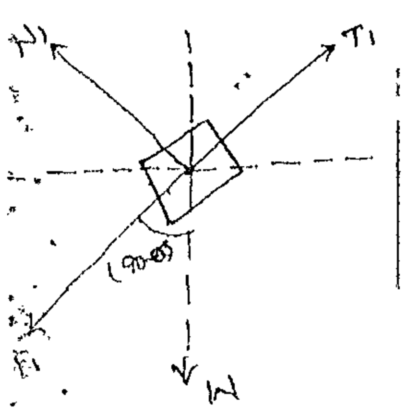


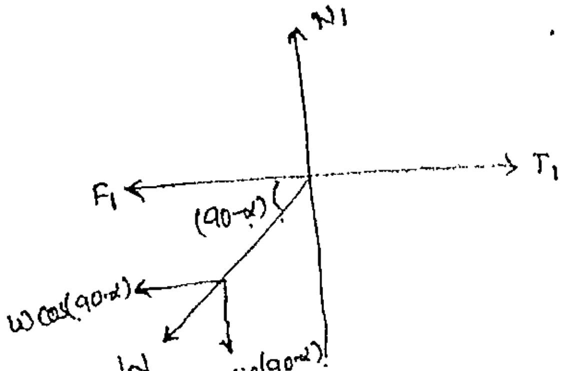
$\tan\alpha = \frac{3}{4}$

$\cos\alpha = \frac{4}{5}$

$\sin\alpha = 3/5$

FBD of block W:





$\Sigma P_x = F_1 - T_1 + w\cos(90-\alpha) = 0 \rightarrow (1)$

$\Sigma P_y = N_1 - w\sin(90-\alpha) = 0$

$N_1 = w\sin(90-\alpha) \rightarrow (2)$

$N_1 = w\cos\alpha$

we have, $F_1 = \mu_c N_1$

$(1) \Rightarrow \mu_c N_1 - T_1 + w\cos(90-\alpha) = 0$

$0.3\,w\cos\alpha - T_1 + w\sin\alpha = 0$

$0.3\,w\,(4/5) - T_1 + w\,(3/5) = 0$

$T_1 = w\left(0.3 \times \frac{4}{5} + \frac{3}{5}\right)$

$\boxed{T_1 = 0.84\,W} \rightarrow (3)$

$W(0.84)$

$W = 5$

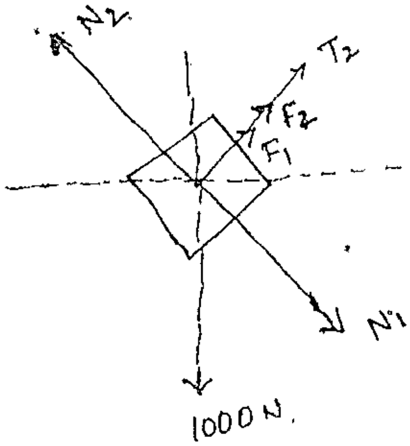
$W =$

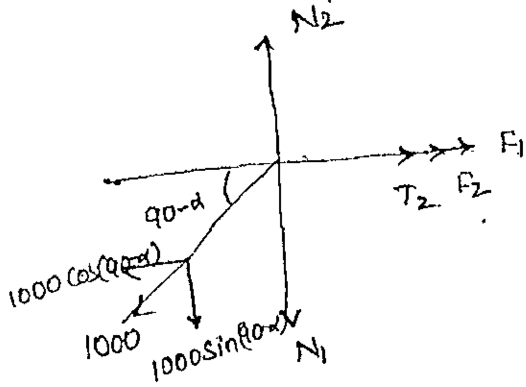
$$\frac{T_2}{T_1} = e^{\mu_d \theta}$$

$$T_2 = T_1 e^{\mu_d \theta}$$

$$T_2 = 0.84W \cdot e^{0.2\pi}$$

$$\boxed{T_2 = 1.57W} \longrightarrow (4)$$





$$\Sigma P_x = F_1 + F_2 + T_2 - 1000\cos(90-\alpha) = 0$$

$$\mu_c N_1 + \mu_c N_2 + 1.57 - 1000\sin\alpha = 0$$

$$\mu_c (N_1 + N_2) + 1.57 - 1000\sin\alpha = 0 \longrightarrow (1)$$

$$\Sigma P_y = N_2 - N_1 - 1000\sin(90-\alpha) = 0$$

$$N_2 - N_1 - 1000\cos\alpha = 0$$

$$N_2 = N_1 + 1000\cos\alpha \longrightarrow (2)$$

$$(1) \Rightarrow 0.3 \times W\cos\alpha + 0.2(N_1 + 1000\cos\alpha) + 1.57W - 1000\sin\alpha = 0$$

$$0.3 \times W\cos\alpha + 0.2(W\cos\alpha + 1000\cos\alpha) + 1.57W - 1000\sin\alpha = 0$$

$$0.3 \times W\left(\frac{4}{5}\right) + 0.2\left(W \cdot \frac{4}{5} + 1000 \cdot \frac{4}{5}\right) + 1.57W - 1000 \cdot \frac{3}{5} = 0$$

$$0.24W + 0.16W + 160 + 1.57 - 600 = 0$$

$$W(0.24 + 0.16) = 600 - 160 - 1.57$$

$$W = \frac{600 - 160 - 1.57}{0.24 + 0.16}$$

$W = 1004.02$ N

$W(0.24) + 240. + 598.43 = 0.$

$W = 598.43 - 240$

$W = 358.43$

$W(0.24 + 0.24 + 1.57) + 240 - 600 = 0.$

$$W = \frac{600 - 240}{0.05}$$

09/01/14

Lifting Machines

Mechanical Advantage :

It is a ratio of load lifted to the effort applied.

i.e., $MA = \frac{W}{P}$

Velocity Ratio :

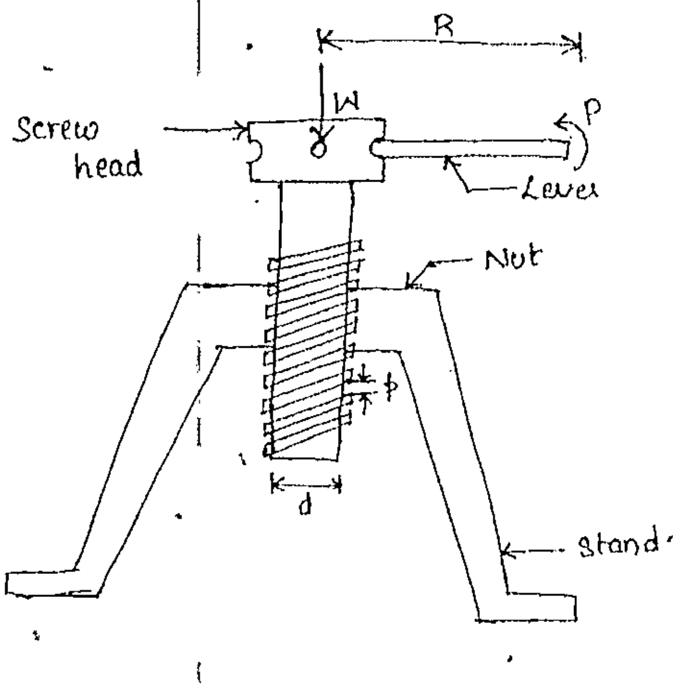
It is a ratio of distance moved by the effort to the distance moved by the load.

i.e., $VR = \frac{D}{d}$.

Efficiency : $\frac{\text{Output}}{\text{Input}} = \frac{Wd}{PD} = \frac{W/P}{D/d} = \frac{MA}{VR}$.

Torque required $(T) = P \times R$.

Screw Jack :

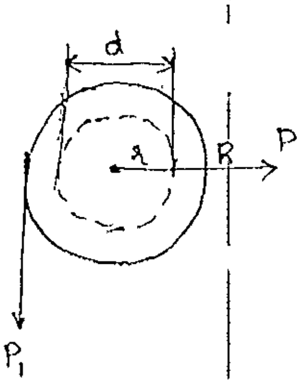


For single thread,

Lead Screw = p (Pitch of the Screw)

For double thread,

Lead Screw = 2p.



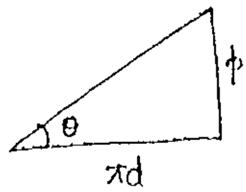
Here, P is the effort applied at the lever end and P_1 is the effort at the screw..

$PR = rP_1$

$P_1 = \frac{PR}{r} = \frac{2PR}{d}$.

Lead of the Screw :

One complete revolution of the screw is called Lead of the screw. The distance b/w the consecutive threads is called PITCH OF A SCREW THREAD.



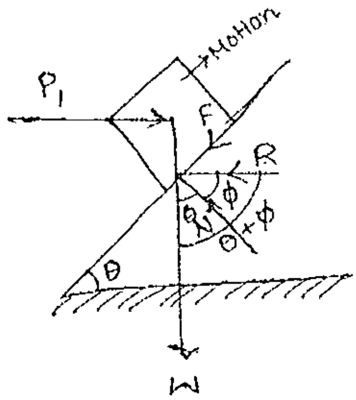
$$\tan\theta = \frac{p}{\pi d}$$

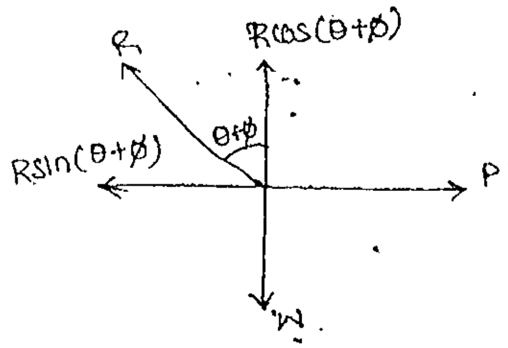
$$\theta = \tan^{-1}(p/\pi d)$$

Here, p = Pitch of the screw

θ = Angle of inclination of the inclined plane.

Load is Ascending :





$$P_1 = R\sin(\theta+\phi)$$

$$W = R\cos(\theta+\phi)$$

$$\frac{P_1}{W} = \tan(\theta+\phi)$$

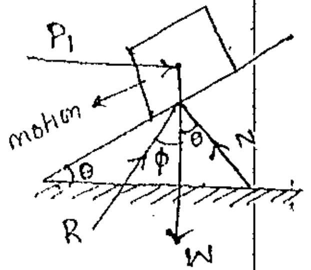
$$\frac{2PR}{Wd} = \tan(\theta+\phi) \qquad \left(\because\ P_1 = \frac{2PR}{d}\right)$$

$$\boxed{P = \frac{Wd}{2R}\tan(\theta+\phi)}$$

$$P = \frac{Wd}{2R}\left(\frac{\tan\theta+\tan\phi}{1-\tan\theta\cdot\tan\phi}\right)$$

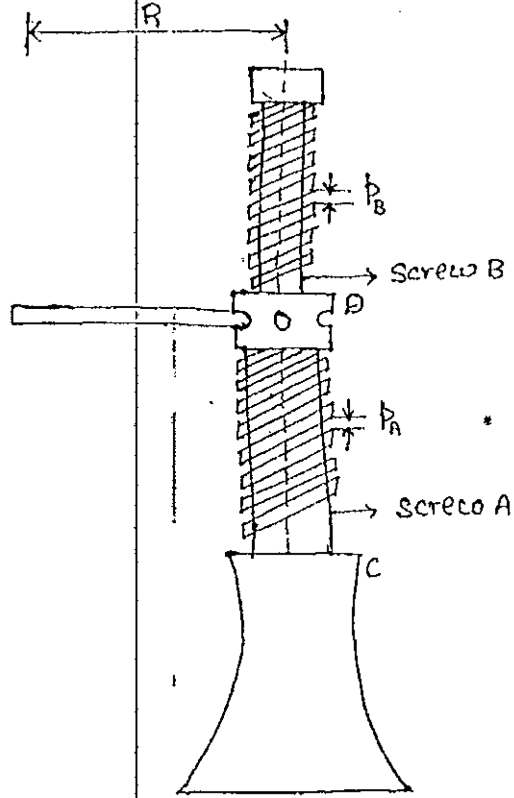
$$\boxed{P = \frac{Wd}{2R}\left(\frac{\tan\theta+\mu}{1-\mu\tan\theta}\right)}$$

Load is Descending :



$$P = \frac{Wd}{2R} \tan(\theta - \phi)$$

Differential Screw Jack (updated form)



The Net height through which the load is lifted $= P_A - P_B$

and $\quad VR = \dfrac{2\pi R}{P_A - P_B}$

Problems :

1. A screw Jack raises a load of 40KN, the screw is square threaded having 3 threads per 20mm length and 40mm in diameter. Calculate the force required at the end of the lever 400mm long measured from the axis of the screw. if the coeff. of friction b/w the screw and nut is 0.12.

lead = (n) pitch

Sol. Given, $W = 40KN$

$d = 40mm$

Lead of screw ... 20 ...

we have,

$$P = \frac{Wd}{2R}\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right);$$

$$= \frac{40\times 40}{2\times 400}\left(\frac{0.05+0.12}{1-0.12\times 0.05}\right)$$

$$= 0.348\,KN.$$

(or)

$$P = 348N$$ //

$\left\{\because \tan\theta = \frac{p}{\pi d}\right.$

$\theta = \tan^{-1}\left(\frac{p}{\pi d}\right)$

$\theta = \tan^{-1}\left(\frac{20}{3\times\pi\times 40}\right)$

$\left.\theta = 3.03\right\}$

$\tan\theta = 0.05$

2. A Screw Jack has a square threads 50mm mean diameter and 10mm pitch. The load on the jack revolves with the screw. The coeff. of friction at the screw thread is 0.05.

(i) Find the tangential force required at the end of 300mm lever to lift a load of 6000N.

(ii) State whether the jack is self locking. if not and find the torque which must be applied to keep the load from descending

Sol:- Given, $d = 50mm$

$p = 10mm$

$\mu = 0.05$

(i) $W = 6000N.$

$R = 300mm$

$P = ?$

we know,

$$P = \frac{Wd}{2R}\left(\frac{\tan\theta + \mu}{1-\mu\tan\theta}\right)$$

Here, $\tan\theta = \frac{p}{\pi d}$

$$\tan\theta = \frac{10}{\pi\times 50}$$

$$\tan\theta = 0.0637$$

Then, $$P = \frac{6000\times 50}{2\times 300}\left(\frac{0.0637+0.05}{1-0.05\times 0.0637}\right)$$

$$= \cancel{56.87N}.$$

$$= 57.03N$$

(ii) We have, Torque (T) = P×R.

As we know the formula for load is descending

i.e., $P = \frac{Wd}{2R} \tan(\theta - \phi)$

$PR = \frac{Wd}{2} \cancel{\tan(\theta - \phi)} \left(\frac{\tan\theta + \mu}{1 + \tan\theta \mu} \right)$

$T = \frac{6000 \times 50}{2} \cancel{\tan} \left(\frac{0.0637 - 0.05}{1 + 0.05 \times 0.0637} \right)$

$= \cancel{19488 N}$

$= 2048.4 N$

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Transmission of Power

Selection of belt drives:

The selection of belt drives as following factors:

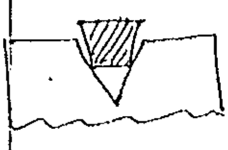
1. Speed of the driving and driven shafts (N)
2. Power to be transmitted (P)
3. The distance b/w the shafts (x) → +ve drive, no slipping.

Types of belt drives:

- Light → ≤ 10 m/s
- Medium → 10 to 22 m/s
- Heavy → > 22 m/s.

Types of Belts:

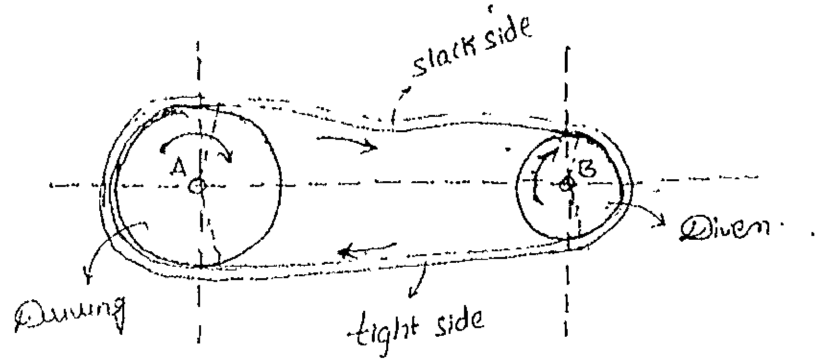
1. Flat belt :  $x < 8m$

2. V-belt :  x in small values

3. Circular/rope belts :  $x > 8m$

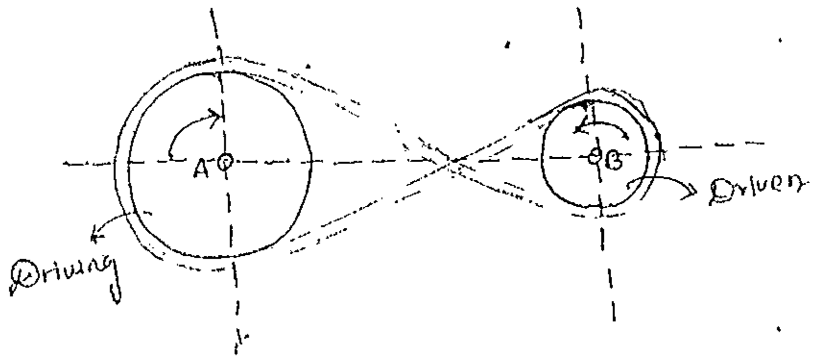
Types of Flat belt drives :

1. Open belt drive :



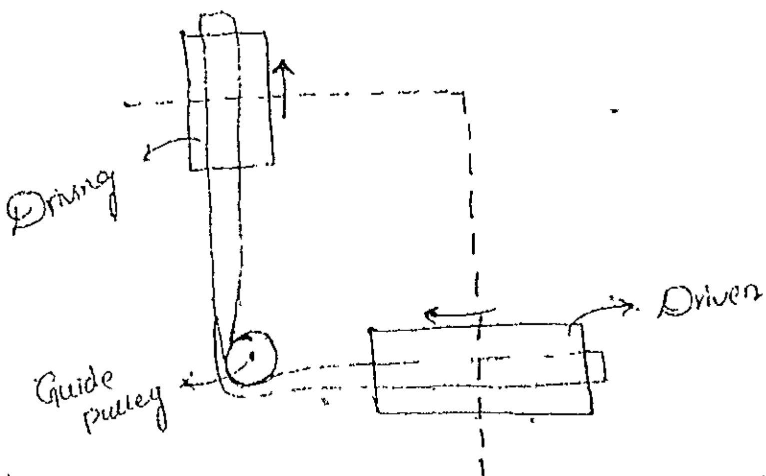
It is used when the shafts are parallel and rotating in the same direction.

2. Crossed / Twisted belt drive :

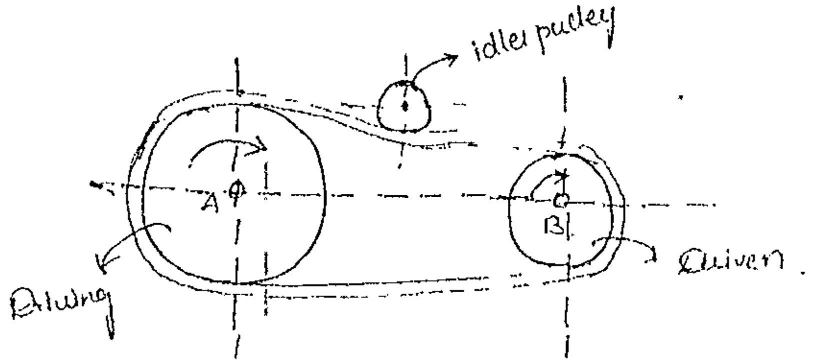


It is used when the shafts are parallel and rotating in the opposite direction.

3. Right angle / Quater turn belt drives.



4. Belt drive with Idler pulley :



It provides high velocity ratios and required belt tensions.

5. Compound belt drives :

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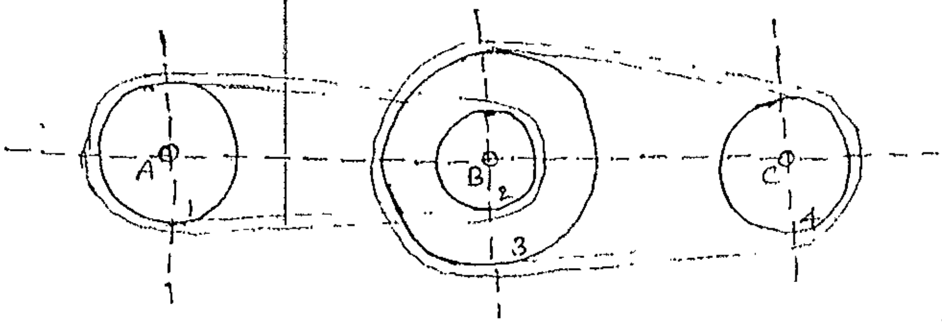
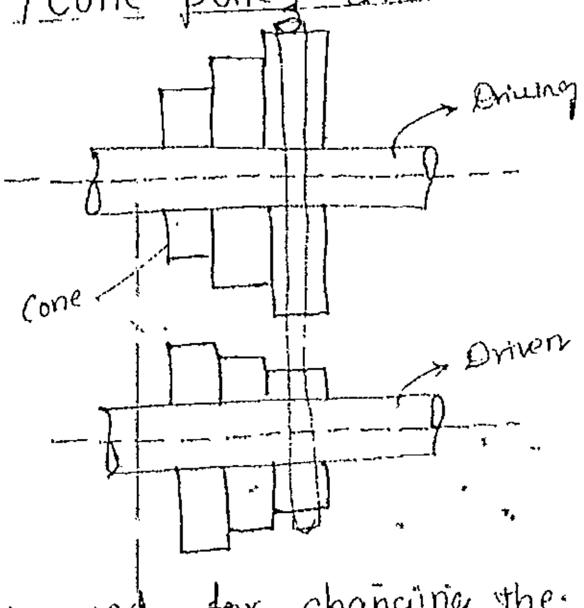


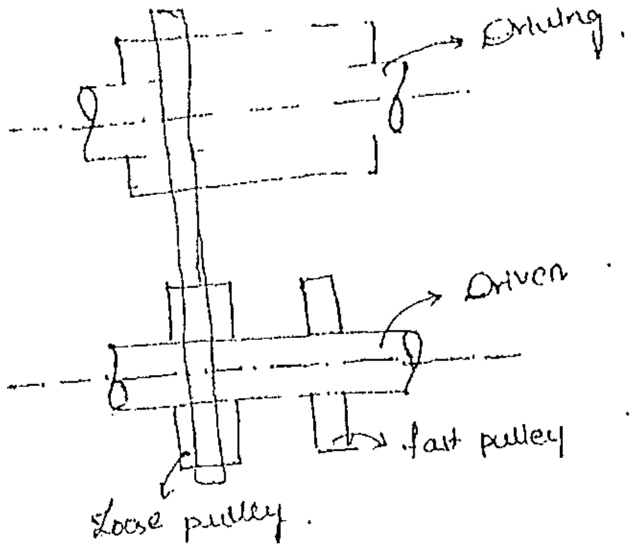
It is used when the power is transmitted from one shaft to another through a no. of pulleys.

6. Stepped / Cone pulley drives.



It is used for changing the speed of the given shafts

7. Fast or Loosed pulley drive :



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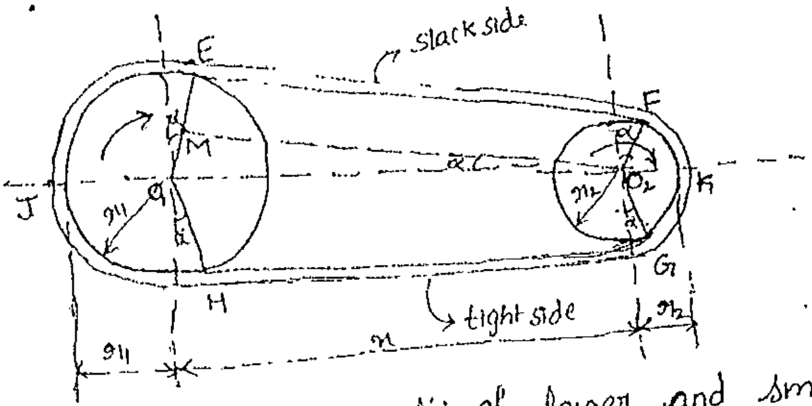
Expressions for the lengths of open & crossed belt drives

(i) Length of open belt drive :



∴ r_1 and r_2 are radii of larger and smaller pulleys.

Let x is the distance b/w the centers of two pulleys.

∴ The length of the belt L

$$L = Arc\ JHE + EF + Arc\ FKG + GH$$

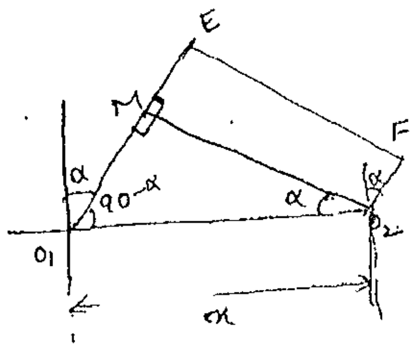
$$L = 2\left(\overset{Arc}{JE} + EF + \overset{Arc}{FK}\right)$$

$$Arc\ JE = r_1(\pi/2 + \alpha)$$

$$Arc\ FK = r_2(\pi/2 - \alpha)$$

$$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2}$$

$$EF = \sqrt{x^2 - (r_1 - r_2)^2}$$



In $\Delta^{le}\ O_1MO_2$

$$O_1O_2^2 = O_1M^2 + O_2M^2$$

$$EF = r\left[1-\left(\frac{r_1-r_2}{x}\right)^2\right]^{1/2}$$

$$EF = x\left[1-\frac{1}{2}\left(\frac{r_1-r_2}{x}\right)^2\right] \quad \{\because \text{from exponential fun.}\}$$

$$EF = \left[x - \frac{(r_1-r_2)^2}{2x}\right]$$

$\therefore$ Length of the belt is

$$L = 2\left[r_1(\pi/2+\alpha) + \left(x - \frac{(r_1+r_2)^2}{2x}\right) + r_2(\pi/2-\alpha)\right]$$

$$= 2\left[\frac{r_1\pi}{2} + r_1\alpha + x - \frac{(r_1+r_2)^2}{2x} + \frac{r_2\pi}{2} - r_2\alpha\right]$$

$$= r_1\pi + 2r_1\alpha + 2x - \frac{(r_1+r_2)^2}{x} + r_2\pi - 2r_2\alpha$$

$$= \pi(r_1+r_2) + 2\alpha(r_1-r_2) + 2x - \frac{(r_1+r_2)^2}{x}$$

$$= \pi(r_1+r_2) + 2\frac{(r_1-r_2)^2}{x} + 2x - \frac{(r_1-r_2)^2}{x}$$

$\left[\because \text{from fig: } \Delta^{le}\, O_1MO_2 \quad \sin\alpha = \frac{r_1-r_2}{x} = \alpha\right]$

$$\boxed{\text{Length } L = \pi(r_1+r_2) + 2x + \frac{(r_1-r_2)^2}{x}}$$

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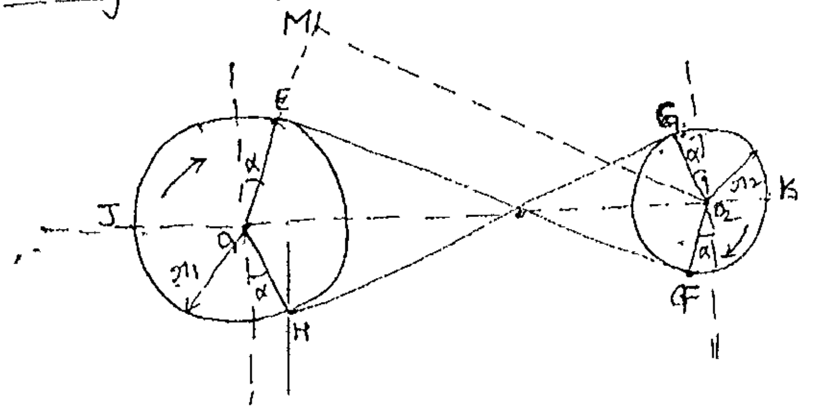
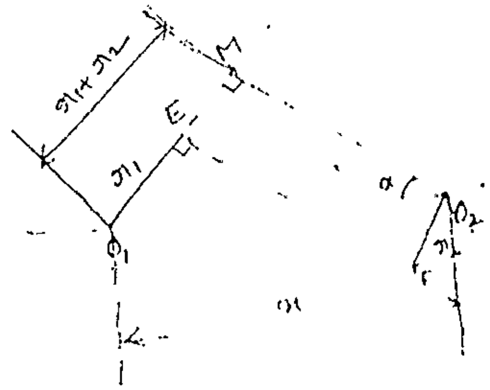
(ii) Length of Cross belt drive:



Length of the belt $L = 2\left[\text{Arc } JE + EF + \text{Arc } FK\right]$

$$\text{Arc } JE = r_1(\pi/2+\alpha)$$



$$EF = MO_2 = \sqrt{O_1O_2^2 - MO_1^2}$$

$$EF = \sqrt{x^2 - (r_1+r_2)^2}$$

$$EF = \left[x - \frac{(r_1+r_2)^2}{2x}\right]$$

In $\triangle^{le}$ O_1MO_2,

$$O_1O_2^2 = MO_2^2 + MO_1^2$$

$$MO_2^2 = \sqrt{O_1O_2^2 - MO_1^2}$$

from $\triangle^{le}$ O_1MO_2

$$\sin\alpha = \frac{r_1+r_2}{x} = \alpha$$

Length of the belt is

$$L = 2\left[r_1(\pi/2+\alpha) + \left(x - \frac{(r_1+r_2)^2}{2x}\right) + r_2(\pi/2+\alpha)\right]$$

$$= 2\left[\frac{r_1\pi}{2} + r_1\alpha + x - \frac{(r_1+r_2)^2}{2x} + \frac{r_2\pi}{2} + r_2\alpha\right]$$

$$= r_1\pi + \frac{2r_1\alpha}{\cancel{2}} + 2x - \frac{(r_1+r_2)^2}{x} + r_2\pi + 2r_2\alpha$$

$$= \pi(r_1+r_2) + 2\alpha(r_1+r_2) + 2x - \frac{(r_1+r_2)^2}{x}$$

$$= \pi(r_1+r_2) + 2\cdot\frac{(r_1+r_2)^2}{x} + 2x - \frac{(r_1+r_2)^2}{x}$$

$$= \pi(r_1+r_2) + 2x + \frac{(r_1+r_2)^2}{x}$$

$$\boxed{\therefore \text{Length } L = \pi(r_1+r_2) + 2x + \frac{(r_1+r_2)^2}{x}}$$

08/01/14

Important Formulas

1. Velocity ratio :

Let V_1 is the velocity of driving pulley $V_1 = \frac{\pi D_1 N_1}{60}$.

Let V_2 is the velocity of driven/follower pulley. $V_2 = \frac{\pi D_2 N_2}{60}$.

If $V_1 = V_2$.

$$D_1N_1 = D_2N_2$$

$$\frac{D_1}{D_2} = \frac{N_2}{N_1} \text{ \{velocity ratio\}}$$

Velocity ratio of compound belt drive :

If pulleys = 6 then, $\frac{N_6}{N_1} = \frac{d_1 d_3 d_5}{d_2 d_4 d_6}$

Speed of last driven pulley. Product of diameter of driving pu

2. <u>Slip of Belt</u> :

Let S_1 = % of slip b/w driving and the belt .

S_2 = % of slip b/w belt and the driven/follower

$$\therefore \quad \frac{N_2}{N_1} = \frac{d_1+t}{d_2+t}\left[1-\frac{S_1-S_2}{100}\right]$$

$$S_1-S_1 = S \text{ (Total \% of slip)}$$

3. <u>Creep of Belt</u> :

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left[\frac{E+\sqrt{\sigma_2}}{E+\sqrt{\sigma_1}}\right]$$

where σ_1 and σ_2 are stresses in the belt on tight and slack sides.

4. <u>Power Transmitted</u> :

Let T_1 and T_2 are tensions in the tight and slack sides of the belt.

Effective turning force $F = (T_1 - T_2)$

Work done / second $W = F \times \frac{S}{t} = FV$

$= (T_1 - T_2) V$ N-m/s

Power transmission = Work done / second

$= (T_1 - T_2) V$ watt.

5. <u>Tight side & Slack side tension</u> :

$$\frac{\{T\cdot 5\}}{\{5\cdot 5\}} \quad \frac{T_1}{T_2} = e^{\mu\theta}$$

where θ is the Angle of Contact / Lap

$\theta = 180 - 2\alpha$ (open Belt)

$180 + 2\alpha$ (Cross Belt).

6. Centrifugal Tension :

$$T_c = mv^2.$$

where m is the mass of the Belt / unit length.

v is the velocity of Belt.

∴ The total tension in the tight side $T_t = T_1 + T_c$.

the total tension in the slack side $T_s = T_2 + T_c$.

and the maximum tension $T_{max} = T_1 + T_c = \sigma bt$.

where σ is the stress induced in the belt.

b is the width of the belt

t is the thickness of the belt.

Conditions for transmission of Max. Power.

$$\boxed{T = 3T_c} \quad (8)$$

$$T_1 + T_c = 3T_c$$

$$\boxed{T_1 = 2T_c = 2/3\,T} \quad (8).$$

$$T = 3T_c = 3mv^2.$$

$$\boxed{v = \sqrt{T/3m}}$$

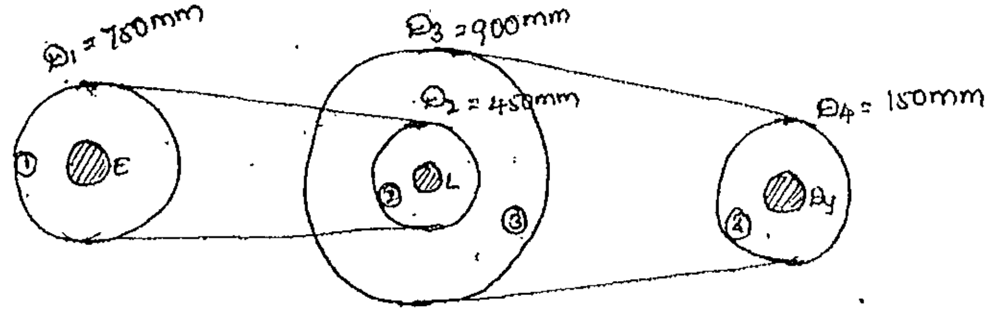
7. Initial tension in the Belt.

$$T_0 = \frac{T_1 + T_2 + 2T_c}{2}.$$

→ An engine running at 150 rpm drives a line shaft by means of a belt the engine pulley is 750mm diameter and pulley on the line shaft is ~~40~~ 450mm. A 900mm diameter pulley on the line shaft drives a 150mm diameter pulley keyed to a dynamo shaft. Find the speed of the dynamo shaft when (i) There is no slip

(ii) There is a slip of 2% at each belt drive.

1:-



(i) $\frac{N_4}{N_1} = \frac{D_1 D_3}{D_2 D_4} = \frac{750 \times 900}{450 \times 150} = 1500 rpm.$

(ii) $\frac{N_4}{N_1} = \frac{D_1 D_3}{D_2 D_4}\left(1 - \frac{S_1}{100}\right)\left(1 - \frac{S_2}{100}\right)$

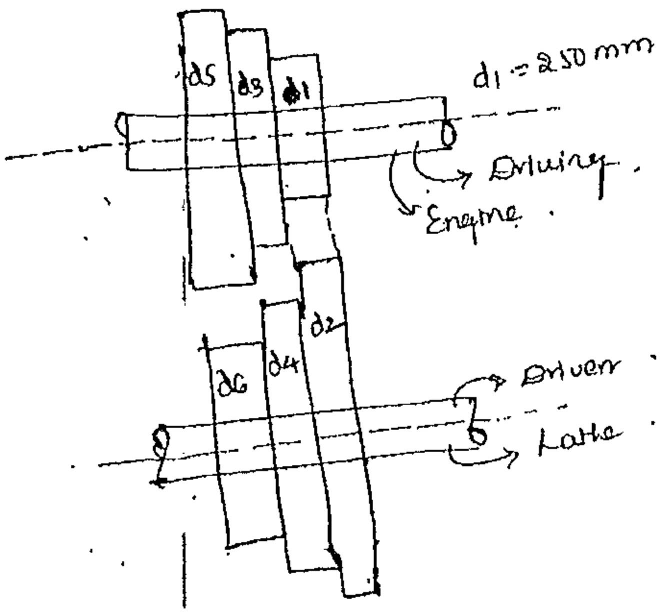
$= 1500\left(1 - \frac{2}{100}\right)\left(1 - \frac{2}{100}\right)$

$= 1440 \text{ rpm}.$

1/14

2. A shaft running at 250 rpm drives a lathe spindle has to run at 100, 125 and 150 rpm. The min. diameter of wheel should be ~~250mm~~ 240mm. Determine the diameters of the other wheels in the system.

Sol:-



$N_1 = 250$ rpm
$N_3 = 250$ rpm
$N_5 = 250$ rpm

$N_2 = 100$ rpm
$N_4 = 125$ rpm
$N_6 = 150$ rpm

$\frac{N_2}{N_1} = \frac{d_1}{d_2} \Rightarrow d_2 = \frac{d_1 \times N_1}{N_2} = \frac{250 \times 240}{\cancel{250}\ 100} = 600 mm$

$\frac{N_4}{N_3} = \frac{d_3}{d_4} \Rightarrow \frac{d_3}{d_4} = \frac{125}{250} \Rightarrow d_4 = 2d_3$

$\frac{N_6}{N_5} = \frac{d_5}{d_6} \Rightarrow \frac{d_5}{d_6} = \frac{150}{250} \Rightarrow d_6 = \frac{5}{3} d_5.$

we know,

$$d_1+d_2 = d_3+d_4 = d_5+d_6 = 840.$$

$$\rightarrow d_3 + 2d_3 = 840$$

$$3d_3 = 840$$

$$d_3 = 840/3 \Rightarrow$$

$$\rightarrow d_5 + \frac{5}{3}d_5 = 840.$$

$$8d_5 = 3\times 840$$

$$d_5 = 315, \quad d_6 = 525$$

→ A shaft running at 200 rpm drives a lathes spindel which is at a distance of 2.5m. The lathe's spindel has to run at 80 rpm. The diameter of the wheel on driver shaft is 240mm and the drive is by cross belt. Determine the length of the belt and assume no slip.

Sol:-

$N_1 = 200$ rpm

$N_2 = 80$ rpm

$d_1 = 240$mm

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}$$

$$\cancel{d_1 = \frac{N_2 \times d_2}{N_1}} = \cancel{\frac{80\times 240}{200}} = 96$$

$$d_2 = \frac{d_1 \times N_1}{N_2} = \frac{240\times 200}{80} = 600\text{ mm}$$

Given, $x = 2.5m = 2500$ mm

$$\text{Length } L = \pi(r_1+r_2) + 2x + \frac{(r_1+r_2)^2}{x}$$

$$= \pi\left(\frac{240}{2} + \frac{600}{2}\right) + 2(2500) + \frac{\left(\frac{240}{2}+\frac{600}{2}\right)^2}{2500}\text{ mm}$$

$$= 6.39m$$

→ An open belt drive 150mm wide and 8mm thick connects two wheels mounted on parallel shafts 3.5m apart. The diameter of the larger pulley is 480mm and that of the smaller is 320mm. if the permissible ... wheel is rotating at 800 rpm ...

(i) Neglect centrifugal tension.

(ii) Consider centrifugal tension if the density of lathers is 1100 kg/m³

Take $\mu = 0.3$

Sol Given data,

$b = 150mm$

$t = 8mm$

$x = 2.5m = 2500mm$

$D_1 = 480mm$

$D_2 = 320mm$

$N_1 = 800$

$\sigma = 3N/mm^2$

$P = ?$

(i) T_c = neglected.

(ii) T_c = considered

$\rho = 110kg/m^3$.

$\mu = 0.3$.

(i) T_c = neglected.

we know, $P = (T_1 - T_2)V$

$V = \frac{\pi D_1 N_1}{60} = \frac{\pi D_2 N_2}{60}$.

$V = \frac{\pi \times 480 \times 800}{60} = 20106 \ mm/sec$.

we know, $\frac{T_2}{T_1} = e^{\mu\theta}$

T_1 is slack side tension

T_2 is tight side tension.

$\theta = (180 - 2\alpha)^\circ$

$\theta = \left[\pi - 2\left(\frac{r_1 - r_2}{x}\right)\right]^\circ$

$\theta = 3.078^\circ$

$T_1 = T_2/(e^{\mu\theta})$

$= T_2 / \cancel{(6.43)}\ 2.517$

$T_2 = \sigma \cdot bt$

$= 3 \times 150 \times 8$

$= 3600N$.

$P = 2 (1430 - 3600)\, 20106$

$P = 4363 \times 10^6$ N-mm/sec

$P = 4363 \times 10^4$ N-m/sec

$P = 4363$ K watt

(ii) Consider centrifugal tension

$T_{max} = T_2 + T_c = \sigma \cdot bt$

$= 3600 N$

$\cancel{T_c = 1100 \times 10^{-9} (150 \times 8 \times 100) \times (20106)^2}$

$T_c = 1100\ m v^2$

$= (\rho v) v^2$

$= \rho (b \times t \times 1) v^2$

$= 1100 \times (0.15 \times 0.08 \times 1) \times (20.106)^2$

$= 533.62$ N.

$T_2 + T_c = 3600$

$T_2 = 3600 - T_c$.

$T_2 = 3066.38$ N.

$T_1 = \dfrac{T_2}{2.517} = 1218$ N.

$\therefore P = (T_2 - T_1) v$

$= (3066.38 - 1218) \times 20.106$.

$= 37166$ watt

$= 37.166$ K watt.

→ Coplanar Concurrent : $R = \sqrt{\Sigma P_x^2 + \Sigma P_y^2}$ $\alpha = \tan^{-1}(\Sigma P_y / \Sigma P_x)$

→ Coplanar Non-Concurrent : $\Sigma M_0 = R r \Rightarrow \Sigma M_0 = P_1 r_1 + P_2 r_2 + P_3 r_3$

$$r = \frac{\Sigma M_0}{R}, \quad y = \frac{\Sigma M_0}{\Sigma P_x}, \quad \frac{\Sigma M_0}{\Sigma P_y}$$

→ Non.coplanar Concurrent : $R = \sqrt{\Sigma P_x^2 + \Sigma P_y^2 + \Sigma P_z^2}$

$$P_x = P\cos\theta_x \left\{ \cos\theta_x = \frac{x_j - x_i}{l} \right\}$$

$$l = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}$$

$$\alpha_x = \cos^{-1}\left(\frac{\Sigma P_x}{R}\right)$$
$$\alpha_y = \cos^{-1}\left(\frac{\Sigma P_y}{R}\right) \qquad \text{|||}^{ly} \quad P_y = P\cos\theta_y$$
$$\alpha_z = \cos^{-1}\left(\frac{\Sigma P_z}{R}\right) \qquad P_z = P\cos\theta_z$$

→ Lami's Theorem : $\frac{P_1}{\sin\alpha} = \frac{P_2}{\sin\beta} = \frac{P_3}{\sin\gamma}$

→ $\mu = \frac{F}{N}$, $\tan\phi = \frac{F}{N} \Rightarrow \phi = \tan^{-1}(F/N)$

$\phi = \tan^{-1}(\mu)$

→ $T_2 / T_1 = e^{\mu\theta}$

→ Mechanical Advantage $MA = \frac{W}{P}$ (W → load lifted, P → effort applied)

→ Velocity ratio $VR = \frac{D}{d}$ (D → distance moved by effort, d → by load)

efficiency $= \frac{\text{out/p}}{\text{I/P}} = \frac{Wd}{PD}$

→ Torque = $P \times R$

→ $PR = rP_1$ where P is the effort applied of the lever end

P_1 is the effort at screw.

$$P_1 = \frac{PR}{r} = \frac{2Pr}{d}$$

→ $\tan\theta = \frac{p}{\pi d}$ where, p is the pitch of the screw

θ is the angle of inclination.

$$\theta = \tan^{-1}(p/\pi d)$$

→ $P = \frac{Wd}{2R}\tan(\theta + \phi)$ {Load is Ascending}

$$P = \frac{Wd}{2R}\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)$$

$P = \frac{Wd}{2R}\tan(\theta - \phi)$ {Load is Descending}

→ In a Screw Jack $VR = \frac{2\pi R}{P}$

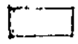
→ Differential Screw Jack $VR = \frac{2\pi R}{P_A - P_B}$

→ $L = \pi(r_1+r_2) + 2x + \frac{(r_1-r_2)^2}{x}$ {for (open belt drive)}

$L = \pi(r_1+r_2) + 2x + \frac{(r_1+r_2)^2}{x}$ {for (cross belt drive)}

→ $x_c = \frac{\int x\,dA}{A}$, $y_c = \frac{\int y\,dA}{A}$

$x_c = \frac{\Sigma ax}{A}$, $y_c = \frac{\Sigma ay}{A}$

→  → $(x_c, y_c) = \frac{b}{2}, \frac{b}{2}$

 → $(x_c, y_c) = \frac{h}{3}, \frac{2h}{3}$

 → " $= 0, \frac{4R}{3\pi}$

 → " $= \frac{4R}{3\pi}, \frac{4R}{3\pi}$

 → " $= \frac{3a}{4}, \frac{3h}{10}$

→ $I_{xx} = \int y^2 dA$

$I_{yy} = \int x^2 dA$ $\qquad I_{AB} = \int R^2 dA$

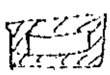
$I_{zz} = \int r^2 dA$

→ ⊥ar axis theorem : $I_{zz} = I_{xx} + I_{yy}$

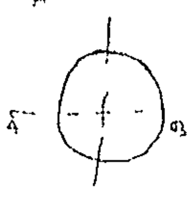
// axis theorem : $I_{AB} = I_{CG} + A y_c^2$

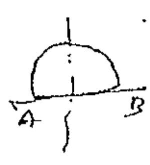
 M.I about centroidal axis

$I_{xG} = \frac{bd^3}{12}$, $I_{yG} = \frac{db^3}{12}$, $I_{AB} = \frac{bd^3}{3}$

 $I_{xG} = \frac{BD^3 - bd^3}{12}$

 $I_{xG} = \frac{bh^3}{36}$ $\quad I_{AB} = \frac{bh^3}{12}$

 $I_{xG} = I_{AB} = \frac{\pi r^4}{4}$

 $I_{AB} = \frac{1}{2}\left(\frac{\pi r^4}{4}\right)$

→ In I - section,

$I_x = \Sigma a y^2 + I_x \text{ self} \rightarrow I_{xx} = I_{CA} = I_x - \Sigma a (y_c^2)$

$I_y = \Sigma a x^2 + I_y \text{ self} \quad I_{yy} = I_y - \Sigma a (x_c^2)$

→ ▭ - $I_{xy} = \frac{(bd)^2}{4}$, $I_{x'y'} = 0$ c.A

△ - $I_{xy} = \frac{(bh)^2}{24}$ about base axis , $I_{x'y'} = \frac{-(bh)^2}{72}$ about centroidal axis

$y = (u\sin\alpha)t - \frac{1}{2}gt^2$

$x = (u\cos\alpha)t$

$h = \frac{1}{2}gt^2$

$R = ut$

$h_{max} = \frac{u^2\sin^2\alpha}{2g}$

$t = \frac{u\sin\alpha}{g}$

$T = \frac{2u\sin\alpha}{g}$

$R_{max} = \frac{u^2}{g} = \frac{u^2\sin 2\alpha}{g}$

BRAKES

→ A brake is a device by means of which artificial frictional resistance is applied to a moving ~~machine~~ machine member, in order to retard (or) stop the motion of a ~~machine~~ machine.

→ The energy absorbed by brakes is dissipated ~~by~~ in the form of heat.

Characteristics of materials used for brake lining

factors

→ coefficient of friction ↑ → but Braking force.

→ wear rate ↓ → velocity of the brake drum.

→ Heat resistance ↑ → projected area.

→ Heat dissipation capacity ↑ → ~~and~~ Energy absorbed.

→ Mechanical strength ↑

→ not affected by moisture/oil.

Types of Brakes :-

I) Hydraulic :-> pumps.

II) Electric :- generators & ~~and~~ eddy current brakes.

III) mechanical brakes :- (Force direction)
acting

① Radial Brakes.

② Axial Brakes.

At the neutral axis, $y = 0$ and hence shear stress is maximum.

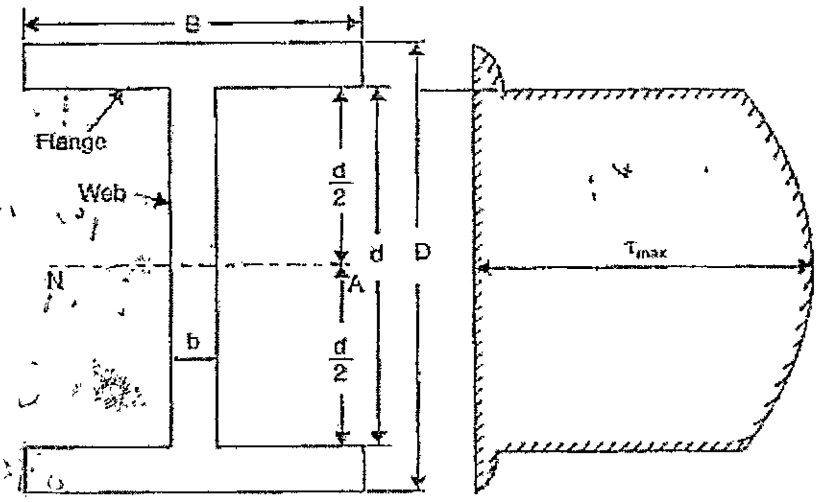
$$\therefore \quad \tau_{max} = \frac{F}{I \times b}\left[\frac{B}{8}(D^2 - d^2) + \frac{b}{2} \times \frac{d^2}{4}\right] = \frac{F}{I \times b}\left[\frac{B(D^2 - d^2)}{8} + \frac{bd^2}{8}\right]$$

At the junction of top of the web and bottom of flange, $y = \frac{d}{2}$

Hence shear stress is given by,

$$\tau = \frac{F}{I \times b}\left[\frac{B}{8}(D^2 - d^2) + \frac{b}{2}\left(\frac{d^2}{4} - \left(\frac{d}{2}\right)^2\right)\right] = \frac{F \times B \times (D^2 - d^2)}{8I \times b}$$

The shear stress distribution for I- section is shown in the below figure.



A steel beam of I-section is 600mm deep. Each flange is 250mm wide and 25mm thick. The web is 15mm thick. The beam section is subjected to a shear force of 500KN. Determine the shear stress distribution for the beam section (i) When the web is vertical and (ii) When the web is horizontal.

ANS:

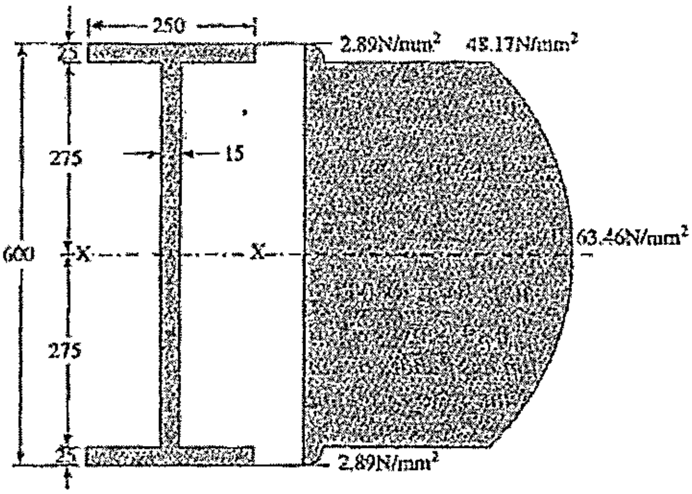
(i) When the web is vertical.

$$I_{xx} = \frac{250 \times 600^3}{12} - \frac{235 \times 550^3}{12} = 1.2418 \times 10^9 \; mm^4$$

Shear stress just above the junction of the flange and web

$$a\bar{y} = 250 \times 25 \times 287.5 = 1796875 \; mm^3$$

$$\text{Shear stress} = \frac{Sa\bar{y}}{Ib} = \frac{500 \times 10^3 \times 796875}{1.2418 \times 10^9 \times 15} = 2.89 \; N/mm^2$$



②

ⓐ Radial Brakes → Internal
→ External.

(i) Block / Shoe Brake.

(ii) Band Brake.

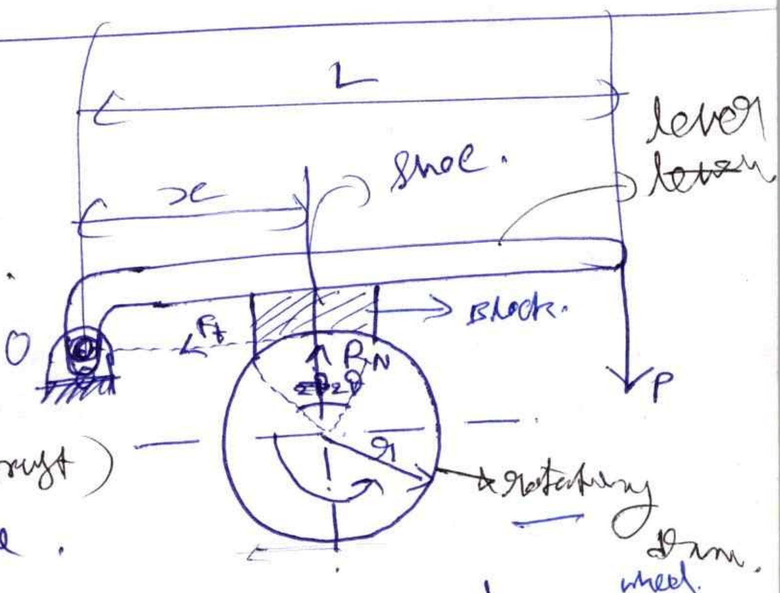
(iii) Band & Block Brake.

(iv) Internal expanding shoe Brake.

ⓑ Axial Brakes → Disc brakes & cone brakes.

Σ Block / Shoe Brake →

It consists of a shoe which is pressed against a rotating drum.



P = Force applied

R_N = normal force. [a side thrust)

2θ = Angle of contact surface.

F_t = Tangential braking force, on the wheel.

$\boxed{2\theta < 60^\circ}$ normal pr. is uniform

$F_t = \mu R_N$

& Braking torque $= F_t \times r = \mu R_N r. = T_B.$

case(i) Line of action of F_t passes through fulcrum of the lever. & ~~wheel~~

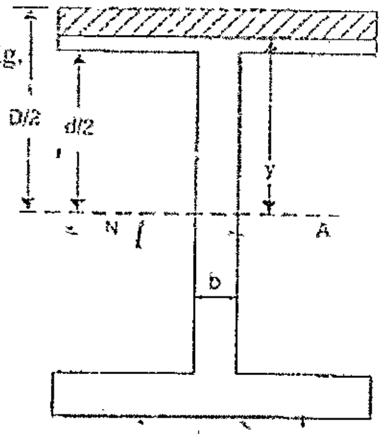
(i) wheel rotates (↻) ⇒ $R_N \times x = P \times L$

(or) (↺) ⇒ $\Rightarrow R_N = \frac{PL}{x}.$

$$T_B = \frac{\mu P L r}{x}.$$

(i) Shear stress distribution in the flange

Consider a section at a distance y from N.A. in the flange as shown in Fig.



Width of the section = B,

Shaded area of flange, $A = B\left(\frac{D}{2} - y\right)$

Distance of the C.G. of the shaded area from neutral axis is given as

$$\bar{y} = y + \frac{1}{2}\left(\frac{D}{2} - y\right) = \frac{1}{2}\left(\frac{D}{2} + y\right)$$

Hence shear stress in the flange becomes,

$$\tau = \frac{F \times A\bar{y}}{I \times B} = \frac{F \times B\left(\frac{D}{2} - y\right) \times \frac{1}{2}\left(\frac{D}{2} + y\right)}{I \times B} = \frac{F}{2I}\left(\frac{D^2}{2} - y^2\right)$$

(a) For the upper edge of the flange, $y = \frac{D}{2}$

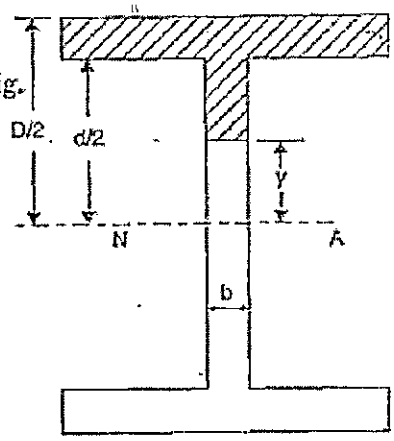
Hence shear stress, $\tau = \frac{F}{2I}\left[\frac{D^2}{4} - \left(\frac{D}{2}\right)^2\right] = 0.$

(b) For the lower edge of the flange, $y = \frac{d}{2}$

Hence $\tau = \frac{F}{2I}\left[\frac{D^2}{4} - \left(\frac{d}{2}\right)^2\right] = \frac{F}{2I}\left(\frac{D^2}{4} - \frac{d^2}{4}\right) = \frac{F}{8I}(D^2 - d^2)$

(ii) Shear stress distribution in the web

Consider a section at a distance y in the web from the N.A. as shown in Fig.



Width of the section = b.

$\therefore A\bar{y}$ = Moment of the flange area about N.A. + moment of the shaded area of web about N.A.

$$= B\left(\frac{D}{2} - \frac{d}{2}\right) \times \frac{1}{2}\left(\frac{D}{2} + \frac{d}{2}\right) + b\left(\frac{d}{2} - y\right) \times \frac{1}{2}\left(\frac{d}{2} + y\right)$$

$$= \frac{B}{8}(D^2 - d^2) + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)$$

Hence the shear stress in the web becomes as

$$\tau = \frac{F \times A\bar{y}}{I \times b} = \frac{F}{I \times b} \times \left[\frac{B}{8}(D^2 - d^2) + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)\right]$$

case (2) :-

The line of action of F_t passes through a distance "a" below the ~~fulcrum~~ fulcrum "O".

(2) $R_N \times x + F_t \times a = P \cdot L \Rightarrow R_N = \frac{PL}{x + \mu a}$ ✓

$\& \; T_B = \frac{\mu P L r}{(x + \mu a)}$

(5) $R_N \times x = PL + F_t a \Rightarrow R_N = \frac{PL}{x - \mu a}$

$\& \; T_B = \frac{\mu P L r}{x - \mu a}$

---

case (3) The line of action of F_t passes through a distance "a" above the fulcrum "O".

(2) $R_N x = PL + F_t a \Rightarrow R_N = \frac{PL}{x - \mu a}$

(5) $R_N x + F_t a = PL \Rightarrow R_N = \frac{PL}{x + \mu a}$

---

case (4) :- For pivoted shoe :- $(2\theta > 60^\circ)$

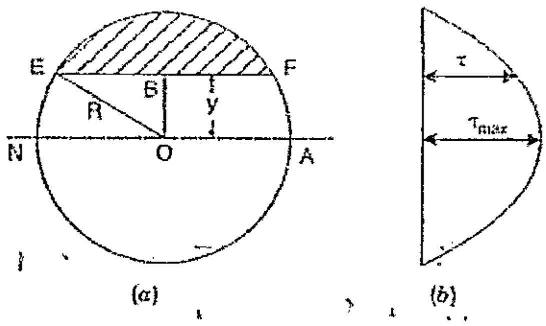
$T_B = F_t \times r = \mu' R_N r.$

μ' = equivalent coeffi. of friction $= \frac{4 \mu \sin\theta}{2\theta + \sin 2\theta}.$

problem (19.1)

Derive an expression for the shear stress at any point in a circular section of a beam, which is subjected to a shear force F. And prove that the maximum shear stress in a circular section of a beam is 4/3 times the average shear stress.

ANS:



The shear stress at the level EF = $\tau = F \cdot \frac{A\bar{y}}{b \times I}$

Consider a strip of thickness dy at a distance y from N.A.

Let dA is the area of strip.

Then $dA = b \times dy = EF \times dy$

$= 2 \times EB \times dy$

$= 2 \times \sqrt{R^2 - y^2} \times dy$

Moment of this area dA about N.A. $= y \times dA = y \times 2\sqrt{R^2 - y^2} \times dy = 2y\sqrt{R^2 - y^2}\, dy.$

Moment of the whole shaded area about the N.A. $= A\bar{y} = \int_y^R 2y\sqrt{R^2 - y^2}\, dy$

$= -\int_y^R (-2y)\sqrt{R^2 - y^2}\, dy.$

$= \frac{2}{3}(R^2 - y^2)^{3/2}$

$b = EF = 2 \times EB = 2 \times \sqrt{R^2 - y^2}$

$$\therefore \tau = F \cdot \frac{A\bar{y}}{b \times I} = \frac{F \times \frac{2}{3}(R^2 - y^2)^{3/2}}{I \times b} = \frac{\frac{2}{3}F(R^2 - y^2)^{3/2}}{I \times 2\sqrt{R^2 - y^2}} = \frac{F}{3I}(R^2 - y^2)$$

$\therefore$ At $y = 0$ i.e., at the neutral axis, the shear stress is maximum and is given by

$$\tau_{max} = \frac{F}{3I}R^2 = \frac{F \times R^2}{3 \times \frac{\pi}{4}R^4} = \frac{4}{3} \times \frac{F}{\pi R^2}$$

But average shear stress,

$$\tau_{avg} = \frac{\text{Shear force}}{\text{Area of circular section}} = \frac{F}{\pi R^2}$$

$$\therefore \tau_{max} = \frac{4}{3} \times \tau_{avg}$$

How will you prove that the shear stress changes abruptly at the junction of the flange and the web of an I-section?

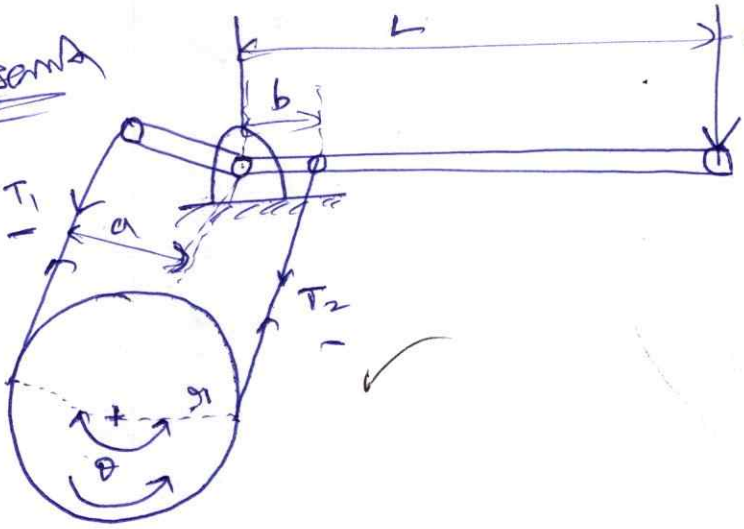
ANS: Let B = overall width of the section,

D = Overall depth of the section,

b = Thickness of the web, and d = Depth of web.

BAND Brakes

differential ~~or~~ band



brake torque on drum.

$T_B = (T_1 - T_2) r$.

~~Slack~~ ~~Tight~~ Tight side tension / ~~Slack~~ ~~Tight~~ Slack slack side tension $= T_1/T_2 = e^{\mu\theta}$

case (i) → $a > b \Rightarrow P \downarrow$

$(\circlearrowleft) \Rightarrow P \times L - T_1 a + T_2 b = 0 \Rightarrow P = \frac{T_1 a - T_2 b}{L}$ $(T_1 > T_2)$

$(\circlearrowright) \Rightarrow P \times L - T_2 a + T_1 b = 0 \Rightarrow P = \frac{T_2 a - T_1 b}{L}$ $(T_2 > T_1)$

case ② → $a < b \Rightarrow P \uparrow$

$(\circlearrowleft) \Rightarrow P \times L + T_1 a - T_2 b = 0 \Rightarrow P = \frac{T_2 b - T_1 a}{L}$

$(\circlearrowright) \Rightarrow P \times L - T_1 b + T_2 a = 0 \Rightarrow P = \frac{T_1 b - T_2 a}{L}$

case ③ $\Rightarrow a = b$ (brake cannot be applied).

simple band brakes ~~a/b = 0~~ a (or) b = 0

$F \downarrow \Rightarrow (\circlearrowleft) F \cdot L - T_1 a = 0 \Rightarrow F = \frac{T_1 a}{L}$

$(\circlearrowright)$ $F = \frac{T_2 a}{L}$.

Consider a level EF at a distance y from the neutral axis.

b = Actual width of the section at the level EF

A = Area of the section above y $= \left(\frac{d}{2} - y\right) \times b$

$\bar{y}$ = Distance of the C.G. of area A from neutral axis $= y + \frac{1}{2}\left(\frac{d}{2} - y\right) = \frac{1}{2}\left(y + \frac{d}{2}\right)$

The shear stress at the level EF = $\tau = F \cdot \frac{A\bar{y}}{b \times I} = \frac{F \cdot \left(\frac{d}{2} - y\right) \times b \times \frac{1}{2}\left(y + \frac{d}{2}\right)}{b \times I} = \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right)$

Therefore the variation of τ with respect to y is parabola.

At the Top edge, y = d / 2 and hence

$$\tau = \frac{F}{2I}\left[\frac{d^2}{4} - \left(\frac{d}{2}\right)^2\right] = \frac{F}{2I} \times 0 = 0$$

At the neutral axis, y = 0 and hence

$$\tau = \frac{F}{2I}\left(\frac{d^2}{4} - 0\right) = \frac{Fd^2}{8I} = \frac{Fd}{8 \times \frac{bd^3}{12}} = 1.5\,\frac{F}{bd} \quad \cdots (i)$$

Now average shear stress, $\tau_{avg} = \frac{\text{Shear force}}{\text{Area of section}} = \frac{F}{b \times d}$.

Substituting the above value in equation (*i*), we get

$$\tau = 1.5 \times \tau_{avg} = \tau_{max}$$

A rectangular beam 100mm wide and 250mm deep is subjected to a maximum shear force of 50KN. Determine: (i) Average shear stress, (ii) maximum shear stress, and (iii) shear stress at a distance of 25mm above the neutral axis.

ANS: Given Data:

Width, $b = 100$ mm

Depth, $d = 250$ mm

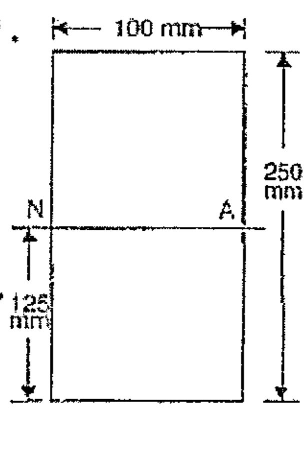
Maximum shear force, $F = 50$ kN $= 50{,}000$ N.

(*i*) Average shear stress $= \tau_{avg} = \frac{F}{\text{Area}} = \frac{50{,}000}{100 \times 250} = 2$ N/mm².

(*ii*) Maximum shear stress $= \tau_{max} = 1.5 \times \tau_{avg} = 3$ N/mm².

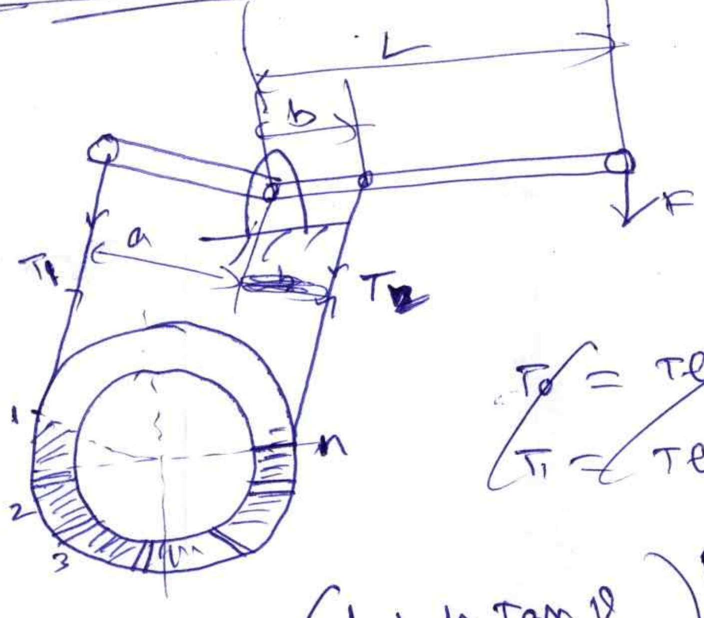
(*iii*) The shear stress at a distance y from N.A. is

$$= \tau = \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right) = \frac{50000}{2I}\left(\frac{250^2}{4} - 25^2\right) = 2.88 \text{ N/mm}^2.$$



M VENKANNA BABU

BAND & BLOCK BRAKE



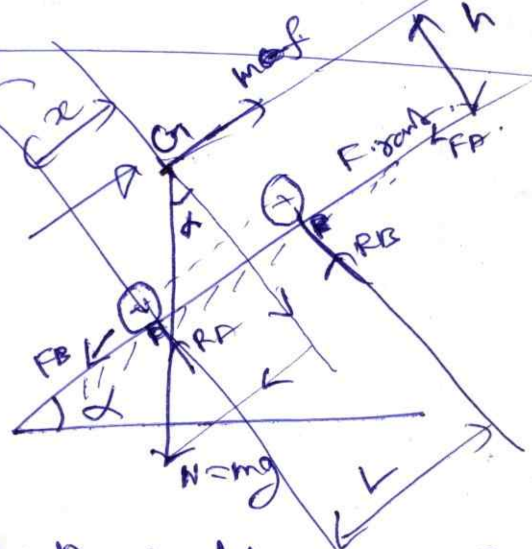
~~T_0 = Tension on the slack side~~

~~T_1 = Tension~~

$$\frac{T_1}{T_2} = \left(\frac{1+\mu\tan\theta}{1-\mu\tan\theta}\right)^n \quad n \rightarrow \text{no. of blocks.}$$

$P = (T_2 - T_1)(v)$ $\qquad v = \frac{\pi D N}{60}$

~~Effect of~~ Effect of Braking:-



~~① $R_A + R_B$ = mg cos α~~

~~② F_B + mg sin α = ma~~

on a level road :- $\boxed{\alpha = 0}$

case① i.e. brake applied to rear wheels :- $\qquad F_B = \mu R_B$

$$a = \text{deceleration} = g\,\frac{\mu(L-x)}{L+\mu h} \quad \& \quad s = \frac{u^2}{2a}.$$

case② → brakes applied to front wheels :- $\qquad \cancel{v^2} - u^2 = 2as.$

$$a = g\,\frac{\mu x}{L-\mu h}$$

case③ :- applied to all the four wheels :-

$$a = g\mu$$

Force on the end of the elemental cylinder on the section AB

$$= \text{Stress} \times \text{Area of elemental cylinder} = \sigma \times dA = \frac{M}{I} \times y \times dA$$

Similarly, force on the elemental cylinder on the section CD

$$= (\sigma + d\sigma)\, dA = \frac{(M + dM)}{I} \times y \times dA$$

$\therefore$ Net unbalanced force on the elemental cylinder $= \frac{dM}{I} \times y \times dA$

$\therefore$ Total unbalanced force $= \int \frac{dM}{I} \times y \times dA = \frac{dM}{I} \int y \times dA = \frac{dM}{I} \times A \times \bar{y}$

Where A = Area of the section above the level EF

$\bar{y}$ = Distance of the C.G. of the area A from the neutral axis.

$\therefore$ Shear resistance (or shear force) at the level EF = Total unbalanced force $= \frac{dM}{I} \times A \times \bar{y}$

Let τ = Intensity of horizontal shear at the level EF

b = Width of beam at the level EF

$\therefore$ Shear force due to τ = Shear stress $\times$ Shear area $= \tau \times b \times dx$

Equating the two values of shear force given by equations

$$\tau \times b \times dx = \frac{dM}{I} \times A \times \bar{y}$$

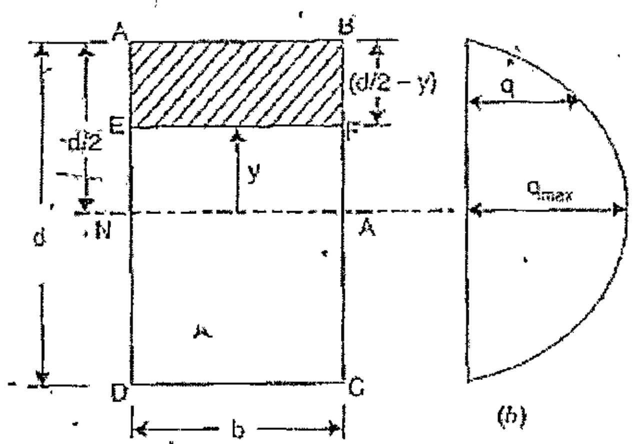
$$\therefore \tau = \frac{dM}{dx} \cdot \frac{A\bar{y}}{I \times b} = F \times \frac{A\bar{y}}{I \times b}$$

- **Prove that the shear stress distribution in a RECTANGULAR section of a beam which is subjected to a shear force F is given by**

$$\tau = \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right)$$

And show that for a rectangular section of the maximum shear stress is 1.5 times the average stress?

ANS: Let the shear force acting at the section = F



(a) (b)

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on a inclined plane

① $a = g \cos\alpha \left[\dfrac{\mu(L-x)}{L+\mu h} \pm \tan\alpha \right]$

② $a = g \cos\alpha \left[\dfrac{\mu x}{L-\mu h} \pm \tan\alpha \right]$

③ $a = g \cos\alpha \, (\mu \pm \tan\alpha)$

(+ve → up the plane
−ve → down the plane)

Internal expanding shoe brakes

$$F_a = \int_{\phi_1}^{\phi_2} R_n^l \, c \sin\theta + \int_{\phi_1}^{\phi_2} \mu R_n^l \, (r - c\cos\theta) = 0$$

$$\int_{\phi_1}^{\phi_2} R_n^l \, c \sin\theta = \frac{r c w P_n^l}{n} (2\phi_2 - 2\phi_1 - \sin 2\phi_2 + \sin 2\phi_1)$$

$$\int_{\phi_1}^{\phi_2} \mu R_n^l (r - c\cos\theta) = \frac{\mu r w P_n^l}{4} \left[4r(\cos\phi_1 - \cos\phi_2) - c(\cos 2\phi_1 - \cos 2\phi_2) \right]$$

$$T_B = r^2 \mu w (P_n^l + P_n^t)(\cos\phi_1 - \cos\phi_2)$$

Shear stress just below the junction of the flange and web = $\frac{250}{15} \times 2.89 = 48.17\ N/mm^2$

Shear stress at the neutral axis $a\bar{y} = (250 \times 25 \times 287.5) + (15 \times 275 \times 137.5) = 2364062.5\ mm^3$

Shear stress $= \frac{Sa\bar{y}}{Ib} = \frac{500\times10^3\times2364062.5}{1.2418\times10^9\times15} = 63.46\ N/mm^2$

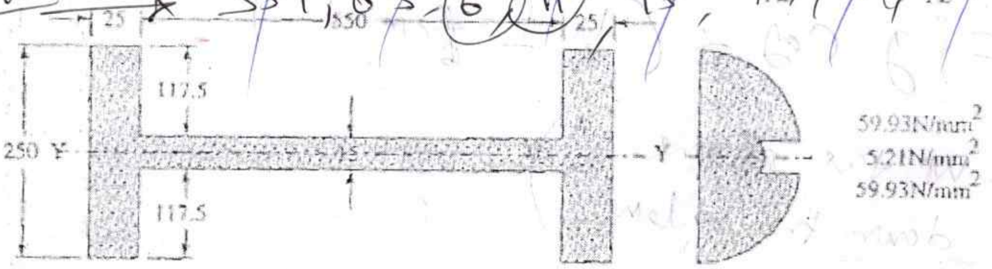
III-A 25/21

A 394, 64, 74, 29, 41, 17, 21,

(ii) When the web is horizontal.

Moment of Inertia of the beam section about the YY axis $= I_{yy} = \frac{2\times25\times250^3}{12} + \frac{550\times15^3}{12} = 6.495\times10^7\ mm^4$

III-B 351, 63, 6, 11, 15, 12, 21



Shear stress in the flange just above the web , $a\bar{y} = 2\times25\times117.5\left(\frac{117.5}{2}+7.5\right) = 389218.75\ mm^3$

Shear stress $= \frac{Sa\bar{y}}{Ib'} = \frac{500\times10^3\times389218.75}{6.495\times10^7\times50} = 59.93\ N/mm^2$

Shear stress in the web at 7.5 *mm* from the N.A. $= \frac{50}{600}\times59.93 = 5\ N/mm^2$

Shear stress at the neutral axis, $a\bar{y} = (2\times25\times125\times62.5) + 550\times7.5\times\frac{7.5}{2} = 406093.75\ mm^3$

Shear stress $q_{na} = \frac{Sa\bar{y}}{Ib'} = \frac{500\times10^3\times406093.75}{6.495\times10^7\times600} = 5.21\ N/mm^2$.

. **The shear force acting on a beam at an I-section with unequal flanges is 50KN. The section is shown in the below figure. The moment of inertia of the section about N.A. is 2.849X10⁴. Calculate the shear stress at the N.A. and also draw the shear stress distribution over the depth of the section.**

ANS:

Given Data:

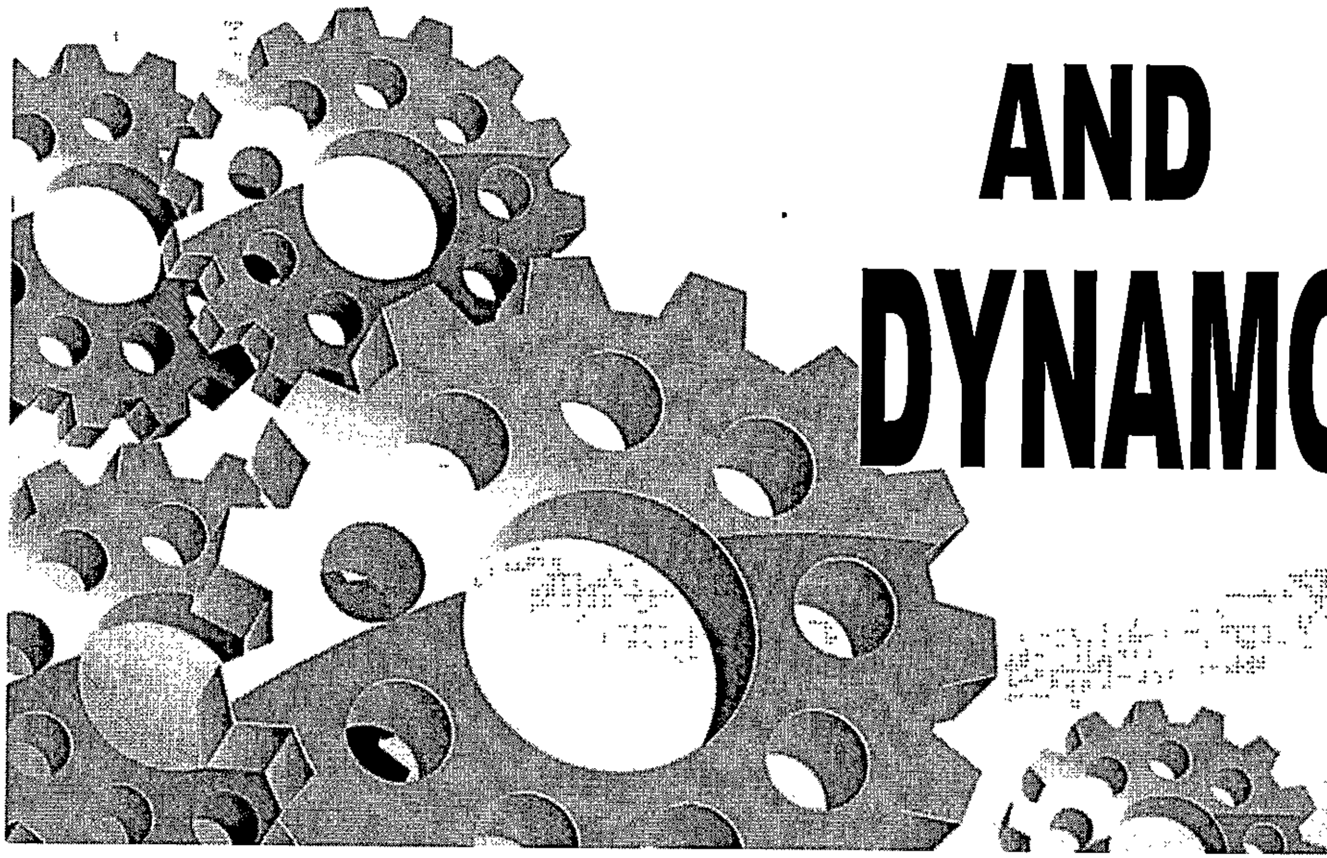
Shear force, $F = 50\ kN = 50{,}000\ N$

Moment of inertia about N.A.,

$I = 2.849\times10^8\ mm^4$.

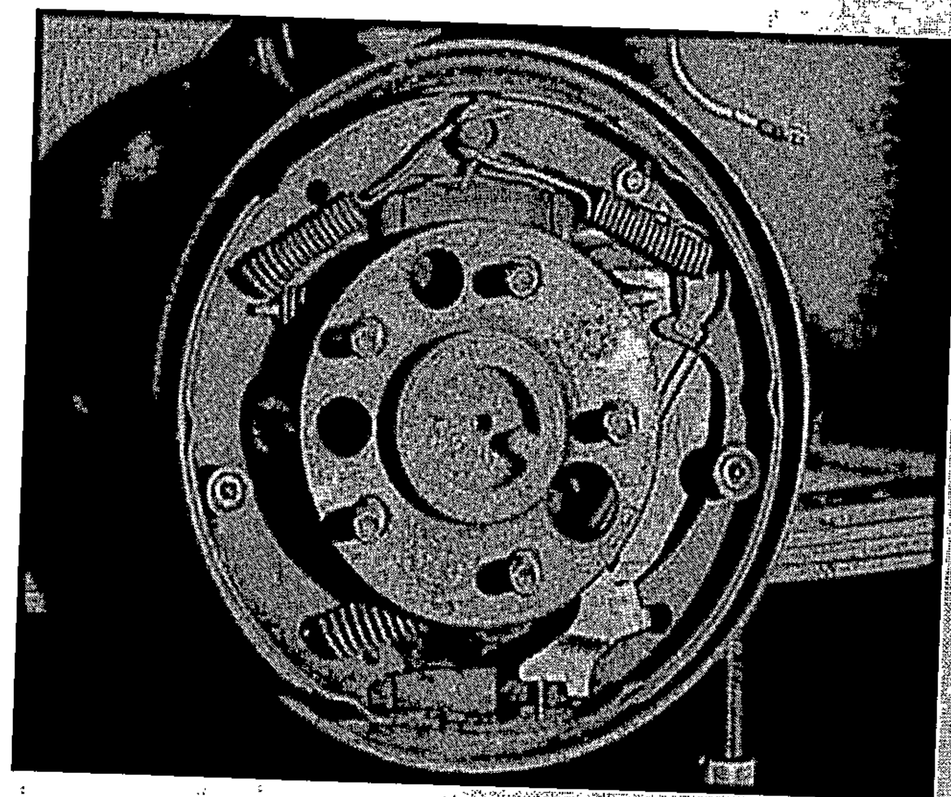
Distance of the center of gravity from the bottom surface $= y^* = \frac{A_1y_1 + A_2y_2 + A_3y_3}{(A_1 + A_2 + A_3)}$

BRAKES AND DYNAMOMETERS



BRAKES

- Brake is an appliance used to apply frictional resistance to a moving body.
- Before applying the brakes, the acceleration is released to stop the fule supply thus the engine develops no more power to run the vehicle, and then the brakes are applied which stop the rolling of the wheels on the road and hence the vehicle is stopped.



TYPES OF BRAKES

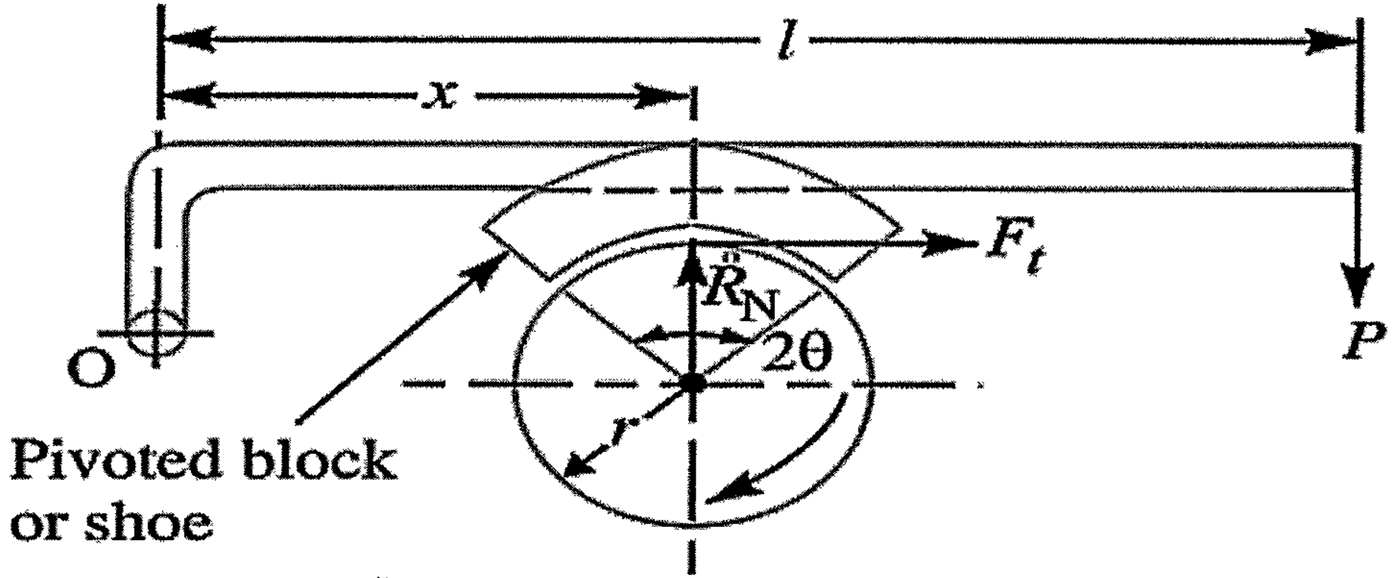
The following are the main types of mechanical brakes:

1. Block or shoe brake
2. Band brake
3. Band and block brake
4. Internal expanding shoe brake

Block (or) Shoe brake:

- A block or shoe brake consists of a block or shoe which is pressed against a rotating drum.
- The force on the drum increased by using a lever.
- If only one block is used for the perpose,a side thrust on the bearing of the shaft supporting the drum will act.
- This can be prevented by using two blocks ontwo side of the drum.

- This also double the braking torque.



Let r = radius of the drum

μ = coefficint of friction

Rn = normal reaction on the block (Fr)

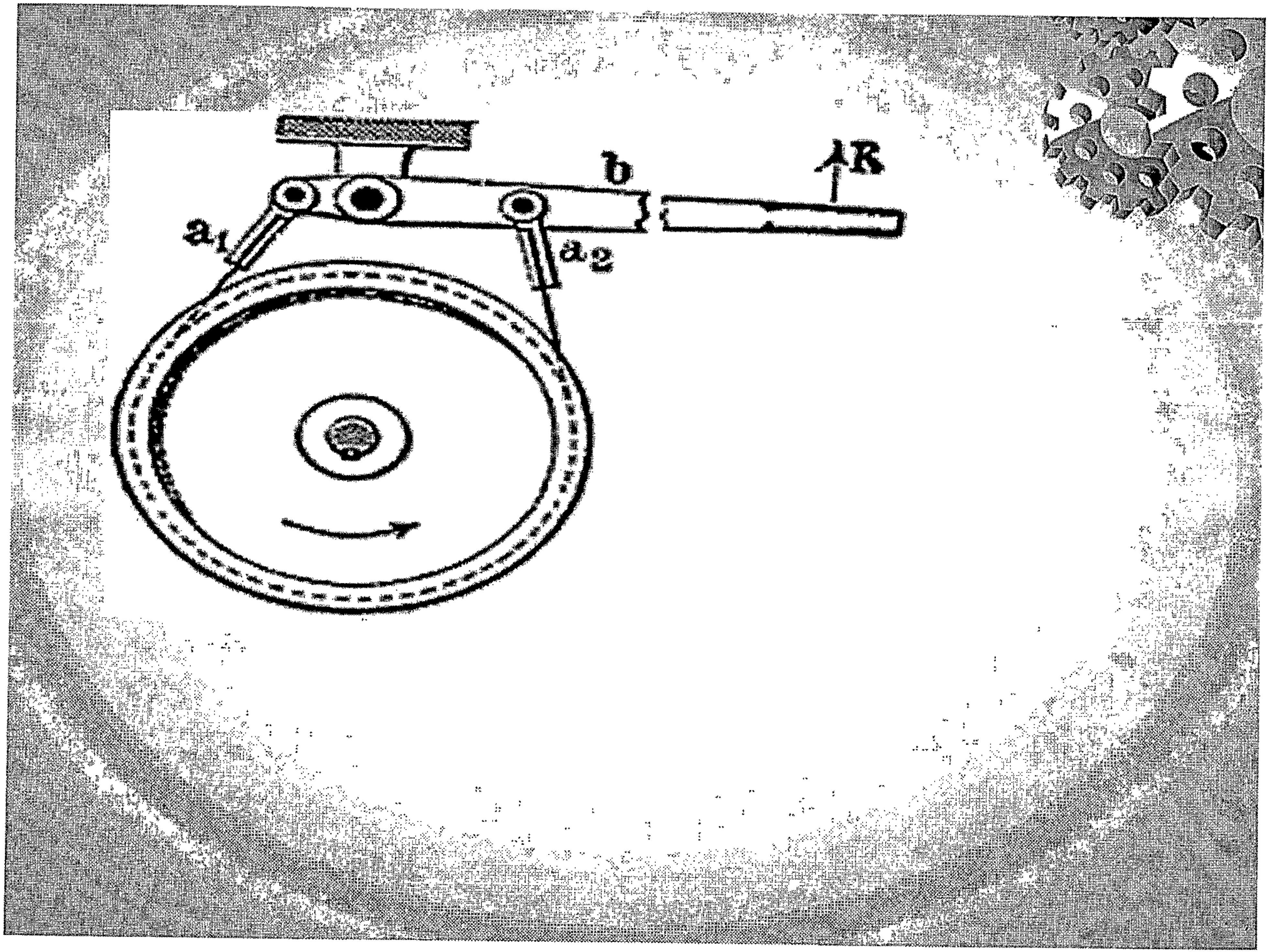
$$T_B = \mathring{F}_t \times r = \mu'.R_N.\, r$$

μ' = Equivalent coefficient of friction = $\dfrac{4\mu \sin\theta}{2\theta + \sin 2\theta}$, and

μ = Actual coefficient of friction.

BAND BRAKE

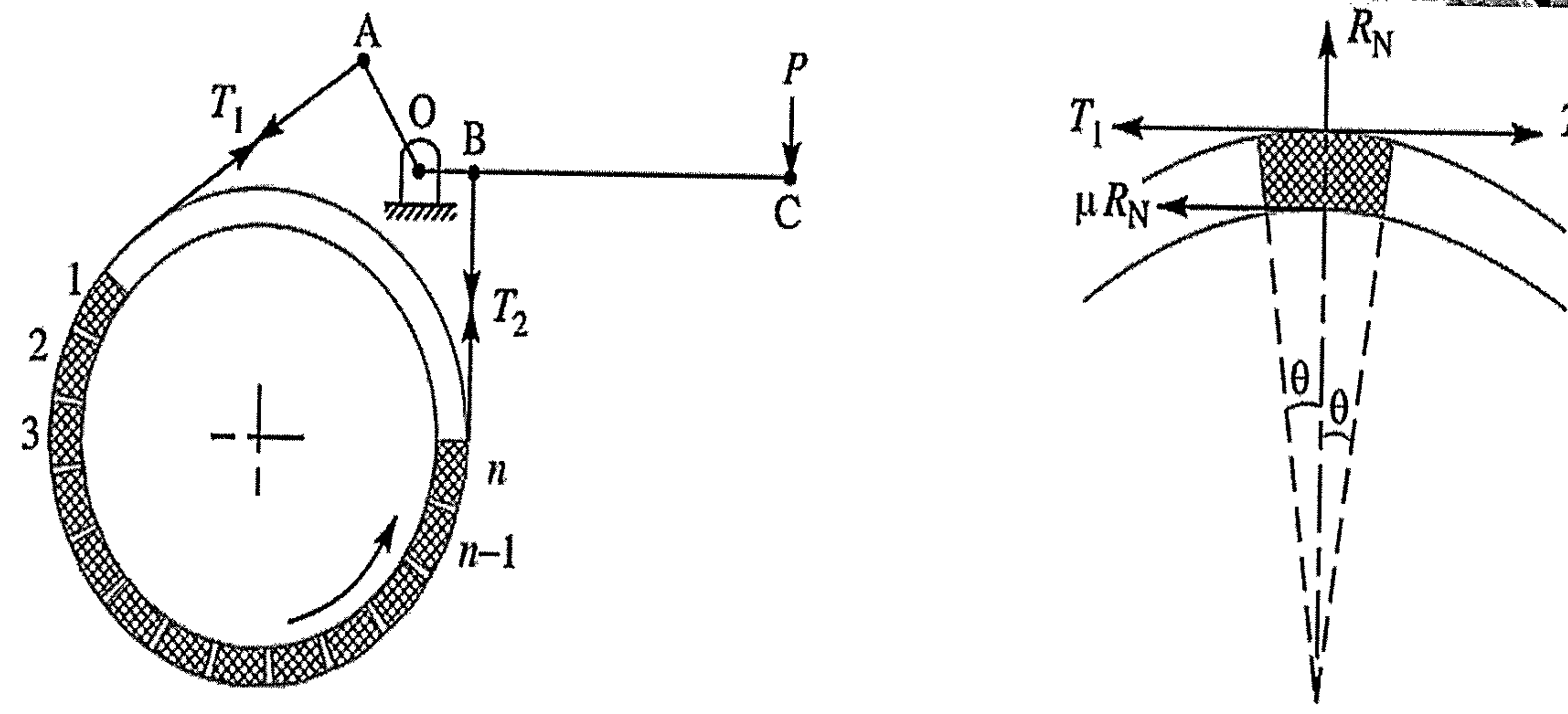
- It consists of a rope, belt or flexible steel band which is pressed against the external surface of acylindrical drum when the brake is applied.
- the force is applied at the free end of a lever.
- The effectiveness of the force F depends upon the
  1. Direction of rotation of the drum
  2. Ratio of lengths a and b
  3. Direction of applied force F



b
R
a1
a2

BAND AND BLOCK BRAKE

- A band and block brake consists of a number of wooden blocks secured inside a flexible steel band.
- When the brake is applied, the blocks are pressed against the drum
- Wooden blocks have a higher coefficient of friction, Thus, increasing the effictiveness of the brake.

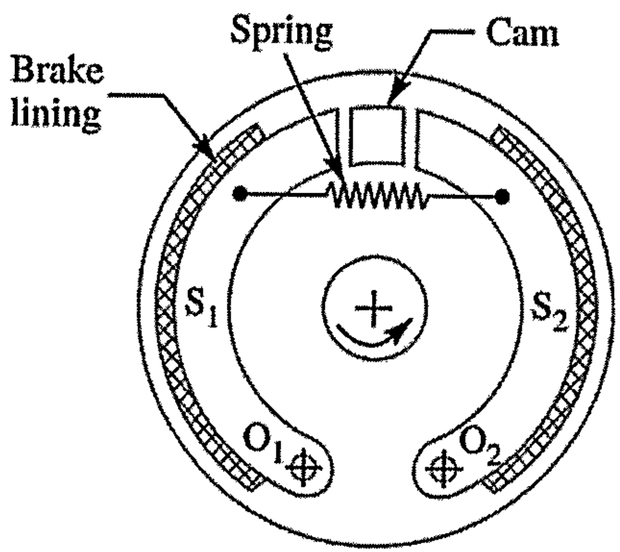


(a)

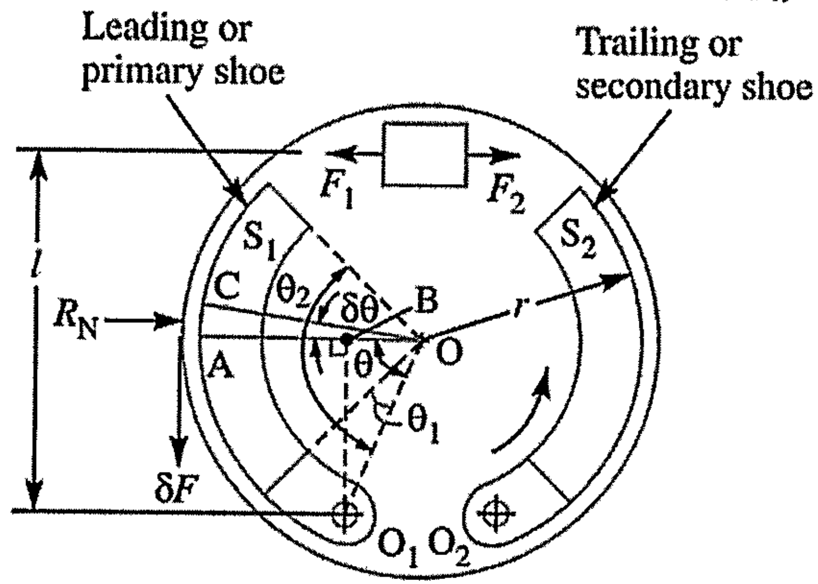
(b)

INTERNAL EXPANDING SHOE BRAKE

- These days, band brakes have been replaced by internal expanding shoe brakes having at least one shoe per a wheel.
- It consists of two semi-circularshoes which are lined with a friction material ferodo.
- the shoes press against the inner flange of the drum when the brakes are applied.
- under normal running of the vehicle, the drum rotates freely as the outer diameter of the shoe is a little less than the internal diameter of the drum.



(*a*) Internal expanding brake.



(*b*) Forces on an internal expanding brake.

DYNAMOMETERS

- It is a device for measuring force, moment of force (torque), or power.
- For example, the power produced by an engine, motor or other rotating prime mover can be calculated torque and rotational speed (rpm)

TYPES OF DYNAMOMETERS

Tere are mainly two types of dynamometers:

1. Absorption Dynamometers

a) Prony brake dynamometer

b) Rope brake dynamometer

2. Transmission dynamometers

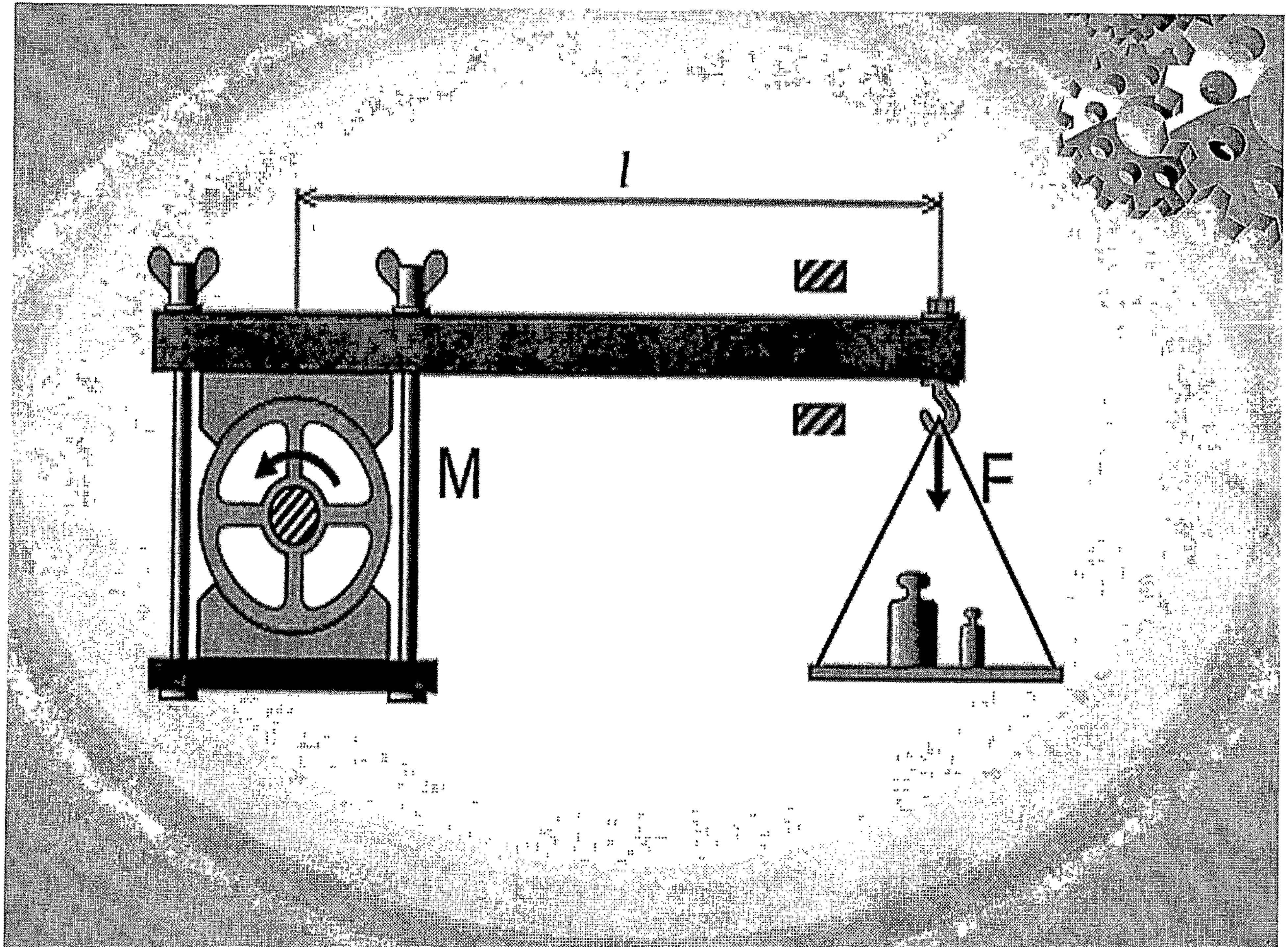
a) Belt transmission dynamometer

b) Epicyclic-Train dynamometer

PRONY BRAKE DYNAMOMETER

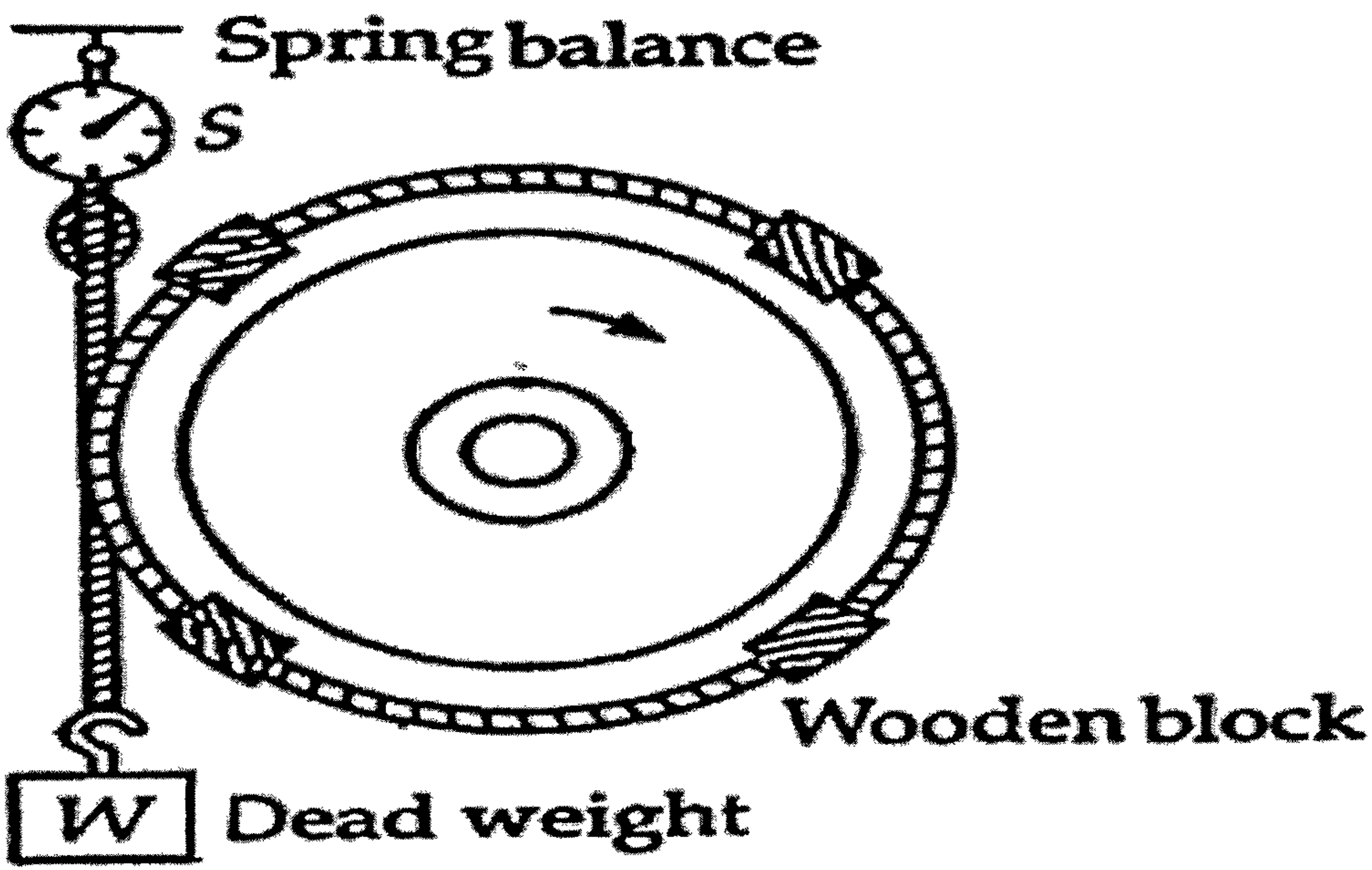
- A prony brake dynamometer consists of two wooden blocks clamped together on a revolving pulley carrying a lever.
- The friction between the blocks and the pully tends to rotate the block in the direction of rotation of the shaft.
- The grip of the blocks over the pulley is adjusted using the blots of the clamp until the engine runs at the required speed.
- The power of the engine is thus absorbed by the friction.

Frictional torque = $Wl = Mgl$


l
M
F

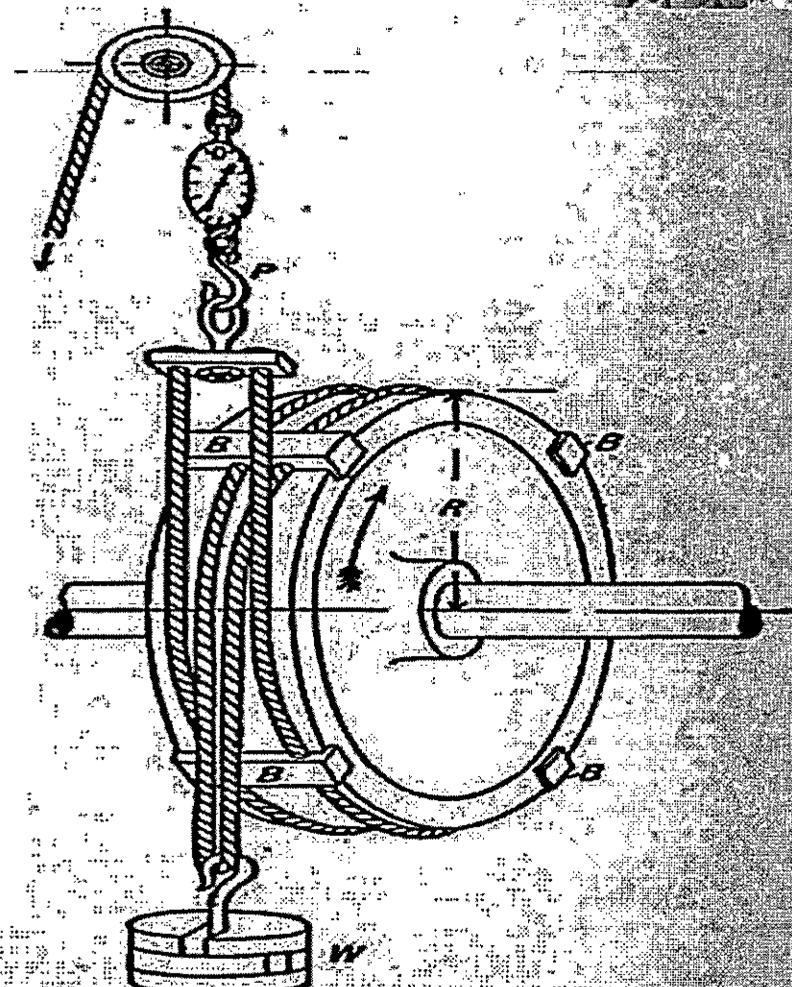
ROPE BRAKE DYNAMOMETER

- In rope brake dynamometer, a rope is wrapped over the rim of a pulley keyed to the shaft of the engine.
- The diameter of the rope depends upon the power of the machine.
- the upper end of the rope is attached to a spring balance whereas the lower end supports the weight of suspended mass.
- a rope brake dynamometer is frequently used to test the power of engine.


Spring balance
S
Wooden block
W
Dead weight

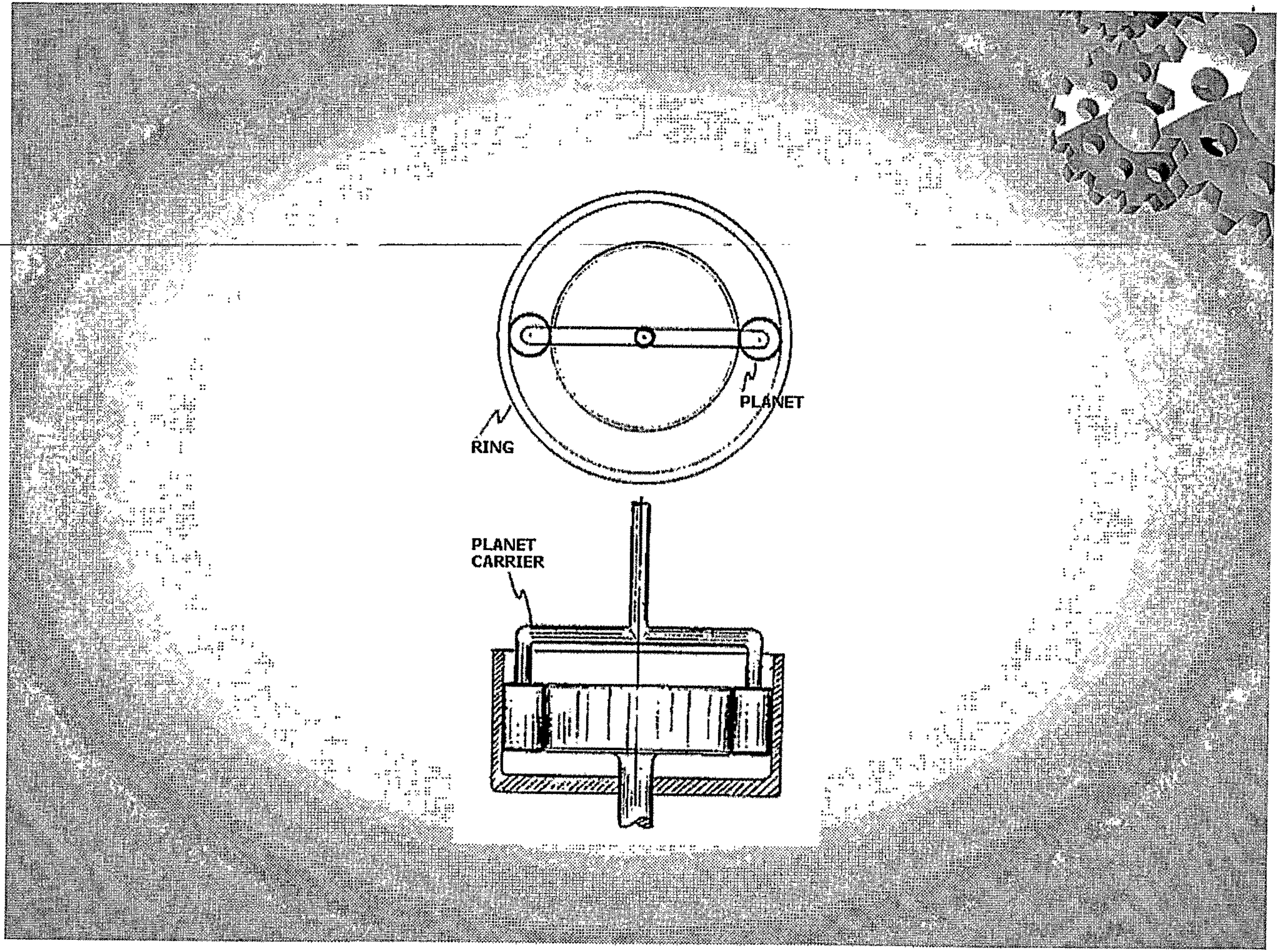
BELT TRANSMISSION DYNAMOMETER

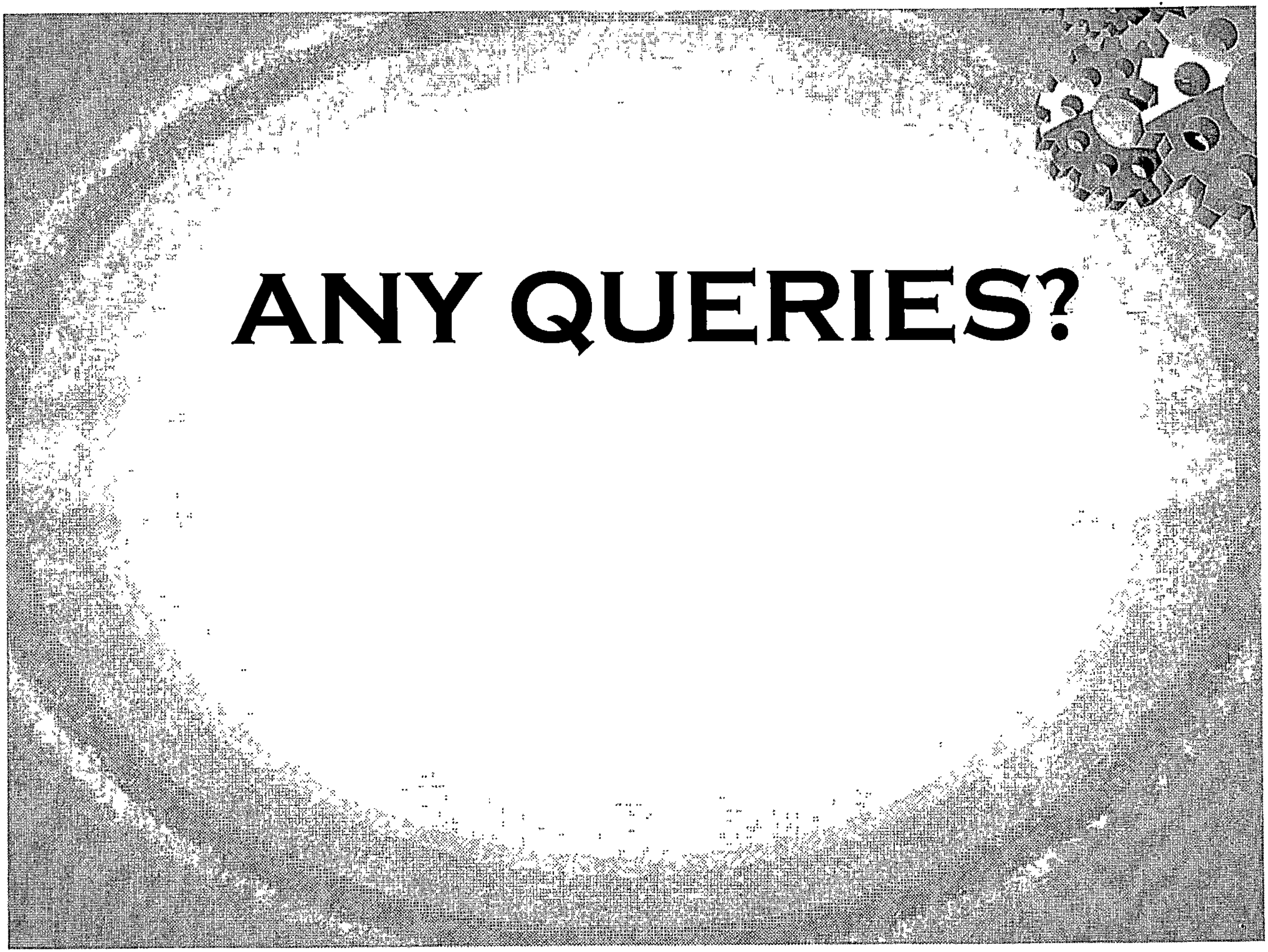
- When a belt transmits power from one pulley to anthoer there exists a difference in tension between the tight and slack sides.
- a dynamometer measures directly the difference in tensions while the belt is running.



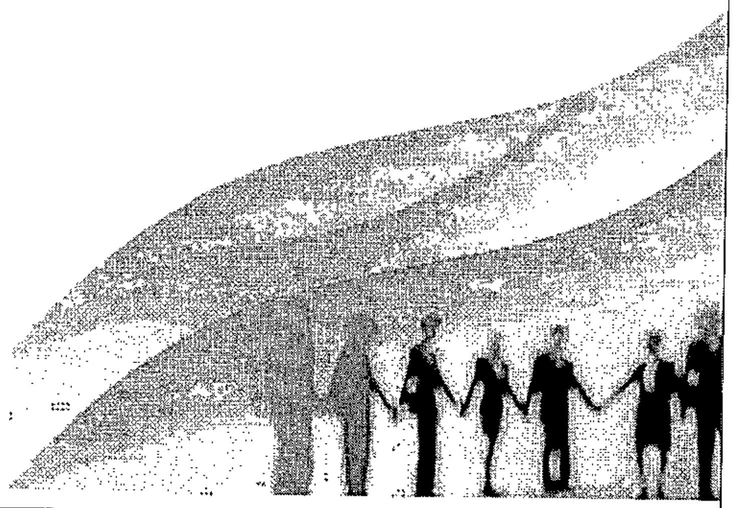
EPICYCLIC-TRAIN DYNAMOMETER

- It consists of a simple epicyclic train of gear.
- A spur gear A is the driving wheel which drives an annular driven wheel B through an intermediate pinion C.
- the intermediate gear C is mounted on a horizontal lever, the weight is balanced by a counterweight at left end when the system is rest.
- when the wheel A rotates counter-clockwise, the wheel B as wellas the wheel C rotates clockwise.


PLANET
RING
PLANET
CARRIER


ANY QUERIES?

THANK YOU



GOVERNORS

Topics to be discussed:

- Pendulum type : Watt governor
- Weight loaded type :
  - Porter governor
  - Proell governor
- Spring loaded type :
  - Hartnell governor
  - Hartung governor
- Sensitiveness
- Isochronisms
- Hunting
- Effort and Power

Introduction :

- The function of the governor is to regulate the mean speed of an engine, when there are variations in the load
- For example, when the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of working fluid.
- On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required.

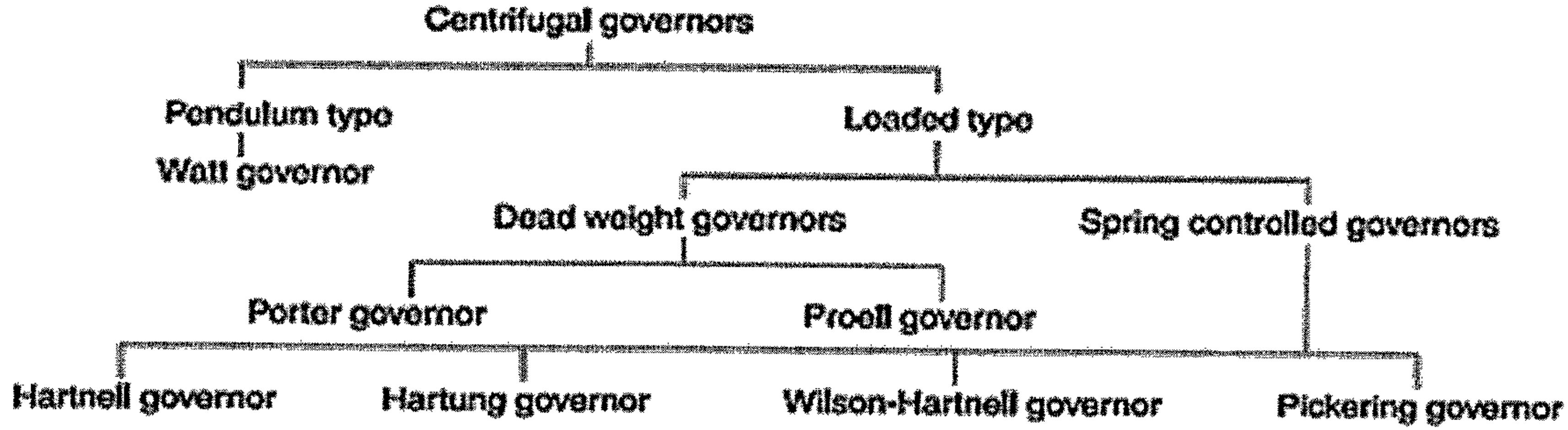
- The governor automatically controls the supply of working fluid to the engine with the varying load conditions and keep the mean speed within certain limits.
- A little consideration will show that, when the load increases, the configuration of the governor changes and a valve is moved to increase the supply of the working fluid ; conversely, when the load decreases, the engine speed increases and the governor decreases the supply of working fluid.

Types of Governors

- The governors may, broadly, be classified as

1. Centrifugal governors
2. Inertia governors.

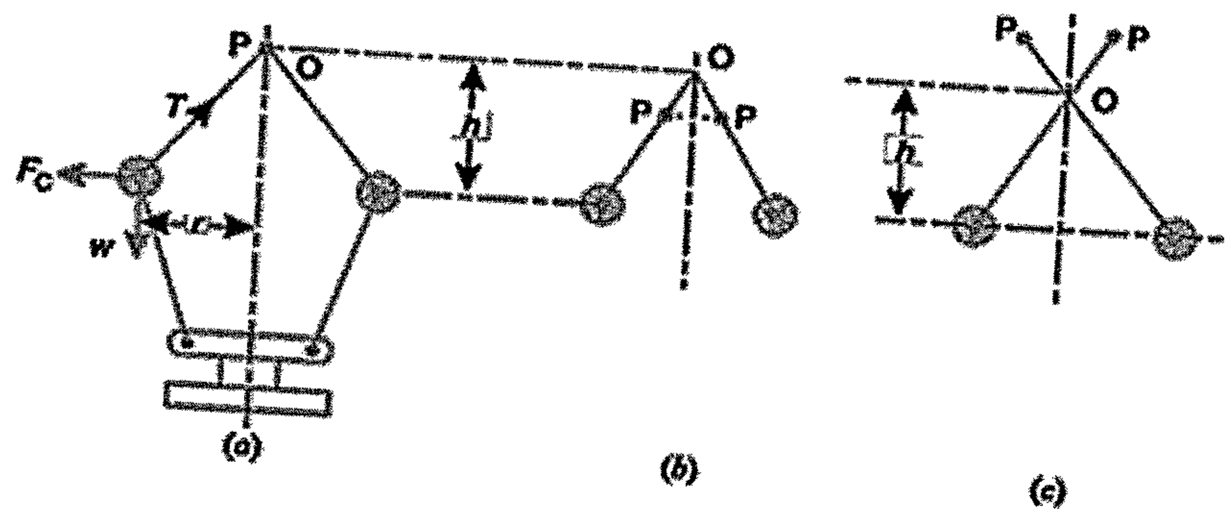
The centrifugal governors, may further be classified as follows :



WATT GOVERNOR

The simplest form of a centrifugal governor is a Watt governor, as shown in fig. It is basically a conical pendulum with links attached to a sleeve of negligible arms. The arms of the governor may be connected to the spindle in the following three ways:

a. The pivot P, may be on the spindle axis
b. The pivot P, may be offset from the spindle axis and the arms when produced intersect at O
c. The pivot P, may be offset, but the arms cross the axis at O



$$F_c \times h = w \times r = m.g.r$$

Where,

m = Mass of the ball in kg,

w = Weight of the ball in newtons = $m.g$,

T = Tension in the arm in newtons,

ω = Angular velocity of the arm and ball about the spindle axis in rad/s,

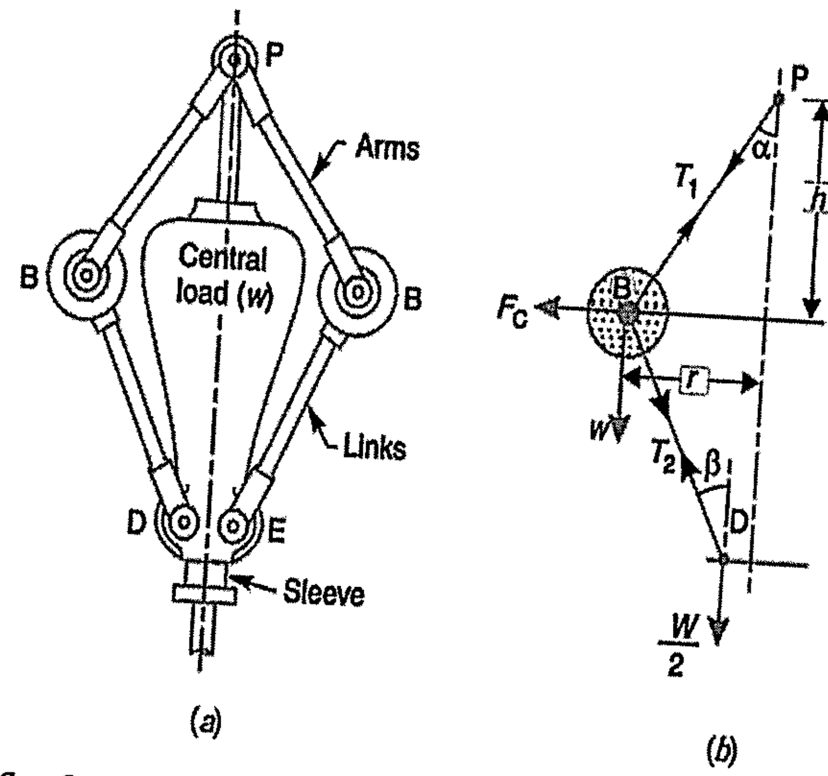
r = Radius of the path of rotation of the ball *i.e.* horizontal distance from the centre of the ball to the spindle axis in metres,

F_C = Centrifugal force acting on the ball in newtons = $m.\omega^2.r$, and

h = Height of the governor in metres.

PORTER GOVERNOR

- If the sleeve of a watt governor loaded with a heavy mass, it becomes a Porter governor.
- In Porter governor central load is attached to the sleeve as shown in fig.
- The load moves up and down the central spindle. This additional downward force increases the speed of revolution required to enable the balls to rise to any predetermined level.



- Consider the force acting on one-half of the governor as shown in fig.

- The relation between height of the governor (h) and the Angular speed of a ball (w) is determined by two methods.

1. Method of resolution of forces:

$$\frac{M.g}{2} + m.g = \frac{F_C}{\tan\alpha} - \frac{M.g}{2} \times \frac{\tan\beta}{\tan\alpha}$$

if, $\tan\alpha = \tan\beta$ or $q = \tan\alpha / \tan\beta = 1$ then,

$$N^2 = \frac{(m+M)}{m} \times \frac{895}{h}$$

2. Instantaneous centre method:

$$F_C \times BM = w \times IM + \frac{W}{2} \times ID$$

$$= m.g \times IM + \frac{M.g}{2} \times ID$$

if, $\tan\alpha = \tan\beta$ or $q = \tan\alpha / \tan\beta = 1$ then,

$$h = \frac{m+M}{m} \times \frac{g}{\omega^2}$$

Where,

m = Mass of each ball in kg,

w = Weight of each ball in newtons = $m.g$,

M = Mass of the central load in kg,

W = Weight of the central load in newtons = $M.g$,

r = Radius of rotation in metres,

h = Height of governor in metres ,

N = Speed of the balls in r.p.m .,

ω = Angular speed of the balls in rad/s = $2\pi N/60$ rad/s,

F_C = Centrifugal force acting on the ball in newtons = $m.\omega^2.r$,

T_1 = Force in the arm in newtons,

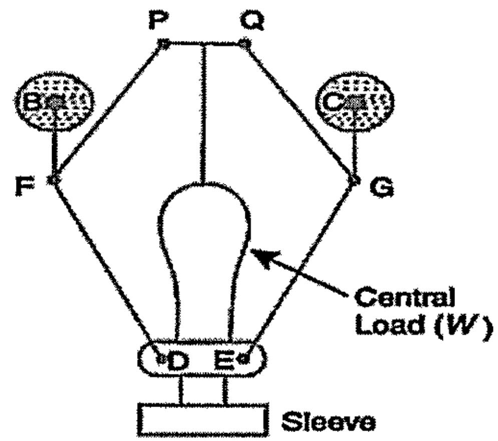
T_2 = Force in the link in newtons,

α = Angle of inclination of the arm (or upper link) to the vertical, and

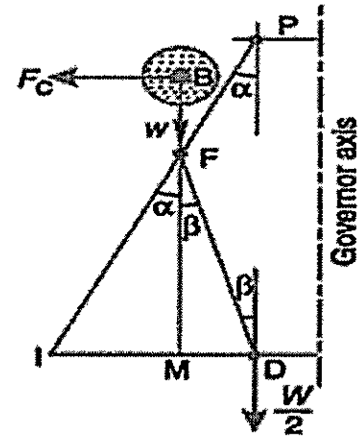
β = Angle of inclination of the link

PROELL GOVERNOR

- The Proell governor has the balls fixed at B and C to the extension of the links DF and EG, as shown in fig. The arms FP and GQ are pivoted at P and Q respectively.



(a)



(b)

we have,

$$F_C = \frac{FM}{BM}\left[m.g\times\frac{IM}{FM}+\frac{M.g}{2}\left(\frac{IM}{FM}+\frac{MD}{FM}\right)\right]$$

HARTNELL GOVERNOR

- A Hartnell governor is a spring loaded governor as shown in fig. It consists of two bell crank levers pivoted at the points O,O to the frame.
- The frame is attached to the governor spindle and therefore rotates with it.
- Each lever carries a ball at the end of the vertical arm OB and a roller at the end of the horizontal arm OR.
- A helical spring in compression provides equal downward forces on the two rollers through a collar on the sleeve.
- The spring force may be adjusted by screwing a nut up or down on the sleeve.

m = Mass of each ball in kg,

M = Mass of sleeve in kg,

r_1 = Minimum radius of rotation in metres,

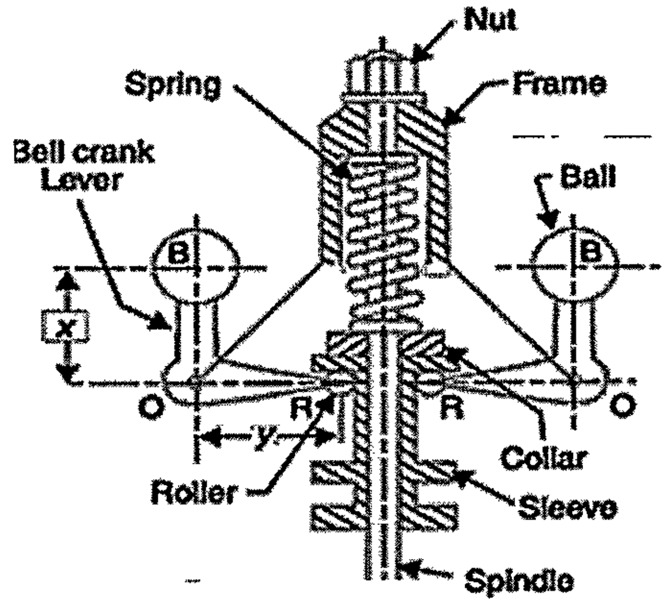
r_2 = Maximum radius of rotation in metres,

ω_1 = Angular speed of the governor at minimum radius in rad/s,

ω_2 = Angular speed of the governor at maximum radius in rad/s,

S_1 = Spring force exerted on the sleeve at ω_1 in newtons,

S_2 = Spring force exerted on the sleeve



F_{C1} = Centrifugal force at ω_1 in newtons = $m(\omega_1)^2 r_1$,

F_{C2} = Centrifugal force at ω_2 in newtons = $m(\omega_2)^2 r_2$,

s = Stiffness of the spring or the force required to compress the spring by one mm,

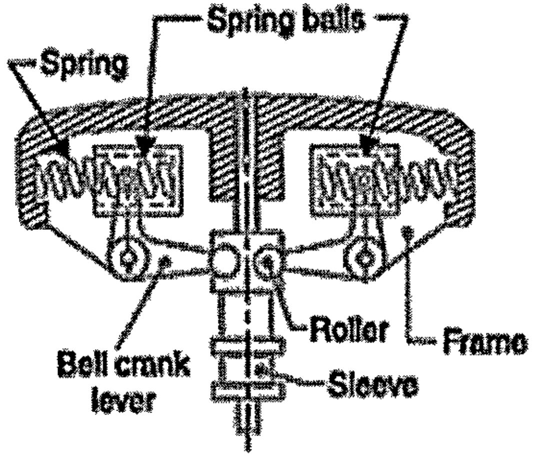
x = Length of the vertical or ball arm of the lever in metres,

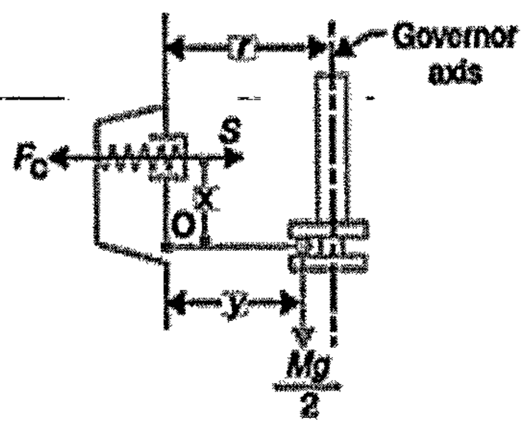
y = Length of the horizontal or sleeve arm of the lever in metres, and

r = Distance of fulcrum O from the governor axis or the radius of rotation when the governor is in mid-position, in metres.

HARTUNG GOVERNOR

- A spring controlled governor of the Hartung type is shown in fig.
- In this type of governor, the vertical arms of the bell crank levers are fitted with spring balls which compress against the frame of the governor when the rollers at the horizontal arm press against the sleeve.





$$F_C \times x = S \times x + \frac{M.g}{2} \times y$$

S = Spring force,

F_C = Centrifugal force,

M = Mass on the sleeve, and

x and y = Lengths of the vertical and horizontal arm of the bell crank lever respectively.

CHARACTERISTICS OF GOVERNOR

Different governors can be compared on the basis of following characteristics:

1. **SENSITIVENESS :**

   The smaller the change in speed from no load to the full load, the more sensitive the governor will be. According to this definition, the sensitiveness of the governor shall be determined by the ratio of speed range to the mean speed. The smaller the ratio more sensitive the governor will be

$$\therefore \quad \text{Sensitiveness} = \frac{N_2 - N_1}{N} = \frac{2(N_2 - N_1)}{(N_2 + N_1)}$$

where N_2-N_1 = speed range from no load to full load.

2. ISOCHRONISM:

A governor is said to be isochronous if equilibrium speed is constant for all the radii of rotation in the working range. Therefore, for an isochronous governor the speed range is zero and this type of governor shall maintain constant speed.

3. HUNTING:

Whenever there is change in speed due to the change in load on the engine, the sleeve moves towards the new position but because of inertia if overshoots the desired position. Sleeve then moves back but again overshoots the desired position due to inertia. This results in setting up of oscillations in engine speed. If the frequency of fluctuations in engine speed coincides with the natural frequency of oscillations due to resonance. The governor, then, tends to intensity the speed variation instead of controlling it. This phenomenon is known as Hunting of the governor. Higher the sensitiveness of the governor, the problem of hunting becomes more acute.

GOVERNOR EFFORT AND POWER

Governor effort and power can be used to compare the effectiveness of different types of governors.

GOVERNOR EFFORT:

It is defined as the mean force exerted on the sleeve during a given change in speed. When governor speed is constant the net force at the sleeve is zero. When governor speed increases, there will be a net force on the sleeve to move it upwards and sleeve starts moving to the new equilibrium position where net force becomes zero.

GOVERNOR POWER:

It is defined as the work done at the sleeve for a given for a given change in speed. Therefore,

Power of governor = Governor effort * Displacement of sleeve

THANK YOU

NARASARAOPETA
NEC ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

MID & ASSIGNMENT EXAMINATION QUESTION PAPERS WITH SCHEME AND SOLUTIONS

NARASARAOPETA ENGINEERING COLLEGE: NARASARAOPET (AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

III B. TECH II – SEMESTER ASSIGNMENT TEST–II, March-2023

SUBJECT: DYNAMICS OF MACHINERY	**DATE:** 02/03/2023
DURATION: 40 MIN	**MAX. MARKS:** 10

Q. No	Questions	Course Outcome (CO)	Knowledge level as per Bloom's Taxonomy	Marks
1	Distinguish between flywheel and governor?	III	Analyzing K4	5
2	Analyse the working of a Hartnell governor With a neat sketch?	III	Analyzing K4	5
3	Analyse the working of a Porter governor With a neat sketch?	III	Applying K3	5
4	Determine the method of balancing of different masses revolving in the same plane.	IV	Evaluating K5	5
5	Analyse the different masses rotating in different planes are balanced?	IV	Analyzing K4	5
6	Analyse the Balancing of a Single Rotating Mass By Two Masses Rotating in Different Planes	IV	Analyzing K4	5

NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

SCHEME OF EVALUATION

III B. TECH II – SEMESTER ASSIGNMENT TEST–II

1. Distinguish between flywheel and governor?

   Diagram ---------------------- 2 M

   Explanation ------------------ 3 M

2. Analyse the working of a Hartnell governor with a neat sketch?

   Types---------------------------- 1 M

   Diagram ----------------------- 2 M

   Explanation ------------------- 2 M

3. Analyse the working of a Porter governor with a neat sketch?

   Types---------------------------- 1 M

   Diagram ----------------------- 2 M

   Explanation ------------------- 2 M

4. Determine the method of balancing of different masses revolving in the same plane.

   Diagram ----------------------- 2 M

   Derivation --------------------- 3 M

5. Analyse the different masses rotating in different planes are balanced?

   Diagram ----------------------- 2 M

   Derivation and Explanation -- 3 M

6. Analyse the Balancing of a Single Rotating Mass By Two Masses Rotating in Different Planes

   Diagram ----------------------- 2 M

   Derivation and Explanation -- 3 M

NARASARAOPETA ENGINEERING COLLEGE (AUTONOMOUS)

Under Graduate | 2022-2023 | III Year II Semester |B.Tech-ME

Assignment-II Exam

R20ME3103 - DYNAMICS OF MACHINERY **Date: 02-03-2023**

S.No.	Roll No./Register No.	Name	Marks (5M)
1	20471A0301	ALAVALA ADITHYA VARA PRASAD	5
2	20471A0302	BATTULA RAJESH	5
3	20471A0303	BHIMAVARAPU HEMANTH KUMAR	5
4	20471A0304	BONAM JAYA PRAKASH	5
5	20471A0305	BOYAPATI PAVAN KUMAR	5
6	20471A0306	DADDANALA VEERANJIREDDY	5
7	20471A0307	DERANGILA PARDHU GANESH	5
8	20471A0308	DOPPALAPUDI S S NAGA RAVITEJA	A
9	20471A0309	EEDARA MOHAN SAI	5
10	20471A0310	GANESH SAI PAVAN	5
11	20471A0312	GERA KOTESWARA RAO	5
12	20471A0313	KARASALA PRASANTH	5
13	20471A0314	KARASANI PAVAN KUMAR REDDY	5
14	20471A0315	KATTA MAHESWAR	5
15	20471A0317	KESARI DHANUNJAYA REDDY	5
16	20471A0318	KOMARAGIRI SASIKUMAR	5
17	20471A0319	KOMERA SIVA NAGARAJU	A
18	20471A0320	KOTHA GOPI	5
19	20471A0321	KUNDURTHI NAVEEN	5
20	20471A0323	MADANU JOSEPH VINAY KUMAR	5
21	20471A0324	MADDUMALA RAMAKRISHNA	5
22	20471A0325	MAGANTI SASI PAVAN	4
23	20471A0326	MAKKENA SAMBASIVA RAO	5
24	20471A0327	MIRIYALA SASHANK	5
25	20471A0328	NALLA ABHIRAM CHOWDARY	5
26	20471A0329	NUTHAKKI RAKESH	5
27	20471A0330	ARAVAPALLI SAI SRINIVAS	4
28	20471A0331	PALETI JOHN HOSANNA	5
29	20471A0332	PERUMAALLA SRIKANTH	5
30	20471A0333	POLURI KRISHNA CHAITHANYA	5
31	20471A0334	PONNAGANTI CHANDU HARSHA VARDHAN	5
32	20471A0336	PATHAN MEERA VALI	A
33	20471A0337	POTTIMURTHI PURNA CHANDRA RAO	5
34	20471A0338	PRUDHVI DURGA BHARATH CHANDAN	5
35	20471A0339	RAMAVATHU BADDUNAIK	5
36	20471A0341	SHAIK APPAPURAM MAHABOOB SUBHANI	5
37	20471A0343	SHAIK GANGARAM ABDUL RAHAMAN	5
38	20471A0344	SHAIK GULLAPALLI NAGURVALI	5

39	20471A0345	SHAIK LAL AHAMAD BASHA	5
40	20471A0346	SHAIK MAHAMMAD FAREED	5
41	20471A0348	SHAIK MANISHA	5
42	20471A0349	SHAIK PARVEZ	5
43	20471A0350	SHAIK SADHIK	5
44	20471A0352	TIPPIREDDY AMARNATHREDDY	5
45	20471A0353	VADLAVALLI GANESH	5
46	20471A0354	VEERAGANDHAM VENKATA MANIKANTA	A
47	20471A0356	ADAKA GOPIRAJU	5

Dr. M. Venkataramana Babu
Name of the Staff Member

Signature of the Staff Member

Signature of the HOD

NARASARAOPETA ENGINEERING COLLEGE (AUTONOMOUS)
Under Graduate | 2022-2023 | III Year II Semester |B.Tech-ME
Assignment-II Exam

R20ME3103 - DYNAMICS OF MACHINERY Date: 02-03-2023

S.No.	Roll No./ Register No.	Name	Marks (5M)
1	20471A0357	ATCHYUTHA PAVAN KUMAR	4
2	20471A0358	BALLE RAMANJANEYULU	5
3	20471A0359	BANDARU SAI GANESH	4
4	20471A0360	BERAM NARENDRA REDDY	2
5	20471A0361	CHEBROLU MANIKANTA SAI NITHIN	5
6	20471A0362	CHENNAMSETTY GOPI	4
7	20471A0363	GANGULA SUNNY	5
8	20471A0364	GANJI HANUMA KOTI GANESH	5
9	20471A0365	GANNNAVARAPU JAYA SRIKANTH	A
10	20471A0366	GUTTIKONDA AYYAPPA REDDY	5
11	20471A0367	MADDINENI AJAY	5
12	20471A0368	MANNEPALLI VEERA NARASIMHA	5
13	20471A0369	MARAGANI NAGA THIRUMALA RAO	3
14	20471A0370	PARELLA BALA GURAVAIAH	3
15	20471A0371	SETLAM RANENDRA VAMSHI	3
16	20471A0372	SHAIK GUTHIKONDA SALIM	5
17	20471A0373	SHAIK JAKIR	5
18	20471A0374	SHAIK MOHAMMAD TAHEER	A
19	20471A0375	THOTA SRIVAMSI NADH	0
20	20471A0376	YAKKANTI SAI KIRAN REDDY	2
21	21475A0301	PALLAPOTHU SAIKIRAN YADAV	5
22	21475A0302	SYED SARDAR VALI	5
23	21475A0303	DERANGULA GOPI KRISHNA	4
24	21475A0304	VADDANI RAKESH	A
25	21475A0305	SHAIK ADIL	5
26	21475A0306	JANAPAREDDI PRASAD	5
27	21475A0307	REPALLE YASHWANTH	2
28	21475A0308	RAMAVATHU PAVAN KUMAR NAIK	2
29	21475A0309	NELAVALLI VIKAS	5
30	21475A0310	DUDDU JOSEPH	4
31	21475A0311	MUNIKOLA SANTHOSH KUMAR	5
32	21475A0312	MORAPAKULA CHARAN TEJA	3
33	21475A0313	GODA SANDEEP	5
34	21475A0314	MOGILI PRAKASH	5
35	21475A0315	SHAIK MABU SUBHANI	5
36	21475A0316	DAGGUPATI VENKATA PRADEEP	5
37	21475A0317	NAGASURENDRA CHARI UPPALAPATI	5

NARASARAOPET ENGINEERING COLLEGE (AUTONOMOUS): NARASARAOPET

DEPARTMENT OF MECHANICAL ENGINEERING

III B.TECH II-SEMESTER MID EXAMINATION-I, FEB-2023

SUBJECT: DYNAMICS OF MACHINERY	**DATE:** 02-02-2023
DURATION: 90 MIN	**MAX MARKS:**25
Sections: A & B	

Q. No	Questions	Course Outcome (CO)	Knowledge Level as Per Bloom's Taxonomy	Marks
1.	The turbine rotor of a ship has a mass of 3500 kg. It has a radius of gyration of 0.45 m and a speed of 3000 r.p.m. clockwise when looking from stern. Determine the gyroscopic couple and its effect upon the ship: 1. when the ship is steering to the left on a curve of 100 m radius at a speed of 36 km/h. 2. when the ship is pitching in a simple harmonic motion, the bow falling with its maximum velocity. The period of pitching is 40 seconds and the total angular displacement between the two extreme positions of pitching is 12 degrees.	CO3	Analyzing (K4)	10
2.	A single plate clutch, effective on both sides, is required to transmit 25 kW at 3000 r.p.m. Determine the outer and inner radii of frictional surface if the coefficient of friction is 0.255, the ratio of radii is 1.25 and the maximum pressure is not to exceed 0.1 N/mm2. Also determine the axial thrust to be provided by springs. Ass ume the theory of uniform wear.	CO4	Applying (K3)	10
3.	**Explain** the working of a watt governer with a neat sketch.	CO5	Applying (K3)	5

NARASARAOPETA
NEC ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

SCHEME OF EVALUATION

III B. TECH II – SEMESTER MID EXAMINATION-I

1. The turbine rotor of a ship has a mass of 3500 kg. It has a radius of gyration of 0.45 m and a speed of 3000 r.p.m. clockwise when looking from stern. Determine the gyroscopic couple and its effect upon the ship: 1. when the ship is steering to the left on a curve of 100 m radius at a speed of 36 km/h. 2. When the ship is pitching in a simple harmonic motion, the bow falling with its maximum velocity. The period of pitching is 40 seconds and the total angular displacement between the two extreme positions of pitching is 12 degrees.

   Given Data------------------- 1 M

   Formulae---------------------2 M

   Diagram --------------------- 2 M

   Problem Solution ------------ 5 M

2. A single plate clutch, effective on both sides, is required to transmit 25 kW at 3000 r.p.m. Determine the outer and inner radii of frictional surface if the coefficient of friction is 0.255, the ratio of radii is 1.25 and the maximum pressure is not to exceed 0.1 N/mm2. Also determine the axial thrust to be provided by springs. Ass ume the theory of uniform wear.

   Given Data------------------- 1 M

   Formulae---------------------2 M

   Diagram --------------------- 2 M

   Problem Solution ------------ 5 M

3. **Explain** the working of a watt governer with a neat sketch.

   Function---------------------- 1 M

   Diagram ---------------------- 5 M

   Explanation ------------------ 4 M

NARASARAOPETA ENGINEERING COLLEGE (AUTONOMOUS) : : NARASARAOPET
(R20) 2020 BATCH III B.TECH II SEM I MID AWARD LIST, FEB - 2023

Branch :ME - A

Subject CODE & NAME:
R20ME3203 - Dynamics of Machinery

Date:
02-02-2023

Sl.No.	H.T.NO.	CO No.	1	2	3	MID - I Total Marks (25M)	MID- I REDUCED MARKS (15 M)	Quiz - I MARKS (10)
		Max.Marks	10	10	5			
		Q.No.	1	2	3			
1	20471A0301		A	A	A	A	A	5
2	20471A0302		9	10	5	24	15	8
3	20471A0303		A	A	A	A	A	10
4	20471A0304		A	A	A	A	A	A
5	20471A0305		5	4	0	9	6	8
6	20471A0306		8	6	4	18	11	5
7	20471A0307		9	10	5	24	15	3
8	20471A0308		9	4	5	18	11	9
9	20471A0309		9	8	0	17	11	7
10	20471A0310		9	7	4	20	12	6
11	20471A0312		9	9	3	21	13	7
12	20471A0313		6	7	4	17	11	8
13	20471A0314		9	9	1	19	12	5
14	20471A0315		9	5	10	24	15	9
15	20471A0317		9	8	5	22	14	10
16	20471A0318		9	5	6	20	12	8
17	20471A0319		8	7	0	15	9	1
18	20471A0320		7	9	5	21	13	8
19	20471A0321		8	6	4	18	11	2
20	20471A0323		9	8	2	19	12	10
21	20471A0324		4	6	7	17	11	8
22	20471A0325		9	5	7	21	13	8
23	20471A0326		8	9	5	22	14	8
24	20471A0327		9	9	5	23	14	9
25	20471A0328		7	7	4	18	11	2
26	20471A0329		8	9	5	22	14	8
27	20471A0330		A	A	A	A	A	A
28	20471A0331		8	6	4	18	11	4

NARASARAOPETA ENGINEERING COLLEGE (AUTONOMOUS) : : NARASARAOPET
(R20) 2020 BATCH III B.TECH II SEM I MID AWARD LIST, FEB - 2023

Branch :ME - B

Subject CODE & NAME:
R20ME3203 - Dynamics of Machinery

Date:
02-02-2023

Sl. No.	H.T.NO.	CO No.	1	2	3	MID - I Total Marks (25M)	MID- I REDUCED MARKS (15 M)	Quiz - I MARKS (10)
		Max.Marks	10	10	5			
		Q.No.	1	2	3			
1	20471A0357		8	9	5	22	14	10
2	20471A0358		10	10	4	24	15	10
3	20471A0359		7	8	2	17	11	9
4	20471A0360		8	8	5	21	13	3
5	20471A0361		9	8	5	22	14	8
6	20471A0362		10	9	5	24	15	10
7	20471A0363		A	A	A	A	A	A
8	20471A0364		9	9	5	23	14	3
9	20471A0365		6	6	0	12	8	2
10	20471A0366		10	6	5	21	13	8
11	20471A0367		A	A	A	A	A	10
12	20471A0368		9	8	4	21	13	9
13	20471A0369		10	9	3	22	14	8
14	20471A0370		4	1		5	3	10
15	20471A0371		10	10	4	24	15	10
16	20471A0372		9	10	5	24	15	8
17	20471A0373		9	9	5	23	14	10
18	20471A0374		9	9	5	23	14	10
19	20471A0375		7	3		10	6	3
20	20471A0376		7	3		10	6	2
21	21475A0301		9	10	5	24	15	1
22	21475A0302		10	10	4	24	15	10
23	21475A0303		10	9	5	24	15	9
24	21475A0304		A	A	A	A	A	A
25	21475A0305		10	10	5	25	15	9
26	21475A0306		10	10	5	25	15	10
27	21475A0307		9	9	5	23	14	4
28	21475A0308		10	9	5	24	15	6
29	21475A0309		10	10	4	24	15	9

30	21475A0310		9	9	5	23	**14**	**9**
31	21475A0311		10	10	4	24	**15**	**9**
32	21475A0312		9	9	5	23	**14**	**10**
33	21475A0313		9	10	5	24	**15**	**7**
34	21475A0314		9	10	5	24	**15**	**7**
35	21475A0315		9	8	5	22	**14**	**10**
36	21475A0316		10	10	4	24	**15**	**9**
37	21475A0317		10	10	5	25	**15**	**10**
38	21475A0318		10	9	5	24	**15**	**9**
39	21475A0319		10	10	4	24	**15**	**9**
40	21475A0320		10	10	4	24	**15**	**10**
41	21475A0321		9	5	9	23	**14**	**7**
42	21475A0322		9	10	5	24	**15**	**10**
43	21475A0323		9	4	10	23	**14**	**8**
44	21475A0324		10	9	5	24	**15**	**10**
45	21475A0325		9	9	5	23	**14**	**9**
46	21475A0326		8	8	5	21	**13**	**10**
47	21475A0327		9	10	5	24	**15**	**10**
48	21475A0328		9	9	5	23	**14**	**6**
49	21475A0329		9	10	5	24	**15**	**10**
50	21475A0330		10	10	4	24	**15**	**10**
51	21475A0331		9	10	5	24	**15**	**9**
52	21475A0332		10	9	5	24	**15**	**10**
53	21475A0333		9	9	5	23	**14**	**10**
54	21475A0334		9	9	5	23	**14**	**9**
55	21475A0335		10	9	5	24	**15**	**4**

Dr. M. Venkataratnam Babu

Name of the Staff Member

Signature of the Staff Member

Signature of the HOD

NARASARAOPET ENGINEERING COLLEGE (AUTONOMOUS): NARASARAOPET

DEPARTMENT OF MECHANICAL ENGINEERING

III B.TECH II-SEMESTER MID EXAMINATION-II, MARCH-2023

SUBJECT: DYNAMICS OF MACHINERY	**DATE:** 29-03-2023
DURATION: 90 MIN	**MAX MARKS:**25
Sections: A & B	

Q. No	Questions	Course Outcome (CO)	Knowledge Level as Per Bloom's Taxonomy	Marks
1.	Compare the Turning Moment Diagram f ning Moment Diagram for a Single Cylinder Double Acting Steam Engine and a Four Stroke Cycle Internal Combustion Engine	CO3	Analyzing (K4)	5
2.	A, B, C and D are four masses carried by a rotating shaft at radii 100, 125, 200 and 150 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the mass of B, C and D are 10 kg, 5 kg, and 4 kg respectively. Determine the required mass A and the relative angular settings of the four masses so that the shaft shall be in complete balance.	CO4	Evaluating (K5)	10
3.	Analysing about the following terms: 1. Primary and Secondary Unbalanced Forces of Reciprocating Masses. 2. Variation of Tractive Force. 3. Swaying Couple. 4. Hammer Blow.	CO5	Analyzing (K4)	10

NARASARAOPETA
NEC ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

SCHEME OF EVALUATION

III B. TECH II – SEMESTER MID EXAMINATION-II

1. Compare the Turning Moment Diagram for a Single Cylinder Double Acting Steam Engine and a Four Stroke Cycle Internal Combustion Engine

   Definition -------------------- 1 M

   Diagrams --------------------- 6 M

   Explanation ------------------3 M

2. A, B, C and D are four masses carried by a rotating shaft at radii 100, 125, 200 and 150 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the mass of B, C and D are 10 kg, 5 kg, and 4 kg respectively. Determine the required mass A and the relative angular settings of the four masses so that the shaft shall be in complete balance.

   Given Data------------------- 1 M

   Formulae----------------------2 M

   Diagram --------------------- 2 M

   Problem Solution ------------ 5 M

3. Analysing about the following terms:
   1. Primary and Secondary Unbalanced Forces of Reciprocating Masses.
   2. Variation of Tractive Force.
   3. Swaying Couple.
   4. Hammer Blow.

   Definitions------------------------------------- 4 M

   Formulae & Explanation ------------------- 6 M

NARASARAOPETA ENGINEERING COLLEGE (AUTONOMOUS) :: NARASARAOPET

(R20) 2020 BATCH III B.TECH II SEM II MID AWARD LIST, March - 2023

Branch :ME - A

Subject CODE & NAME:
R20ME3203 - Dynamics of Machinery

Date:29-03-2023

Sl.No.	H.T.NO.	CO No.	3	4	5	MID - I Total Marks (25M)	MID- I REDUCED MARKS (15 M)	Quiz - I MARKS (10)
		Max.Marks	5	10	10			
		Q.No.	1	2	3			
1	20471A0301		4	8	7	19	12	10
2	20471A0302		5	9	10	24	15	10
3	20471A0303		5	8	6	19	12	4
4	20471A0304		5	10	8	23	14	A
5	20471A0305		2	6	7	15	9	9
6	20471A0306		4	8	7	19	12	10
7	20471A0307		5	9	7	21	13	10
8	20471A0308		5	8	9	22	14	10
9	20471A0309		5	9	9	23	14	10
10	20471A0310		A	A	A	A	A	9
11	20471A0312		5	9	9	23	14	9
12	20471A0313		5	9	8	22	14	9
13	20471A0314		5	10	9	24	15	10
14	20471A0315		5	9	9	23	14	10
15	20471A0317		5	10	7	22	14	9
16	20471A0318		5	9	9	23	14	10
17	20471A0319		A	A	A	A	A	A
18	20471A0320		5	10	9	24	15	10
[illegible]	20471A0321		5	8	8	21	13	7
20	20471A0323		5	9	10	24	15	10
21	20471A0324		5	9	9	23	14	10
22	20471A0325		5	7	7	19	12	10
23	20471A0326		4	9	9	22	14	8
24	20471A0327		5	10	9	24	15	10
25	20471A0328		5	8	10	23	14	5
26	20471A0329		5	4	10	19	12	10
27	20471A0330		4	10	8	22	14	10
28	20471A0331		A	A	A	A	A	10
29	20471A0332		5	9	9	23	14	9

NARASARAOPETA ENGINEERING COLLEGE (AUTONOMOUS) : : NARASARAOPET

(R20) 2020 BATCH III B.TECH II SEM II MID AWARD LIST, March - 2023

Branch :ME - B

Subject CODE & NAME: R20ME3203 - Dynamics of Machinery

Date:29-03-2023

Sl. No.	H.T.NO.	CO No.	3	4	5	MID - I Total Marks (25M)	MID- I REDUCED MARKS (15 M)	Quiz - I MARKS (10)
		Max.Marks	5	10	10			
		Q.No.	1	2	3			
1	20471A0357		5	7	7	19	12	10
2	20471A0358		5	5	9	19	12	10
3	20471A0359		3	7	7	17	11	10
4	20471A0360		5	8	5	18	11	10
5	20471A0361		5	10	9	24	15	10
6	20471A0362		5	9	7	21	13	10
7	20471A0363		5	10	9	24	15	10
8	20471A0364		5	10	0	15	9	10
9	20471A0365		A	A	A	A	A	A
10	20471A0366		5	10	9	24	15	10
11	20471A0367		5	10	9	24	15	10
12	20471A0368		A	A	A	A	A	10
13	20471A0369		0	7	6	13	8	7
14	20471A0370		9	5	9	23	14	10
15	20471A0371		5	8	9	22	14	10
16	20471A0372		5	10	9	24	15	10
17	20471A0373		4	8	8	20	12	10
18	20471A0374		5	9	6	20	12	10
19	20471A0375		4	9	5	18	11	A
20	20471A0376		3	9	7	19	12	10
21	21475A0301		5	8	10	23	14	8
22	21475A0302		5	10	9	24	15	10
23	21475A0303		5	10	7	22	14	10
24	21475A0304		A	A	A	A	A	A
25	21475A0305		5	9	10	24	15	10
26	21475A0306		5	10	9	24	15	10
27	21475A0307		5	9	7	21	13	10
28	21475A0308		5	7	8	20	12	8
29	21475A0309		5	9	10	24	15	10

30	21475A0310		5	8	9	22	14	10
31	21475A0311		5	9	9	23	14	10
32	21475A0312		5	9	8	22	14	10
33	21475A0313		5	8	10	23	14	10
34	21475A0314		5	8	6	19	12	9
35	21475A0315		5	9	8	22	14	10
36	21475A0316		5	9	7	21	13	10
37	21475A0317		5	9	9	23	14	10
38	21475A0318		5	8	9	22	14	10
39	21475A0319		5	5	10	20	12	9
40	21475A0320		5	10	9	24	15	10
41	21475A0321		4	8	8	20	12	10
42	21475A0322		5	9	10	24	15	9
43	21475A0323		5	8	9	22	14	10
44	21475A0324		5	9	9	23	14	10
45	21475A0325		4	9	9	22	14	9
46	21475A0326		3	6	6	15	9	10
47	21475A0327		5	9	8	22	14	10
48	21475A0328		4	7	6	17	11	10
49	21475A0329		5	8	0	13	8	10
50	21475A0330		5	9	10	24	15	10
51	21475A0331		4	9	9	22	14	10
52	21475A0332		5	9	10	24	15	10
53	21475A0333		5	9	10	24	15	10
54	21475A0334		5	9	6	20	12	10
55	21475A0335		5	9	9	23	14	8

Dr. M. VenkataBabu
Name of the Staff Member

Signature of the Staff Member

Signature of the HOD

NARASARAOPETA ENGINEERING COLLEGE: NARASARAOPET (AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

III B. TECH II – SEMESTER ASSIGNMENT TEST–I, January-2023

SUBJECT: DYNAMICS OF MACHINERY	**DATE:** 05/01/2023
DURATION: 40 MIN	**MAX. MARKS:** 10

Q. No	Questions	Course Outcome (CO)	Knowledge level as per Bloom's Taxonomy	Marks
1	Explain about Gyroscopic Couple & Precessional Angular Motion?	I	Analyzing K5	5
2	Explain about the terminology of an Aeroplane and the effect of Gyroscopic Couple on an Aeroplane?	I	Analyzing K5	5
3	The turbine rotor of a ship has a mass of 3500kg .It has a radius of gyration of 0.45m and a speed of 3000 r.p.m. clockwise when looking from stern. Determine the gyroscopic couple and its effect upon the ship, when the ship is steering to the left on a curve of 100m radius at a speed of 36km/h?	I	Applying K3	5
4	Explain about the terminology of a naval ship and the effect of Gyroscopic Couple on naval ship during different movements?	I	Analyzing K5	5
5	An aeroplane makes a complete half circle of 50m radius, towards left, when flying at 200km per hr. The rotary engine and propeller of the plane has a mass of 400kg and a radius of gyration of 0.3m. The engine rotates at 200 r.p.m. clockwise when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it?	I	Applying K3	5
6	A uniform disc of diameter 300mm and of mass 5kg is mounted one end of an arm of length 600mm. The other end of the arm is free to rotate in a universal bearing. If the disc rotates about the arm with a speed of 300 r.p.m. clockwise, looking from the front, with what speed will it precess about the vertical axis?	I	Applying K3	5

NEC NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

SCHEME OF EVALUATION

III B. TECH II – SEMESTER ASSIGNMENT TEST–I

1. Explain about Gyroscopic Couple & Precessional Angular Motion?

   Definition---------------------- 1 M
   Diagram ---------------------- 2 M
   Explanation ------------------ 2 M

2. Explain about the terminology of an Aeroplane and the effect of Gyroscopic Couple on an Aeroplane?

   Terminology--------------------- 2 M
   Diagram ----------------------- 1 M
   Explanation ------------------ 2 M

3. The turbine rotor of a ship has a mass of 3500kg .It has a radius of gyration of 0.45m and a speed of 3000 r.p.m. clockwise when looking from stern. Determine the gyroscopic couple and its effect upon the ship, when the ship is steering to the left on a curve of 100m radius at a speed of 36km/h?

   Given Data--------------------- 1 M
   Formulae-----------------------2 M
   Problem Solution -------------- 2 M

4. Explain about the terminology of a naval ship and the effect of Gyroscopic Couple on naval ship during different movements?

   Terminology-------------------- 2 M
   Diagram ----------------------- 1 M
   Explanation ------------------- 2 M

5. An aeroplane makes a complete half circle of 50m radius, towards left, when flying at 200km per hr. The rotary engine and propeller of the plane has a mass of 400kg and a radius of gyration of 0.3m. The engine rotates at 200 r.p.m. clockwise when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it?

   Given Data--------------------- 1 M
   Formulae-----------------------2 M
   Problem Solution -------------- 2 M

6. A uniform disc of diameter 300mm and of mass 5kg is mounted one end of an arm of length 600mm. The other end of the arm is free to rotate in a universal bearing. If the disc rotates about the arm with a speed of 300 r.p.m. clockwise, looking from the front, with what speed will it precess about the vertical axis?

   Given Data--------------------- 1 M
   Formulae-----------------------2 M
   Problem Solution -------------- 2 M

NARASARAOPETA ENGINEERING COLLEGE (AUTONOMOUS)

Under Graduate | 2022-2023 | III Year II Semester |B.Tech-ME

Assignment-I Exam

R20ME3103 - DYNAMICS OF MACHINERY **Date: 05-01-2023**

S.No.	Roll No./Register No.	Name	Marks (5M)
1	20471A0301	ALAVALA ADITHYA VARA PRASAD	A
2	20471A0302	BATTULA RAJESH	5
3	20471A0303	BHIMAVARAPU HEMANTH KUMAR	3
4	20471A0304	BONAM JAYA PRAKASH	A
5	20471A0305	BOYAPATI PAVAN KUMAR	4
6	20471A0306	DADDANALA VEERANJIREDDY	5
7	20471A0307	DERANGILA PARDHU GANESH	5
8	20471A0308	DOPPALAPUDI S S NAGA RAVITEJA	4
9	20471A0309	EEDARA MOHAN SAI	3
10	20471A0310	GANESH SAI PAVAN	5
11	20471A0312	GERA KOTESWARA RAO	5
12	20471A0313	KARASALA PRASANTH	3
13	20471A0314	KARASANI PAVAN KUMAR REDDY	3
14	20471A0315	KATTA MAHESWAR	5
15	20471A0317	KESARI DHANUNJAYA REDDY	5
16	20471A0318	KOMARAGIRI SASIKUMAR	4
17	20471A0319	KOMERA SIVA NAGARAJU	A
18	20471A0320	KOTHA GOPI	5
19	20471A0321	KUNDURTHI NAVEEN	4
20	20471A0323	MADANU JOSEPH VINAY KUMAR	5
21	20471A0324	MADDUMALA RAMAKRISHNA	3
22	20471A0325	MAGANTI SASI PAVAN	5
23	20471A0326	MAKKENA SAMBASIVA RAO	5
24	20471A0327	MIRIYALA SASHANK	4
25	20471A0328	NALLA ABHIRAM CHOWDARY	4
26	20471A0329	NUTHAKKI RAKESH	4
27	20471A0330	ARAVAPALLI SAI SRINIVAS	4
28	20471A0331	PALETI JOHN HOSANNA	A
29	20471A0332	PERUMAALLA SRIKANTH	4
30	20471A0333	POLURI KRISHNA CHAITHANYA	5
31	20471A0334	PONNAGANTI CHANDU HARSHA VARDHAN	4
32	20471A0336	PATHAN MEERA VALI	A
33	20471A0337	POTTIMURTHI PURNA CHANDRA RAO	5
34	20471A0338	PRUDHVI DURGA BHARATH CHANDAN	5
35	20471A0339	RAMAVATHU BADDUNAIK	4
36	20471A0341	SHAIK APPAPURAM MAHABOOB SUBHANI	5
37	20471A0343	SHAIK GANGARAM ABDUL RAHAMAN	4
38	20471A0344	SHAIK GULLAPALLI NAGURVALI	3

NARASARAOPETA ENGINEERING COLLEGE (AUTONOMOUS)
Under Graduate | 2022-2023 | III Year II Semester |B.Tech-ME
Assignment-I Exam

R20ME3103 - DYNAMICS OF MACHINERY Date: 05-01-2023

S.No.	Roll No./ Register No.	Name	Marks (5M)
1	20471A0357	ATCHYUTHA PAVAN KUMAR	5
2	20471A0358	BALLE RAMANJANEYULU	5
3	20471A0359	BANDARU SAI GANESH	4
4	20471A0360	BERAM NARENDRA REDDY	5
5	20471A0361	CHEBROLU MANIKANTA SAI NITHIN	5
6	20471A0362	CHENNAMSETTY GOPI	5
7	20471A0363	GANGULA SUNNY	5
8	20471A0364	GANJI HANUMA KOTI GANESH	4
9	20471A0365	GANNNAVARAPU JAYA SRIKANTH	3
10	20471A0366	GUTTIKONDA AYYAPPA REDDY	5
11	20471A0367	MADDINENI AJAY	5
12	20471A0368	MANNEPALLI VEERA NARASIMHA	4
13	20471A0369	MARAGANI NAGA THIRUMALA RAO	A
14	20471A0370	PARELLA BALA GURAVAIAH	4
15	20471A0371	SETLAM RANENDRA VAMSHI	4
16	20471A0372	SHAIK GUTHIKONDA SALIM	5
17	20471A0373	SHAIK JAKIR	4
18	20471A0374	SHAIK MOHAMMAD TAHEER	A
19	20471A0375	THOTA SRIVAMSI NADH	5
20	20471A0376	YAKKANTI SAI KIRAN REDDY	5
21	21475A0301	PALLAPOTHU SAIKIRAN YADAV	5
22	21475A0302	SYED SARDAR VALI	5
23	21475A0303	DERANGULA GOPI KRISHNA	5
24	21475A0304	VADDANI RAKESH	A
25	21475A0305	SHAIK ADIL	5
26	21475A0306	JANAPAREDDI PRASAD	5
27	21475A0307	REPALLE YASHWANTH	5
28	21475A0308	RAMAVATHU PAVAN KUMAR NAIK	5
29	21475A0309	NELAVALLI VIKAS	5
30	21475A0310	DUDDU JOSEPH	5
31	21475A0311	MUNIKOLA SANTHOSH KUMAR	5
32	21475A0312	MORAPAKULA CHARAN TEJA	5
33	21475A0313	GODA SANDEEP	5
34	21475A0314	MOGILI PRAKASH	3
35	21475A0315	SHAIK MABU SUBHANI	3
36	21475A0316	DAGGUPATI VENKATA PRADEEP	5
37	21475A0317	NAGASURENDRA CHARI UPPALAPATI	5

38	21475A0318	NALLURI NAVEEN	5
39	21475A0319	ORCHU VENKATA RAVINDRA	5
40	21475A0320	NELLURI YASWANTH	5
41	21475A0321	PENUMALA PAVAN KUMAR	4
42	21475A0322	BAANANA PRADEEP KUMAR	5
43	21475A0323	BOJANKI DEMUDU BABU	5
44	21475A0324	DATTI CHANDU	5
45	21475A0325	BORUGADDA NITHIN	5
46	21475A0326	VARIKUTI KARTHIK VENKATA RAM	4
47	21475A0327	GOLLA SUNDARA SAMRAJYA SUGNAN	5
48	21475A0328	CHATTA VENKATRAMAIAH	5
49	21475A0329	KSHATRIYA JITHENDRA SINGH	5
50	21475A0330	BOMMALI BALA SIVA YOGENDRA SAI NANDU	5
51	21475A0331	REVALLA SAI	5
52	21475A0332	BANDI SRINIVAS	5
53	21475A0333	GURRAM SIVA GANESH	5
54	21475A0334	EMANI LEELA SHANKAR	4
55	21475A0335	KUPPALA SRINU	5

Dr. M. Venkanna Babu
Name of the Staff Member

Signature of the Staff Member

Signature of the HOD

NARASARAOPETA
NEC ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

UNIT WISE IMPORTANT QUESTIONS

DYNAMICS OF MACHINERY
Important Questions

UNIT-I:

1. Explain about Gyroscopic Couple & Precessional Angular Motion?
2. Explain about the terminology of an Aeroplane and the effect of Gyroscopic Couple on an Aeroplane?
3. Explain about the terminology of a Naval Ship and the effect of Gyroscopic Couple on Naval Ship?
4. Explain the effect of the gyroscopic couple on the reaction of the four wheels of a vehicle negotiating a curve?
5. Discuss the effect of the gyroscopic couple on a two wheeled vehicle when taking a turn?
6. **Examples: 14.2 , 14.3, 14.6, 14.10, 14.11, 14.15**

UNIT-II:

1. Describe with a neat sketch the working of a single plate friction clutch?
2. Describe with a neat sketch the working of a multi plate friction clutch.
3. Describe with a neat sketch the working of a Cone clutch and Centrifugal Clutch?
4. Describe with the help of a neat sketch the principles of operation of an internal expanding shoe. Derive the expression for the braking torque?
5. **Examples: 19.1, 19.5, 19.6, 19.7, 19.9, 19.10**

UNIT-III:

1. With a neat sketch, explain the working of a Watt Governor, Porter Governor, Proell Governor, Hartnell governor and Hartung Governor?
2. Differences between flywheel and governor?
3. Compare the Turning Moment Diagram for a Single Cylinder Double Acting Steam Engine and a Four Stroke Cycle Internal Combustion Engine
4. **Examples: 18.1, 18.2, 16.1, 16.2**

UNIT-IV:

1. Explain the method of balancing of different masses revolving in the same plane?
2. How the different masses rotating in different planes are balanced??
3. Explain about the Balancing of a Single Rotating Mass By a Single Mass Rotating in the Same Plane?
4. Explain about the Balancing of a Single Rotating Mass By Two Masses Rotating in Different Planes?
5. **Examples: 21.1, 21.2, 21.3, 21.4, 21.5**

UNIT-V:

1. Analysing about the following terms:
    a. Primary and Secondary Unbalanced Forces of Reciprocating Masses.
    b. Variation of Tractive Force.
    c. Swaying Couple.
    d. Hammer Blow
2. Analyze the Balancing of V- engines?
3. **Examples: 22.1, 22.2, 22.3, 22.7, 22.8,**

MACHINE DYNAMICS AND VIBRATIONS

IMP QUESTIONS

UNIT-I FRICTION

1. State the laws of i) Static friction ii) Dynamic friction iii) Solid friction iv) Fluid friction.
2. Explain the following: (i) Limiting friction, (ii) Angle of friction, and (iii) Coefficient of friction.
3. Discuss briefly the various types of friction experienced by a body.
4. An expression for efficiency of an inclined plane when a body moves up the plane?
5. Derive from first principles an expression for the effort required to raise a load with a screw jack taking friction into consideration.
6. Problem - Friction of a Inclined Plane

Example: An effort of 1500 N is required to just move a certain body up an inclined plane of angle 12°, force acting parallel to the plane. If the angle of inclination is increased to 15°, then the effort required is 1720 N. Find the weight of the body and the coefficient of friction.

7. Problem - Friction of a Screw Jack:

Example: The pitch of 50 mm mean diameter threaded screw of a screw jack is 12.5mm. The coefficient of friction between the screw and the nut is 0.13. Determine the torque required on the screw to raise a load of 25 kN, assuming the load to rotate with the screw. Determine the ratio of the torque required to raise the load to the torque required to lower the load and also the efficiency of the machine.

UNIT-II FRICTION CLUTCHES

1. Write a short note on journal bearing.
2. What is meant by the expression 'friction circle'? Deduce an expression for the radius of friction circle in terms of the radius of the journal and the angle of friction?
3. Derive an expression for the friction moment for a flat collar bearing in terms of the inner radius $r1$, outer radius $r2$, axial thrust W and coefficient of friction μ. Assume uniform intensity of pressure.
4. Describe with a neat sketch the working of a single plate friction clutch.
5. Describe with a neat sketch the working of a cone clutch.

6. Describe with a neat sketch a centrifugal clutch and deduce an equation for the total torque transmitted.
7. Problem – Single Plate Clutches:

   Example: A single plate clutch, with both sides effective, has outer and inner diameters 300 mm and 200 mm respectively. The maximum intensity of pressure at any point in the contact surface is not to exceed 0.1 N/mm2. If the coefficient of friction is 0.3, determine the power transmitted by a clutch at a speed 2500 r.p.m.

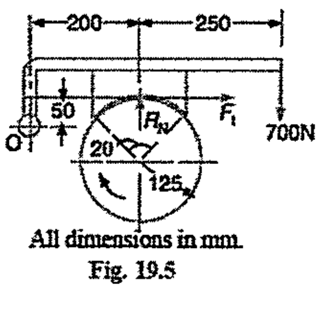
8. Problem – Cone Clutch:

Example: A conical friction clutch is used to transmit 90 kW at 1500 r.p.m. The semi-cone angle is 20° and the coefficient of friction is 0.2. If the mean diameter of the bearing surface is375 mm and the intensity of normal pressure is not to exceed 0.25 N/mm2, find the dimensions of the conical bearing surface and the axial load required.

UNIT-III – BRAKES AND DYNAMOMETERS

1. Distinguish between brakes and dynamometers.
2. Discuss the various types of the brakes.
3. What is the difference between absorption and transmission dynamometers? What are torsion dynamometers?
4. Describe with the help of a neat sketch the principles of operation of an internal expanding shoe. Derive the expression for the braking torque.
5. Describe the construction and operation of a prony brake or rope brake absorption dynamometer.
6. Problem - A single block brake:

   Example: A single block brake is shown in Fig. 19.5.The diameter of the drum is 250 mm and the angle of contact is90°. If the operating force of 700 N is applied at the end of a lever and the coefficient of friction between the drum and the lining is0.35, determine the torque that may be transmitted by the block brake.

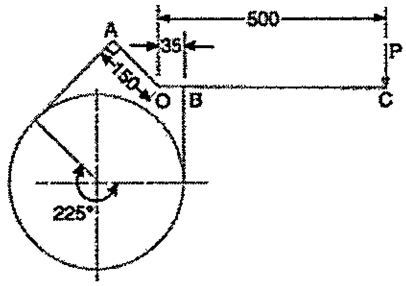


All dimensions in mm.

Fig. 19.5

7. Problem - A differential band brake

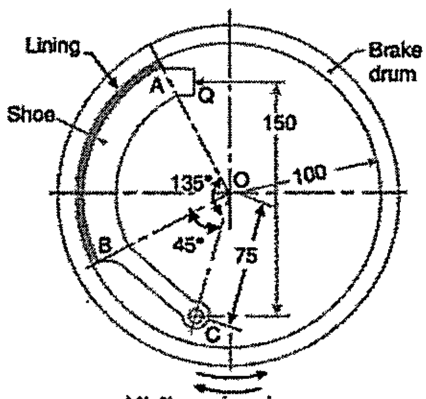
Example: A differential band brake, as shown in Fig. 19.17, has an angle of contact of225°. The band has a compressed woven lining and bears against a cast iron drum of 350 mm diameter. The brake is to sustain a torque of 350 N-m and the coefficient of friction between the band and the drum is 0.3. Find : 1. The necessary force (P) for the clockwise and anticlockwise rotation of the drum; and 2. The value of 'OA' for the brake to be self locking, when the drum rotates clockwise.



All dimensions in mm.
Fig. 19.17

8. Problem – An Internal Expanding friction brake:

Example: The arrangement of an internal expanding friction brake, in which the brake shoe is pivoted at 'C' is shown in Fig. 19.26. The distance 'CO' is 75 mm, O being the centre of the drum. The internal radius of the brake drum is100 mm. The friction lining extends over an arc AB, such that the angle AOC is 135° and angle BOC is 45°. The brake is applied by means of a force at Q, perpendicular to the line CQ, the distance CQ being 150 mm. The local rate of wear on the lining may be taken as proportional to the normal pressure on an element at an angle of 'θ' with OC and may be taken as equal to p1 sin θ, where p1 is the maximum intensity of normal pressure. The coefficient of friction may be taken as 0.4 and the braking torque required is 21 N-m. Calculate the force Q required to operate the brake when 1. The drum rotates clockwise, and 2. The drum rotates anticlockwise.



All dimensions in mm
Fig. 19.26

UNIT-IV – BALANCING

1. Explain clearly the terms 'static balancing' and 'dynamic balancing'. State the necessary conditions to achieve them. (OR) Discuss how a single revolving mass is balanced by two masses revolving in different planes.
2. Explain the method of balancing of different masses revolving in the same plane.
3. How the different masses rotating in different planes are balanced?
4. Write a short note on primary and secondary balancing.
5. Derive the following expressions, for an uncoupled two cylinder locomotive engine : (a)Variation is tractive force; (b) Swaying couple; and (c) Hammer blow.
6. Discuss the balancing of V-engines.
7. Problem - balancing of different masses revolving in the same plane:

   Example: Four masses m1, m2, m3 and m4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required, if its radius of rotation is 0.2 m.
8. Problem - balancing of different masses revolving in different planes:

   Example: A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg,400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.
9. Problem - balancing of Inside Cylinder Locomotive:

   Example: An inside cylinder locomotive has its cylinder centre lines 0.7 m apart and has a stroke of 0.6 m. The rotating masses per cylinder are equivalent to 150 kg at the crank pin, and the reciprocating masses per cylinder to 180 kg. The wheel centre lines are 1.5 m apart. The cranks are at right angles. The whole of the rotating and 2/3 of the reciprocating masses are to be balanced by masses placed at a radius of 0.6 m. Find the magnitude and direction of the balancing masses. Find the fluctuation in rail pressure under one wheel, variation of tractive effort and the magnitude of swaying couple at a crank speed of 300 r.p.m

10. Problem – Multi Cylinder In-line Engine.

Example: A four cylinder vertical engine has cranks 150 mm long. The planes of rotation of the first, second and fourth cranks are 400 mm, 200 mm and 200 mm respectively from the third crank and their reciprocating masses are 50 kg, 60 kg and 50 kg respectively. Find the mass of the reciprocating parts for the third cylinder and the relative angular positions of the cranks in order that the engine may be in complete primary balance.

UNIT-V – MECHANICAL VIBRATIONS

1. Define, in short, free vibrations, forced vibrations and damped vibrations.
2. Discuss briefly with neat sketches the longitudinal, transverse and torsional free vibrations.
3. Derive an expression for the natural frequency of free transverse and longitudinal vibrations by Equilibrium method.
4. Derive the differential equation characterizing the motion of an oscillation system subject to viscous damping and no periodic external force. Assuming the solution to the equation, find the frequency of oscillation of the system. And Explain the terms 'under damping, critical damping' and 'over damping'
5. Explain the term 'Logarithmic decrement' as applied to damped vibrations.
6. Establish an expression for the amplitude of forced vibrations and explain the term 'dynamic magnifier'.
7. What do you understand by Vibration Isolation and Transmissibility?
8. Problem – Natural Frequency of Free Longitudinal And Transverse Vibrations :

   Example: A cantilever shaft 50 mm diameter and 300 mm long has a disc of mass100kg at its free end. The Young's modulus for the shaft material is 200 GN/m2. Determine the frequency of longitudinal and transverse vibrations of the shaft.
9. Problem – Vibratory System With Viscous Damping:

   Example: The measurements on a mechanical vibrating system show that it has amass of 8 kg and that the springs can be combined to give an equivalent spring of stiffness5.4 N/mm. If the vibrating system have a dashpot attached which exerts a force of 40 N when the mass has a velocity of 1 m/s, find: 1. critical damping coefficient, 2. damping factor, 3. Logarithmic decrement, and 4. ratio of two consecutive amplitudes.
10. Problem – Forced Vibrations:

Example: A mass of 10 kg is suspended from one end of a helical spring, the other end being fixed. The stiffness of the spring is 10 N/mm. The viscous damping causes the amplitude to decrease to one-tenth of the initial value in four complete oscillations. If a periodic force of 150 cos 50 t N is applied at the mass in the vertical direction, find the amplitude of the forced vibrations. What is its value of resonance?

UNIT-VI – TRANSVERSE AND TORSIONAL VIBRATIONS

1. Deduce an expression for the natural frequency of free transverse vibrations for a simply supported shaft carrying uniformly distributed mass of m kg per unit length.
2. Establish an expression for the natural frequency of free transverse vibrations for a simply supported beam carrying a number of point loads, by (a) Energy method; and (b) Dunkerley's method.
3. Explain the term 'whirling speed' or 'critical speed' of a shaft. Prove that the whirling spèed for a rotating shaft is the same as the frequency of natural transverse vibration.
4. Derive an expression for the frequency of free torsional vibrations for a shaft fixed at one end and carrying a load on the free end.
5. How the natural frequency of torsional vibrations for a two rotor system is obtained?
6. Describe the method of finding the natural frequency of torsional vibrations for a three rotor system.
7. Establish the expression to determine the frequency of torsional vibrations of a geared system.
8. Problem – Simlpy Supported Beam Carrying Several Loads:

   Example: A shaft 50 mm diameter and 3 metres long is simply supported at the ends and carries three loads of 1000 N, 1500 N and 750 N at 1 m, 2 m and 2.5 m from the left support. The Young's modulus for shaft material is 200 GN/m2. Find the frequency of transverse vibration.
9. Problem – Whirling Speed of the Shaft:

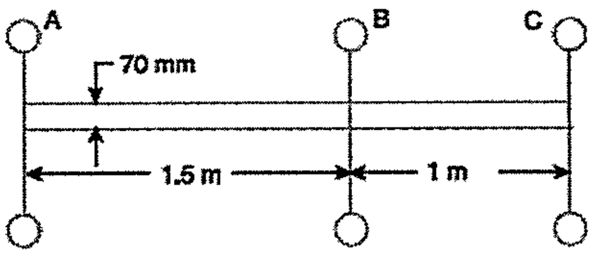
   Example: A shaft 1.5 m long, supported in flexible bearings at the ends carries two wheels each of 50 kg mass. One wheel is situated at the centre of the shaft and the other at a distance of 375 mm from the centre towards left. The shaft is hollow of external diameter 75 mm and internal diameter 40 mm. The density of the shaft material is 7700 kg/m3 and its modulus of elasticity is 200 GN/m2. Find the lowest whirling speed of the shaft, taking into account the mass of the shaft.

10. Problem – Two Rotor System:

Example: A steel shaft 1.5 m long is 95 mm in diameter for the first 0.6 m of its length, 60 mm in diameter for the next 0.5 m of the length and 50 mm in diameter for the remaining 0.4 m of its length. The shaft carries two flywheels at two ends, the first having a mass of 900 kg and 0.85 m radius of gyration located at the 95 mm diameter end and the second having a mass of 700 kg and 0.55 m radius of gyration located at the other end. Determine the location of the node and the natural frequency of free torsional vibration of the system. The modulus of rigidity of shaft material may be taken as 80 GN/m .

11. Problem – Three Rotor System:

Example: A single cylinder oil engine drives directly a centrifugal pump The rotating mass of the engine flywheel and the pump with the shaft is equivalent to a three rotor system as shown in Fig 24 11. The mass moment of inertia of the rotors A B and C are 0 15 0 .3 and 0 09 kg-m2 Find the natural frequency of the torsional vibration The modulus of rigidity for the shaft material is 84 kNImm2.



12. Problem – Geared System:

Example: A motor drives a centrifugal pump through gearing, the pump speed being one-third that of the motor. The shaft from the motor to the pinion is 60 mm diameter and 300 mm long. The moment of inertia of the motor is 400 kg-m2. The impeller shaft is 100 mm diameter and 600 mm long. The moment of inertia of the impeller is 1500 kg-m2. Neglecting inertia of the gears and the shaft, determine the frequency of torsional vibration of the system. The modulus of rigidity of the shaft material is 80 GNIm2?

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ALL THE BEST

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MACHINE DYNAMICS MULTIPLE CHOICE QUESTIONS

1. Which of the following statements is correct about the balancing of a mechanical system?

a) If it is under static balance, then there will be dynamic balance also

b) If it is under dynamic balance, then there will be static balance also

c) Both static as well as dynamic balance have to be achieved separately

d) None of the mentioned **Answer: c**

2. In a locomotive, the ratio of the connecting rod length to the crank radius is kept very large in order to

a) Minimize the effect of primary forces b) Minimize the effect of secondary forces

c) Have perfect balancing d) Start the locomotive conveniently **Answer: b**

3. The magnitude of swaying couple due to partial balance of the primary unbalancing force in locomotive is

a) Inversely proportional to the reciprocating mass

b) Directly proportional to the square of the distance between the centre lines of the two cylinders

c) Inversely proportional to the distance between the centerlines of the two cylinders

d) Directly proportional to the distance between the centerlines of the two cylinders **Answer: d**

4. Which of the following statement is correct?

a) In any engine, 100% of the reciprocating masses can be balanced dynamically

b) In the case of balancing of multicylinder engine, the value of secondary force is higher than the value of the primary force

c) In the case of balancing of multimass rotating systems, dynamic balancing can be directly started without static balancing done to the system

d) None of the mentioned **Answer: c**

5. The partial balancing means

a) Balancing partially the revolving masses b) Balancing partially the reciprocating masses

c) Best balancing of engines d) All of the mentioned **Answer: b**

6. In order to facilitate the starting of locomotive in any position, the cranks of a locomotive, with two cylinders, are placed at ____________ to each other.

a) 45° b) 90° c) 120° d) 180° **Answer: b**

7. If c be the fraction of the reciprocating parts of mass m to be balanced per cyclinder of a steam locomotive with crank radius r, angular speed ω, distance between centre lines of two cylinders a, then the magnitude of the maximum swaying couple is given by

a) 1 – c / 2 mrω2a b) 1 – c / √2mrω2a c) √2(1 – c)mrω2a d) None of the mentioned **Answer: b**

8. The tractive force is maximum or minimum when the angle of inclination of the crank to the line of stroke (θ) is equal to

a) 90° and 225° b) 135° and 180° c) 180° and 225° d) 135° and 315° **Answer: d**

9. The swaying couple is due to the

a) Primary unbalanced force b) Secondary unbalanced force

c) Two cylinders of locomotive d) Partial balancing **Answer: a**

10. In a locomotive, the maximum magnitude of the unbalanced force along the perpendicular to the line of stroke, is known as

a) Tractive force b) Swaying couple c) Hammer blow d) None of the mentioned **Answer: c**

11. The effect of hammer blow in a locomotive can be reduced by

a) Decreasing the speed b) Using two or three pairs of wheels coupled together

c) Balancing whole of the reciprocating parts d) Both (a) and (b) **Answer: d**

12. The unbalanced force caused due to reciprocating mass is given by the equation

a. mrω2 sin θ + mrω2 (sin 2θ/n) b. mrω2 sin θ + mrω2 (cos 2θ/n)

c. mrω2 cos θ + mrω2 (cos 2θ/n) d. mrω2 (sin θ + sin 2θ/n) **Answer: c**

13. At which angle primary unbalanced force in reciprocating engine mechanism is maximum?

a. 0° b. 90° c. 360° d. All of the above **Answer: a**

14. Which of the following conditions should be satisfied for complete balancing of multi-cylinder inline engines?

a. Primary couples should be balanced b. Secondary couples should be balanced

c. Both a. and b. d. None of the above **Answer: c**

15. Hammer blow in locomotives results in

A. Pulsating torque B. Uniform torque

C. Tendency to lift wheel from rails D. Uneven speed **Answer: C**

16. When there is a reduction in amplitude over every cycle of vibration, then the body is said to have

a) Free vibration b) Forced vibration c) Damped vibration d) None of the mentioned **Answer: c**

17. Longitudinal vibrations are said to occur when the particles of a body moves

a) Perpendicular to its axis b) Parallel to its axis

c) In a circle about its axis d) None of the mentioned **Answer: b**

18. During transverse vibrations, shaft is subjected to which type of stresses?

a. Tensile stresses b. Torsional shear stress c. Bending stresses d. All of the above **Answer: c**

19. The natural frequency (in Hz) of free longitudinal vibrations is equal to

a) $1/2\pi\sqrt{s/m}$ b) $1/2\pi\sqrt{g/\delta}$ c) $0.4985/\delta$ d) all of the mentioned **Answer: d**

20. The equation of motion for a vibrating system with viscous damping is

$d^2x/dt^2 + c/m \text{ X } dx/dt + s/m \text{ X } x = 0$ If the roots of this equation are real, then the system will be

a) Over damped b) Under damped c) Critically damped d) None of the mentioned **Answer: a**

21. In under damped vibrating system, if x1 and x2 are the successive values of the amplitude on the same side of the mean position, then the logarithmic decrement is equal to

a) x1/x2 b) Log (x1/x2) c) Loge (x1/x2) d) Log (x1.x2) **Answer: b**

22. The ratio of the maximum displacement of the forced vibration to the deflection due to the static force, is known as

a) Damping factor b) Damping coefficient

c) Logarithmic decrement d) Magnification factor **Answer: d**

23. According to which method, maximum kinetic energy at mean position is equal to maximum potential energy at extreme position?

a. Energy method b. Rayleigh's method c. Equilibrium method d. All of the above **Answer: b**

24. Which among the following is the value of static deflection (δ) for a fixed beam with central point load?

a. $(Wl^3)/(192\ EI)$ b. $(Wl^2)/(192\ EI)$ c. $(Wl^3)/(384\ EI)$ d. None of the above **Answer: a**

25. δ = (W a2b2) / (3 EIl) is the value of deflection for ______

a. simply supported beam which has central point load

b. simply supported beam which has eccentric point load

c. simply supported beam which has U.D.L. point load per unit length

d. fixed beam which has central point load **Answer: b**

26. Determine natural frequency of a system, which has equivalent spring stiffness of 30000 N/m and mass of 20 kg?

a. 12.32 Hz b. 4.10 Hz c. 6.16 Hz d. None of the above **Answer: c**

27. Magnification factor is the ratio of ______

a. zero frequency deflection and amplitude of steady state vibrations

b. amplitude of steady state vibrations and zero frequency deflection

c. amplitude of unsteady state vibrations and zero frequency distribution

d. none of the above **Answer: b**

28. Which type of vibrations are also known as transient vibrations?

a. Undamped vibrations b. Damped vibrations c. Torsional vibrations d. Transverse vibrations

Answer: b

29. Which of the following relations is true for viscous damping?

a. Force α relative displacement b. Force α relative velocity c. Force α (1 / relative velocity)

d. None of the above **Answer: b**

30. In damped free vibrations, which parameters indicate vibrations?

a. Natural frequency b. Rate of decay of amplitude c. Both a. and b. d. None of the above
Answer: c

31. At which frequency ratio, phase angle increases as damping factor increases?

a. When frequency ratio is less than unity b. When frequency ratio is more than unity

c. When frequency ratio is zero d. All of the above **Answer: a**

32. When frequency ratio ($\omega/\omega n$) is greater than unity, phase angle decreases as ______

a. damping factor increases b. damping factor decreases

c. both a. and b. d. none of the above **Answer: a**

33. What is the effect of damping on phase angle at resonance frequency?

a. Phase angle increases as damping increases b. Damping has no effect on phase angle

c. Phase angle increases as damping decreases d. None of the above **Answer: b**

34. Which of the following instruments measure amplitude of a vibrating body?

a. Vibrometers b. Seismometer c. Both a. and b. d. None of the above **Answer: c**

35. In vibration isolation system, if $\omega/\omega n$, then the phase difference between the transmitted force and the disturbing force is

a) 0° b) 90° c) 180° d) 270° **Answer: c**

36. In steady state forced vibrations, the amplitude of vibrations at resonance is ____________ damping coefficient.

a) Equal to b) Directly proportional to c) Inversely proportional to d) Independent of **Answer: c**

37. Critical speed is expressed as

a) Rotation of shaft in degrees b) Rotation of shaft in radians

c) Rotation of shaft in minutes d) Natural frequency of the shaft **Answer: d**

38. he natural frequency of a spring-mass system on earth is ωn. The natural frequency of this system on the moon (gmoon = gearth/6) is

a) ω_n b) $0.408\omega_n$ c) $0.204\omega_n$ d) $0.167\omega_n$ **Answer: a**

39. Which of the following instruments may be used in vibration measurements?

A. Photometer B. Accelerometer C. Dynamometer D. Balometer **Answer: B**

40. A vibrating system with unity as damping factor will be

A. Critically damped B. Damped to safe limits C. Partly damped D.Free from vibrations

Answer: A

41. In S.H.M., acceleration is proportional to

(a) Velocity (b) Displacement (c) Rate of change of velocity (d) All of the above

Ans: b

42. As amplitude of resonant vibrations decreases, degree of damping

a. Increases b. Remains same c. Decreases d. Varies **Answer: c**

43. Oscillations become damped due to

a. Normal force b. Friction c. Tangential force d. Parallel force **Answer: b**

44. Potential energy of system is maximum at

a. Extreme position b. Mean position c. In between extreme and mean position

d. Moderate position **Answer: a**

45. Number of oscillations per unit time is

a. Amplitude b. Wavelength c. Frequency d. Period **Answer: c**

46. Maximum displacement from equilibrium position is

a. Frequency b. Amplitude c. Wavelength d. Period **Answer: b**

47. Over-damping results in

a. Slower return to equilibrium b. Faster return to equilibrium

c. Equilibrium is never achieved d. Arrhythmic return to equilibrium **Answer: a**

48. Our eyes detect oscillations up to

a. 8 Hz b. 9 Hz c. 6 Hz d. 5 Hz **Answer: d**

49. A force that acts to return mass to it's equilibrium position is called

a. Frictional force b. Restoring force c. Normal force d. Contact force **Answer: b**

50. If time period of an oscillation is 0.40 s, then it's frequency is

a. 2 Hz b. 2.5 Hz c. 3 Hz d. 3.5 Hz **Answer: b**

1. The axis of precession is ___________ to the plane in which the axis of spin is going to rotate

Select one:

- a. parallel
- b. perpendicular
- c. spiral
- d. none of the mentioned

2. Which of the following represent the maximum angular acceleration

Select one:

- a. $\alpha_{max} = \Phi(\omega_1)^2$
- b. $\alpha_{max} = \Phi 2\ (\omega_1)^2$
- c. $\alpha_{max} = (\omega_1)^2/\ \Phi$
- d. $\alpha_{max} = (\omega_1)^2/\ \Phi^2$

3. The pitching of the ship is assumed to take place with which of the following motion

Select one:

- a. Constant acceleration
- b. Simple Harmonic Motion
- c. Cycloid
- d. Uniform Velocity

4. Maximum couple during pitching is given by : (ω: angular velocity of rotor & ω_{pmax}: Maximum angular velocity of precession)

Select one:

- a. $C_{max} = I.(\omega p_{max})^2$
- b. $C_{max} = I.(\omega)^2$
- c. $C_{max} = I.\ \omega p_{max}$
- d. $C_{max} = I^2.\ \omega.\ \omega_{pmax}$

5. The friction moment in a clutch with uniform wear as compared to friction moment with uniform pressure is

- a. More
- b. Equal
- c. Less

d. More or less depending on speed

6. The material used for lining of friction surfaces of a clutch should have ____________ coefficient of friction.

Select one:

a. low

b. high

c. medium

d. none of the mentioned

7. The clutch used in scooters is

Select one:

a. Multi-plate clutch

b. Single plate clutch

c. Centrifugal clutch

d. Cone clutch

8. The torque developed by a disc clutch is given by

Select one:

a. $T = 0.25\ \mu.W.R$

b. $T = 0.5\ \mu.W.R$

c. $T = 0.75\ \mu.W.R$

d. $T = \mu.W.R$

9. In case of a multiple disc clutch, if n_1 are the number of discs on the driving shaft and n_2 are the number of the discs on the driven shaft, then the number of pairs of contact surfaces will be

Select one:

a. $n_1 + n_2$

b. $n_1 + n_2 - 1$

c. $n_1 + n_2 + 1$

d. none of the mentioned

10. The cone clutches have become obsolete because of

Select one:

a. small cone angles

b. exposure to dirt and dust

c. difficulty in disengaging

d. all of the mentioned

11. The axial force (W_e) required for engaging a cone clutch is given by

Select one:

a. $W_n \sin \alpha$

b. $W_n (\sin \alpha + \mu \cos \alpha)$

c. $W_n (\sin \alpha + 0.25 \mu \cos \alpha)$

d. none of the mentioned

12. A disc is a spinning with an angular velocity ω rad/s about the axis of spin. The couple applied to the disc causing precession will be

Select one:

a. $1/2 I\omega^2$

b. $I\omega^2$

c. $1/2 I\omega\omega_p$

d. $I\omega\omega_p$

13. . In a centrifugal clutch, the force with which the shoe presses against the driven member is the __________ of the centrifugal force and the spring force

Select one:

a. difference

b. sum

c. ratio

d. none of the mentioned

14. Brakes is absorb .... types of energy.

Select one:

a. Potential energy

b. Kinetic energy

c. either A or B

d. Strain energy

15. When brakes are applied on a moving vehicle; the kinetic energy is converted to

Select one:

- a. Mechanical energy
- b. Heat energy
- c. Electrical energy
- d. Potential energy.

16. Hydraulic brakes function on the principle of

Select one:

- a. Law of conservation of momentum
- b. Law of conservation of energy
- c. Pascal's law
- d. None of the above.

17. The following factor(s) contribute to the effectiveness of the brakes

Select one:

- a. Area of brake linings
- b. Radius of car wheel
- c. Amount of pressure applied to shoe brakes
- d. All of the above

18. The torque which a clutch can transmit, depends upon the

Select one:

- a. coefficient of friction
- b. spring force
- c. contact surfaces
- d. all of the above

19. Disc brakes are .... brake

Select one:

- a. Axial
- b. Radial
- c. either A or B
- d. None of the above.

20. Which of the following brake is not a types of drum breaks

Select one:

- a. Internal expanding brake
- b. External expanding brake
- c. Disc brake
- d. All of above these

21. In brake friction and wear occurs on ....

Select one:

- a. Friction lining
- b. Brake drum
- c. Both A and B
- d. None of these

22. In railway trains which types of brakes used

Select one:

- a. Band brake
- b. Block brake
- c. Internal expanding brake
- d. External expanding brake

23. The engine of an aero plane rotates in clockwise direction when seen from the tail end and the aero plane takes a turn to the left. The effect of gyroscopic couple on the aero plane will

Select one:

- a. to dip the nose and tail
- b. to raise the nose and tail
- c. to raise the nose and dip of the tail
- d. to dip the nose and raise the tail

24.Band brake system generally used in..........

Select one:

- a. Hoists
- b. Chain saws
- c. Bucket conveyor
- d. All of above these

25. Which of the following is a characteristic feature of a dynamometer?

Select one:

- a. It can measure torque
- b. It can measure frictional resistance
- c. It can measure the balancing force
- d. It can act as a speedometer.

26. In which of the following dynamometers does the entire energy or power produced by the engine is absorbed by the friction resistances of the brake?

Select one:

- a. Prony brake dynamometer
- b. Torsional dynamometer
- c. Epicyclic train dynamometer
- d. Belt transmission dynamometer

27. In which of the following dynamometers does the energy produced by the engine is used for doing work?

Select one:

- a. Prony brake dynamometer
- b. Rope brake dynamometer
- c. Absorption dynamometer
- d. Epicyclic train dynamometer

28. For a rope brake dynamometer, the flywheel is cooled with soapy water because________

Select one:

- a. Energy is absorbed by the dynamometer
- b. Energy is used to do work
- c. Energy is provided by the motor
- d. Energy supplied is more

29. Belt transmission dynamometer is an absorption dynamometer

Select one:

- a. True
- b. False
- c. neither a nor b

d. either a or b

30. The height of a Watt's governor is

Select one:

a. directly proportional to speed

b. inversely proportional to speed

c. neither a nor b

d. independent of speed

31. When the sleeve of a porter governor moves upwards, the governor speed

Select one:

a. increases b

b. decreases

c. remains unaffected

d. first increases and then decreases

32. A Hartnell governor is a

Select one:

a. dead weight governor

b. pendulum type governor

c. spring loaded governor

d. inertia governor

33. The height of a Watt's governor is equal to

Select one:

a. $8.95/N^2$

b. $89.5/N^2$

c. $895/N^2$

d. $8950/N^2$

34. The engine of an aero plane rotates in clockwise direction when seen from the tail end and the aero plane takes a turn to the right. The effect of gyroscopic couple on the aero plane will be to dip the nose and raise the tail.

1)True 2) False

Select one:

- a. 1
- b. 2
- c. 1&2
- d. None

35. For isochronouns, spring controlled governor, the controlling force with increase in radius of rotation.......

Select one:

- a. increase
- b. decreases
- c. remains constant
- d. behaves in unpredictable way

36. . For a machine to be self-locking, its efficiency should be.......

Select one:

- a. 100%
- b. less than 67%
- c. less than 50%
- d. more than 50%

37. Hartnell governor could be classified under the head of.......

Select one:

- a. inertia type governors
- b. pendulum type governors
- c. centrifugal type governors
- d. dead weight type governors

38. Effect of friction, at the sleeve of a centrifugal governor is to make it

Select one:

- a. More sensitive
- b. More stable
- c. Insensitive over a small range of speed
- d. Unstable

39. When the speed of the governor decreases, which of the following effect does friction causes?

Select one:

- a. Prevents downward movement of sleeve
- b. Prevents upward movement of sleeve
- c. Prevents radial outward movement of balls
- d. Increases downward movement of sleeve

40. If the controlling force line for a spring controlled governor when produced intersects the Y-axis at the origin, then the governor is said to be

Select one:

- a. stable
- b. unstable
- c. isochronous
- d. none of the mentioned

41. The relation between the controlling force (F_c) and radius of rotation (r) for a stable spring controlled governor is

Select one:

- a. $F_c = ar + b$
- b. $F_c = ar - b$
- c. $F_c = ar$
- d. $F_c = a/r + b$

42. A Hartnell governor has its controlling force (F_c) given by $F_c = ar + b$, where r is the radius of rotation and a and b are constants. The governor becomes isochronous when

Select one:

- a. a is + ve and b = 0
- b. a = 0 and b is +ve
- c. a is +ve and b is –ve
- d. a is +ve and b is also +ve

43. Calculate the vertical height of a Watt governor when it rotates at 60 r.p.m.

Select one:

- a. 0.548 m

b. 0.428 m

c. 0.448 m

d. 0.248 m

44. In a governor, if the equilibrium speed is constant for all radii of rotation of balls, the governor is said to be

Select one:

a. Stable

b. unstable

c. inertial

d. isochronous

45. The steering of a ship means

Select one:

a. movement of a complete ship up and down in vertical plane about transverse axis

b. turning of a complete ship in a curve towards right or left, while it moves forward

c. rolling of a complete ship side-ways

d. none of the mentioned

46. In a spring controlled governor, when the controlling force ____________ as the radius of rotation increases, it is said to be a stable governor.

Select one:

a. remains constant

b. decreases

c. increases

d. none of the mentioned

47. The rotor of a ship rotates in clockwise direction when viewed from stern and the ship takes a left turn. The effect of gyroscopic couple acting on it will be

Select one:

a. to raise the bow and stern

b. to lower the bow and stern

c. to raise the bow and lower the stern

d. to raise the stern and lower the bow

NEC NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

PREVIOUS QUESTION PAPERS

NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

III B.Tech II Semester Regular Examinations

Sub Code: R20ME3203 SUBJECT: DYNAMICS OF MACHINERY

(ME)

MODEL PAPER-I

Time: 3 hours | Max. Marks: 70

Note: Answer All **FIVE** Questions.
All Questions Carry Equal Marks (5 X 14 = 70M)

Q. NO		QUESTION	KL	CO	MARKS
		Unit - I			
1	a	The rotor of the turbine of a ship makes 1500 rpm clockwise when viewed from the stern. The rotor has a mass of 800 kg and its radius of gyration is 300 mm. Find the maximum gyro-couple transmitted to the hull when the ship pitches with maximum angular velocity of 1 rad/s.	K4	CO1	[10M]
		Explain about Gyroscopic Couple & Precessional Angular Motion?	K3	CO1	[4M]
		OR			
	b	Identify the physical terms used to explain gyroscopic action on the Aero plane.	K3	CO1	[14M]
		Unit - II			
2	a	Describe with a neat sketch the working of a single plate friction clutch.	K3	CO2	[14M]
		OR			
	b	Describe with the help of a neat sketch the principles of operation of an internal expanding shoe. Derive the expression for the braking torque.	K4	CO2	[10M]
		What are the various types of the brakes?	K2	CO2	[4M]
		Unit - III			
3	a	With a neat sketch, explain the working of a Hartnell governor.	K3	CO3	[7M]
		A Porter governor carries a central load of 30Kgf and each ball weighs 4.5kgf. The upper links are 20cm long and the lower links are 30cm long. The points of suspension of upper and lower links are 5cm from axis of spindle. Calculate: i. The speed of the governor in rpm if the radius of revolution of the governor ball is 12.5cm, and ii. The effort of the governor for increase of speed of 1%.	K3	CO3	[7M]
		OR			
	b	Differences between flywheel and governor?	K3	CO3	[7M]
		Explain about the turning moment diagrams	K3	CO3	[7M]
		Unit - IV			
4	a	A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg,400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X	K4	CO4	[14M]

		and Y is 400mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.			
		OR			
	b	Explain the method of balancing of different masses revolving in the same plane.	K3	CO4	[7M]
		How the different masses rotating in different planes are balanced?	K3	CO4	[7M]
5		Unit - V			
	a	Analyze the Balancing of V- engines	K4	CO5	[7M]
		Write a short note on primary and secondary balancing.	K3	CO5	[7M]
		OR			
	b	A four cylinder vertical engine has cranks 150 mm long. The planes of rotation of the first, second and fourth cranks are 400 mm, 200 mm and 200 mm respectively from the third crank and their reciprocating masses are 50 kg, 60 kg and 50 kg respectively. Find the mass of the reciprocating parts for the third cylinder and the relative angular positions of the cranks in order that the engine may be in complete primary balance	K4	CO5	[14M]

NEC NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

III B.Tech II Semester Regular Examinations

Sub Code: R20ME3203 **SUBJECT NAME: DYNAMICS OF MACHINERY**

(ME)

MODEL PAPER-II

Time: 3 hours Max. Marks: 70

Note: Answer All **FIVE** Questions.
All Questions Carry Equal Marks (5 X 14 = 70M)

Q. NO		QUESTION	KL	CO	MARKS
1		Unit - I			
	a	Explain about the terminology of an Aeroplane and the effect of Gyroscopic Couple on an Aeroplane?	K3	CO1	[7M]
		Explain the gyroscopic effect on four wheeled vehicles.	K3	CO1	[7M]
		OR			
	b	An aeroplane makes a complete half circle of 50m radius, towards left, when flying at 200km per hr. The rotary engine and propeller of the plane has a mass of 400kg and a radius of gyration of 0.3m. The engine rotates at 200 r.p.m. clockwise when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it?	K4	CO1	[14M]
2		Unit - II			
	a	Describe with a neat sketch the working of a multi plate friction clutch.	K3	CO2	[7M]
		A car engine has its rated output of 12 kW. The maximum torque developed is 100 N-m. The clutch used is of single plate type having two active surfaces. The axial pressure is not to exceed 85 KN/m2.The external diameter of the friction plate is 1.25 times the internal diameter. Determine the dimensions of the friction plate and the axial force exerted by the springs. Coefficient of friction =	K4	CO2	[7M]

		0.3.?			
		OR			
	b	A simple band brake operates on a drum of 600 mm in diameter that is running at 200 RPM. The coefficient of friction is 0.25. The brake band has a contact of 270°, one end is fastened to a fixed pin and the other end to the brake arm 125 mm from the fixed pin. The straight brake arm is 750 mm long and placed perpendicular to the diameter that bisects the angle of contact. a) What is the pull necessary on the end of the brake arm to stop the wheel if 35 kW is being absorbed? What is the direction for this minimum pull? b) What width of steel band of 2.5 mm thick is required for this brake if the maximum tensile stress is not to exceed 50 N/mm2?	K4	CO2	[14M]
3		**Unit - III**			
	a	Explain the difference in the construction features of a Watt governor, Porter governor, and Proell governor.	K3	CO3	[14M]
		OR			
	b	In a turning moment diagram the areas above and below the mean torque line taken in order are 5.81, 3.23, 3.87, 5.16, 1.94, 3.87, 2.58, and 1.94 sq-cm respectively. The scales of the turning moment diagram are: Turning moment 1 cm is equal to 700 kg/m Crank angle 1 cm is equal to 60 deg. The mean speed of engine is 1200 rpm and the variation of speed must not exceed ± 3 percent of the mean speed. Assuming the radius of gyration of the flywheel to be 106.67 cm, find the weight of flywheel to keep the speed within the given limits..	K4	CO3	[14M]
4		**Unit - IV**			
	a	Three masses P, Q and R with masses 12 kg, 11 kg and 18 kg respectively revolve in the same plane at radii 120 mm, 144 mm and 70 mm respectively. The angular position of Q and R are 600 and 1350 from P. Determine the position and magnitude of mass S at radius 152 mm to balance the system.	K3	CO4	[14M]
		OR			
	b	Four masses m1, m2, m3 and m4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required, if its radius of rotation is 0.2 m.	K4	CO4	[14M]
5		**Unit - V**			
	a	Explain the terms: variation of tractive force, swaying couple, and hammer blow	K3	CO5	[14M]
		OR			
	b	An inside cylinder locomotive has its cylinder centre lines 0.7 m apart and has a stroke of 0.6 m. The rotating masses per cylinder are equivalent to 150 kg at the crank pin, and the reciprocating masses per cylinder to 180 kg. The wheel centre lines are 1.5 m apart. The cranks are at right angles. The whole of the rotating and 2/3 of the reciprocating masses are to be balanced by masses placed at a radius of 0.6 m. Find the magnitude and direction of the balancing masses. Find the fluctuation in rail pressure under one wheel, variation of tractive effort and the magnitude o sfwaying couple at a crank speed of 300 r.p.m.	K4	CO5	[14M]

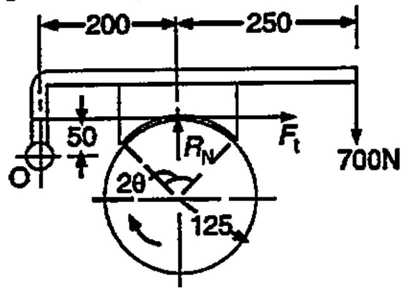
NEC **NARASARAOPETA ENGINEERING COLLEGE**
(AUTONOMOUS)

III B.Tech II Semester Regular Examinations, April-2023

Sub Code: R20ME3203 **DYNAMICS OF MACHINERY** (ME)

Time: 3 hours Max. Marks: 70

Note: Answer All **FIVE** Questions.
All Questions Carry Equal Marks (5 X 14 = 70M)

Q.No		Questions	KL	CO	M
		Unit-I			
1	a	The turbine rotor of a ship has a mass of 2000 kg and rotates at a speed of 3000 r.p.m. clockwise when looking from a stern. The radius of gyration of the rotor is 0.5 m. Determine the gyroscopic couple and its effects upon the ship when the ship is steering to the right in a curve of 100 m radius at a speed of 16.1 knots (1 knot = 1855 m/hr). Calculate also the torque and its effects when the ship is pitching in simple harmonic motion, the bow falling with its maximum velocity. The period of pitching is 50 seconds and the total angular displacement between the two extreme positions of pitching is 12°. Find the maximum acceleration during pitching motion.	K3	CO1	14M
		OR			
	b	Define gyroscopic effect. With usual notation and diagram, derive an expression for gyroscopic couple produced by a rotating disc.	K1	CO1	14M
		Unit-II			
2	a	I) Describe with a neat sketch the working of a single plate friction clutch	K3	CO1	7M
		ii) Discuss the various types of brakes with neat sketches.	K2	CO1	7M
		OR			
	b	i) A single block brake is shown in **Figure.1**. The diameter of the drum is 250 mm and the angle of contact is 90°. If the operating force of 700 N is applied at the end of a lever and the coefficient of friction between the drum and the lining is 0.35, determine the torque that may be transmitted by the block brake.  **All dimensions in mm.** ***Figure.1.***	K3	CO2	7M
		ii) A single plate clutch, with both sides effective, has outer and inner diameters 300 mm and 200 mm respectively. The maximum intensity of pressure at any point in the contact surface is not to exceed 0.1 N/mm^2. If the coefficient of friction is 0.3, determine the power transmitted by a clutch at a speed 2500 r.p.m.	K2	CO2	7M
3		**Unit-III**			
	a	i) Define and explain the following terms relating to governors: 1. Stability, 2. Sensitiveness, 3. Isochronism, and 4. Hunting	K2	CO3	7M
		ii) A Porter governor has all four arms 250 mm long. The upper arms are attached on the axis of rotation and the lower arms are attached to the sleeve at a distance of 30 mm from the axis. The mass of each ball is 5 kg and the sleeve has a mass of 50kg. The extreme radii of rotation are 150 mm and 200 mm. Determine the range of speed of the governor.	K3	CO3	7M

		OR			
	b	i) Compare the differences between governors and flywheel	**K3**	**CO3**	**7M**
		ii) Explain the turning moment diagram of a four-stroke cycle internal combustion engine.	**K2**	**CO3**	**7M**
		Unit-IV			
4	**a**	A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.	**K5**	**CO4**	**14M**
		OR			
	b	i) Explain the method of balancing of different masses revolving in the same plane.	**K2**	**CO4**	**7M**
		ii) Justify the need of balancing of rotating parts for high-speed engine. What is the difference between static and dynamic balancing?	**K2**	**CO4**	**7M**
		Unit-V			
5	**a**	i) A single cylinder reciprocating engine has speed 240 r.p.m., stroke 300 mm, mass of reciprocating parts 50 kg, mass of revolving parts at 150 mm radius 37 kg. If **two-third** of the reciprocating parts and all the revolving parts are to be balanced, Determine the followings: **1.** The balance mass required at a radius of 400 mm, and **2.** The residual unbalanced force when the crank has rotated 60° from top dead centre	**K3**	**CO5**	**7M**
		ii) The reciprocating mass per cylinder in a 60° V-twin engine is 1.5 kg. The stroke and connecting rod length are 100 mm and 250 mm respectively. If the engine runs at 2500 r.p.m., determine the maximum and minimum values of the primary and secondary forces. Also find out the crank position corresponding these values.	**K4**	**CO5**	**7M**
		OR			
	b	The following data refer to two-cylinder locomotive with cranks at 90° : Reciprocating mass per cylinder = 300 kg Crank radius = 0.3 m Driving wheel diameter = 1.8 m Distance between cylinder centre lines = 0.65 m Distance between the driving wheel central planes = 1.55 m. Determine the followings: **1.** the fraction of the reciprocating masses to be balanced if the hammer blow is not to exceed 46 kN at 96.5 km. p.h. **2.** the variation in tractive effort; and **3.** the maximum swaying couple.	**K3**	**CO5**	**14M**

KL: Blooms Taxonomy Knowledge Level CO: Course Outcome M:Marks

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NARASARAOPETA ENGINEERING COLLEGE (AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

III BTech II Sem Regular Exam, April-2023

DYNAMICS OF MACHINERY

PART-A

1. (a) ANS:

Solution. Given : $m = 2000$ kg ; $N = 3000$ r.p.m. or $\omega = 2\pi \times 3000/60 = 314.2$ rad/s ; $k = 0.5$ m ; $R = 100$ m ; $v = 16.1$ knots $= 16.1 \times 1855 / 3600 = 8.3$ m/s

Gyroscopic couple

We know that mass moment of inertia of the rotor,

$$I = m.k^2 = 2000\ (0.5)^2 = 500 \text{ kg-m}^2$$

and angular velocity of precession,

$$\omega_p = v/R = 8.3/100 = 0.083 \text{ rad/s}$$

$\therefore$ Gyroscopic couple,

$$C = I.\omega.\omega_p = 500 \times 314.2 \times 0.083 = 13\ 040 \text{ N-m} = 13.04 \text{ kN-m}$$

We have discussed in Art. 14.6, that when the rotor rotates clockwise when looking from a stern and the ship steers to the right, the effect of the reactive gyroscopic couple is to raise the stern and lower the bow. Ans.

Torque during pitching

Given : $t_p = 50$ s ; $2\phi = 12°$ or $\phi = 6° \times \pi/180 = 0.105$ rad

We know that angular velocity of simple harmonic motion,

$$\omega_1 = 2\pi / t_p = 2\pi / 50 = 0.1257 \text{ rad/s}$$

and maximum angular velocity of precession,

$$\omega_{Pmax} = \phi.\omega_1 = 0.105 \times 0.1257 = 0.0132 \text{ rad/s}$$

$\therefore$ Torque or maximum gyroscopic couple during pitching,

$$C_{max} = I.\omega.\omega_{P\,max} = 500 \times 314.2 \times 0.0132 = 2074 \text{ N-m Ans.}$$

We have discussed in Art. 14.7, that when the pitching is downwards, the effect of the reactive gyroscopic couple is to turn the ship towards port side.

Maximum acceleration during pitching

We know that maximum acceleration during pitching

$$\alpha_{max} = \phi\ (\omega_1)^2 = 0.105\ (0.1257)^2 = 0.00166 \text{ rad/s}^2 \text{ Ans.}$$

(b) ANS:

Consider a disc spinning with an angular velocity ω rad/s about the axis of spin *OX*, in anticlockwise direction when seen from the front, as shown in Fig. 14.2 (*a*). Since the plane in which the disc is rotating is parallel to the plane *YOZ*, therefore it is called *plane of spinning*. The plane *XOZ* is a horizontal plane and the axis of spin rotates in a plane parallel to the horizontal plane about an axis *OY*. In other words, the axis of spin is said to be rotating or processing about an axis *OY*. In other words, the axis of spin is said to be rotating or processing about an axis *OY* (which is perpendicular to both the axes *OX* and *OZ*) at an angular velocity ω_P rap/s. This horizontal plane *XOZ* is called *plane of precession* and *OY* is the *axis of precession*.

Let I = Mass moment of inertia of the disc about *OX*, and

ω = Angular velocity of the disc.

∴ Angular momentum of the disc

$= I.\omega$

Since the angular momentum is a vector quantity, therefore it may be represented by the vector $\vec{ox}$, as shown in Fig. 14.2 (*b*). The axis of spin *OX* is also rotating anticlockwise when seen from the top about the axis *OY*. Let the axis *OX* is turned in the plane *XOZ* through a small angle δθ radians to the position OX', in time δ*t* seconds. Assuming the angular velocity ω to be constant, the angular momentum will now be represented by vector ox'.

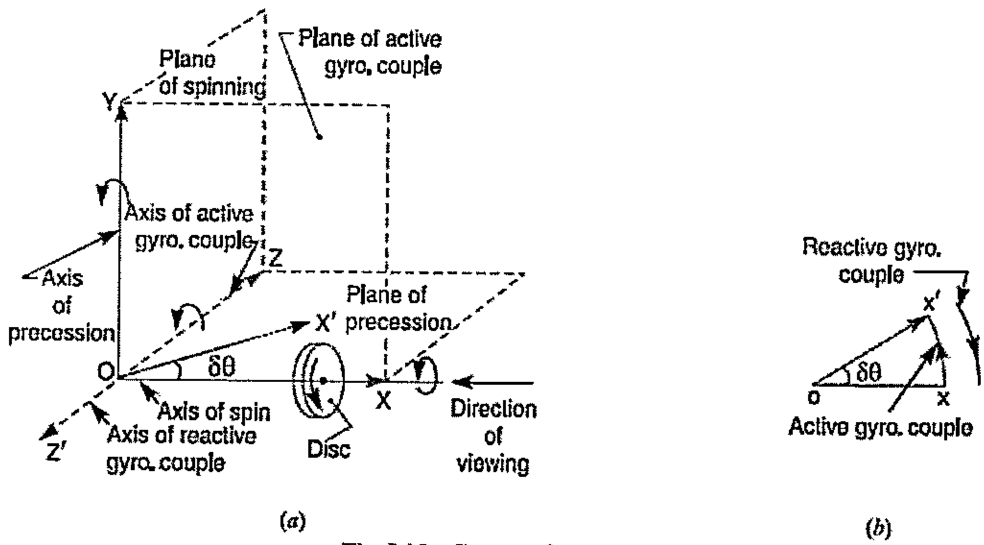


Fig. 14.2. Gyroscopic couple.

∴ Change in angular momentum

$$= \vec{ox'} - \vec{ox} = \vec{xx'} = \vec{ox}.\delta\theta \qquad \text{...(in the direction of } \vec{xx'})$$

$$= I.\omega.\delta\theta$$

and rate of change of angular momentum

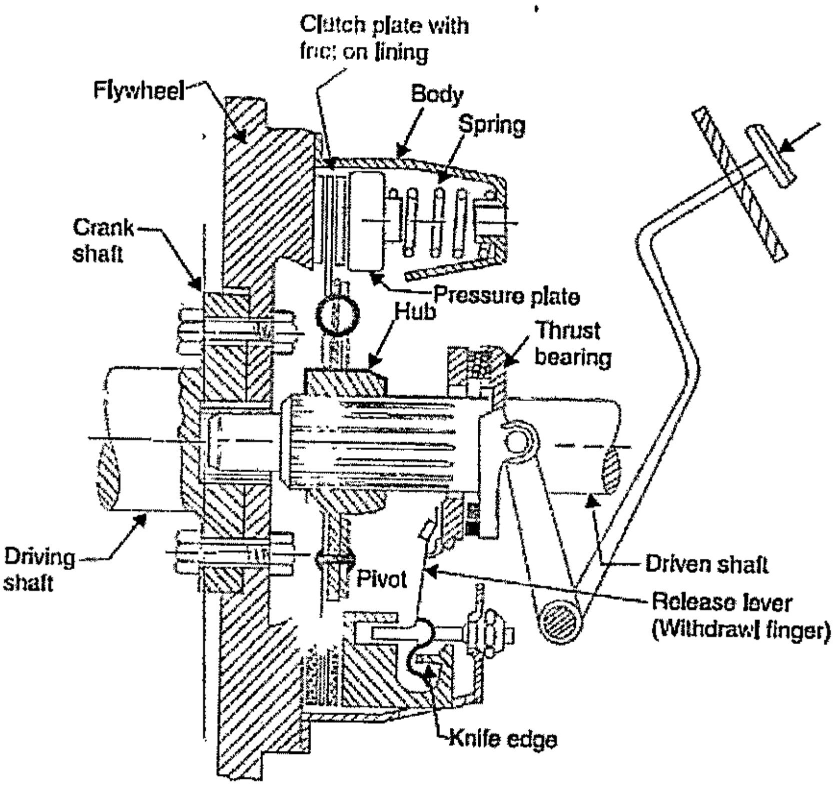
$$= I.\omega \times \frac{\delta\theta}{dt}$$

Since the rate of change of angular momentum will result by the application of a couple to the disc, therefore the couple applied to the disc causing precession,

$$C = \underset{\delta t \to 0}{\text{Lt}}\; I.\omega \times \frac{\delta\theta}{\delta t} = I.\omega \times \frac{d\theta}{dt} = I.\omega.\omega_P \qquad \ldots\left(\because \frac{d\theta}{dt} = \omega_P\right)$$

2. (a) ANS:

A single disc or plate clutch, as shown in Fig. 10.21, consists of a clutch plate whose both sides are faced with a friction material (usually of Ferrodo). It is mounted on the hub which is free to move axially along the splines of the driven shaft. The pressure plate is mounted inside the clutch body which is bolted to the flywheel. Both the pressure plate and the flywheel rotate with the engine



The axial pressure exerted by the spring provides a frictional force in the circumferential direction when the relative motion between the driving and driven members tends to take place. If the torque due to this frictional force exceeds the torque to be transmitted, then no slipping takes place and the power is transmitted from the driving shaft to the driven shaft.

1. Hydraulic brakes *e.g.* pumps or hydrodynamic brake and fluid agitator,
2. Electric brakes *e.g.* generators and eddy current brakes, and
3. Mechanical brakes.

The hydraulic and electric brakes cannot bring the member to rest and are mostly used where large amounts of energy are to be transformed while the brake is retarding the load such as in laboratory dynamometers, high way trucks and electric locomotives. These brakes are also used for retarding or controlling the speed of a vehicle for down-hill travel.

The mechanical brakes, according to the direction of acting force, may be divided into the following two groups :

(*a*) ***Radial brakes.*** In these brakes, the force acting on the brake drum is in radial direction. The radial brakes may be sub-divided into ***external brakes*** and ***internal brakes***. According to the shape of the friction elements, these brakes may be ***block*** or ***shoe brakes*** and ***band brakes.***

(*b*) ***Axial brakes.*** In these brakes, the force acting on the brake drum is in axial direction. The axial brakes may be disc brakes and cone brakes. The analysis of these brakes is similar to clutches.

Since we are concerned with only mechanical brakes, therefore, these are discussed, in detail, in the following pages.

2. (b) ANS:

Solution. Given : $d = 250$ mm or $r = 125$ mm ; $2\theta = 90° = \pi/2$ rad ; $P = 700$ N ; $\mu = 0.35$

Since the angle of contact is greater than 60°, therefore equivalent coefficient of friction,

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.35 \times \sin 45°}{\pi/2 + \sin 90°} = 0.385$$

Let R_N = Normal force pressing the block to the brake drum, and

F_t = Tangential braking force = $\mu' . R_N$

Taking moments about the fulcrum O, we have

$$700(250 + 200) + F_t \times 50 = R_N \times 200 = \frac{F_t}{\mu'} \times 200 = \frac{F_t}{0.385} \times 200 = 520\, F_t$$

or $520\, F_t - 50 F_t = 700 \times 450$ or $F_t = 700 \times 450/470 = 670$ N

We know that torque transmitted by the block brake,

$$T_B = F_t \times r = 670 \times 125 = 8\,3750 \text{ N-mm} = 83.75 \text{N-m Ans.}$$

b(iv)

Solution. Given : $d_1 = 300$ mm or $r_1 = 150$ mm ; $d_2 = 200$ mm or $r_2 = 100$ mm ; $p = 0.1$ N/mm^2 ; $\mu = 0.3$; $N = 2500$ r.p.m. or $\omega = 2\pi \times 2500/60 = 261.8$ rad/s

Since the intensity of pressure (p) is maximum at the inner radius (r_2), therefore for uniform wear,

$$p.r_2 = C \quad \text{or} \quad C = 0.1 \times 100 = 10 \text{ N/mm}$$

We know that the axial thrust,

$$W = 2\pi C (r_1 - r_2) = 2\pi \times 10 (150 - 100) = 3142 \text{ N}$$

and mean radius of the friction surfaces for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{150 + 100}{2} = 125 \text{ mm} = 0.125 \text{ m}$$

We know that torque transmitted,

$$T = n.\mu.W.R = 2 \times 0.3 \times 3142 \times 0.125 = 235.65 \text{ N-m}$$

...($\because n = 2$, for both sides of plate effective)

$\therefore$ Power transmitted by a clutch,

$$P = T.\omega = 235.65 \times 261.8 = 61\,693 \text{ W} = 61.693 \text{ kW Ans.}$$

3. (a) ANS:

Stability of Governors

A governor is said to be *stable* when for every speed within the working range there is a definite configuration *i.e.* there is only one radius of rotation of the governor balls at which the governor is in equilibrium. For a stable governor, if the equilibrium speed increases, the radius of governor balls must also increase.

Sensitiveness of Governors

In general, the greater the lift of the sleeve corresponding to a given fractional change in speed, the greater is the sensitiveness of the governor. It may also be stated in another way that for a given lift of the sleeve, the sensitiveness of the governor increases as the speed range decreases. This definition of sensitiveness may be quite satisfactory when the governor is considered as an independent mechanism. But when the governor is fitted to an engine, the practical requirement is simply that the change of equilibrium speed from the full load to the no load position of the sleeve should be as small a fraction as possible of the mean equilibrium speed. The actual displacement of the sleeve is immaterial, provided that it is sufficient to change the energy supplied to the engine by the required amount. For this reason, the sensitiveness is defined as the *ratio of the difference between the maximum and minimum equilibrium speeds to the mean equilibrium speed.*

$\therefore$ Sensitiveness of the governor

$$= \frac{N_2 - N_1}{N} = \frac{2(N_2 - N_1)}{N_1 + N_2}$$

$$= \frac{2(\omega_2 - \omega_1)}{\omega_1 + \omega_2}$$

Isochronous Governors

A governor is said to be *isochronous* when the equilibrium speed is constant (*i.e.* range of speed is zero) for all radii of rotation of the balls within the working range, neglecting friction. The isochronism is the stage of infinite sensitivity.

Let us consider the case of a Porter governor running at speeds N_1 and N_2 r.p.m. We have discussed in Art. 18.6 that

$$(N_1)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_1} \quad \ldots (i)$$

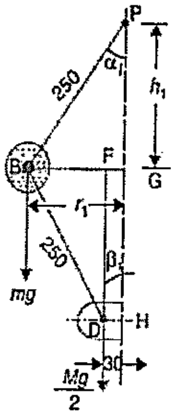
Hunting

A governor is said to be *hunt* if the speed of the engine fluctuates continuously above and below the mean speed. This is caused by a too sensitive governor which changes the fuel supply by a large amount when a small change in the speed of rotation takes place. For example, when the load on the engine increases, the engine speed decreases and, if the governor is very sensitive, the governor sleeve immediately falls to its lowest position. This will result in the opening of the

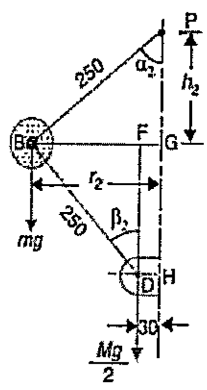
Such a governor may admit either the maximum or the minimum amount of fuel. The effect of this will be to cause wide fluctuations in the engine speed or in other words, the engine will hunt.

Solution. Given : $BP = BD = 250$ mm ; $DH = 30$ mm ; $m = 5$ kg ; $M = 50$ kg ; $r_1 = 150$ mm ; $r_2 = 200$ mm

First of all, let us find the minimum and maximum speed of the governor. The minimum and maximum position of the governor is shown in Fig. 18.8 (*a*) and (*b*) respectively.



(*a*) Minimum position.



(*b*) Maximum position.

Fig. 18.8

Let N_1 = Minimum speed when $r_1 = BG = 150$ mm ; and

N_2 = Maximum speed when $r_2 = BG = 200$ mm.

From Fig. 18.8 (*a*), we find that height of the governor,

$$h_1 = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(250)^2 - (150)^2} = 200 \text{ mm} = 0.2 \text{ m}$$

$$BF = BG - FG = 150 - 30 = 120 \text{ mm} \quad \ldots (\because FG = DH)$$

and $$DF = \sqrt{(DB)^2 - (BF)^2} = \sqrt{(250)^2 - (120)^2} = 219 \text{ mm}$$

$\therefore$ $$\tan \alpha_1 = BG/PG = 150/200 = 0.75$$

and $$\tan \beta_1 = BF/DF = 120/219 = 0.548$$

$\therefore$ $$q_1 = \frac{\tan \beta_1}{\tan \alpha_1} = \frac{0.548}{0.75} = 0.731$$

We know that $$(N_1)^2 = \frac{m + \frac{M}{2}(1 + q_1)}{m} \times \frac{895}{h_1} = \frac{5 + \frac{50}{2}(1 + 0.731)}{5} \times \frac{895}{0.2} = 432$$

$\therefore$ $$N_1 = 208 \text{ r.p.m.}$$

From Fig. 18.8(*b*), we find that height of the governor,

$$h_2 = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(250)^2 - (200)^2} = 150 \text{ mm} = 0.15 \text{ m}$$

$$BF = BG - FG = 200 - 30 = 170 \text{ mm}$$

and $$DF = \sqrt{(DB)^2 - (BF)^2} = \sqrt{(250)^2 - (170)^2} = 183 \text{ mm}$$

$\therefore$ $$\tan \alpha_2 = BG/PG = 200/150 = 1.333$$

and $$\tan \beta_2 = BF/DF = 170/183 = 0.93$$

$\therefore$ $$q_2 = \frac{\tan \beta_2}{\tan \alpha_2} = \frac{0.93}{1.333} = 0.7$$

We know that

$$(N_2)^2 = \frac{m + \frac{M}{2}(1 + q_2)}{m} \times \frac{895}{h_2} = \frac{5 + \frac{50}{2}(1 + 0.7)}{5} \times \frac{895}{0.15} = 56\,683$$

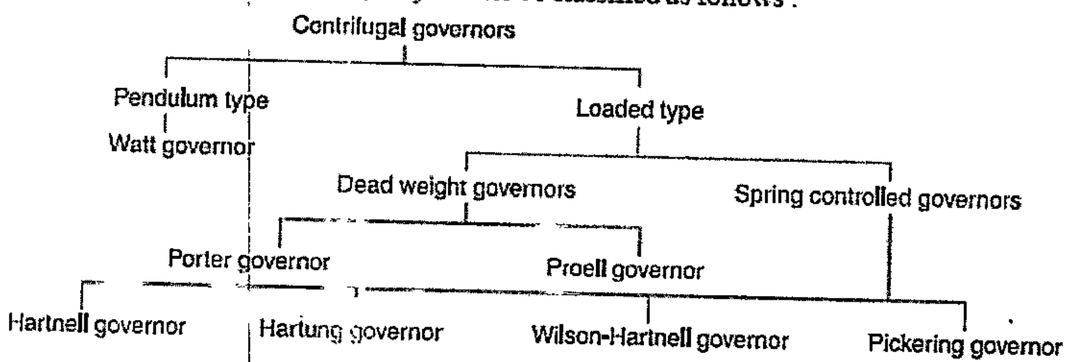
$\therefore$ $$N_2 = 238 \text{ r.p.m.}$$

We know that range of speed

$$= N_2 - N_1 = 238 - 208 = 30 \text{ r.p.m. Ans.}$$

3 (b) ANS:

The centrifugal governors, may further be classified as follows :



The function of a governor is to regulate the mean speed of an engine, when there are variations in the load *e.g.* when the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of working fluid. On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required. The governor automatically controls the supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limits.

A little consideration will show, that when the load increases, the configuration of the governor changes and a valve is moved to increase the supply of the working fluid ; *conversely*, when the load decreases, the engine speed increases and the governor decreases the supply of working fluid.

A flywheel used in machines serves as a reservoir, which stores energy during the period when the supply of energy is more than the requirement, and releases it during the period when the requirement of energy is more than the supply.

In case of steam engines, internal combustion engines, reciprocating compressors and pumps, the energy is developed during one stroke and the engine is to run for the whole cycle on the energy produced during this one stroke. For example, in internal combustion engines, the energy is developed only during expansion or power stroke which is much more than the engine load and no energy is being developed during suction, compression and exhaust strokes in case of four stroke engines and during compression in case of two stroke engines. The excess energy developed during power stroke is absorbed by the flywheel and releases it to the crankshaft during other strokes in which no energy is developed, thus rotating the crankshaft at a uniform speed. A little consideration will show that when the flywheel absorbs energy, its speed increases and when it releases energy, the speed decreases. Hence a flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed. In other words, *a flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation.*

Turning Moment Diagram for a Four Stroke Cycle Internal Combustion Engine

A turning moment diagram for a four stroke cycle internal combustion engine is shown in Fig. 16.2. We know that in a four stroke cycle internal combustion engine, there is one working stroke after the crank has turned through two revolutions, *i.e.* 720° (or 4 π radians).

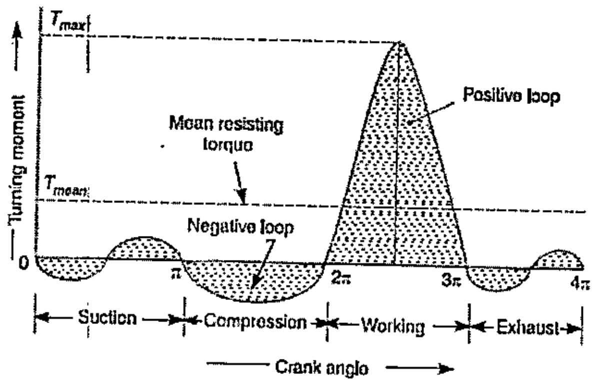


Fig. 16.2. Turning moment diagram for a four stroke cycle internal combustion engine.

Since the pressure inside the engine cylinder is less than the atmospheric pressure during the suction stroke, therefore a negative loop is formed as shown in Fig. 16.2. During the compression stroke, the work is done on the gases, therefore a higher negative loop is obtained. During the expansion or working stroke, the fuel burns and the gases expand, therefore a large positive loop is obtained. In this stroke, the work is done by the gases. During exhaust stroke, the work is done on the gases, therefore a negative loop is formed. It may be noted that the effect of the inertia forces on the piston is taken into account in Fig. 16.2.

4. (a) ANS:

Solution. Given : $m_A = 200$ kg ; $m_B = 300$ kg ; $m_C = 400$ kg ; $m_D = 200$ kg ; $r_A = 80$ mm $= 0.08$m ; $r_B = 70$ mm $= 0.07$ m ; $r_C = 60$ mm $= 0.06$ m ; $r_D = 80$ mm $= 0.08$ m ; $r_X = r_Y = 100$ mm $= 0.1$ m

Let m_X = Balancing mass placed in plane *X*, and

m_Y = Balancing mass placed in plane *Y*.

The position of planes and angular position of the masses (assuming the mass A as horizontal) are shown in Fig. 21.8 (*a*) and (*b*) respectively.

Assume the plane *X* as the reference plane (*R.P.*). The distances of the planes to the right of plane *X* are taken as + ve while the distances of the planes to the left of plane *X* are taken as – ve. The data may be tabulated as shown in Table 21.2.

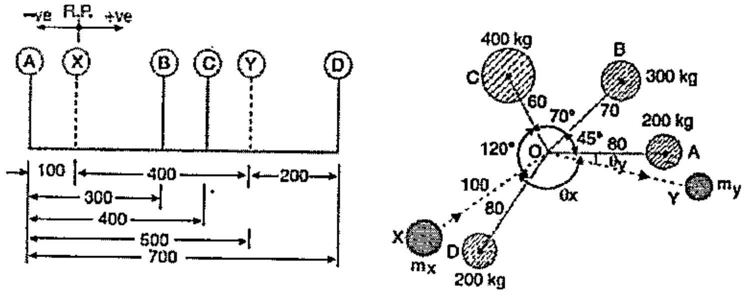
Table 21.2

Plane (1)	*Mass (m)* kg (2)	*Radius (r)* m (3)	*Cent.force* $\div \omega^2$ *(m.r) kg-m* (4)	*Distance from Plane x(l) m* (5)	*Couple* $\div \omega^2$ *(m.r.l) kg-m²* (6)
A	200	0.08	16	– 0.1	– 1.6
X(R.P.)	m_X	0.1	$0.1\, m_X$	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_Y	0.1	$0.1\, m_Y$	0.4	$0.04\, m_Y$
D	200	0.08	16	0.6	9.6

The balancing masses m_X and m_Y and their angular positions may be determined graphically as discussed below :

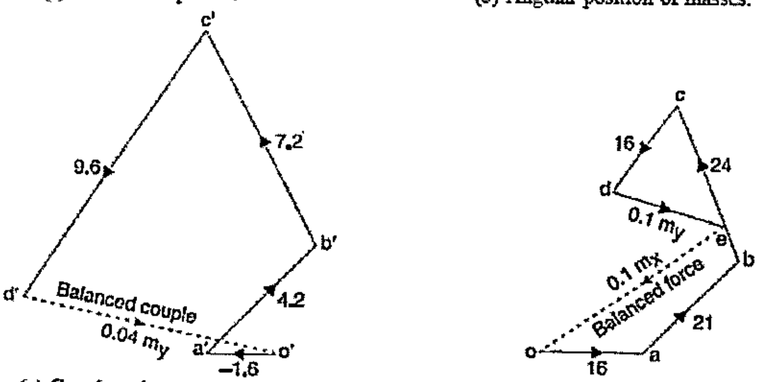
1. First of all, draw the couple polygon from the data given in Table 21.2 (column 6) as shown in Fig. 21.8 (*c*) to some suitable scale. The vector $d'o'$ represents the balanced couple. Since the balanced couple is proportional to $0.04\, m_Y$, therefore by measurement,

$$0.04\, m_Y = \text{vector } d'o' = 7.3 \text{ kg-m}^2 \quad \text{or} \quad m_Y = 182.5 \text{ kg Ans.}$$



All dimensions in mm.

(*a*) Position of planes. (*b*) Angular position of masses.



(*c*) Couple polygon. (*d*) Force polygon.

Fig. 21.8

The angular position of the mass m_Y is obtained by drawing Om_Y in Fig. 21.8 (*b*), parallel to vector $d'o'$. By measurement, the angular position of m_Y is $\theta_Y = 12°$ in the clockwise direction from mass m_A (*i.e.* 200 kg). Ans.

2. Now draw the force polygon from the data given in Table 21.2 (column 4) as shown in Fig. 21.8 (*d*). The vector *eo* represents the balanced force. Since the balanced force is proportional to $0.1\, m_X$, therefore by measurement,

$$0.1\, m_X = \text{vector } eo = 35.5 \text{ kg-m} \quad \text{or} \quad m_X = 355 \text{ kg Ans.}$$

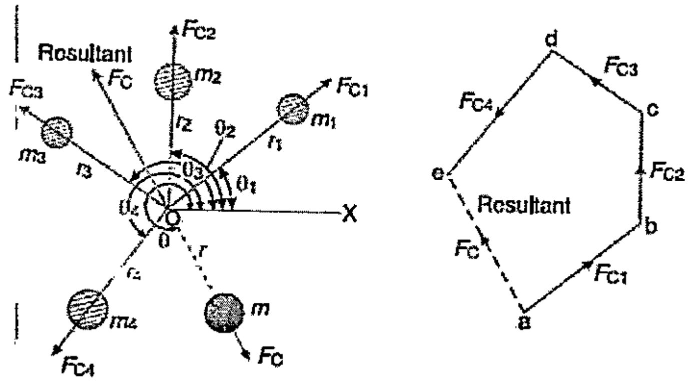
The angular position of the mass m_X is obtained by drawing Om_X in Fig. 21.8 (*b*), parallel to vector *eo*. By measurement, the angular position of m_X is $\theta_X = 145°$ in the clockwise direction from mass m_A (*i.e.* 200 kg). Ans.

4. (b) ANS:

Balancing of Several Masses Rotating in the Same Plane

Consider any number of masses (say four) of magnitude m_1, m_2, m_3 and m_4 at distances of r_1, r_2, r_3 and r_4 from the axis of the rotating shaft. Let $\theta_1, \theta_2, \theta_3$ and θ_4 be the angles of these masses with the horizontal line *OX*, as shown in Fig. 21.4 (*a*). Let these masses rotate about an axis through *O* and perpendicular to the plane of paper, with a constant angular velocity of ω rad/s.

The magnitude and position of the balancing mass may be found out analytically or graphically as discussed below :



(*a*) Space diagram. (*b*) Vector diagram.

Fig. 21.4. Balancing of several masses rotating in the same plane.

5. (a) ANS:

Solution. Given : $N = 240$ r.p.m. or $\omega = 2\pi \times 240/60 = 25.14$ rad/s ; Stroke = 300 mm = 0.3 m; $m = 50$ kg ; $m_1 = 37$ kg ; $r = 150$ mm = 0.15 m ; $c = 2/3$

1. *Balance mass required*

Let B = Balance mass required, and

b = Radius of rotation of the balance mass = 400 mm = 0.4 m ... (Given)

We know that

$$B.b = (m_1 + c.m)\, r$$

$$B \times 0.4 = \left(37 + \frac{2}{3} \times 50\right) 0.15 = 10.55 \quad \text{or} \quad B = 26.38 \text{ kg Ans.}$$

2. *Residual unbalanced force*

Let θ = Crank angle from top dead centre = 60° ... (Given)

We know that residual unbalanced force

$$= m \cdot \omega^2 \cdot r \sqrt{(1-c)^2 \cos^2\theta + c^2 \sin^2\theta}$$

$$= 50(25.14)^2 0.15 \sqrt{\left(1-\frac{2}{3}\right)^2 \cos^2 60° + \left(\frac{2}{3}\right)^2 \sin^2 60°} \text{ N}$$

$= 4740 \times 0.601 = 2849$ N **Ans.**

Solution. Given $2\alpha = 60°$ or $\alpha = 30°$, $m = 1.5$ kg ; Stroke = 100 mm or $r = 100/2$ = 50 mm = 0.05 m ; $l = 250$ mm = 0.25 m ; N = 250 r.p.m. or $\omega = 2\,\pi \times 2500 / 60 = 261.8$ rad/s

Maximum and minimum values of primary forces

We know that the resultant primary force,

$$F_P = 2m.\omega^2 r\sqrt{(\cos^2\alpha \cdot \cos\theta)^2 + (\sin^2\alpha \cdot \sin\theta)^2}$$

$$= 2m.\omega^2.r\sqrt{(\cos^2 30°\cos\theta)^2 + (\cos^2 30°\sin\theta)^2}$$

$$= 2m\omega^2 r\sqrt{\left(\frac{3}{4}\cos\theta\right)^2 + \left(\frac{1}{4}\sin\theta\right)^2}$$

$$= \frac{m}{2}\times\omega^2 r\sqrt{9\cos^2\theta + \sin^2\theta} \qquad \ldots(i)$$

The primary force is maximum, when $\theta = 0°$. Therefore substituting $\theta = 0°$ in equation (*i*), we have maximum primary force,

$$F_{P(max)} = \frac{m}{2}\times\omega^2 r\times 3 = \frac{1.5}{2}(261.8)^2 0.05\times 3 = 7710.7 \text{ N Ans.}$$

The primary force is minimum, when $\theta = 90°$. Therefore substituting $\theta = 90°$ in equation (*i*), we have minimum primary force,

$$F_{P(min)} = \frac{m}{2}\times\omega^2 r = \frac{1.5}{2}(261.8)^2 0.05 = 2570.2 \text{ N Ans.}$$

Maximum and minimum values of secondary forces

We know that resultant secondary force.

$$F_S = \frac{2m}{n}\times\omega^2\sqrt{(\cos\alpha\cos 2\alpha\cos 2\theta)^2 + (\sin\alpha\sin 2\alpha\sin 2\theta)^2}$$

$$= \frac{2m}{n}\times\omega^2 r\sqrt{(\cos 30°\cos 60°\cos 2\theta)^2 + (\sin 30°\sin 60°\sin 2\theta)^2}$$

$$= \frac{2m}{n}\times\omega^2 r\sqrt{\left(\frac{\sqrt{3}}{2}\times\frac{1}{2}\cos 2\theta\right)^2 + \left(\frac{1}{2}\times\frac{\sqrt{3}}{2}\sin 2\theta\right)^2}$$

$$= \frac{\sqrt{3}}{2}\times\frac{m}{n}\times\omega^2 r$$

$$= \frac{\sqrt{3}}{2}\times\frac{1.5}{0.25/0.05}(261.8)^2 0.05 \qquad \ldots(\because n = l/r)$$

$$= 890.3 \text{ N Ans.}$$

5. (b) ANS:

Solution. Given : $m = 300$ kg ; $r = 0.3$ m ; $D = 1.8$ m or $R = 0.9$ m ; $a = 0.65$ m ; Hammer blow = 46 kN = 46×10^3 N ; $v = 96.5$ km/h = 26.8 m/s

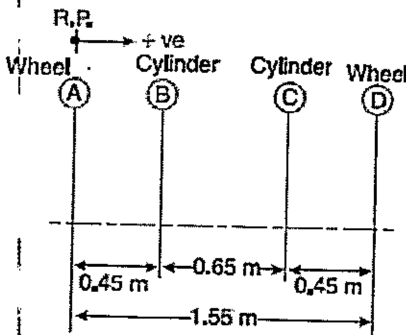
1. *Fraction of the reciprocating masses to be balanced*

Let c = Fraction of the reciprocating masses to be balanced, and

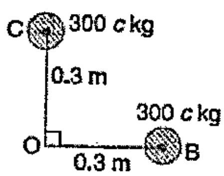
B = Magnitude of balancing mass placed at each of the driving wheels at radius b.

We know that the mass of the reciprocating parts to be balanced

$$= c.m = 300c \text{ kg}$$



(*a*) Position of planes.



(*b*) Position of cranks.

Fig. 22.9

The position of planes of the wheels and cylinders is shown in Fig. 22.9 (*a*), and the position of cranks is shown in Fig 22.9 (*b*). Assuming the plane of wheel *A* as the reference plane, the data may be tabulated as below :

Table 22.3

Plane (1)	*Mass* (*m*) *kg* (2)	*Radius* (*r*) *m* (3)	*Cent. force* ÷ ω^2 (*m.r*) *kg-m* (4)	*Distance from plane A* (*l*)*m* (5)	*Couple* ÷ ω^2 (*m.r.l*) *kg-m*^{2} (6)
A (*R.P.*)	*B*	*b*	*B.b*	0	0
B	300 *c*	0.3	90 *c*	0.45	40.5 *c*
C	300 *c*	0.3	90 *c*	1.1	99 *c*
D	*B*	*b*	*B.b*	1.55	1.55 *B.b*

Now the couple polygon, to some suitable scale, may be drawn with the data given in Table 22.3 (column 6), as shown in Fig. 22.10. The closing side of the polygon (vector $c'o'$) represents the balancing couple and is proportional to 1.55 *B.b*.

From the couple polygon,

$$1.55\, B.b = \sqrt{(40.5c)^2 + (99c)^2} = 107c$$

$\therefore$ $B.b = 107\, c / 1.55 = 69\, c$

We know that angular speed,

$$\omega = v/R = 26.8/0.9 = 29.8 \text{ rad/s}$$

$\therefore$ Hammer blow,

$$46 \times 10^3 = B.\ \omega^2 .b$$
$$= 69\, c\, (29.8)^2 = 61\ 275\, c$$

$\therefore$ $c = 46 \times 10^3 / 61\ 275 = 0.751$ Ans.

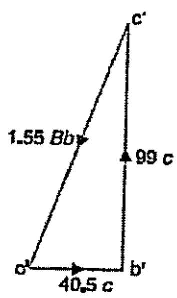


Fig. 22.10

2. Variation in tractive effort

We know that variation in tractive effort

$$= \pm\sqrt{2}(1-c)m.\omega^2.r = \pm\sqrt{2}(1-0.751)\,300\,(29.8)^2\,0.3$$

$$= 28\ 140\ \text{N} = 28.14\ \text{kN Ans.}$$

Maximum swaying couple

We know the maximum swaying couple

$$= \frac{a(1-c)}{\sqrt{2}}\times m.\omega^2.r = \frac{0.65(1-0.751)}{\sqrt{2}}\times 300\,(29.8)^2\,0.3 = 9148\ \text{N-m}$$

$$= 9.148\ \text{kN-m Ans.}$$

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III BTech I Sem Regular Exam, March-2020

MACHINE DYNAMICS AND VIBRATIONS

PART-A **[2+2+2+2+2+2 = 12M]**

1. (a) ANS:

Static friction is the friction that keeps an object in place. This friction must be overcome by energy to move the object. Because of a combination of surface irregularities and electrical charges between the object and the surface, the energy required to initially move the static object is greater than the energy required to keep it in motion. Because it takes more energy to move the static object, physicists say that static friction is greater than kinetic friction.

(b) ANS:

When selecting a clutch or brake for an application, a torque service factor must be taken into consideration. A service or safety factor will assure the clutch or brake will have the required torque when new to drive or stop as anticipated. A service factor of 1.5 to 2 (50% to 100% more torque than required) is recommended by most clutch and brake manufacturers.

(c) ANS:

A clutch is a transmission and control device that provides for energy transfer from the driver to the driven shaft. A brake is a transmission and control device that stops a moving load, regulates movement, or holds a load at rest by transforming kinetic energy into heat.

(d) ANS:

Static balancing definition refers to the ability of a stationary object to its balance. However, the dynamic balance definition is the ability of an object to balance in motion or when switching between positions. Condition for static balancing is equilibrium of forces and Condition for dynamic balancing is equilibrium of forces and moments.

(e) ANS:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{s}{m}}, \quad \frac{1}{s} = \frac{1}{s_1} + \frac{1}{s_1} \Rightarrow S = 3.08 \text{ N/mm} = 3.08X10^3 \text{ N/m}$$

$\therefore$ Natural Frequency = f_n = 3 Hz

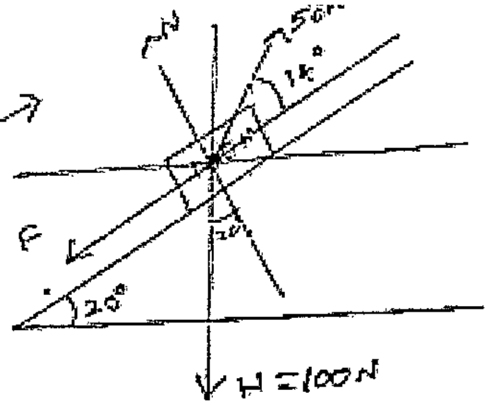
(f) ANS:

The ratio of the force transmitted (F_T) to the force applied (F) is known as the ***isolation factor*** or ***transmissibility ratio*** of the spring support.

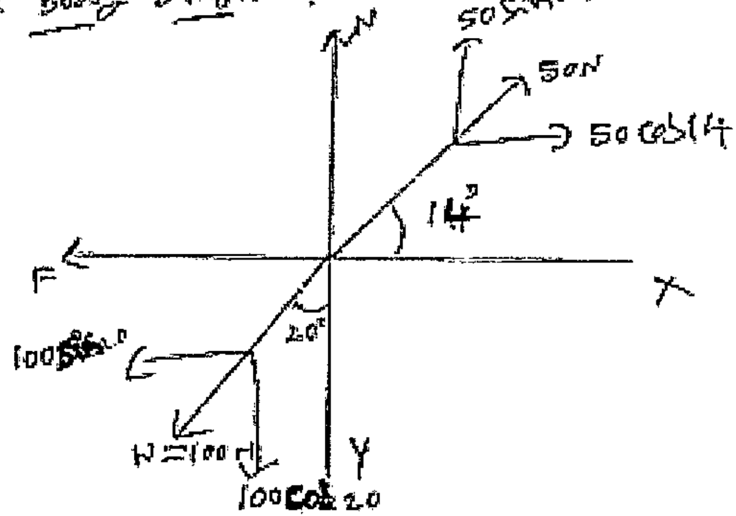
PART-B **[4X12 = 48M]**

2. (a) ANS:

2)(a)



∴ Free Body Diagram:-



At Equilibrium,

$$\Sigma F_x = 0$$

$$\Rightarrow 50\cos 14 - F - 100\sin 20 = 0$$

$$\Rightarrow F = 14.31\,N$$

$$\Sigma F_y = 0$$

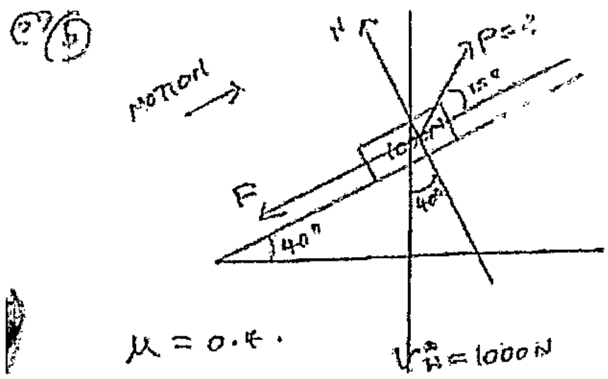
$$\Rightarrow 50\sin 14 + N - 100\cos 20 = 0$$

$$\Rightarrow N = 81.87\,N$$

∴ coefficient of Friction $\mu = F/N$

$$\Rightarrow \mu = 0.17$$

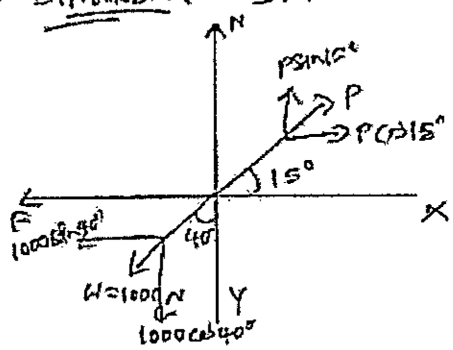
2. (b) ANS:



$\mu = 0.4$.

Let P = force required to move the block up the plane.

FREE BODY DIAGRAM (FBD):-



$\mu = \frac{F}{N} \Rightarrow 0.4 = F/N \Rightarrow F = 0.4N \rightarrow (1)$

At equilibrium,

$\Sigma F_x = 0$

$\Rightarrow P\cos 15° - F - 1000\sin 40 = 0$

$\Rightarrow P\cos 15 = F + 1000\sin 40$

$\Rightarrow F = P\cos 15 - 1000\sin 40 \rightarrow (2)$

$\Sigma F_y = 0$

$\Rightarrow P\sin 15 + N - 1000\cos 40 = 0$

$\Rightarrow N = 1000\cos 40 - P\sin 15 \rightarrow (3)$

From (2) & (3)

(2) $\Rightarrow 0.4N = P\cos 15 - 1000\sin 40$

$\Rightarrow 0.4(1000\cos 40 - P\sin 15) = P\cos 15 - 1000\sin 40$

$\Rightarrow 400\cos 40 - 0.4P\sin 15 = P\cos 15 - 1000\sin 40$

$\Rightarrow P = 895\,N$

3. ANS:

A single disc or plate clutch, as shown in Fig. , consists of a clutch plate whose both sides are faced with a friction material (usually of Ferrodo).

It is mounted on the hub which is free to move axially along the splines of the driven shaft. The pressure plate is mounted inside the clutch body which is bolted to the flywheel.

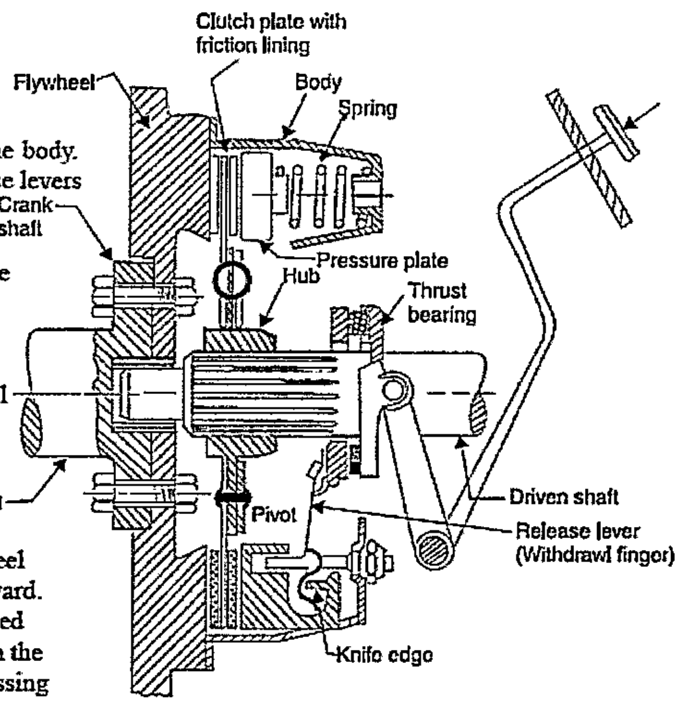
Both the pressure plate and the flywheel rotate with the enginecrankshaft or the driving shaft.

The pressure plate pushes the clutch plate towards the flywheel by a set of strong springs which are arranged radially inside the body.

The three levers (also known as release levers or fingers) are carried on pivots suspended from the case of the body. These are arranged in such a manner so that the pressure plate moves away from the flywheel by the inward movement of a thrust bearing.

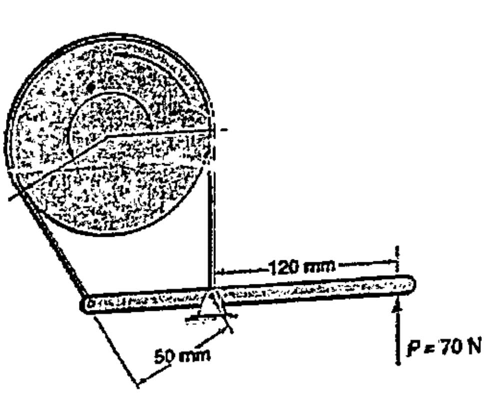
The bearing is mounted upon a forked shaft and moves forward when the clutch pedal s pressed.

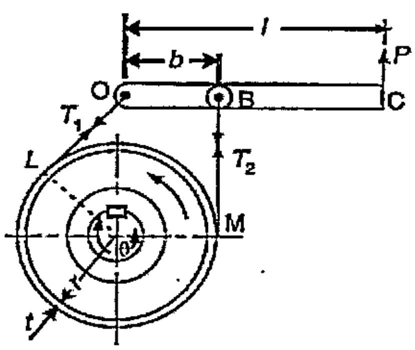
When the clutch pedal is pressed down, its linkage forces the thrust release bearing to move in towards the flywheel and pressing the longer ends of the levers inward. The levers are forced to turn on their suspended pivot and the pressure plate moves away from the flywheel by the knife edges, there by compressing the clutch springs. This action removes the pressure from the clutch plate and thus moves back from the flywheel and the driven shaft becomes stationary.



Single disc or plate clutch

4. a) ANS:





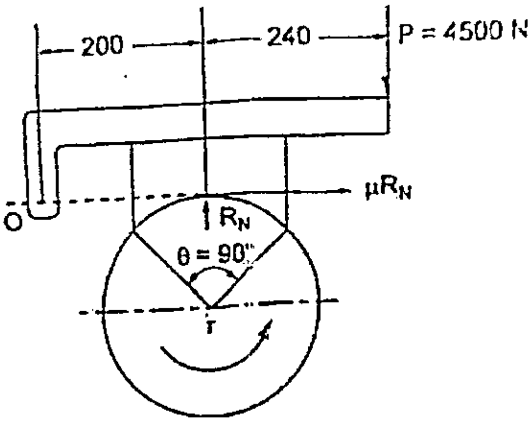
$$\frac{T_1}{T_2} = e^{\mu\theta} \qquad \text{or} \qquad 2.3\log\left(\frac{T_1}{T_2}\right) = \mu.\theta$$

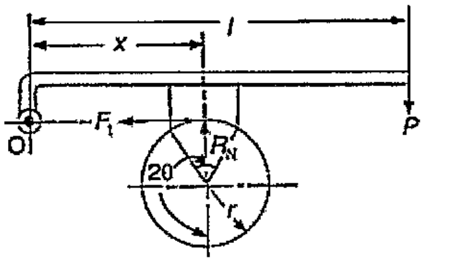
∴ Braking torque on the drum,

$$T_B = (T_1 - T_2)\, r$$

$P.l = T_2.b$ ... (For anticlockwise rotation of the drum)

4. b) ANS:





(*b*) Anticlockwise rotation of brake wheel.

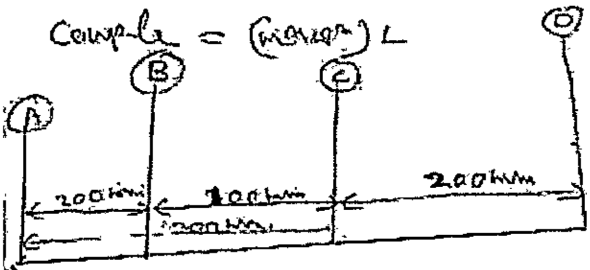
$$T_B = \mu R_N r = \frac{\mu P l r}{x}$$

5. ANS:

Plane	Mass (kg)	Radius r (m)	$F/\omega^2 = m.r$	Distance from 'A' L (m)	Couple/ω^2 = m.r.L
A	ma	0.1	0.1ma	0	0
B	9	0.15	1.35	0.2	0.27
C	5	0.15	0.75	0.3	0.225
D	4	0.2	0.8	0.5	0.4

Centrifugal force F = $m r \omega^2$

Couple = $(m r \omega^2)$ L



Linear position Diagram.

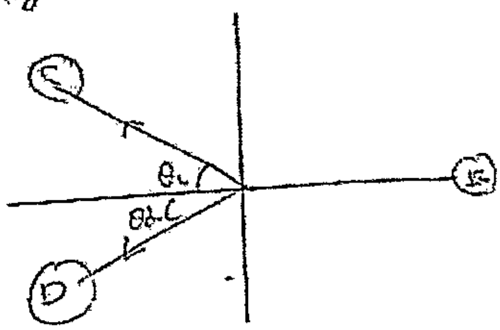
Couple polygon:->

B -> 0.27

C -> 0.225

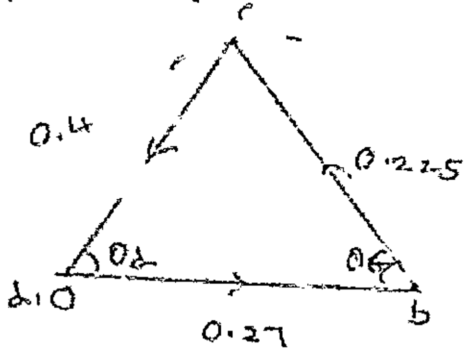
D -> 0.4

Let B is horizontal



Angular position Diagram

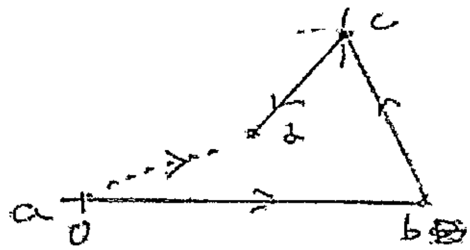
(iv) graphical method.



measure θa & θb.

Force polygon

(A) → 0.1ma
(B) → 1.35
(C) → 0.75
(D) → 0.8



measure ad =

6 ANS:

1. *Critical damping coefficient*

We know that critical damping coefficient,

$$c_c = 2m.\omega_n$$

2. *Damping factor*

We know that damping factor

$$= \frac{c}{c_c}$$

3. *Logarithmic decrement*

We know that logarithmic decrement,

$$\delta = \frac{2\pi c}{\sqrt{(c_c)^2 - c^2}}$$

4. *Ratio of two consecutive amplitudes*

Let x_n and x_{n+1} = Magnitude of two consecutive amplitudes,

We know that logarithmic decrement,

$$\delta = \log_e \left[\frac{x_n}{x_{n+1}}\right] \text{ or } \frac{x_n}{x_{n+1}} = e^{\delta}$$

7. ANS:

Solution. Given : $d = 50$ mm $= 0.05$ m ; $l = 3$ m, $W_1 = 1000$ N ; $W_2 = 1500$ N ; $W_3 = 750$ N; $E = 200$ GN/m^2 $= 200 \times 10^9$ N/m^2

The shaft carrying the loads is shown in Fig. 23.13

We know that moment of inertia of the shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.05)^4 = 0.307 \times 10^{-6}\,\text{m}^4$$

and the static deflection due to a point load W,

$$\delta = \frac{Wa^2b^2}{3EIl}$$

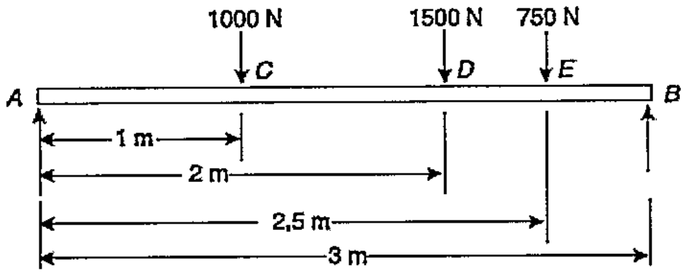


Fig. 23.13

$\therefore$ Static deflection due to a load of 1000 N,

$$\delta_1 = \frac{1000 \times 1^2 \times 2^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3} = 7.24 \times 10^{-3}\ \text{m}$$

... (Here $a = 1$ m, and $b = 2$ m)

Similarly, static deflection due to a load of 1500 N,

$$\delta_2 = \frac{1500 \times 2^2 \times 1^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3} = 10.86 \times 10^{-3}\ \text{m}$$

... (Here $a = 2$ m, and $b = 1$ m)

and static deflection due to a load of 750 N,

$$\delta_3 = \frac{750\,(2.5)^2\,(0.5)^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3} = 2.12 \times 10^{-3}\ \text{m}$$

... (Here $a = 2.5$ m, and $b = 0.5$ m)

We know that frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3}} = \frac{0.4985}{\sqrt{7.24 \times 10^{-3} + 10.86 \times 10^{-3} + 2.12 \times 10^{-3}}}$$

$$= \frac{0.4985}{0.1422} = 3.5 \text{ Hz} \quad \textbf{Ans.}$$

Prepared

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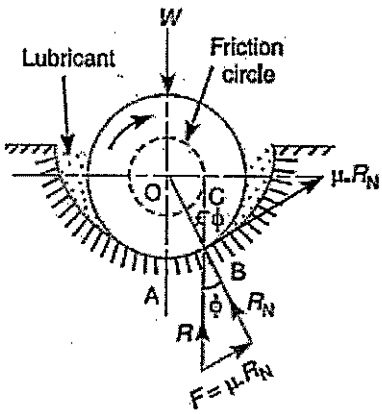
DEPARTMENT OF MECHANICAL ENGINEERING

III BTech I Sem Supplementary Exam, May-2019

MACHINE DYNAMICS AND VIBRATIONS

PART-A [2+2+2+2+2+2=12M]

1. (a) ANS:



If a circle is drawn with centre O and radius OC = r sin φ, then this circle is called the **friction circle** of a bearing. In order that the rotation may be maintained, there must be a couple rotating the shaft is called **friction couple.**

(b) ANS:

In case of a new clutch, the intensity of pressure is approximately uniform but in an old clutch the uniform wear theory is more approximate. **The uniform pressure** theory gives a higher frictional torque than the **uniform wear theory**. Therefore in case of friction clutches, uniform wear should be considered, unless otherwise stated.

(c) ANS:

In the **absorption dynamometers**, the entire energy or power produced by the engine is absorbed by the friction resistances of the brake and is transformed into heat, during the process of measurement. But in the **transmission dynamometers**, the energy is not wasted in friction but is used for doing work.

(d) ANS:

Two balancing masses are placed in two different planes, parallel to the plane of rotation of the disturbing mass, in such a way that they satisfy the following two conditions of equilibrium.

1. The net dynamic force acting on the shaft is equal to zero. This requires that the line of action of three centrifugal forces must be the same. In other words, the centre of the masses of the system must lie on the axis of rotation. This is the condition for **Static balancing.**

2. The net couple due to the dynamic forces acting on the shaft is equal to zero. In other words, the algebraic sum of the moments about any point in the plane must be zero. The conditions (1) and (2) together give **dynamic balancing.**

(e) ANS:

FREE OR NATURAL VIBRATIONS:

When no external force acts on the body, after giving it an initial displacement, then the body is said to be under free or natural vibrations. The frequency of the free vibrations is called free or natural frequency.

DAMPED VIBRATIONS:

When there is a reduction in amplitude over every cycle of vibration, the motion is said to be damped vibration.

(f) ANS:

Logarithmic decrement is defined as the natural logarithm of the amplitude reduction factor. The amplitude reduction factor is the ratio of any two successive amplitudes on the same side of the mean position.

$\therefore$ Logarithmic decrement,

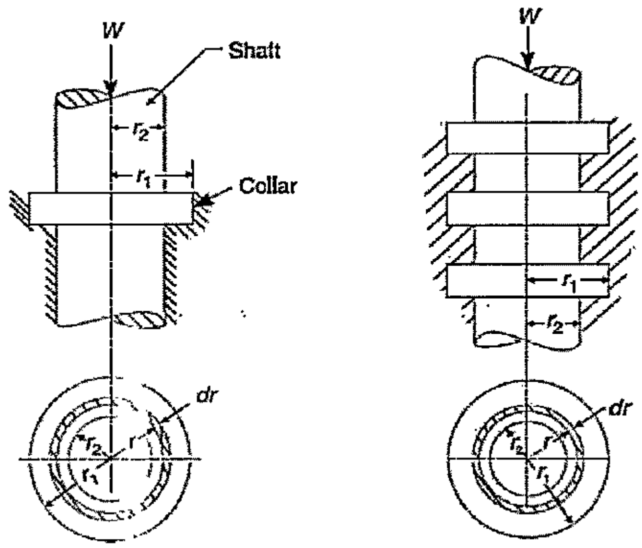
$$\delta = \log_e\left(\frac{x_n}{x_{n+1}}\right) = a t_p = \frac{2\pi \times c}{\sqrt{(c_c)^2 - c^2}}$$

PART-B **[4X12 = 48M]**

2. ANS:

FLAT COLLAR BEARINGS:

We have already discussed that collar bearings are used to take the axial thrust of the rotating shafts. There may be a single collar or multiple collar bearings as shown in Fig. (*a*) and (*b*) respectively. The collar bearings are also known as *thrust bearings.*



(*a*) Single collar bearing (*b*) Multiple collar bearing.

Fig. Flat collar bearings.

1. *Considering uniform pressure*

When the pressure is uniformly distributed over the bearing surface, then the intensity of pressure,

$$p = \frac{W}{A} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \qquad ...(i)$$

We have seen in Art. 10.25, that the frictional torque on the ring of radius and thickness *dr*,

$$T_r = 2\pi\mu . p r^2 . dr$$

Integrating this equation within the limits from r_2 to r_1 for the total frictional torque on the collar.

∴ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\mu . p r^2 dr = 2\pi\mu . p\left[\frac{r^3}{3}\right]_{r_2}^{r_1} = 2\pi\mu . p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p from equation (*i*),

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]}\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

2. *Considering unifrom wear*

We have seen in Art. 10.25 that the load transmitted on the ring, considering uniform wear is,

$$\delta W = p_r.2\pi r.dr = \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

$\therefore$ Total load transmitted to the collar,

$$W = \int_{r_2}^{r_1} 2\pi C.dr = 2\pi C[r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

We also know that frictional torque on the ring,

$$T_r = \mu.\delta W.r = \mu \times 2\pi C.dr.r = 2\pi\mu.C.r.dr$$

$\therefore$ Total frictional torque on the bearing,

$$T = \int_{r_2}^{r_1} 2\pi\mu.C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\mu.C[(r_1)^2 - (r_2)^2]$$

Substituting the value of C from equation (*ii*),

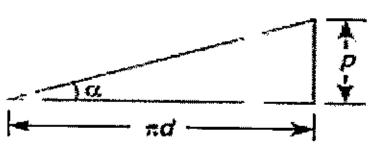
$$T = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)}[(r_1)^2 - (r_2)^2] = \frac{1}{2} \times \mu W(r_1 + r_2)$$

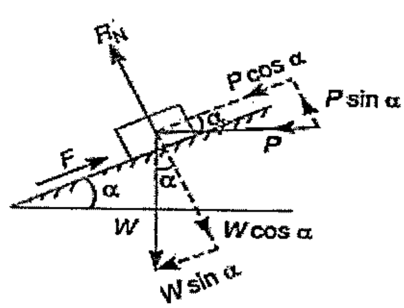
3. (a) ANS: **[7M]**

TORQUE TRANSMITTED FOR A BODY MOVING DOWN AN INCLENED PLAE:

Torque Required to Lower the Load by a Screw Jack

Let
- p = Pitch of the screw,
- d = Mean diameter of the screw,
- α = Helix angle,
- P = Effort applied at the circumference of the screw to lower the load,
- W = Weight to be lowered, and
- μ = Coefficient of friction between the screw and nut = tan ϕ, where ϕ is the friction angle.





From the geometry of the figure, we find that

$$\tan\alpha = p/\pi d$$

Since the load is being lowered, therefore the force of friction ($F = \mu.R_N$) will act upwards. All the forces acting on the screw are shown in Fig. 10.13 (*b*).

Resolving the forces along the plane,

$$P\cos\alpha = F - W\sin\alpha = \mu.R_N - W\sin\alpha \quad ...(i)$$

and resolving the forces perpendicular to the plane,

$$R_N = W\cos\alpha - P\sin\alpha \quad ...(ii)$$

Substituting this value of R_N in equation (*i*),

$$P\cos\alpha = \mu(W\cos\alpha - P\sin\alpha) - W\sin\alpha$$

$$= \mu.W\cos\alpha - \mu.P\sin\alpha - W\sin\alpha$$

or $$P\cos\alpha + \mu.P\sin\alpha = \mu.W\cos\alpha - W\sin\alpha$$

or $$P(\cos\alpha + \mu\sin\alpha) = W(\mu\cos\alpha - \sin\alpha)$$

$$\therefore \quad P = W \times \frac{(\mu\cos\alpha - \sin\alpha)}{(\cos\alpha + \mu\sin\alpha)}$$

Substituting the value of $\mu = \tan\phi$ in the above equation, we get

$$P = W \times \frac{(\tan\phi\cos\alpha - \sin\alpha)}{(\cos\alpha + \tan\phi\sin\alpha)}$$

Multiplying the numerator and denominator by $\cos\phi$,

$$P = W \times \frac{(\sin\phi\cos\alpha - \sin\alpha\cos\phi)}{(\cos\alpha\cos\phi + \sin\phi\sin\alpha)} = W \times \frac{\sin(\phi - \alpha)}{\cos(\phi - \alpha)}$$

$$= W\tan(\phi - \alpha)$$

$\therefore$ Torque required to overcome friction between the screw and nut,

$$T = P \times \frac{d}{2} = W\tan(\phi - \alpha)\frac{d}{2}$$

3. (b) ANS: **[5M]**

Given : $p = 10$ mm ; $d = 50$ mm ; $W = 20$ kN $= 20 \times 10^3$ N ; $D_2 = 60$ mm or $R_2 = 30$ mm ; $D_1 = 10$ mm or $R_1 = 5$ mm ; $\mu = \tan\phi = \mu_1 = 0.08$

We know that $$\tan\alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 50} = 0.0637$$

$\therefore$ Force required at the circumference of the screw to lift the load,

$$P = W\tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha\tan\phi}\right]$$

$$= 20 \times 10^3\left[\frac{0.0637 + 0.08}{1 - 0.0637 \times 0.08}\right] = 2890 \text{ N}$$

and torque required to overcome friction at the screw,

$$T = P \times d/2 = 2890 \times 50/2 = 72250 \text{ N-mm} = 72.25 \text{ N-m}$$

Since the load is lifted through a vertical distance of 170 mm and the distance moved by the screw in one rotation is 10 mm (equal to pitch), therefore number of rotations made by the screw,

$$N = 170/10 = 17$$

1. *When the load rotates with the screw*

We know that work done in lifting the load

$$= T \times 2\pi N = 72.25 \times 2\pi \times 17 = 7718 \text{ N-m} \quad \text{Ans.}$$

and efficiency of the screw jack,

$$\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)} = \frac{\tan\alpha(1-\tan\alpha\tan\phi)}{\tan\alpha+\tan\alpha}$$

$$= \frac{0.0637(1-0.0637\times0.08)}{0.0637 + 0.08} = 0.441 \text{ or } 44.1\% \text{ Ans.}$$

4. (a) ANS:

Band and Block Brake

The band brake may be lined with blocks of wood or other material, as shown in Fig. (*a*). The friction between the blocks and the drum provides braking action. Let there are '*n*' number of blocks. each subtending an angle 2θ at the centre and the drum rotates in anticlockwise direction.

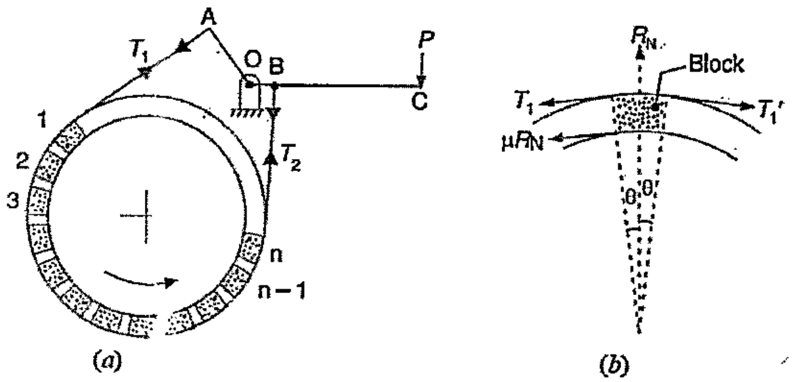


Fig. Band and block brake.

Let T_1 = Tension in the tight side,

T_2 = Tension in the slack side,

μ = Coefficient of friction between the blocks and drum,

T_1' = Tension in the band between the first and second block,

Consider one of the blocks (say first block) as shown in Fig. This is in equilibrium under the action of the following forces :

1. Tension in the tight side (T_1),
2. Tension in the slack side (T_1') or tension in the band between the first and second block,
3. Normal reaction of the drum on the block (R_N), and
4. The force of friction ($\mu.R_N$).

Resolving the forces radially, we have

$$(T_1 + T_1')\sin\theta = R_N \quad \text{... (i)}$$

Resolving the forces tangentially, we have

$$(T_1 + T_1')\cos\theta = \mu.R_N \quad \text{... (ii)}$$

Dividing equation (*ii*) by (*i*), we have

$$\frac{(T_1 - T_1')\cos\theta}{(T_1 + T_1')\sin\theta} = \frac{\mu.R_N}{R_N}$$

or

$$(T_1 - T_1') = \mu\tan\theta(T_1 + T_1')$$

$$\therefore \quad \frac{T_1}{T_1'} = \frac{1+\mu\tan\theta}{1-\mu\tan\theta}$$

4. (b) ANS:

. Rope Brake Dynamometer

It is another form of absorption type dynamometer which is most commonly used for measuring the brake power of the engine. It consists of one, two or more ropes wound around the flywheel or rim of a pulley fixed rigidly to the shaft of an engine. The upper end of the ropes is attached to a spring balance while the lower end of the ropes is kept in position by applying a dead weight as shown in Fig.

In order to prevent the slipping of the rope over the flywheel, wooden blocks are placed at intervals around the circumference of the flywheel.

In the operation of the brake, the engine is made to run at a constant speed. The frictional torque, due to the rope, must be equal to the torque being transmitted by the engine.

Let W = Dead load in newtons,

S = Spring balance reading in newtons,

D = Diameter of the wheel in metres,

d = diameter of rope in metres, and

N = Speed of the engine shaft in r.p.m.

$\therefore$ Net load on the brake

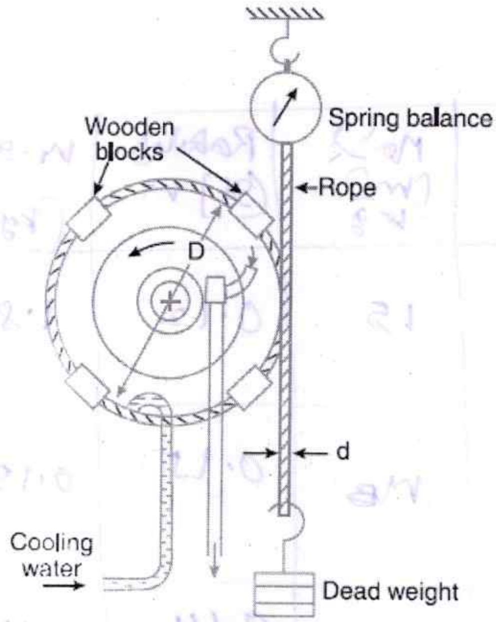
$$= (W - S)\text{ N}$$

We know that distance moved in one revolution

$$= \pi(D+d)\text{m}$$

$\therefore$ Work done per revolution

$$= (W - S)\pi(D+d)\text{ N-m}$$



5. ANS:

Let

mass of A = m_A = 15 kg

mass of B = m_B = ?

mass of C = m_C = 10 kg

mass of D = m_D = 8 kg

radii of A = r_A = 12 cm = 0.12 m

radii of B = r_B = 15 cm = 0.15 m

radii of C = r_C = 14 cm = 0.14 m

radii of D = r_D = 18 cm = 0.18 m

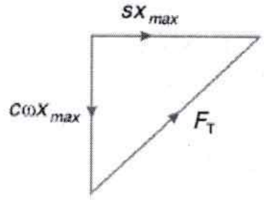
6. (a) ANS: **[4M]**

The ratio of the force transmitted (F_T) to the force applied (F) is known as the *isolation factor* or *transmissibility ratio* of the spring support.

We have discussed above that the force transmitted to the foundation consists of the following two forces :

1. Spring force or elastic force which is equal to $s.\,x_{max}$, and
2. Damping force which is equal to $c.\,\omega\,x_{max}$.

Since these two forces are perpendicular to one another , as shown in Fig. therefore the force transmitted,



$$F_T = \sqrt{(s.x_{max})^2 + (c.\omega.x_{max})^2}$$

$$= x_{max}\sqrt{s^2 + c^2.\omega^2}$$

$\therefore$ Transmissibility ratio,

$$\varepsilon = \frac{F_T}{F} = \frac{x_{max}\sqrt{s^2 + c^2.\omega^2}}{F}$$

$$\varepsilon = \frac{\sqrt{1+\left(\dfrac{2c.\omega}{c_c.\omega_n}\right)^2}}{\sqrt{\left(\dfrac{2c.\omega}{c_c.\omega_n}\right)^2+\left(1-\dfrac{\omega^2}{(\omega_n)^2}\right)^2}} \quad \ldots (i)$$

6. (b) ANS:

Given : m = 2 kg ; F = 25 N ; Resonant x_{max} = 12.5 mm = 0.0125 m ; t_p = 0.2 s ; f = 4 Hz

Damping coefficient

Let c = Damping coefficient in N/m/s.

We know that natural circular frequency of the exicting force,

$$\omega_n = 2\pi / t_p = 2\pi / 0.2 = 31.42 \text{ rad/s}$$

We also know that the maximum amplitude of vibration at resonance (x_{max}),

$$0.0125 = \frac{F}{c.\omega_n} = \frac{25}{c \times 31.42} = \frac{0.796}{c} \quad \text{or } c = 63.7 \text{ N/m/s } \textbf{Ans.}$$

Percentage increase in amplitude

Since the system is excited by a harmonic force of frequency (f) = 4 Hz, therefore corresponding circular frequency

$$\omega = 2\pi \times f = 2\pi \times 4 = 25.14 \text{ rad/s}$$

We know that maximum amplitude of vibration with damping,

$$x_{max} = \frac{F}{\sqrt{c^2.\omega^2 + (s - m.\omega^2)^2}}$$

$$= \frac{25}{\sqrt{(63.7)^2 (25.14)^2 + [2(31.42)^2 - 2(25.14)^2]^2}}$$

$$\ldots \left[\because (\omega_n)^2 = s/m \quad \text{or} \quad s = m(\omega_n)^2\right]$$

$$= \frac{25}{\sqrt{2.56 \times 10^6 + 0.5 \times 10^6}} = \frac{25}{1749} = 0.0143 \text{ m} = 14.3 \text{ mm}$$

and the maximum amplitude of vibration when damper is removed,

$$x_{max} = \frac{F}{m\left[(\omega_n)^2 - \omega^2\right]} = \frac{25}{2[(31.42)^2 - (25.14)^2]} = \frac{25}{710} = 0.0352 \text{ m}$$

$$= 35.2 \text{ mm}$$

$\therefore$ Percentage increase in amplitude

$$= \frac{35.2 - 14.3}{14.3} = 1.46 \quad \text{or} \quad 146\% \textbf{ Ans.}$$

7. ANS:

Solution. Given : d = 50 mm = 0.05 m ; l = 3 m, W_1 = 1000 N ; W_2 = 1500 N ; W_3 = 750 N; E = 200 GN/m^2 = 200 × 10^9 N/m^2 Take $g \approx 10$ m/s^2

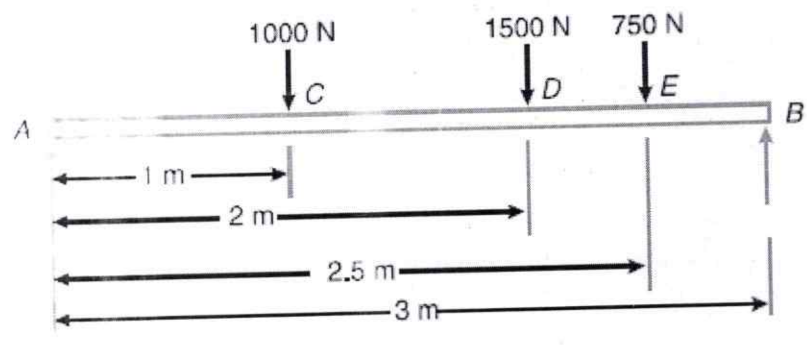
The shaft carrying the loads is shown in Fig. 23.13

We know that moment of inertia of the shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.05)^4 = 0.307 \times 10^{-6} \text{m}^4$$

and the static deflection due to a point load W,

$$\delta = \frac{Wa^2b^2}{3EIl}$$



$\therefore$ Static deflection due to a load of 1000 N,

$$\delta_1 = \frac{1000 \times 1^2 \times 2^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3} = 7.24 \times 10^{-3} \text{ m}$$

... (Here $a = 1$ m, and $b = 2$ m)

Similarly, static deflection due to a load of 1500 N,

$$\delta_2 = \frac{1500 \times 2^2 \times 1^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3} = 10.86 \times 10^{-3} \text{ m}$$

... (Here $a = 2$ m, and $b = 1$ m)

and static deflection due to a load of 750 N,

$$\delta_3 = \frac{750 (2.5)^2 (0.5)^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3} = 2.12 \times 10^{-3} \text{ m}$$

... (Here $a = 2.5$ m, and $b = 0.5$ m)

We know that frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3}} = \frac{0.4985}{\sqrt{7.24 \times 10^{-3} + 10.86 \times 10^{-3} + 2.12 \times 10^{-3}}}$$

$$= \frac{0.4985}{0.1422} = 3.5 \text{ Hz } \textbf{Ans.}$$

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Subject Code: R16ME3103

III B.Tech I Semester Regular Examinations, November-2018.

MACHINE DYNAMICS AND VIBRATIONS

(ME)

Time: 3 hours **Max Marks: 60**

Question Paper Consists of **Part-A** and **Part-B.**

Answering the question in **Part-A** is Compulsory & Four Questions should be answered from Part-B

All questions carry equal marks of 12.

PART-A

1. (a) What are the various types of friction experienced by a body?
   (b) Which of the two assumptions-uniform intensity of pressure or uniform rate of wear, would you make use of in designing friction clutch and why?
   (c) What are the various types of the brakes.?
   (d) Why is balancing of rotating parts necessary for high speed engines?
   (e) What is the difference between free and forced vibrations.
   (f) What is meant by a torsionally equivalent length of a shaft as referred to stepped shaft?

[2+2+2+2+2+2]

PART-B

4 X 12 = 48

2. (a) Derive from first principles an expression for the effort required to raise a load with a screw jack taking friction into consideration. 6
   (b) A square threaded screw of mean diameter 25mm and pitch of thread 6mm is utilised to lift a weight of 10kN by a horizontal force applied at the circumference of the screw. Find the magnitude of the force if the coefficient of friction between the nut and screw is 0.12. 6

3. A car engine has its rated output of 12 kW. The maximum torque developed is 100 N-m. The clutch used is of single plate type having two active surfaces. The axial pressure is not to exceed 85 kN/m^2.The external diameter of the friction plate is 1.25 times the internal diameter. Determine the dimensions of the friction plate and the axial force exerted by the springs. Coefficient of friction = 0.3. 12

4. A simple band brake operates on a drum of 600 mm in diameter that is running at 200 RPM. The coefficient of friction is 0.25. The brake band has a contact of 270°, one end is fastened to a fixed pin and the other end to the brake arm 125 mm from the fixed pin. The straight brake arm is 750 mm long and placed perpendicular to the diameter that bisects the angle of contact. 6
   a) What is the pull necessary on the end of the brake arm to stop the wheel if 35 kW is being absorbed? What is the direction for this minimum pull?
   b) What width of steel band of 2.5 mm thick is required for this brake if the maximum tensile stress is not to exceed50 N/mm^2? 6

5. A rotating shaft carries four masses *A*, *B*, *C* and *D* which are radially attached to it. The mass centres are 30 mm, 38 mm, 40 mm and 35 mm respectively from the axis of rotation. The masses *A*, *C* and *D* are 7.5 kg, 5 kg and 4 kg respectively. The axial distances between the planes of rotation of *A* and *B* is 400 mm and between *B* and *C* is 500 mm. The masses *A* and *C* are at right angles to each other. Find for a complete balance,

   a) The angles between the masses *B* and *D* from mass *A*, 4
   b) The axial distance between the planes of rotation of *C* and *D*, 4
   c) The magnitude of mass *B*. 4

6. a) What are the various types of vibrations? 6
   b) Derive the natural frequency of free longitudinal vibrations of systems having single degree of freedom by equilibrium method. 6

7. A steel shaft ABCD 1.5m long has flywheel at its ends A and D. The mass of the flywheel A is 600kg and has a radius of gyration of o.6 m. The mass of the flywheel D is 800 kg and has a radius gyration of 0.9m.The connecting shaft has a diameter of 50mm for the portion AB which is 0.4m long :and has a diameter of 60mm for the portion BC which is 0.5m long: and has a diameter of d for the portion CD which is 0.6m long. Determine
   a) The diameter d of the portion CD so that the node of the torsional vibration of the system will be at the centre of the length BC and 4
   b) The natural frequency of the torsional vibrations. 4
   c) Take modulus of rigidity for the shaft material as 60 GN/m² 4

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R16ME3103

NARASARAOPETA ENGINEERING COLLEGE (AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

III BTech I Sem Regular Exam, NOV-2018

MACHINE DYNAMICS AND VIBRATIONS

PART-A

1. (a) What are the various types of friction experienced by a body?

ANS:

In general, the friction is of the following two types:

a. Static Friction

b. Dynamic Friction

Dynamic Friction is of the following three types:

i. Sliding friction

ii. Rolling friction

iii. Pivot friction.

(b) Which of the two assumptions- uniform intensity of pressure or uniform rate of wear, would you make use of in designing friction clutch and why?

ANS:

In case of a new clutch, the intensity of pressure is approximately uniform but in an old clutch the uniform wear theory is more approximate. The uniform pressure theory gives a higher frictional torque than the uniform wear theory. Therefore in case of friction clutches, uniform wear should be considered, unless otherwise stated.

(c) What are the various types of the brakes?

ANS:

The brakes are mainly classified as:

a. Hydraulic brakes

b. Electric brakes

c. Mechanical brakes.

The mechanical brakes may be divided into the following two groups:

i. Radial Brakes i.e. External Brakes, Internal Brakes, Block/Shoe Brakes and Band Brakes.

ii. Axial Brakes i.e. Disc brakes and Cone brakes.

(d) Why is balancing of rotating parts necessary for high speed engines?

ANS:

Balancing of rotating parts is necessary for every engine; only in high speed engines it becomes very important. If these parts are not properly balanced, the dynamic forces are set up. These forces not only increase the loads on bearings and stresses in the various members, but also produce unpleasant and even dangerous vibrations. The balancing of unbalanced forces is caused by rotating masses, in order to minimize pressure on the main bearings when an engine is running.

(e) What is the difference between free and forced vibrations?

ANS:

FREE OR NATURAL VIBRATIONS:

When no external force acts on the body, after giving it an initial displacement, then the body is said to be under free or natural vibrations. The frequency of the free vibrations is called free or natural frequency.

FORCED VIBRATIONS:

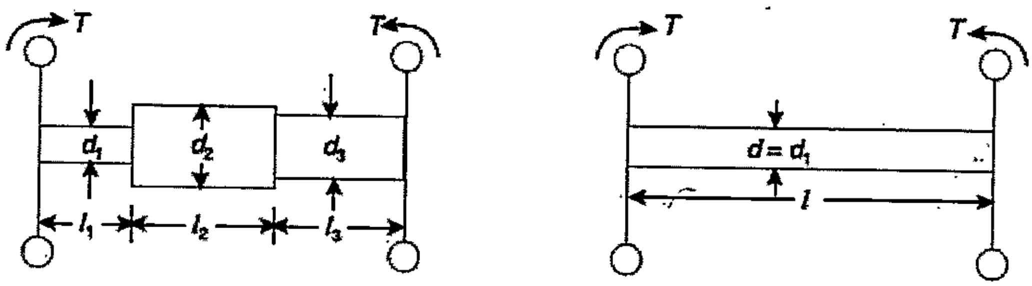
When the body vibrates under the influence of external force, then the body is said to be under forced vibrations. The external force applied to the body is a periodic disturbing force created by unbalance.

(f) What is meant by a torsionally equivalent length of a shaft as referred to stepped shaft?

ANS:

The shaft may have variable diameter for different lengths such shaft may, theoretically, be replaced by an equivalent shaft of uniform diameter.

The following two shafts must have the same total angle of twist when equal opposing torques T is applied at their opposite ends.



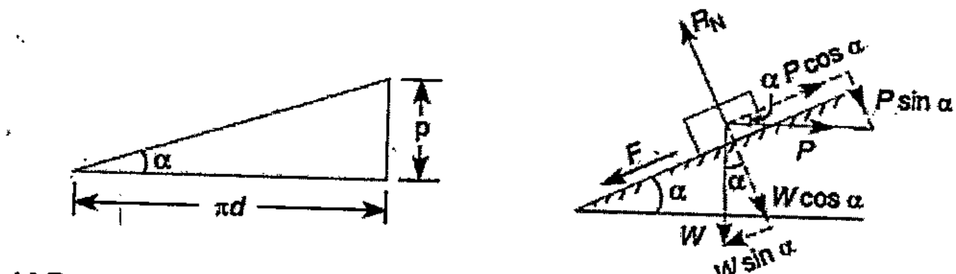
(*a*) Shaft of varying diameters. (*b*) Torsionally equivalent shaft.

PART-B **[4X12 = 48M]**

2. (a) Derive from first principles an expression for the effort required to raise a load with a screw jack taking friction into consideration?

ANS:

If one complete turn of a screw thread by imagined to be unwound, from the body of the screw and developed, it will form an inclined plane as shown in the below figure.



(a) Development of a screw. (b) Forces acting on the screw.

Let, p = Pitch of the screw,

d = Mean diameter of the screw,

α = Helix angle,

P = Effort applied at the circumference of the screw to lift the load,

W = Load to be lifted,

μ = Coefficient of friction, between the screw and nut = tan φ,

From the geometry of the above figure, we find that:

$$\tan \alpha = p/\pi d$$

Since the principle on which a screw jack works is similar to that of an inclined plane,

∴ Resolving the forces along the plane,

$$\Rightarrow P \cos \alpha = W \sin \alpha + F = W \sin \alpha + \mu . R_N \quad \text{.......................... (i)}$$

And resolving the forces perpendicular to the plane,

$$\therefore R_N = P \sin \alpha + W \cos \alpha \quad \text{.......................... (ii)}$$

From Equations (i) and (ii),

$$\therefore \quad P = W \times \frac{\sin \alpha + \mu \cos \alpha}{\cos \alpha - \mu \sin \alpha}$$

Substituting the value of μ = tan φ in the above equation, we get

$$\therefore \quad P = W \tan (\alpha + \Phi)$$

2. (b) A square threaded screw of mean diameter 25mm and pitch of thread 6mm is utilised to lift a weight of 10KN by a horizontal force applied at the circumference of the screw. Find the magnitude of the force if the coefficient of friction between the nut and screw is 0.12?

ANS: Given Data: Mean Diameter = D = 25mm

Pitch of thread = p = 6mm

Lifting Weight = 10KN

Coefficient of Friction = μ = 0.12 = tan Φ

$$\therefore \ \text{Tan}\ \alpha = \frac{p}{\pi d} = \frac{6}{\pi(25)} = 0.0764$$

Let, P = Force applied at the circumference of the screw,

$$\therefore \ P = W \tan(\alpha + \Phi) = W\left[\frac{\tan\alpha + \tan\emptyset}{1 - \tan\alpha\tan\emptyset}\right]$$

$$\therefore \ P = 10 \text{ X } 103 \left[\frac{0.0764 + 0.12}{1 - 0.0764 \times 0.12}\right] = 1982\text{N}$$

3. A car engine has its rated output of 12 kW. The maximum torque developed is 100 N-m. The clutch used is of single plate type having two active surfaces. The axial pressure is not to exceed 85 KN/m2.The external diameter of the friction plate is 1.25 times the internal diameter. Determine the dimensions of the friction plate and the axial force exerted by the springs. Coefficient of friction = 0.3.?

ANS: Given Data: Power Output = 12KW

The maximum Torque Developed = T = 100N-m

No. Of Pairs of Contact Surfaces = n = 2

The axial pressure = P = 85 KN/m2

External Diameter = 1.25 Internal Diameter

$$\therefore \ r_1 = 1.25\, r_2$$

Coefficient of Friction = μ = 0.3

Let, r_1 and r_2=Outer and inner radii of frictional surfaces, and T= Torque transmitted.

Since the intensity of pressure is maximum at the inner radius (r_2),

$$\therefore \ P.\, r_2 = C$$

$$\therefore \ C = 85X10^3\, r_2 \text{ N/mm}$$

The axial thrust transmitted to the frictional surface,

$$W = 2\pi C (r_1 - r_2) = 2\pi \times 85X10^3\, r_2 (1.25\, r_2 - r_2) = 133.52\ X\ 10^3\, (r_2)^2$$

We know that mean radius of the frictional surface for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{1.25\, r_2 + r_2}{2} = 1.125\, r_2$$

We know that, Torque transmitted = T = n.μ.W.R

$$\Rightarrow\ 100 = 2\ X\ 0.3\ X\ 133.52\ X\ 10^3\, (r_2)^2\ X\ 1.125\, r_2$$

$$\therefore\ r_2 = 0.1036\ m = 103.6\ mm\ \&\ r_1 = 1.25\, r_2 = 129.5\ mm$$

The axial force exerted by the springs = $W = 2\pi C (r_1 - r_2) = 133.52\ X\ 10^3\, (r_2)^2 = 1433\ N$

4. A simple band brake operates on a drum of 600 mm in diameter that is running at 200 RPM. The coefficient of friction is 0.25. The brake band has a contact of 270°, one end is fastened to a fixed pin and the other end to the brake arm 125 mm from the fixed pin. The straight brake arm is 750 mm long and placed perpendicular to the diameter that bisects the angle of contact. a) What is the pull necessary on the end of the brake arm to stop the wheel if 35 kW is being absorbed? What is the direction for this minimum pull? b) What width of steel band of 2.5 mm thick is required for this brake if the maximum tensile stress is not to exceed 50 N/mm²?

ANS: Given Data: Diameter of the Drum = d = 600 mm

Running Speed = N = 200 RPM

Coefficient of friction = μ = 0.25

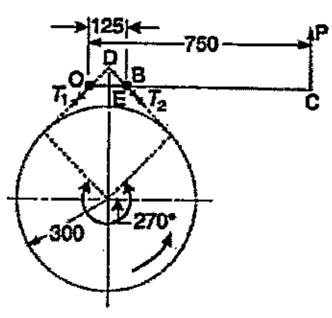
Angle of contact = θ = 270^0

Power = P = 35KW

Thickness of the Band = t = 2.5 mm

Maximum Tensile Stress = 50 N/mm^2

Since for minimum pull P on the end of the brake arm will act upward and when the wheel rotates anticlockwise. Therefore the end of the band attached to O will be tight with tension T_1 and the end of the band attached to B will be slack with tension T_2.



All Dimensions in mm

We know that,

$$2.3\log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 4.713$$

$\therefore \quad \frac{T_1}{T_2} = 3.25 \quad \text{... }(i)$

Let T_B = Braking torque.

We know that power absorbed,

$$35 \times 10^3 = \frac{2\pi \times N.T_B}{60}$$

$\therefore \quad T_B = 1667 \text{ N-m} = 1667 \times 10^3 \text{ N-mm}$

We also know that braking torque (T_B),

$$1667 \times 10^3 = (T_1 - T_2)\ r = (T_1 - T_2)\ 300$$

$\therefore \quad T_1 - T_2 = 5556 \text{ N} \quad \text{...}(ii)$

From equations (*i*) and (*ii*), we find that

$$T_1 = 8025 \text{ N}; \quad \text{and} \quad T_2 = 2469 \text{ N}$$

Now taking moments about O, we have

$$P \times 750 = T_2 \times \ OD = T_2 \times 62.5\sqrt{2}$$

$\therefore \quad P = 291 \text{ N}$

Let $\quad w$ = Width of steel band in mm.

We know that maximum tension in the band (T_1),

$$8025 = \sigma.w.t = 50 \times w \times 2.5$$

$\therefore \quad w = 64.2 \text{ mm}$

5. A rotating shaft carries four masses *A*, *B*, *C* and *D* which are radially attached to it. The mass centres are 30 mm, 38 mm, 40 mm and 35 mm respectively from the axis of rotation. The masses *A*, *C* and *D* are 7.5 kg, 5 kg and 4 kg respectively. The axial distances between the planes of rotation of *A* and *B* is 400 mm and between *B* and *C* is 500 mm. The masses *A* and *C* are at right angles to each other. Find for a complete balance,

a. The angles between the masses *B* and *D* from mass *A*,

b. The axial distance between the planes of rotation of *C* and *D*,

c. The magnitude of mass *B*.

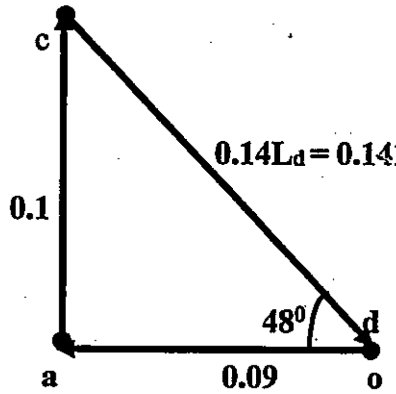
ANS:

Assume the plane of mass B as the reference plane (R.P.) and the given data is tabulated as below:

Let, m_b = Mass of B and L_d = Axial distance of mass D from B

Plane	Mass (m) Kg	Radius (r) M	Centrifugal Force / ω^2 (m r) Kg-m	Distance From R.P. (L) (m)	Couple / ω^2 (m r L) Kg-m^2
A	7.5	0.03	0.225	- 0.4	- 0.09
B (RP)	m_b	0.038	0.038 m_b	0	0
C	5	0.04	0.2	0.5	0.1
D	4	0.035	0.14	L_d	0.14 L_d

- **Couple Polygon:**



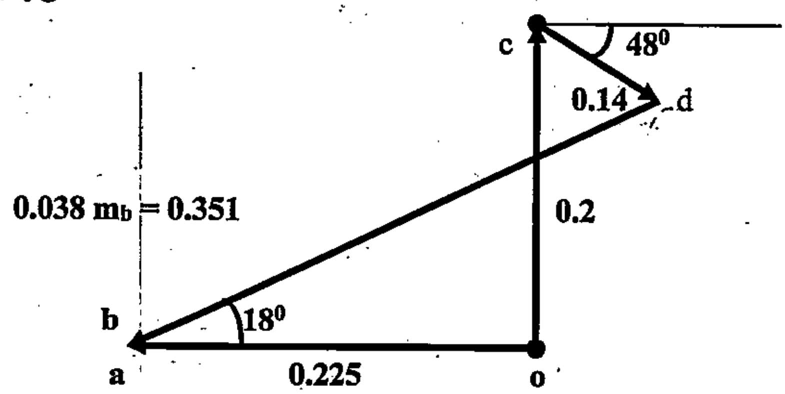
$\therefore$ From the above Couple Polygon:

oc = $0.14L_d = 0.141$ (Approximately)

$\Rightarrow L_d = 0.101$

Angle of Inclination of mass D wit Horizontal is 48^0

- **Force Polygon:**



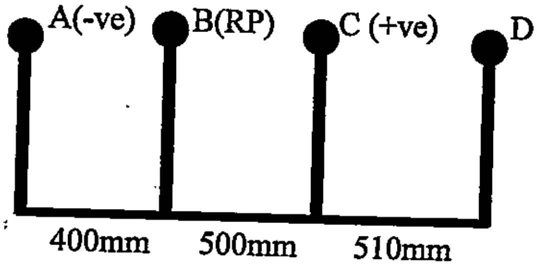
$\therefore$ From the above Force Polygon:

db = $0.038\ m_b = 0.351$ (approximately)

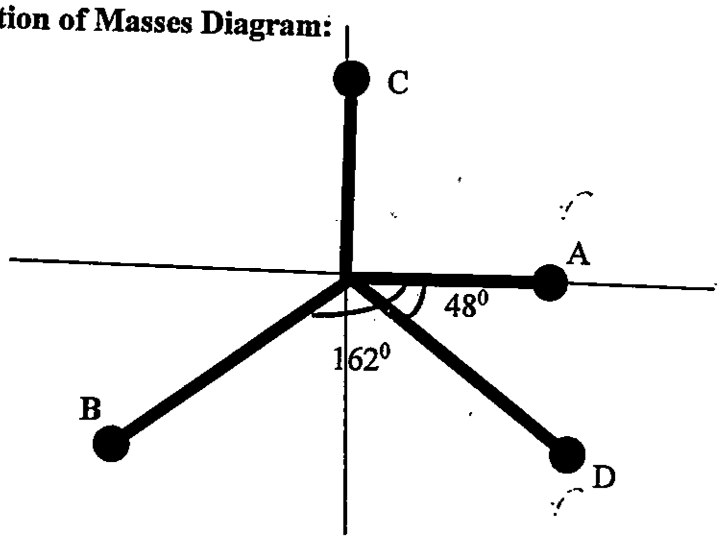
$\Rightarrow m_b = 920$ Kg

And Angle of Inclination of mass B with Horizontal is 72^0

Position of Planes Diagram:



Angular Position of Masses Diagram:



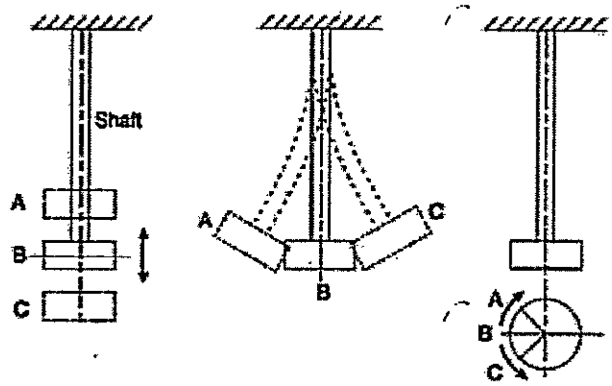
6. **a) What are the various types of vibrations?**

ANS: Types of Vibratory Motion:

1. **Free or natural vibrations:** When no external force acts on the body, after giving it an initial displacement, then the body is said to be under free or natural vibrations.
2. **Forced vibrations:** When the body vibrates under the influence of external force, then the body is said to be under forced vibrations.
3. **Damped vibrations:** When there is a reduction in amplitude over every cycle of vibration, the motion is said to be damped vibration.

Types of Free Vibrations:

1. **Longitudinal vibrations:** When the particles of the shaft or disc moves parallel to the axis of the shaft
2. **Transverse vibrations:** When the particles of the shaft or disc move approximately perpendicular to the axis of the shaft
3. **Torsional vibrations:** When the particles of the shaft or disc move in a circle about the axis of the shaft.



(a) Longitudinal vibrations. (b) Transverse vibrations. (c) Torsional vibrations.

6. b) Derive the natural frequency of free longitudinal vibrations of systems having single degree of freedom by equilibrium method?

ANS:

Equilibrium Method

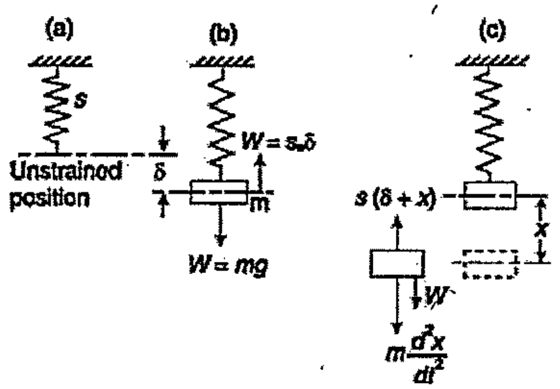
Let s = Stiffness of the constraint.

m = Mass of the body suspended from the constraint in kg,

W = Weight of the body in newtons = $m.g$,

δ = Static deflection of the spring in metres due to weight W newtons, and

x = Displacement given to the body by the external force, in metres.



In the equilibrium position, as shown in Fig., the gravitational pull $W = m.g$, is balanced by a force of spring, such that $W = s.\delta$.

Restoring force $= W - s(\delta + x) = s.\delta - s.\delta - s.x = -s.x$ ... (i)

and Accelerating force $= m \times \dfrac{d^2x}{dt^2}$ ... (ii)

Equating equations (i) and (ii),

$$\therefore \quad \frac{d^2x}{dt^2} + \frac{s}{m} \times x = 0 \qquad \text{... (iii)}$$

We know that the fundamental equation of simple harmonic motion is

$$\frac{d^2x}{dt^2} + \omega^2.x = 0 \qquad \text{... (iv)}$$

Comparing equations (*iii*) and (*iv*), we have $\omega = \sqrt{\frac{s}{m}}$

$\therefore$ Time period, $t_p = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{s}}$

and natural frequency, $f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$

7. **A steel shaft ABCD 1.5 m long has flywheel at its ends A and D. The mass of the flywheel A is 600 kg and has a radius of gyration of 0.6 m. The mass of the flywheel D is 800 kg and has a radius of gyration of 0.9 m. The connecting shaft has a diameter of 50 mm for the portion AB which is 0.4 m long; and has a diameter of 60 mm for the portion BC which is 0.5 m long; and has a diameter of d mm for the portion CD which is 0.6 m long. Determine :**

   a) The diameter d of the portion CD so that the node of the torsional vibration of the system will be at the centre of the length BC; and

   b) The natural frequency of the torsional vibrations.

   c) The modulus of rigidity for the shaft material is 60 GN/m².

ANS: Given Data: Length of the shaft = L = 1.5 m

Mass of the flywheel A = m_A = 600 kg & Radius of Gyration of A = k_A = 0.6 m

Mass of the Flywheel D = m_D = 800 kg & Radius of Gyration of D = k_D = 0.9 m

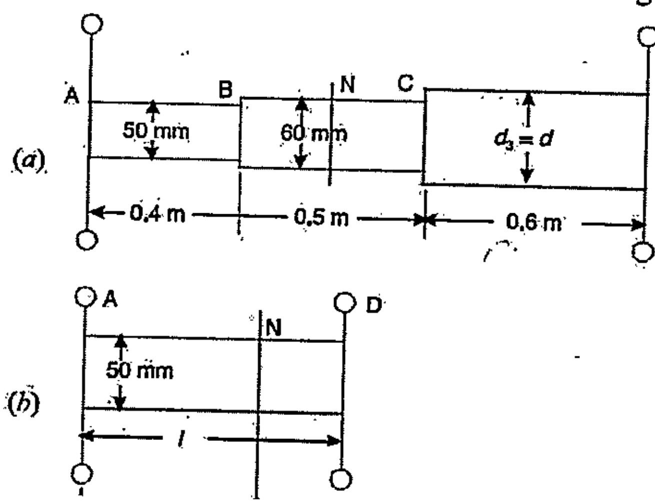
AB Portion Diameter = d_1 = 50 mm = 0.05 m & Length of AB = L_1 = 0.4 m

BC Portion Diameter = d_2 = 60 mm = 0.06 m & Length of BC L_2 = 0.5 m

CD Portion Diameter = d_3 = d & Length of CD = L_3 = 0.6 m

C = 60 GN/m² = 60×10^9 N/m²

The actual shaft is shown is Figure (a). First of all, let us find the length of the equivalent shaft, assuming its diameter as d_1 = 50 mm, as shown in Figure (b).



Length of Equivalent Shaft,

$$\Rightarrow L = L_1 + L_2 \frac{d^4{}_1}{d_2} + L_3 \frac{d^4{}_1}{d_3} = 0.4 + 0.5 \frac{0.05^4}{0.06} + 0.6 \frac{0.05^4}{d} = 0.64 \text{ X } \frac{3.7510^6}{d^4} \text{ m} \quad \ldots\ldots \text{(i)}$$

a) Diameter d of the shaft CD:

Let, L_A = Distance of the node from Flywheel A, and

L_D = Distance of the node from Flywheel D, and

∴ Mass Moment of Inertia of Flywheel A = $I_A = m_A (k_A)2 = 216$ kg-m²

∴ Mass Moment of Inertia of Flywheel D = $I_D = m_D (k_D)2 = 648$ kg-m²

We know that, $L_A I_A = L_D I_D$

$$\Rightarrow L_D = \frac{L_A}{3} \quad \ldots\ldots\ldots\ldots \text{(ii)}$$

∴ Equivalent Length from Rotor A = $L_A = L_1 + \frac{L_1}{2}\frac{d_1}{d_2} = 0.52$m

(ii) ⟹ $L_D = 0.173$m

We know that, $L = L_A + L_D$

$$\Rightarrow 0.64 \text{ X } \frac{3.7510^6}{d^4} = 0.52 + 0.173$$

$$\Rightarrow \mathbf{d = 0.0917m = 91.7mm}$$

b) Natural frequency of Torsional Vibrations:

Polar Moment of Inertia of the Equivalent Shaft = $J = \frac{\pi}{32}(d_1)^4 = 0.614 \text{ X } 10^{-6}\text{ m}^4$

Natural frequency of Torsional Vibrations,

$$f_n = f_{nA} = f_{nD}$$

$$\Rightarrow f_n = \frac{1}{2\pi}\sqrt{\frac{CJ}{L_A I_A}} = \frac{1}{2\pi}\sqrt{\frac{60\times10^9\times0.614\times10^{-6}}{0.52\times216}} = 2.88 \text{ Hz}$$

Prepared

By

Babu 15/11/18

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Subject Code: R16ME3103

III B.Tech I Semester Supple Examinations, March-2022

MACHINE DYNAMICS AND VIBRATIONS

(ME)

Time: 3 hours **Max Marks: 60**

Question Paper Consists of **Part-A** and **Part-B.**

Answering the question in **Part-A** is Compulsory & Four Questions should be answered from Part-B

All questions carry equal marks of 12.

PART-A

1. (a)A 60 mm diameter shaft running in a bearing carries a load of 1000 N. If the coefficient of friction between the shaft and bearing is 0.03, find the power transmitted when it runs at 1200 r.p.m.

(b) Define uniform wear theory for single plate clutch.

(c)List the applications of brakes.

(d)State any two diferrences between static balancing and dynamic balancing.

(e)What do you mean by vibrations? Explain with any two examples in your daily life.

(f)Exlain about torsional vibrations.

[2+2+2+2+2+2]

PART-B

4 X 12 = 48

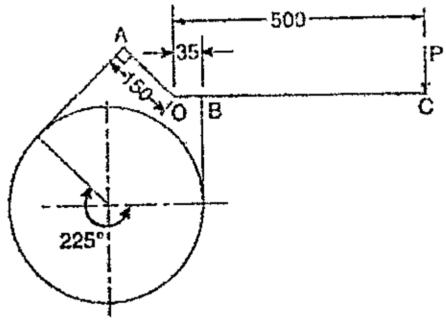
2. (a) A body, resting on a rough horizontal plane required a pull of 180 N inclined at 30° to the plane just to move it. It was found that a push of 220 N inclined at 30° to the plane just moved the body. Determine the weight of the body and the coefficient of friction **8M**

(b) State the diferrenceses between Boundary friction and film friction. **4M**

3. (a) A single plate clutch, with both sides effective, has outer and inner diameters 200 mm and 100 mm respectively. The maximum intensity of pressure at any point in the contact surface is not to exceed 0.1 N/mm^2. If the coefficient of friction is 0.3, determine the power transmitted by a clutch ~~at a speed 2000 r.p.m.~~ 4M

(b) Explain the working of clone clutch with a neat sketch. **8M**

4. A differential band brake, as shown in figure, below has an angle of contact of 225°. The band has a compressed woven lining and bears against a cast iron drum of 350 mm diameter. The brake is to sustain a torque of 350 N-m and the coefficient of friction between the band and the drum is 0.3. Find i)The necessary force (P) for the clockwise and anticlockwise rotation of the drum; and ii) The value of 'OA' for the brake to be self locking, when the drum rotates clockwise.**12M**



All dimensions in mm.

5. (a)Why is balancing necesscary for high speed engines ? **4M**

(b) A, B, C and D are four masses carried by a rotating shaft at radii 100, 125, 200 and 150 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the mass of B, C and D are 10 kg, 5 kg, and 4 kg respectively.Find the required mass A and the relative angular settings of the four masses so that the shaft shall be in complete balance. **8M**

6. The mass of a single degree damped vibrating system is 7.5 kg and makes 24 free oscillations in 14 seconds when disturbed from its equilibrium position. The amplitude of vibration reduces to 0.25 of its initial value after five oscillations. Determine

1. stiffness of the spring
2. logarithmic decrement, and
3. damping factor, i.e. the ratio of the system damping to critical damping. **12M**

7. (a)Determine the natural frequency of free transverse vibration due to uniformly distributed load acting over a simply supported shaft. **5 M**

(b) A shaft 50mm diameter and 3 m long is simply suppported at the ends and carries three loads of 1250N,950N,650N at 3m,5m,7m from the left support. The Young's modulus for shaft material is 200 GN/m^2. Find the frequency of transverse vibration. **7M**

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Narasaraopeta Engineering College (Autonomous)
Kotappakonda Road, Yellamanda (P.O), Narasaraopet- 522601, Guntur District, AP.

Subject Code: R16MMD104

M.Tech - I Semester Regular and Supplementary Examinations, Dec – 2018.

MECHANICAL VIBRATIONS

(MD)

Time: 3 hours **Max Marks: 60**

Answer any FIVE questions.
All questions carry EQUAL marks

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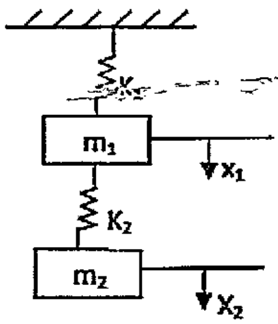
1. (a) Define the following terms? 4M
   i) Periodic motion
   ii) Fundamental mode of vibration
   iii) Degree of freedom
   iv) SHM (simple harmonic motion)

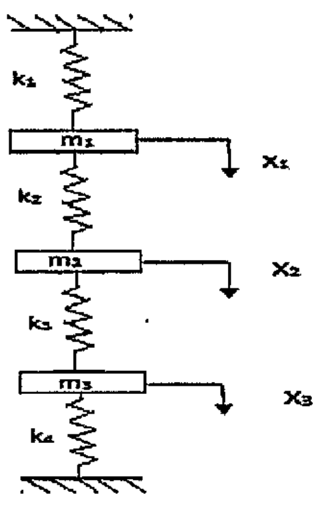
   b) Derive an expression for vibration response of a single degree of freedom system if the damping provided is under damped system. 8M

2. (a) Differentiate between free vibrations and forced vibrations? 4M

   (b) Find the normal modes of the system shown in figure below. Assume that $k_1 = k_2 = k$ and $m_1 = m_2 = m$ 8M



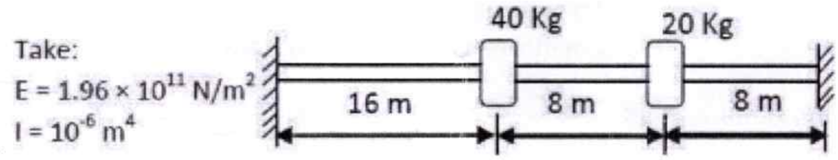
3. Determine the natural frequencies of the system shown in the Fig.2. Assume $m_1 = m_2 = m_3 = m$ and $k_1 = k_2 = k_3 = k_4 = k$ 12M



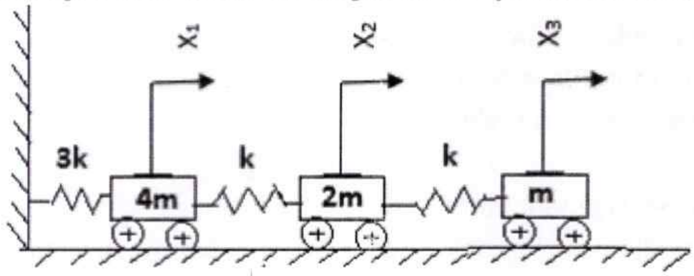
4. Write short on any three of the following: a) Damping ratio b) Undamped system (no damped) c) Under damped d) Critical damped e) Logarithmic decrement. 12M

5. Prove that the critical speed of whirling speed for a rotating shaft is same as the frequency of natural transverse vibration. 12M

6. Find the natural frequency of transverse vibrations for the system shown below by Rayleigh Method. 12M



7. Determine the natural frequencies and mode shapes of the system shown in figure below. 12M



8. a) What are the principles on which a Vibrometer and an accelerometer are based? Explain. 6M

b) Discuss Seismic instrument with help of a sketch? 6M

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PSTE → 16

Narasaraopeta Engineering College (Autonomous)

Kotappakonda Road, Yellamanda (P.O), Narasaraopet- 522601, Guntur District, AP.

Subject Code: R16ME3103

III B.Tech I Semester Regular & Supple Examinations, March-2021

MACHINE DYNAMICS AND VIBRATIONS

(ME)

Time: 3 hours **Max Marks: 60**

Question Paper Consists of **Part-A** and **Part-B.**

Answering the question in **Part-A** is Compulsory & Four Questions should be answered from Part-B

All questions carry equal marks of 12.

PART-A

1. (a) Why static coefficient of friction is always greater than kinetic coefficient of friction?
   (b) Why a service factor is used for calculating the design capacity of a clutch? √
   (c) Differentiate between a brake and a clutch.
   (d) State the conditions for static and dynamic balancing.
   (e) Determine the natural frequency of mass of 10kg suspended at the bottom of two springs stiffness: 5N/mm and 8N/mm in series.
   (f) Define transmissibility ratio or isolation factor

[2+2+2+2+2+2]

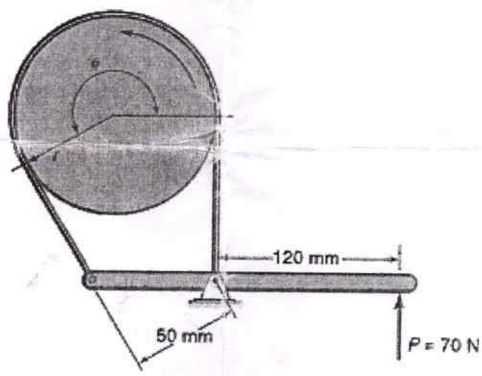
PART-B

4 X 12 = 48

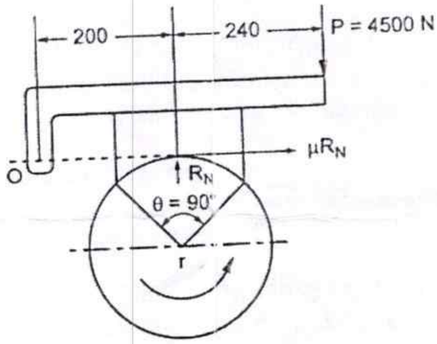
2. (a) A block weighing 100 N is resting on a rough plane inclined 20^0 to the horizontal. It is acted upon by a force of 50N directed upward at angle of 14^0 above the plane. Determine the friction. If the block is about to move up the plane, determine the co-efficient of friction.
   (b) A block weighing 1000N is kept on a rough plane inclined at 40^0 to the horizontal. The coefficient of friction between the block and the plane is 0.4. Determine the smallest force inclined at 15^0 to the plane required just to move the block up the plane.

3. Explain about the single plate clutch with neat diagram

4. (a) The band brake as shown in fig. has wrap angle of 215^0 and drum radius 60mm. Calculate the brake torque when coefficient of friction is 0.25.



(b) Calculate the average bearing pressure and the initial and average braking powers for the block shoe shown in fig.. The diameter of the drum is 400 mm and it rotates at 200 rpm. Coefficient of friction is 0.2 and drum width is 75 mm.



5. A, B, C and D are four masses carried by a rotating shaft at radii 100mm, 150mm, 150 mm and 200mm respectively. The planes of masses B, C and D are at 200mm, 300mm and 500mm respectively from plane of mass A. The magnitude *of* the masses B, C and D are 9kg, 5 kg and 4 kg respectively. Find the required mass A and the relative angular settings, *of* the 4 masses so that the shaft shall be in complete balance.

6. A vibrating system consists of a mass 50 kg, a spring of stiffness 30 KN/m and a damper. The damping provided is only 20% of the critical value. Determine:
(a) The damping factor.
(b) The critical damping coefficient.
(c) The logarithmic decrement.
(d) The natural frequency of damped vibrations.
(e) The ratio of two consecutive amplitudes.

7. A shaft, 50 mm diameter and 3 m long, is simply supported at the ends. It carries three masses 100 kg, 120 kg and 80 kg at 1.0 m, 1.75 m and 2.5 m respectively from the left support. Taking E = 20 GN/m^2, find the frequency of transverse vibrations using Dunkerley's method.

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Narasaraopeta Engineering College (Autonomous)

Kotappakonda Road, Yellamanda (P.O), Narasaraopet- 522601, Guntur District, AP.

Subject Code: R16ME3103

III B.Tech I Semester Supple Examinations, October-2021

MACHINE DYNAMICS AND VIBRATIONS

(ME)

Time: 3 hours **Max Marks: 60**

Question Paper Consists of **Part-A** and **Part-B.**

Answering the question in **Part-A** is Compulsory & Four Questions should be answered from Part-B

All questions carry equal marks of 12.

PART-A

1. (a) Discuss briefly the various types of friction experienced by a body
   (b) What are the advantages of centrifugal clutch?
   (c) How does the function of a brake differ from that of a clutch?
   (d) Discuss in brief about the terms 'static balancing' and 'dynamic balancing'.
   (e) Explain the term 'Logarithmic decrement' as applied to damped vibrations.
   (f) Discuss the effect of inertia of a shaft on the free torsional vibrations.

[2+2+2+2+2+2]

PART-B

4 X 12 = 48

2. A vertical screw with single start square threads 50 mm mean diameter and 12.5 mm pitch is raised against a load of 10 kN by means of a hand wheel, the boss of which is threaded to act as a nut. The axial load is taken up by a thrust collar which supports the wheel boss and has a mean diameter of 60 mm. If the coefficient of friction is 0.15 for the screw and 0.18 for the collar, and the tangential force applied by each hand to the wheel is 100 N; find suitable diameter of the hand wheel.

3. a) What are the materials used for lining of friction surfaces? [3]
   b) A disc clutch with a single friction surface has coefficient of friction equal to 0.3. The maximum pressure which can be imposed on the friction material is 1.5 MPa. The outer diameter of the clutch plate is 200 mm and its internal diameter is 100 mm. Assuming uniform wear theory for the clutch plate, Determine the maximum torque (in Nm) that can be transmitted [9]

4. a) What is a self-energizing brake ? When a brake becomes self-locking. [3]
   b) A differential band brake shown in Fig. 1 is operated by a lever of length 500 mm. The brake drum has a diameter of 500 mm and the maximum torque on the drum is 1000 N-m. The band brake embraces $2/3^{rd}$ of the circumference. One end of the band is attached to a pin 100 mm from the fulcrum and the other end to another pin 80 mm from the fulcrum and on the other side of it when the operating force is also acting. If the band brake is lined with asbestos fabric having a coefficient of friction 0.3, find the operating force required. Design the steel band, shaft, key, lever and fulcrum pin. The permissible stresses may be taken as 70 MPa in tension, 50 MPa in shear and 20 MPa in bearing. The bearing pressure for the brake lining should not exceed 0.2 N/ mm^2. [9]

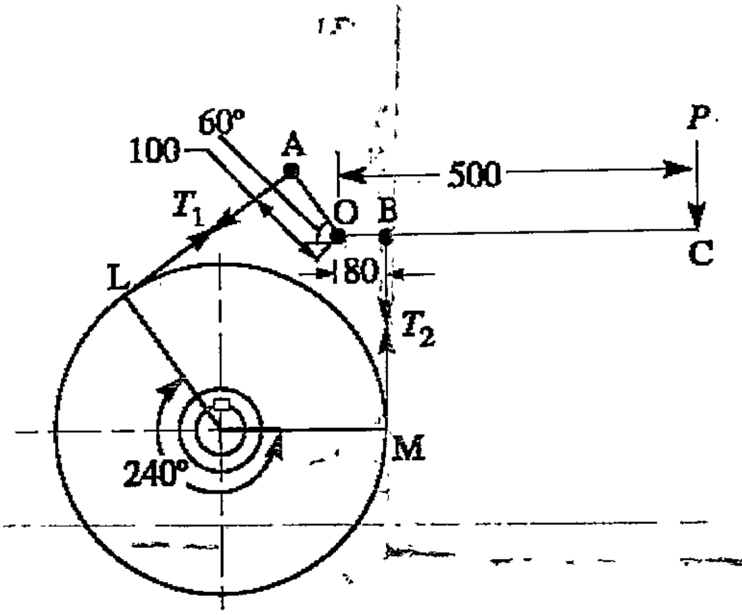

Figure. 1 (All dimensions are in mm)

5. A shaft carries four masses in parallel planes A, B, C and D in this order along its length. The masses at B and C are 18 kg and 12.5 kg respectively, and each has an eccentricity of 60 mm. The masses at A and D have an eccentricity of 80 mm. The angle between the masses at B and C is 100° and that between the masses at B and A is 190°, both being measured in the same direction. The axial distance between the planes A and B is 100 mm and that between B and C is 200 mm. If the shaft is in complete dynamic balance, determine:
   (a) the magnitude of the masses at A and D; (b) the distance between planes A and D; and
   (c) the angular position of the mass at D.

6. A machine mounted on springs and fitted with a dashpot has a mass of 60 kg. There are three springs, each of stiffness 12 N/mm. The amplitude of vibrations reduces from 45 to 8 mm in two complete oscillations. Assuming that the damping force varies as the velocity, determine the
   (i) Damping coefficient (ii) ratio of frequencies of damped and undamped vibrations
   (iii) periodic time of damped vibrations.

7. The shaft showing in Fig. 2 carries two masses. The mass A is 300 kg with a radius of gyration of 0.75 m and the mass B is 500 kg with a radius of gyration of 0.9 m. Determine the frequency of the torsional vibrations. It is desired to have the node at the mid-section of the shaft of 120-mm diameter by changing the diameter of the section having a 90mm diameter. What will be the new diameter?

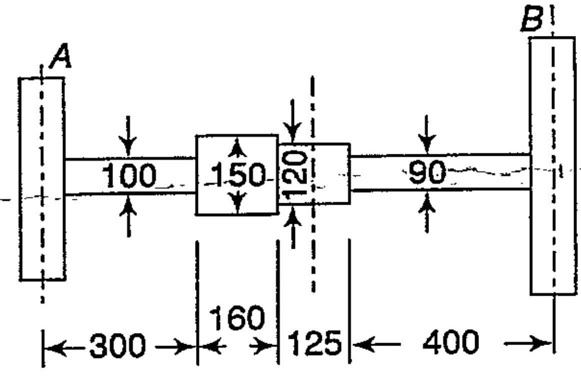

Figure. 2

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Narasaraopeta Engineering College (Autonomous)
Kotappakonda Road, Yellamanda (P.O), Narasaraopet- 522601, Guntur District, AP.

Subject Code: R16ME3103

III B.Tech I Semester (Regular/supple) Examinations, (Month, Year)

MACHINE DYNAMICS AND VIBRATIONS

(ME)

Time: 3 hours **Max Marks: 60**

Question Paper Consists of **Part-A** and **Part-B.**

Answering the question in **Part-A** is Compulsory & Four Questions should be answered from Part-B

All questions carry equal marks of 12.

PART-A

1. a) Explain limiting friction?
   b) Explain angle of friction?
   c) Why is balancing of rotating parts necessary for high speed engines
   d) What are the differences between brakes and dynamometers?
   e) Expalin the terms 'under damping',critical and over damping ?
   f) Explain resonance condition?

[2+2+2+2+2+2]

PART –B

4 X 12 = 48

2. Derive an expression for efficiency of an inclined plane when a body moves up the plane?

3. A shaft carries four balls A,B,C and D having mass,respectively,15kg,25kg,35kg and 22kg spaced 50cm apart. The radii of rotation of A,B,C and D are 30cm,25cm,20cm and 50cm respectively. The angular positions of the masses B,C and D, measured anticlockwise from A, are 200 degrees, 120 degrees and 290 degrees respectively. The speed of rotation of the shaft is 200 rpm .Determine the magnitude and direction relative to A of unbalanced force and couple about a plane midway between A and B.

4. Three rotors A,B, and C having moment of inertia of 2000:6000: and 3500 kg-m^2 respectively are carried on a uniform shaft of 0.35m diameter. The length of the shaft between the rotors A and B is 6m and between B and Cis 32m .find the natural frequency of the torsional vibrations .the modulus of rigidity for the shaft material is 80GN/m.

5. What is the difference between absorption and transmission dynamometers? What are torsion dynamometers.

6. Describe with a neat sketch the working of a single plate friction clutch.

7. What are the laws of i)static friction ii)Dynamic friction iii) solid friction iv)fluid friction.

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Narasaraopeta Engineering College (Autonomous)

Kotappakonda Road, Yellamanda (P.O), Narasaraopet- 522601, Guntur District, AP.

Subject Code: R16ME3103

III B.Tech I Semester Regular/supple Examinations, November-2019

MACHINE DYNAMICS AND VIBRATIONS (ME)

Time: 3 hours **Max Marks: 60**

Question Paper Consists of **Part-A** and **Part-B.**

Answering the question in **Part-A** is Compulsory & Four Questions should be answered from Part-B

All questions carry equal marks of 12.

PART-A

1. (a) Explain the terms friction circle and friction couple?
   (b) What is meant by friction axis of a link?
   (c) Give the classification of brakes.
   (d) What is meant by balancing of reciprocating masses?
   (e) Define damping factor.
   (f) What are sources of vibrations and how they can be eliminated?

[2+2+2+2+2+2]

PART-B 4 X 12 = 48

2. A thrust shaft of a ship has 6 collars of 600 mm external diameter and 300 mm internal diameter. The total thrust from the propeller is 100 kN. If the coefficient of friction is 0.12 and speed of the engine is 90 rpm. Find the power absorbed in friction at the thrust block, assuming (a) uniform pressure and (b) uniform wear.

3. Explain the working of a single plate clutch with neat sketch.

4. a) Describe the construction working of a prony brake dynamometer.

b)In a laboratory experiment, the following data were recorded with rope brake: diameter of the flywheel 1.2 m; diameter of the rope 12.5 mm; speed of the engine 200 r.p.m.; dead load on the brake 600 N; spring balance reading 150 N. Calculate the brake power of the engine.

5. Four masses m_1, m_2, m_3 and m_4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required, if its radius of rotation is 0.2 m.

6.a) Derive an expression for the natural frequency for free longitudinal vibrations

b)An instrument vibrates with a frequency of 1 Hz when there is no damping. When the damping is provided, the frequency of damped vibrations was observed to be 0.9 Hz. Find (i) The damping factor and (ii) Logarithmic decrement.

7. Determine an expression for the natrual frequency of free transverse vibrations for a simply supported beam carrying a number of point loads by Dunkerley's method

NEC NARASARAOPETA ENGINEERING COLLEGE
(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

CO-POs & CO-PSOs ATTAINMENT

Course Code: C323	Course Name: DYNAMICS OF MACHINERY	Year/Sem: III/II

External Examination Assessment

S.No	Q.No	1	1	2	2	2	2	3	3	3	3	4	4	4	5	5	5
		a	b	a		b		a		b		a	b		a		b
		i	i	i	ii	i	ii	i	ii	i	ii	i	i	ii	i	ii	i
	COs	I	I	II	II	II	II	III	III	III	III	IV	IV	IV	V	V	V
	Max. Marks	14	14	7	7	7	7	7	7	7	7	14	7	7	7	7	14
1			4	4	4					6	6	10					6
2			2	7	7					6	6	10					
3		8		4	4					4	4	10					6
4		10				5	5			7	7	14					8
5		4		6	2			4	4			6					4
6		8		6	2			4	4			10			4	4	
7		12		6	2						6	6					10
8		6		2	4					4	2						8
9		4				6	6			7	7	10			2		
10		10		4	4					4	6	8				2	
11		8		7	7					6	6	10					4
12		14		7	6					6	6	14					12
13		5		7	3					4	6	14					10
14		2				2	2	4	2			10			2	2	
15		8		7	4					6	6	12				2	
16		8		4	4					6	6	12					2
17		10		7	7					6	6	14			2	4	
18		10				4	6			6	6	12			4	4	
19		10		7	4					6	6	12					8
20		10				2	2			5	5	10			2	2	
21		10		6	6					6	6	14			4		
22		10							6			6			4		
23		6		2	2					6	6	8					2
24			6	4	2					5	4	10					6
25		10		7	6					5	5		5	5			8
26		14		7	5					7	7	14					10
27		8		6						6	6	12					
28		4		4	2					5	5	8					8
29			2	5	3					4	2	6					4
30			8	4	4					4	4	8			4	4	
31			6	5	1					5	6	12					8
32		12		6	6					5	5	12					
33				3	3					2	4	10					4
34				2													
35			10	5	2					2	6	10					
36		12				4	4			6	6	8			2	2	
37			12			4	4			5	7	6					10
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40		4		5	2					4	6	2					4

41				8	6	4					7	5	6					6
42				8	4	4						4	10			2	2	
43					6	2					4	4	12					
44				8	5							5	8					2
45					2								2					
46					7	3					2	4		4	4			2
47			8		2							4						8
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53			8		7	3					5	5	10					10
54				7	7	4			2	2			12					10
55				14	7	2					4	4	10					2
56				4	7	4			4	4			12			2	2	
57			4		2	2												
58					2	2												
59							2								2			
60			2		3	6			4	3			4					4
61			10		6	2					6	6	12					4
62			6		6							6	2					4
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66			2		5	4					4	6	10					
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68				4	7	5					4	2	2					
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72				14	4	2					6	6	2					4
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89				4	2	4			2	6			8					2
90				10	4	4					2	4	10					8

91			6		4						6	6	12					6
92			6		2	2					4	6	4					2
93			4				2	2			6	6	4					4
94				2	2	2					2							
95				4	6	2					4	2		4	4			
96																		
97				2			2	2					2					
98				2	2						2	4	4					
99					6	2					5	5		2	2			4
No. of Students answered			48	38	82	72	15	14	9	10	75	77	77	11	13	18	16	56
50% of Max.Marks			7	7	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	7	3.5	3.5	3.5	3.5	7
No. of Students crossed 50% of Max Marks			29	16	65	39	7	7	6	6	65	71	54	8	10	9	8	23
% of Students crossed 50% of Max Marks			60	42	79	54	47	50	67	60	87	92	70	73	77	50	50	41
Attainment Level			2	0	3	1	0	1	2	2	3	3	3	3	3	1	1	0
Course Outcome			I	II	III	IV	V											
No.of Times CO Repeated			2	4	4	3	3											
Final CO Attainment Level			1	1	3	3	1											

Rubrics:

If 50% of the students crossed 50% of the marks: Attainment Level 1
If 60% of the students crossed 50% of the marks: Attainment Level 2
If 70% of the students crossed 50% of the marks: Attainment Level 3

1. Enter the question wise marks.
2. Identify the CO of each question.
3. Calculate the maximum marks of each CO.
4. Calculate the CO wise marks obtained by each student.
5. Calculate 50% of maximum marks of each CO.
6. Find number of students crossed 50% of maximum marks for each CO.
7. Find percentage of students crossed 50% of maximum marks for each CO.
8. Find the attainment level of each CO as per the above Rubrics.

Course Code: C323	Course Name: DYNAMICS OF MACHINERY	Year/Sem: III/II

Internal Examination Assessment

S.No	Roll. No	Test	Mid1					A1	Quiz 1	Mid2					A2	Quiz 2	CO I	CO II	CO III	CO IV	CO V
		Q.No	1		2		3			1	2		3				Max. Marks	Max. Marks	Max. Marks	Max. Marks	Max. Marks
			1.a	1.b	2.a	2.b	3.a			1.a	2.a	2.b	3.a	3.b							
		COs	I	I	II	II	III	I		III	IV	IV	V	V							
		Max. Marks	5	5	5	5	5	5	10	5	5	5	5	5	5	10	25	20	35	25	20
1	20471A0301								5	4	3	5	4	4	5	10	5	5	24	23	18
2	20471A0302		5	5	5	5	5	5	8	5	5	5	5	5	5	10	23	18	33	25	20
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5	20471A0305		2	2	2	2	2	4	8	2	3	2	4	4	5	9	16	12	26	19	17
6	20471A0306		5	4	3	3	3	5	5	4	3	4	5	4	5	10	19	11	27	22	19
7	20471A0307		5	5	5	5	5	5	3	4	5	5	4	4	5	10	18	13	27	25	18
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9	20471A0309		5	5	3	2	3	3	7	3	5	5	5	5	5	10	20	12	28	25	20
10	20471A0310		5	4	3	4	4	5	6						5	9	20	13	24	14	9
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14	20471A0315		5	5	5	5	5	5	9	5	5	5	3	5	5	10	24	19	34	25	18
15	20471A0317		5	5	5	5	3	5	10	4	5	5	5	4	5	9	25	20	31	24	18
16	20471A0318		4	3	3	5	5	4	8	4	5	5	5	4	5	10	19	16	32	25	19
17	20471A0319		4	3	3	3	2		1								8	7	3	0	0
18	20471A0320		5	5	4	4	4	5	8	5	5	5	5	5	5	10	23	16	32	25	20
19	20471A0321		5	5	3	2	3	4	2	5	4	5	4	4	5	7	16	7	22	21	15
20	20471A0323		5	4	3	4	4	5	10	5	5	5	5	5	5	10	24	17	34	25	20
21	20471A0324		3	3	4	4	4	3	8	5	5	5	3	5	5	10	17	16	32	25	18
22	20471A0325		3	4	5	5	5	5	8	4	4	4	4	4	4	10	20	18	31	22	18
23	20471A0326		4	4	5	5	5	5	8	4	5	4	5	5	5	8	21	18	30	22	18

24	20471A0327		5	5	4	5	4	4	9	5	5	5	5	5	5	10	**23**	**18**	**33**	**25**	**20**
25	20471A0328		4	3	5	3	3	4	2	5	5	5	3	5	5	5	**13**	**10**	**20**	**20**	**13**
26	20471A0329		4	5	5	4	5	4	8	3	5	3	5	4	5	10	**21**	**17**	**31**	**23**	**19**
27	20471A0330							4		4	5	5	4	5	4	10	**4**	**0**	**18**	**24**	**19**
28	20471A0331		5	5	3	2	3		4						5	10	**14**	**9**	**22**	**15**	**10**
29	20471A0332							4		5	5	4	4	5	5	9	**4**	**0**	**19**	**23**	**18**
30	20471A0333		5	5	5	5	5	5	7	4	5	5	5	4	5	9	**22**	**17**	**30**	**24**	**18**
31	20471A0334		3	4	2	1	3	4	10	3	3	3	3		5	10	**21**	**13**	**31**	**21**	**13**
32	20471A0336		3	2	4	2	2		1	2	3	3	2				**6**	**7**	**5**	**6**	**2**
33	20471A0337		5	5	5	5	5	5	8	5	5	5	5	5	5	10	**23**	**18**	**33**	**25**	**20**
34	20471A0338		5	5	5	5	5	5	10	5	5	5	5	5	5	10	**25**	**20**	**35**	**25**	**20**
35	20471A0339		5	5	5	5	5	4	6	4	3	4	3	4	5	9	**20**	**16**	**29**	**21**	**16**
36	20471A0341		4	4	4	5	5	5	6	4	4	4	5	5	5	9	**19**	**15**	**29**	**22**	**19**
37	20471A0343		4	5	5	5	4	4	4	4	3	3	4	3	5	10	**17**	**14**	**27**	**21**	**17**
38	20471A0344		5	5	5	5	3	3	9	2	2	3	3		5	10	**22**	**19**	**29**	**20**	**13**
39	20471A0345		4	2	3	2	2		3	4	4	4	3	3	5	10	**9**	**8**	**24**	**23**	**16**
40	20471A0346		5	4	3	4	4	4	10	5	5	4	4	4	5	10	**23**	**17**	**34**	**24**	**18**
41	20471A0348		5	5	5	5	5	5	8	5	5	5	5		5	10	**23**	**18**	**33**	**25**	**15**
42	20471A0349		5	4	3	3	3	5	10	5	4	4	3	4	5	10	**24**	**16**	**33**	**23**	**17**
43	20471A0350		3	5	5	5	5	4	6	3		3	4	3	5	10	**18**	**16**	**29**	**18**	**17**
44	20471A0352		4	5	3	5	5	3	9	3	4	4	5	4	5	10	**21**	**17**	**32**	**23**	**19**
45	20471A0353		3	3	3	3	3		2	2	2		3	3	5	10	**8**	**8**	**22**	**17**	**16**
46	20471A0354		5	5	5	5	5	4	9	5	5	5	5			10	**23**	**19**	**29**	**20**	**15**
47	20471A0356		5	5	5	5	5	5	3	5	4	5	4	5	5	10	**18**	**13**	**28**	**24**	**19**
48	20471A0357		5	5	5	3	5	5	10	3	3	5	4	5	4	10	**25**	**18**	**32**	**22**	**19**
49	20471A0358		5	5	5	5	5	5	10	4	4	5	3	4	5	10	**25**	**20**	**34**	**24**	**17**
50	20471A0359		4	3	5	3	3	4	9	3	3	5	3	4	4	10	**20**	**17**	**29**	**22**	**17**
51	20471A0360		3	5	5	4	5	5	3	5	4	3	3	3	2	10	**16**	**12**	**25**	**19**	**16**
52	20471A0361		5	5	5	5	3	5	8	5	5	5	5		5	10	**23**	**18**	**31**	**25**	**15**
53	20471A0362		5	5	5	5	5	5	10	5	5	4	5	3	4	10	**25**	**20**	**34**	**23**	**18**
54	20471A0363							5		5	5	5	5		5	10	**5**	**0**	**20**	**25**	**15**
55	20471A0364		5	4	5	4	5	4	3	3	3	3	3	3	5	10	**16**	**12**	**26**	**21**	**16**
56	20471A0365		3	3	2	3	2	3	2								**11**	**7**	**4**	**0**	**0**

57	20471A0366		4	5	5	4	4	5	8	5	5	5	5		5	10	22	17	32	25	15
58	20471A0367							5	10	5	5	5	5		5	10	15	10	30	25	15
59	20471A0368		4	4	4	5	5	4	9						5	10	21	18	29	15	10
60	20471A0369		3	5	5	5	5		8		4	3	3	3	3	7	16	18	23	17	13
61	20471A0370		1	2	2			4	10	5	4	5	4	5	3	10	17	12	28	22	19
62	20471A0371		5	5	5	5	5	4	10	4	5	5	5	4	3	10	24	20	32	23	19
63	20471A0372		5	5	5	5	5	5	8	5	5	5	5		5	10	23	18	33	25	15
64	20471A0373		4	4	5	5	5	4	10	4	4	4	4	4	5	10	22	20	34	23	18
65	20471A0374		3	5	5	5	5		10	4	5	4	3	4		10	18	20	29	19	17
66	20471A0375		3	2	2	1	2	5	3	3	4	5	3	3	0		13	6	8	9	6
67	20471A0376		3	1	4		2	5	2	3	4	5	4	4	2	10	11	6	19	21	18
68	21475A0301		5	5	5	5	5	5	9	4	4	5	5	5	5	8	24	19	31	22	18
69	21475A0302		5	5	5	5	5	5	10	5	5	5	5		5	10	25	20	35	25	15
70	21475A0303		5	5	5	5	5	5	9	5	5	5	4	4	4	10	24	19	33	24	18
71	21475A0304																0	0	0	0	0
72	21475A0305		5	5	5	5	5	5	9	5	5	5	5		5	10	24	19	34	25	15
73	21475A0306		5	5	5	5	5	5	10	5	5	5	5		5	10	25	20	35	25	15
74	21475A0307		4	4	5	5	5	5	4	5	5	4	5	3	2	10	17	14	26	21	18
75	21475A0308		5	5	5	5	5	5	6	4	4	4	4	4	2	8	21	16	25	18	16
76	21475A0309		5	5	5	5	5	5	9	5	5	5	5		5	10	24	19	34	25	15
77	21475A0310		5	5	5	5	3	5	9	5	5	5	3	5	4	10	24	19	31	24	18
78	21475A0311		5	5	5	5	5	5	9	3	5	5	5	5	5	10	24	19	32	25	20
79	21475A0312		5	5	5	5	3	5	10	4	5	5	5	4	3	10	25	20	30	23	19
80	21475A0313		5	5	5	5	5	5	7	5	3	5	5	5	5	10	22	17	32	23	20
81	21475A0314		5	5	5	5	5	3	7	3	4	5	5	3	5	9	20	17	29	23	17
82	21475A0315		4	4	5	5	5	3	10	5	5	5	5	3	5	10	21	20	35	25	18
83	21475A0316		5	5	5	5	5	5	9	4	4	4	5	5	5	10	24	19	33	23	20
84	21475A0317		5	5	5	5	5	5	10	5	5	5	3	5	5	10	25	20	35	25	18
85	21475A0318		5	5	5	5	5	5	9	5	5	3	5	5	5	10	24	19	34	23	20
86	21475A0319		5	5	5	5	5	5	9	4	4	4	4	4	5	9	24	19	32	22	17
87	21475A0320		5	5	5	5	5	5	10	5	5	5	5		5	10	25	20	35	25	15
88	21475A0321		5	5	5	3	5	4	7	3	3	5	5	4		10	21	15	25	18	19
89	21475A0322		5	5	5	5	5	5	10	5	5	5	5		5	9	25	20	34	24	14
90	21475A0323		4	4	5	5	5	5	8	5	5	5	3	5	5	10	21	18	33	25	18

91	21475A0324		5	5	5	5	5	5	10	4	5	5	5	4	5	10	25	20	34	25	19
92	21475A0325		5	5	5	5	3	5	9	5	5	5	3	5	5	9	24	19	31	24	17
93	21475A0326		4	4	4	5	5	4	10	3	3	3	3	3	5	10	22	19	33	21	16
94	21475A0327		5	5	5	5	5	5	10	4	5	5	5	4	5	10	25	20	34	25	19
95	21475A0328		4	4	5	5	5	5	6	5	4	3	3	3	5	10	19	16	31	22	16
96	21475A0329		5	5	5	5	5	5	10	3	3	3	4		4	10	25	20	32	20	14
97	21475A0330		5	5	5	5	5	5	10	5	5	5	5		5	10	25	20	35	25	15
98	21475A0331		5	5	5	5	5	5	9	5	5	5	3	5	5	10	24	19	34	25	18
99	21475A0332		5	5	5	5	5	5	10	5	5	5	5		5	10	25	20	35	25	15
100	21475A0333		5	5	5	5	3	5	10	5	5	5	5		3	10	25	20	31	23	15
101	21475A0334		4	4	5	5	5	4	9	3	4	5	3	5	5	10	21	19	32	24	18
102	21475A0335		5	5	5	5	5	5	4	4	5	5	5	4	5	8	19	14	26	23	17
50% of maximum marks																	12.5	10	17.5	12.5	10
No. of Students crossed 50% of max. marks																	90	87	96	97	95
% of students crossed 50% of max. marks																	88	85	94	95	93
Attainment Level																	3	3	3	3	3

Rubrics:
If 50% of the students crossed 50% of the marks: Attainment Level 1
If 60% of the students crossed 50% of the marks: Attainment Level 2
If 70% of the students crossed 50% of the marks: Attainment Level 3

1. **Enter the question wise marks for mid examinations, assignments & quiz.**
2. **Identify the CO of each question.**
3. **Calculate the maximum marks of each CO based mid exams, assignments and quiz.**
4. **Calculate the CO wise marks obtained by each student.**
5. **Calculate 50% of maximum marks of each CO.**
6. **Find number of students crossed 50% of maximum marks for each CO.**
7. **Find percentage of students crossed 50% of maximum marks for each CO.**
8. **Find the attainment level of each CO as per the above Rubrics.**

Course Code: C323	Course Name: DYNAMICS OF MACHINERY	Year/Sem: III/I

CO-PO & CO-PSO Mapping

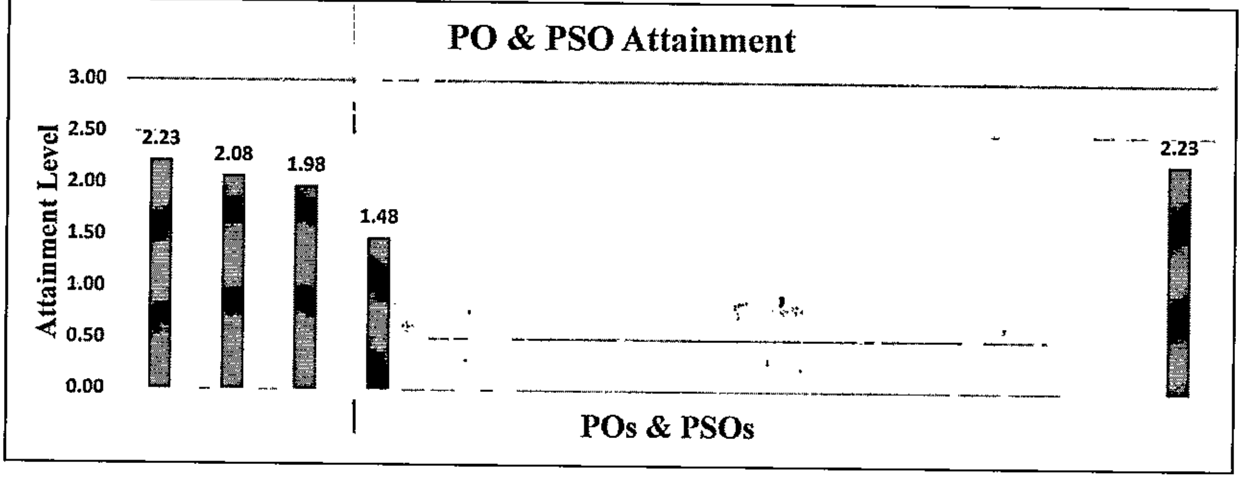
COs	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3
	POs & PSOs														
C323.1	3	2	2	-	-	-	-	-	-	-	-	-	-	-	3
C323.2	3	3	-	-	-	-	-	-	-	-	-	-	-	-	3
C323.3	3	3	-	-	-	-	-	-	-	-	-	-	-	-	3
C323.4	3	3	3	2	-	-	-	-	-	-	-	-	-	-	3
C323.5	3	3	3	2	-	-	-	-	-	-	-	-	-	-	3
C323	3.00	2.80	2.67	2.00	-	-	-	-	-	-	-	-	-	-	3.00

Total CO Attainment through Direct & Indirect Assessment

CO Attainment	2.23

PO & PSO Attainment

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3
PO Attainment	2.23	2.08	1.98	1.48	-	-	-	-	-	-	-	-	-	-	2.23



1. Copy CO - PO matrix and CO attainment matrix from previous pages and find PO attainment.
2. PO attainment is calculated as per the following formula:
   POi * Total CO attainment Level / 3 where 'i' ranges from 1 to 12

1. Copy CO - PSO matrix and CO attainment matrix from previous pages and find PSO attainment.
2. PSO attainment is calculated as per the following formula:
   PSOi * Total CO attainment Level / 3 where 'i' ranges from 1 to 3

Course Code: C323	Course Name: DYNAMICS OF MACHINERY				Year/Sem: III/II
CO Attainment					
CO	CO Attainment Level (Internal)	CO Attainment Level (External)	Direct CO Attainment Level (Internal * 30%) + (External * 70%)	Indirect CO Attainment Level	Total CO Attainment Level (Direct CO Attainment * 90% + Indirect CO Attainment * 10%)
C323.1	3	1	1.60	2.84	1.72
C323.2	3	1	1.60	2.68	1.71
C323.3	3	3	3.00	2.76	2.98
C323.4	3	3	3.00	2.92	2.99
C323.5	3	1	1.60	2.86	1.73
C323					2.23

1. Copy the Direct CO Attainment Level (Internal) and Direct CO Attainment Level (External) from the previous
   sheets and then find the Direct CO Attainment Level.
2. Find Direct CO attainment level using the formula:
   CO Attainment Level (Internal) * 30% + CO Attainment Level (External) * 70%
3. Copy Indirect CO Attainment Level.
4. Find the CO attainment level using the formula:
   Direct CO Attainment Level *90% + Indirect CO Attainment Level * 10%