STRENGTH OF MATERIALS- II

UNIT- I Principal Stresses And Strains And Theories Of Failures: Introduction – Stresses on an inclined section of a bar under axial loading – compound stresses – Normal and tangential stresses on an inclined plane for biaxial stresses – Two perpendicular normal stresses accompanied by a state of simple shear – Mohr's circle of stresses – Principal stresses and strains – Analytical and graphical solutions.
Theories Of Failures: Introduction – Various Theories of failures like Maximum Principal stress theory – Maximum Principal strain theory – Maximum shear stress theory – Maximum strain energy theory – Maximum shear strain energy theory.

UNIT – II Torsion Of Circular Shafts And Springs: Theory of pure torsion – Derivation of Torsion equations: $T/J = q/r = N\phi/L$ – Assumptions made in the theory of pure torsion – Torsional moment of resistance – Polar section modulus – Power transmitted by shafts – Combined bending and torsion and end thrust – Design of shafts according to theories of failure.
Springs: Introduction – Types of springs – deflection of close and open coiled helical springs under axial pull and axial couple – springs in series and parallel – Carriage or leaf springs.

UNIT – III Columns And Struts: Introduction – Types of columns – Short, medium and long columns – Axially loaded compression members – Crushing load – Euler's theorem for long columns- assumptions- derivation of Euler's critical load formulae for various end conditions – Equivalent length of a column – slenderness ratio – Euler's critical stress – Limitations of Euler's theory – Rankine – Gordon formula – Long columns subjected to eccentric loading – Secant formula – Empirical formulae – Straight line formula – Prof. Perry's formula. Laterally loaded struts – subjected to uniformly distributed and concentrated loads – Maximum B.M. and stress due to transverse and lateral loading.

UNIT – IV Direct And Bending Stresses: Stresses under the combined action of direct loading and B.M. Core of a section – determination of stresses in the case of chimneys, retaining walls and dams – conditions for stability – stresses due to direct loading and B.M. about both axis.

UNIT – V Unsymetrical Bending: Introduction – Centroidal principal axes of section – Graphical method for locating principal axes – Moments of inertia referred to any set of rectangular axes – Stresses in beams subjected to unsymmetrical bending – Principal axes – Resolution of bending moment into two rectangular axes through the centroid – Location of neutral axis Deflection of beams under unsymmetrical bending.

UNIT – VI Analysis Of Pin-Jointed Plane Frames: Determination of Forces in members of plane pin-jointed perfect trusses by (i) method of joints and (ii) method of sections. Analysis of various types of cantilever and simply supported trusses by method of joints, method of sections.

Principal Stresses & Strains & Theories of failures

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Definition: The planes which have no shear stress are known as principal planes. These planes carry only normal stresses. The normal stresses acting on a principal plane are known as principal stresses.

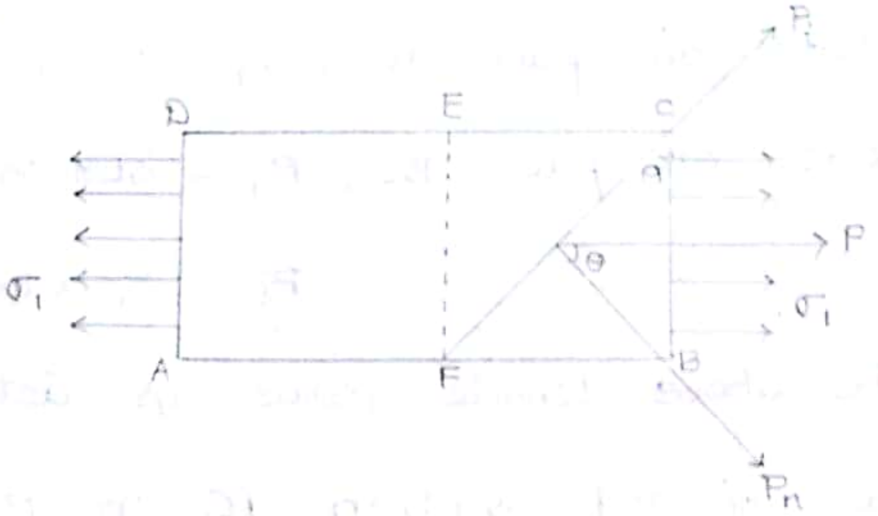
Methods for determining stresses:

① Analytical method

② Graphical method

Analytical methods

A member subjected to stress in one plane:



* Figure shows a rectangular member of uniform cross-sectional area (A) and

2 unit thickness.

* The bar is subjected to a principal tensile stress (σ_1) on the faces AB and BC.

* ∴ Area of cross-section, $A = BC \times 1$

* Let these stresses on this oblique plane FC are to be calculated.

* The plane FC is inclined at an angle θ with the normal cross-section BC or EF.

* This can be done by converting this stress (σ_1) acting on the face BC into equivalent force.

* Then this force will be resolved along the inclined planes FC and perpendicular to FC.

* Stress on face $BC = \sigma_1$

Force on face BC, P_1 = stress × Area

$$P_1 = \sigma_1 \times BC \times 1$$

* The above tensile force is acting on the inclined section FC in the axial direction.

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3 * This force is resolved into two components i.e., one normal to the plane FC, and other along the plane FC.

* Let P_n = normal force to the section FC.

P_n = normal force.

$= P_1 \cos\theta$

$= \sigma_1 \times BC \times 1 \times \cos\theta$

Normal stress on FC, $\sigma_n = \dfrac{P_n}{Area}$

$$= \frac{\sigma_1 \times BC \times 1 \times \cos\theta}{FC \times 1}$$

$$= \sigma_1 \times \cos\theta \times \cos\theta$$

$$= \sigma_1 \cos^2\theta$$

$$\boxed{\sigma_n = \sigma_1 \cos^2\theta} \longrightarrow (1)$$

* Let P_t = tangential force to the section FC

$P_t = P_1 \sin\theta$

$= \sigma_1 \times BC \times 1 \times \sin\theta.$

Tangential / shear stress on face FC,

$$\sigma_t = \tau = \frac{P_t}{Area}$$

$$= \frac{\sigma_1 \times BC \times 1 \times \sin\theta}{FC \times 1}$$

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$$= \sigma_1 \times \cos\theta \times \sin\theta$$

$$\tau = \frac{\sigma_1}{2}(2\cos\theta . \sin\theta)$$

$$\boxed{\tau = \frac{\sigma_1}{2}\sin 2\theta} \longrightarrow (2)$$

* From eqn (1) it is seen that the normal stress on the face BC will be maximum when $\cos\theta$ is maximum.

* And $\cos\theta$ will be maximum when $\theta = 0$

But when $\theta = 0$, the section FC will coincide with face BC.

$\therefore$ Max. normal stress, $\sigma_n = \sigma_1 \cos^2\theta$.

$$\cos\theta = max.$$

$$\cos\theta = 1$$

$$\theta = 0$$

$$\sigma_n = \sigma_1 \cos^2\theta$$

$$\Rightarrow \boxed{\sigma_n = \sigma_1}$$

* From eqn (2) it is observed that the tangential stress across the section FC will be maximum. when $\sin 2\theta$ is maximum.

$\therefore$ Max. tangential stress, $\tau = \frac{\sigma_1}{2}\sin 2\theta$.

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$\sin 2\theta = max$

$\sin 2\theta = 1$

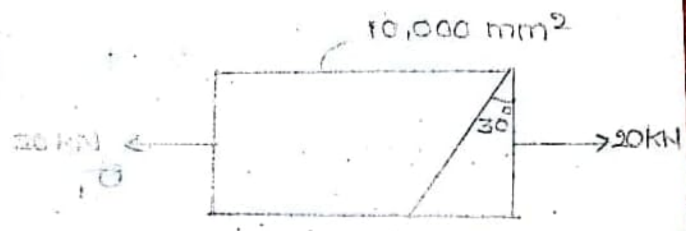
$2\theta = 90^\circ$

$\theta = 45^\circ$

$\tau_{max} = \frac{\sigma_1}{2} \sin 2\theta$.

$$\boxed{\sigma_t = \tau_{max} = \frac{\sigma_1}{2}}$$

* A rectangular bar of cross-sectional area 10,000 mm² is subjected to axial load of 20 KN. Determine the normal and shear stress on a section which is inclined at an angle of 30° with normal cross-section of the bar.



Sol Area, $A = 10,000 \text{ mm}^2$

Load, $P = 20$ KN

$= 20 \times 10^3$ N

$\theta = 30^\circ$

Stress, $\sigma_1 = \frac{P}{A} = \frac{20 \times 10^3}{10,000} = 2 \frac{N}{mm^2}$

Normal stress, $\sigma_n = \sigma_1 \cos^2\theta$

$= 2 \times (\cos 30^\circ)^2$

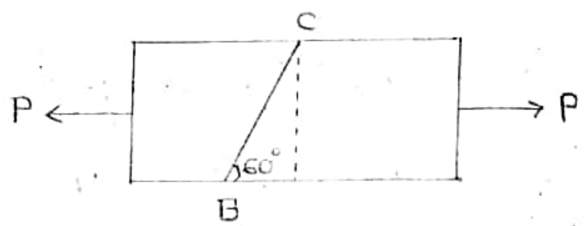
$= 1.5 \text{ N/mm}^2$

Tangential stress, $\sigma_t = \tau = \frac{\sigma_1}{2} \sin 2\theta$.

$= \frac{2}{2} \sin 2(30) = 0.866 \frac{N}{mm^2}$

A rectangular bar of cross-sectional area of 11000 mm² is subjected to tensile load P as shown in figure. The permissable normal and shear stress on an oblique plane BC are given as 7 N/mm² and 3.5 N/mm². Determine the safe load.

Area = 11,000 mm²



$\theta = 30°$

$\sigma_n = 7 \dfrac{N}{mm^2}$

$\tau = 3.5 \dfrac{N}{mm^2}$

Normal stress, $\sigma_n = \sigma_1 \cos^2\theta$.

$$\Rightarrow 7 = \frac{\cancel{\sigma}_1}{\cancel{A}} \cos^2 30$$

$$\cancel{\sigma}_1 = \frac{7 \times \cancel{11000}}{\cos^2 30°}$$

$$\cancel{\sigma}_1 = 9.33 \frac{N}{mm^2}$$

Tangential stress, $\tau = \dfrac{\sigma_1}{2} \sin 2\theta$

$$3.5 = \frac{\sigma_1}{2} \sin 2(30)$$

$$\sigma_1 = \frac{3.5 \times 2}{\sin 60}$$

$$\sigma_1 = \frac{7}{0.866}$$

$$\sigma_1 = 8.08 \frac{N}{mm^2}$$

Safe stress = $8.08 \frac{N}{mm^2}$

Safe load, P = stress × Area.

$$P = \sigma \times A$$

$$= 8.08 \times 11{,}000$$

$$P = 88.88 \text{ KN.}$$

Find the diameter of a circular bar which is subjected to an axial pull of 160 KN. If the maximum allowable shear stress on any section is 65 N/mm^2

Let Dia of circular bar = D

Max. shear stress, $\tau_{max} = 65 \ N/mm^2$.

Load, P = 160 KN.

Area, $A = \frac{\pi}{4} D^2$

$$\text{Stress, } \sigma = \frac{P}{A} = \frac{160 \times 10^3}{\frac{\pi}{4} \times D^2}$$

$$= \frac{203718.3}{D^2}$$

$$\text{Max. shear stress, } \tau = \frac{\sigma}{2}$$

$$= \frac{203718.3}{2D^2}$$

$$\tau = \frac{101859.2}{D^2}$$

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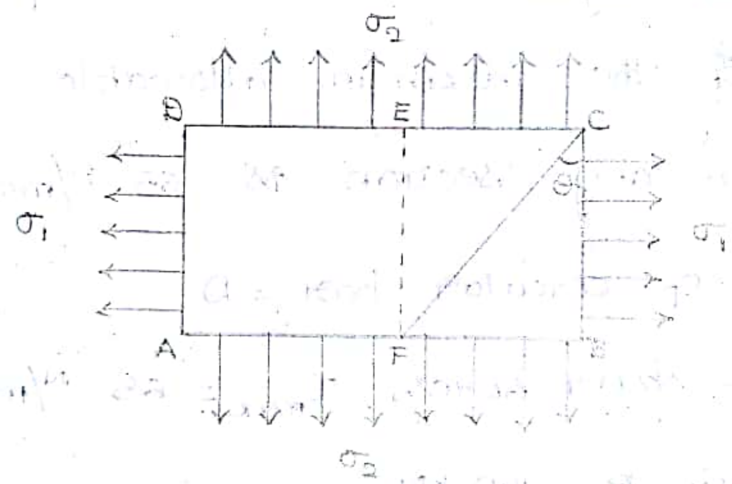
But $\tau = 65 \ \frac{N}{mm^2}$

$$65 = \frac{101859.2}{D^2}$$

$$D^2 = 1567.06$$

$$D = 39.58 \ mm$$

A member subjected to like direct stresses in two mutually perpendicular directions:



* Figure shows a rectangular bar ABCD of uniform cross-sectional area (A) and unit thickness.

* The bar is subjected to two direct tensile stresses (two principal tensile stresses)

* Let FC be the oblique section on which stresses are to be calculated.

* This can be done by converting the

stresses σ_1 and σ_2 into equivalent forces.

* Consider the forces acting on wedge FBC.

* Let θ be the angle made by oblique section FC with the normal cross-section BC.

σ_1 = major tensile stress

σ_2 = minor tensile stress

P_1 = tensile force on face BC

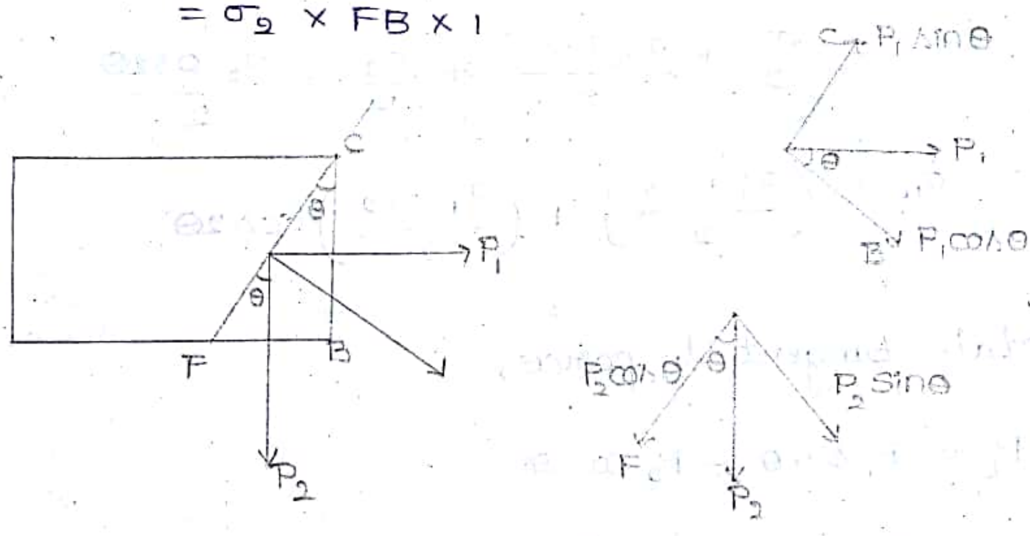
P_1 = stress × Area

$= \sigma_1 \times BC \times 1$

P_2 = tensile force on face FB

P_2 = stress × Area

$= \sigma_2 \times FB \times 1$



* The tensile forces P_1 and P_2 are also acting on the oblique section FC.

* The force P_1 is acting in the axial

direction whereas the force P_2 is acting downwards as shown in figure.

* Total normal force, $P_n = P_1 \cos\theta + P_2 \sin\theta$

$$P_n = \sigma_1 \times BC \times 1 \times \cos\theta + \sigma_2 \times FB \times 1 \times \sin\theta$$

$\therefore$ Normal stress, $\sigma_n = \frac{P_n}{A}$

$$\sigma_n = \frac{\sigma_1 \times BC \times \cos\theta + \sigma_2 \times FB \times \sin\theta}{FC \times 1}$$

$$= \sigma_1 \times \left(\frac{BC}{FC}\right) \times \cos\theta + \sigma_2 \times \left(\frac{FB}{FC}\right) \times \sin\theta$$

$$= \sigma_1 \times \cos\theta \times \cos\theta + \sigma_2 \times \sin\theta \cdot \times \sin\theta$$

$$= \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta$$

$$= \sigma_1 \left(\frac{1+\cos 2\theta}{2}\right) + \sigma_2 \left(\frac{1-\cos 2\theta}{2}\right)$$

$$= \frac{\sigma_1}{2} + \frac{\sigma_1 \cos 2\theta}{2} + \frac{\sigma_2}{2} - \frac{\sigma_2 \cos 2\theta}{2}$$

$$\sigma_n = \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \left(\frac{\sigma_1 - \sigma_2}{2}\right) \cos 2\theta$$

*
Total tangential force,

$$P_t = P_1 \sin\theta - P_2 \cos\theta$$

$$= \sigma_1 \times BC \times 1 \times \sin\theta - \sigma_2 \times FB \times 1 \times \cos\theta$$

$\therefore$ Tangential stress, $\sigma_t = \tau = \frac{P_t}{\text{Area}}$

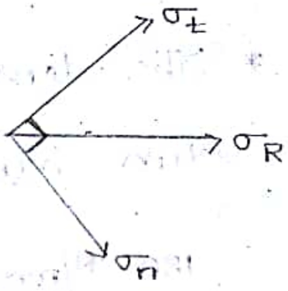
$$\tau = \frac{\sigma_1 \times BC \times \sin\theta - \sigma_2 \times FB \times \cos\theta}{FC \times 1}$$

$$= \sigma_1 \times \left(\frac{BC}{FC}\right) \times \sin\theta - \sigma_2 \times \left(\frac{FB}{FC}\right) \times \cos\theta$$

$$= \sigma_1 \times \cos\theta \times \sin\theta - \sigma_2 \times \sin\theta \times \cos\theta$$

$$= \frac{\sigma_1}{2} \sin 2\theta - \frac{\sigma_2}{2} \sin 2\theta$$

$$\tau = \left(\frac{\sigma_1 - \sigma_2}{2}\right) \sin 2\theta$$



Resultant stress, $\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$

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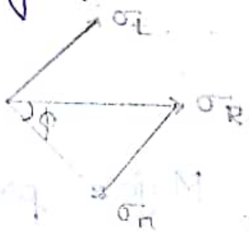
Obliquity:

The angle made by the resultant stress with the normal of the oblique plane is called obliquity. It is denoted by ϕ.

$$\tan\phi = \frac{\sigma_t}{\sigma_n}$$



Maximum shear stress:

The shear stress is given by equation

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta$$

∴ Shear stress will be maximum when

$$\sin 2\theta = 1$$

$$\sin 2\theta = \sin 90^\circ$$

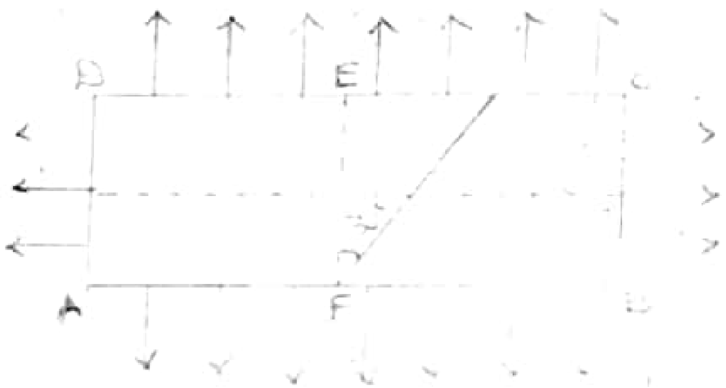
$$2\theta = 90^\circ$$

$$\theta = 45^\circ$$

∴ Maximum shear stress / Tangential stress

$$\sigma_{t_{max}} = \tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$$

The tensile stresses at a point across two mutually perpendicular planes are 180 N/mm^2 and 60 N/mm^2. Determine the normal, tangential and resultant stresses on a plane inclined at 30° to the axis of minor ~~axis~~ stress.



Major principal stress, $\sigma_x = 180$ N/mm^2

60 N/mm^2

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$\sigma_n = 105 \ N/mm^2$

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Tangential stress, $\sigma_t = \left(\frac{\sigma_1 - \sigma_2}{2}\right) \sin 2\theta$

$= \left(\frac{120-60}{2}\right) \sin 2(30)$

$= 30 \sin 60^\circ$

$= 25.98 \ N/mm^2$

Resultant stress, $\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$

$= \sqrt{(105)^2 + (25.98)^2}$

$= 108.16 \ N/mm^2$

Obliquity, $\tan \phi = \frac{\sigma_t}{\sigma_n}$

$\tan \phi = \frac{25.98}{105}$

$\phi = \tan^{-1}(0.247)$

$\phi = 13.87^\circ$

* The stresses at a point in a bar are. 200 N/mm² (tensile) and 100 N/mm² (Compressive). Determine the resultant stress in magnitude and direction on a plane inclined at 60° to the axis of the major stress. Also determine the maximum shear stress in the material at that point.

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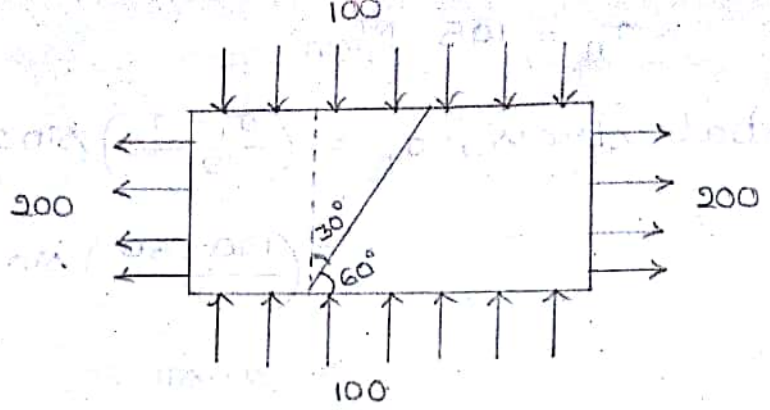


Major principal stress = 200 N/mm²

Minor principal stress = 100 N/mm²

Angle made by minor axis = 90° − 60°

= 30°

Normal stress, $\sigma_n = \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \left(\frac{\sigma_1 - \sigma_2}{2}\right) \cos 2\theta$

$= \left(\frac{200 - 100}{2}\right) + \left(\frac{200 + 100}{2}\right) \cos 2(30)$

$= 50 + 150 \cos 60$

$= 50 + 150 \times \frac{1}{2}$

$= 50 + 75$

$= 125$ N/mm²

Shear stress, $\sigma_t = \left(\frac{\sigma_1 - \sigma_2}{2}\right) \sin 2\theta$

$= \left(\frac{200 + 100}{2}\right) \sin 2(30)$

$= 150 \sin 60$

$= 129.9$ N/mm²

Resultant stress, $\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$

$$\sigma_R = \sqrt{125^2 + 129.9^2}$$

$$= 180.27 \text{ N/mm}^2$$

Obliquity, $\tan\phi = \frac{\sigma_t}{\sigma_n}$

$$= \frac{129.9}{125}$$

$$\phi = \tan^{-1}\left(\frac{129.9}{125}\right)$$

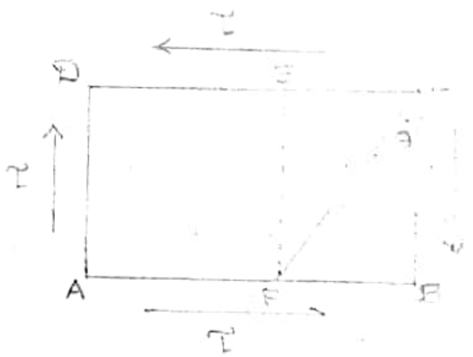
$$\phi = 46.1^\circ$$

Maximum shear stress, $\sigma_{t_{max}} = \frac{\sigma_1 - \sigma_2}{2}$

$$= \frac{200 - (-100)}{2}$$

$$= 150 \text{ N/mm}^2$$

A member subjected to a simple shear stress:



* Figure shows a rectangular bar ABCD of uniform cross-sectional area (A) and unit thickness.

* The bar is subjected to a simple shear stress across the faces BC & AD

* Let FC be the oblique section on which normal and tangential stresses are to be calculated.

θ = angle made by oblique section FC. with cross-section BC.

* Let τ = shear stress

Q_1 = shear force on face BC.

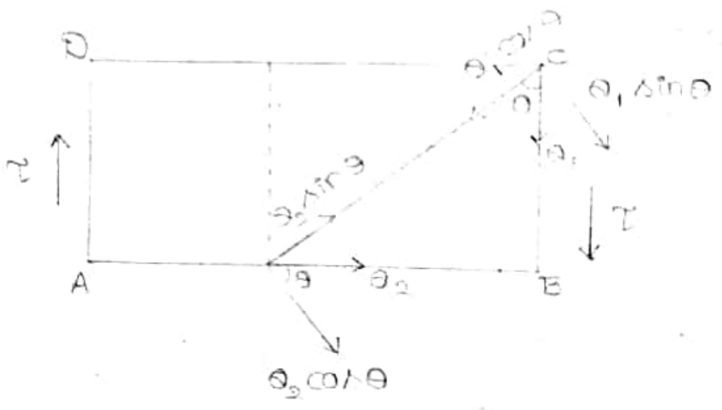
Q_1 = stress x Area

$= \tau \times BC \times 1$

Q_2 = shear force on face FB

Q_2 = stress x Area

$= \tau \times FB \times 1$



* P_n be the total normal force on section FC.

* P_t be the total tangential force on section FC.

* The force Q_1 is resolved into two

components i.e., $Q_1 \cos\theta$ and $Q_1 \sin\theta$ along the plane and normal to the plane respectively.

* The force Q_2 is resolved into two components i.e., $Q_2 \cos\theta$ and $Q_2 \sin\theta$ along the plane and normal to the plane. respectively.

* $\therefore P_n = Q_1 \sin\theta + Q_2 \cos\theta$

$$= \tau \times BC \times \sin\theta + \tau \times FB \times \cos\theta$$

Normal stress, $\sigma_n = \dfrac{P_n}{\text{Area}}$

$$\sigma_n = \frac{\tau \times BC \times \sin\theta + \tau \times FB \times \cos\theta}{FC \times 1}$$

$$= \tau \times \left(\frac{BC}{FC}\right) \times \sin\theta + \tau \times \left(\frac{FB}{FC}\right) \times \cos\theta$$

$$= \tau \times \cos\theta \times \sin\theta + \tau \times \sin\theta \times \cos\theta$$

$$= 2\tau \cos\theta \cdot \sin\theta$$

$$\sigma_n = \tau \sin 2\theta.$$

* $\therefore P_t = Q_2 \sin\theta - Q_1 \cos\theta$

$$= \tau \times FB \times \sin\theta - \tau \times BC \times \cos\theta$$

Tangential stress, $\sigma_t = \dfrac{P_t}{\text{Area}}$

$$\sigma_t = \tau \times FB \times \sin\theta - \tau \times BC \times \cos\theta$$

$$\sigma_t = \tau \times \left(\frac{FB}{FC}\right) \times \sin\theta - \tau \times \left(\frac{BC}{FC}\right) \times \cos\theta$$

$$= \tau \times \sin\theta \times \sin\theta - \tau \times \cos\theta \times \cos\theta$$

$$= \tau \sin^2\theta - \tau \cos^2\theta$$

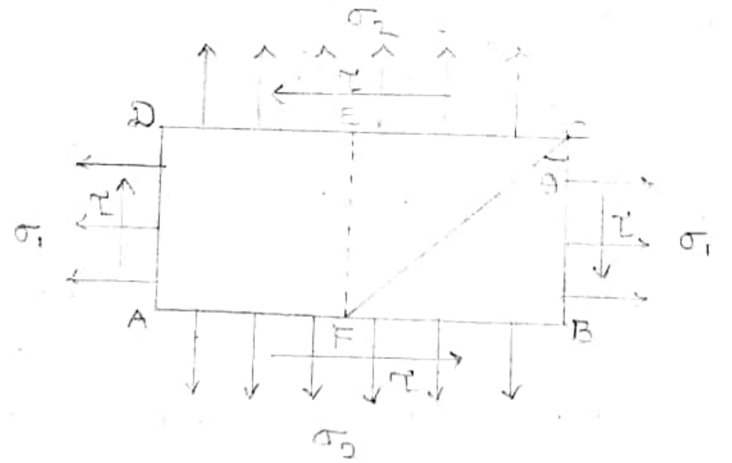
$$= -\tau (\cos^2\theta - \sin^2\theta)$$

$$= -\tau \cos 2\theta . \qquad \cos^2\theta - \sin^2\theta = \cos 2\theta$$

Here $-$sign indicates σ_t will be acting downwards.

Resultant stress, $\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$

A member subjected to direct stresses in two mutually perpendicular directions accompanied by a simple shear stress:



* Figure shows a rectangular bar ABCD of uniform cross-sectional area (A) and unit thickness.

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* This bar is subjected to tensile stress (σ_1) on the face BC and AD, tensile stress (σ_2) on the face AB and CD. and a simple shear stress (τ) on face BC and AD.

* Let FC be the oblique section on which normal and tangential stresses are to be calculated.

* The given stresses are converted into equivalent forces.

* The forces acting on the wedge FBC.

P_1 = tensile force on face BC

= stress × Area

$= \sigma_1 \times BC \times 1$

P_2 = tensile force on face FB

= stress × Area

$= \sigma_2 \times FB \times 1$

Q_1 = shear force on face BC.

= stress × Area

$= \tau \times BC \times 1$

Q_2 = shear force on face FB

$= \tau \times FB \times 1$

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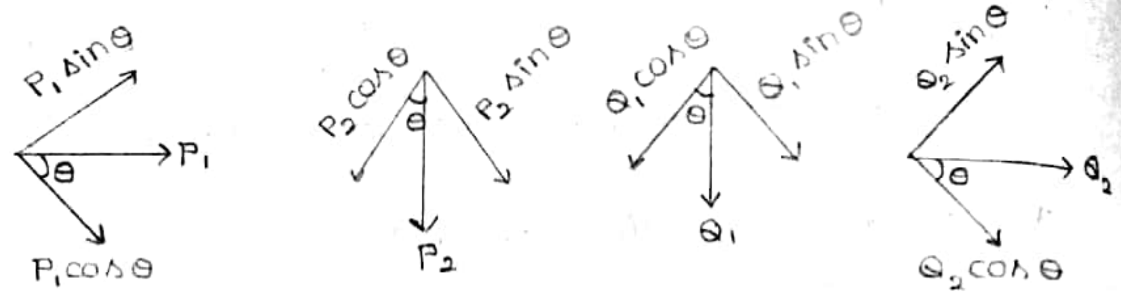
* Resolving the above four forces normal to the oblique section.



*∴ Total normal force,

$$P_n = P_1 \cos\theta + P_2 \sin\theta + Q_1 \sin\theta + Q_2 \cos\theta$$

$$= \sigma_1 \times BC \times \cos\theta + \sigma_2 \times FB \times \sin\theta +$$

$$\tau \times BC \times \sin\theta + \tau \times FB \times \cos\theta$$

Normal stress, $\sigma_n = \dfrac{P_n}{Area}$

$$\sigma_n = \frac{\sigma_1 \times BC \times \cos\theta + \sigma_2 \times FB \times \sin\theta + \tau \times BC \times \sin\theta + \tau \times FB \times \cos\theta}{FC \times 1}$$

$$= \sigma_1 \times \left(\frac{BC}{FC}\right) \times \cos\theta + \sigma_2 \times \left(\frac{FB}{FC}\right) \times \sin\theta +$$

$$\tau \times \left(\frac{BC}{FC}\right) \times \sin\theta + \tau \times \left(\frac{FB}{FC}\right) \times \cos\theta$$

$$= \sigma_1 \times \cos\theta \times \cos\theta + \sigma_2 \times \sin\theta \times \sin\theta +$$

$$\tau \times \cos\theta \times \sin\theta + \tau \times \sin\theta \times \cos\theta$$

$$= \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta + 2\tau \sin\theta \cos\theta$$

$$= \sigma_1 \left(\frac{1 + \cos 2\theta}{2}\right) + \sigma_2 \left(\frac{1 - \cos 2\theta}{2}\right) + 2\tau \sin 2$$

21 $= \frac{\sigma_1}{2} + \frac{\sigma_1 \cos 2\theta}{2} + \frac{\sigma_2}{2} - \frac{\sigma_2 \cos 2\theta}{2} + \tau \sin 2\theta$.

$$\boxed{\sigma_n = \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \left(\frac{\sigma_1 - \sigma_2}{2}\right)\cos 2\theta + \tau \sin 2\theta.}$$

*∴ Total tangential force,

$$P_t = P_1 \sin\theta - P_2 \cos\theta - Q_1 \cos\theta + Q_2 \sin\theta$$

$$= \sigma_1 \times BC \times \sin\theta - \sigma_2 \times FB \times \cos\theta - \tau \times BC \times \cos\theta + \tau \times FB \times \sin\theta.$$

Tangential stress, $\sigma_t = \frac{P_t}{Area}$

$$\sigma_t = \frac{\sigma_1 \times BC \times \sin\theta - \sigma_2 \times FB \times \cos\theta - \tau \times BC \times \cos\theta + \tau \times FB \times \sin\theta}{FC \times 1}$$

$$= \sigma_1 \times \left(\frac{BC}{FC}\right) \times \sin\theta - \sigma_2 \times \left(\frac{FB}{FC}\right)\cos\theta - \tau \times \left(\frac{BC}{FC}\right) \times \cos\theta + \tau \times \left(\frac{FB}{FC}\right) \times \sin\theta$$

$$= \sigma_1 \times \cos\theta \times \sin\theta - \sigma_2 \times \sin\theta \times \cos\theta - \tau \times \cos\theta \times \cos\theta + \tau \times \sin\theta \times \sin\theta$$

$$= \frac{\sigma_1}{2}\sin 2\theta - \frac{\sigma_2}{2}\sin 2\theta - \tau\cos^2\theta + \tau\sin^2\theta.$$

$$= \left(\frac{\sigma_1 - \sigma_2}{2}\right)\sin 2\theta - \tau\left(\cos^2\theta - \sin^2\theta\right)$$

$$\boxed{\sigma_t = \left(\frac{\sigma_1 - \sigma_2}{2}\right)\sin 2\theta - \tau\cos 2\theta}$$

Position of principal planes:

* The planes on which shear stress is zero are known as principal planes.
* And the stresses acting on these planes are known as principal stresses.
* The position of principal planes are obtained by equating shear stress eqn to zero.

$$\therefore \sigma_t = 0$$

$$\left(\frac{\sigma_1-\sigma_2}{2}\right)\sin 2\theta - \tau\cos 2\theta = 0$$

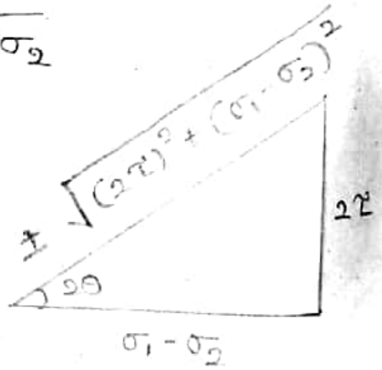
$$\left(\frac{\sigma_1-\sigma_2}{2}\right)\sin 2\theta = \tau\cos 2\theta.$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \frac{2\tau}{\sigma_1-\sigma_2}$$

$$\tan 2\theta = \frac{2\tau}{\sigma_1-\sigma_2}$$

$$\cos 2\theta = \frac{\sigma_1-\sigma_2}{\sqrt{(2\tau)^2+(\sigma_1-\sigma_2)^2}}$$

$$\sin 2\theta = \frac{2\tau}{\sqrt{(2\tau)^2+(\sigma_1-\sigma_2)^2}}$$



* The value of major principal stress is obtained by substituting the values of $\sin 2\theta$ and $\cos 2\theta$ in the normal

stress equation.

$$\sigma_n = \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \left(\frac{\sigma_1 - \sigma_2}{2}\right)\cos 2\theta + \tau \sin 2\theta$$

$$= \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \left(\frac{\sigma_1 - \sigma_2}{2}\right)\left(\frac{\sigma_1 - \sigma_2}{\sqrt{(2\tau)^2 + (\sigma_1 - \sigma_2)^2}}\right)$$

$$+ \tau \cdot \left(\frac{2\tau}{\sqrt{(2\tau)^2 + (\sigma_1 - \sigma_2)^2}}\right)$$

$$= \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \frac{(\sigma_1 - \sigma_2)^2}{2\sqrt{4\tau^2 + (\sigma_1 - \sigma_2)^2}} + \frac{2\tau^2}{\sqrt{4\tau^2 + (\sigma_1 - \sigma_2)^2}}$$

$$= \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \frac{(\sigma_1 - \sigma_2)^2 + 4\tau^2}{2\sqrt{4\tau^2 + (\sigma_1 - \sigma_2)^2}}$$

$$= \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

$$\boxed{\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}.$$

* The value of minor principal stress is obtained by substituting the values of $\sin 2\theta$ and $\cos 2\theta$ in the normal stress equation.

$$\cos 2\theta = \frac{\sigma_1 - \sigma_2}{-\sqrt{(2\tau)^2 + (\sigma_1 - \sigma_2)^2}}$$

$$\sin 2\theta = \frac{2\tau}{\;}$$

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$$\sigma_n = \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \left(\frac{\sigma_1 - \sigma_2}{2}\right) \cos 2\theta + \tau \sin 2\theta$$

$$= \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \left(\frac{\sigma_1 - \sigma_2}{2}\right)\left(\frac{\sigma_1 - \sigma_2}{-\sqrt{4\tau^2 + (\sigma_1 - \sigma_2)^2}}\right)$$

$$+ \tau \left(\frac{2\tau}{-\sqrt{4\tau^2 + (\sigma_1 - \sigma_2)^2}}\right)$$

$$= \left(\frac{\sigma_1 + \sigma_2}{2}\right) - \frac{(\sigma_1 - \sigma_2)^2}{2\sqrt{4\tau^2 + (\sigma_1 - \sigma_2)^2}}$$

$$- \frac{2\tau^2}{\sqrt{4\tau^2 + (\sigma_1 - \sigma_2)^2}}$$

$$= \left(\frac{\sigma_1 + \sigma_2}{2}\right) - \frac{(\sigma_1 - \sigma_2)^2 + 4\tau^2}{2\sqrt{4\tau^2 + (\sigma_1 - \sigma_2)^2}}$$

$$= \left(\frac{\sigma_1 + \sigma_2}{2}\right) - \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

$$\boxed{\sigma_n = \frac{\sigma_1 + \sigma_2}{2} - \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

Maximum shear stress:

* The shear stress will be maximum when $\frac{\partial}{\partial \theta}(\sigma_t) = 0$

$$\frac{\partial}{\partial \theta}\left\{\left(\frac{\sigma_1 - \sigma_2}{2}\right) \sin 2\theta - \tau \cos 2\theta\right\} = 0$$

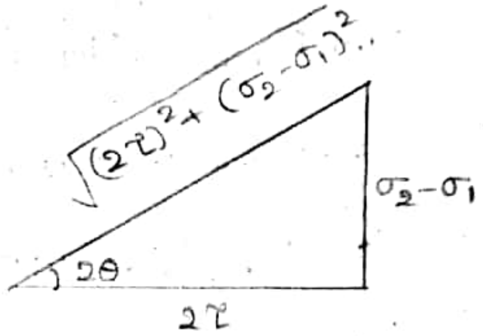
$$\Rightarrow \left(\frac{\sigma_1 - \sigma_2}{\cancel{2}}\right) \cos 2\theta \, (\cancel{2}) - \tau \, (-\sin 2\theta)(2) = 0$$

$$\Rightarrow (\sigma_1 - \sigma_2) \cos 2\theta + 2\tau \sin 2\theta = 0$$

$$2\tau \sin 2\theta = (\sigma_2 - \sigma_1)\cos 2\theta.$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \frac{\sigma_2 - \sigma_1}{2\tau}$$

$$\tan 2\theta = \frac{\sigma_2 - \sigma_1}{2\tau}$$



$$\cos 2\theta = \frac{2\tau}{\sqrt{(2\tau)^2 + (\sigma_2-\sigma_1)^2}}$$

$$\sin 2\theta = \frac{\sigma_2 - \sigma_1}{\sqrt{(2\tau)^2 + (\sigma_2-\sigma_1)^2}}$$

$\therefore$ Maximum shear stress,

$$(\sigma_t)_{max} = \left(\frac{\sigma_1 - \sigma_2}{2}\right)\sin 2\theta - \tau\cos 2\theta$$

$$= \left(\frac{\sigma_1 - \sigma_2}{2}\right)\left(\frac{\sigma_2 - \sigma_1}{\sqrt{4\tau^2 + (\sigma_2-\sigma_1)^2}}\right) - \tau\left(\frac{2\tau}{\sqrt{(4\tau)^2 + (\sigma_2-\sigma_1)^2}}\right)$$

$$= \frac{-(\sigma_2-\sigma_1)^2}{2\sqrt{4\tau^2 + (\sigma_2-\sigma_1)^2}} - \frac{2\tau^2}{\sqrt{4\tau^2 + (\sigma_2-\sigma_1)^2}}$$

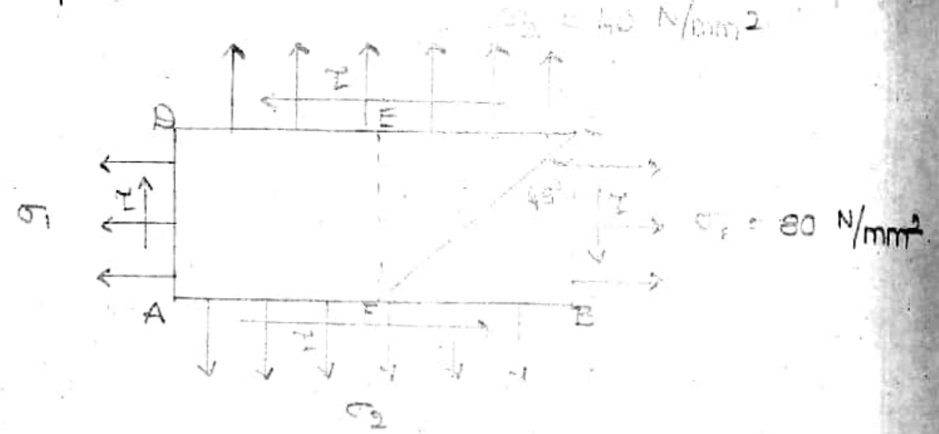
$$= \frac{-(\sigma_2-\sigma_1)^2 - 4\tau^2}{2\sqrt{4\tau^2 + (\sigma_2-\sigma_1)^2}}$$

$$= \frac{(\sigma_2-\sigma_1)^2 + 4\tau^2}{2\sqrt{4\tau^2 + (\sigma_2-\sigma_1)^2}}$$

$$= \frac{1}{2}\sqrt{(\sigma_2-\sigma_1)^2 + 4\tau^2}.$$

$$= \sqrt{\left(\frac{\sigma_2-\sigma_1}{2}\right)^2 + \tau^2}.$$

At a point within a body subjected to two mutually perpendicular directions. These stresses are 80 N/mm2 (tensile) and 40 N/mm2 (tensile). Each of the above stresses are accompanied by a shear stress of 60 N/mm2. Determine the normal stress, tangential stress and resultant stress on an oblique plane at an angle 45° with the axis of minor stress.



Given:

Major principal stress $(\sigma_1) = 80$ N/mm^2

Minor principal stress $(\sigma_2) = 40$ N/mm^2

Shear stress $(\tau) = 60$ N/mm^2

Angle made with minor axis, $\theta = 45°$

Normal stress, $\sigma_n = \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \left(\frac{\sigma_1 - \sigma_2}{2}\right) \cos 2\theta +$

$\tau . \sin 2\theta$

27 $\sigma_n = \left(\frac{80+40}{2}\right) + \left(\frac{80-40}{2}\right) \cos 2(45^\circ) + 60 \sin 2(45^\circ)$

$= 60 + 20 \cos 90 + 60 \sin 90$

$= 60 + 0 + 60$

$= 120 \ N/mm^2$

Tangential stress, $\sigma_t = \left(\frac{\sigma_1 - \sigma_2}{2}\right) \sin 2\theta - \tau \cos 2\theta$

$\sigma_t = \left(\frac{80-40}{2}\right) \sin 2(45^\circ) - 60 \cos 2(45^\circ)$

$= 20 \sin 90^\circ - 60 \cos 90^\circ$

$= 20 \ N/mm^2$.

Resultant stress, $\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$

$= \sqrt{120^2 + 20^2}$

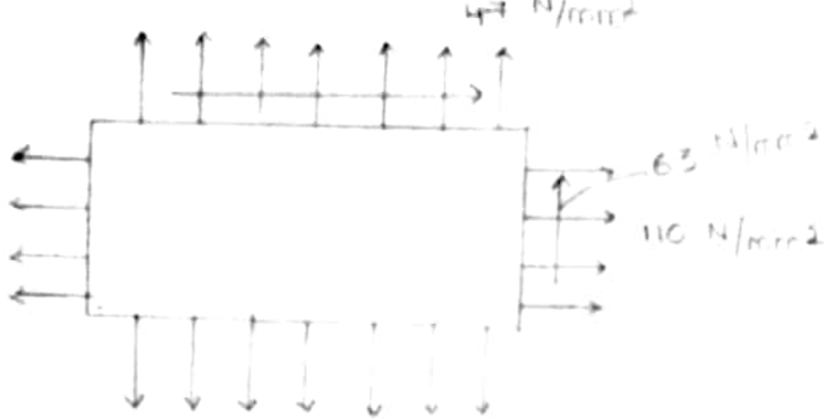
$\sigma_R = 121.65 \ N/mm^2$

4/12

* A rectangular block of material is subjected to a tensile stress of 110 N/mm^2 on one plane and a tensile stress of 47 N/mm^2 on the plane at right angles to the formal stress. Each of the above stresses is accompanied by a shear stress of 63 N/mm^2. Find

① the magnitude and direction of each of the principal stress.

(iii) Magnitude of the greatest shear stress.



Given:

Major principal stress $(\sigma_1) = 110 \ N/mm^2$

Minor principal stress $(\sigma_2) = 47 \ N/mm^2$

Shear stress $(\tau) = 63 \ N/mm^2$

(i) Principal stress.

Major principal stress

$$= \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{110 + 47}{2} + \sqrt{\left(\frac{110 - 47}{2}\right)^2 + 63^2}$$

$$= 78.5 + 70.4$$

$$= 148.9 \ N/mm^2$$

Minor principal stress

$$= -\sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2} + \left(\frac{\sigma_1 + \sigma_2}{2}\right)$$

29

$= 78.5 - 70.4$

$= 8.1 \ N/mm^2$

Direction, $\tan 2\theta = \dfrac{2\tau}{\sigma_1 - \sigma_2}$

$= \dfrac{2(63)}{110-47}$

$= 2$

$2\theta = 63.43$

$\theta = 31.715^\circ$

(ii) Greatest shear stress.

$$\tau = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \sqrt{\left(\frac{110-47}{2}\right)^2 + 63^2}$$

$$= \sqrt{\cancel{78.5}\ \cancel{31.5} + \cancel{3969}}$$

$= \cancel{63.24} \ N/mm^2$

$$= \sqrt{992.25 + 3969}$$

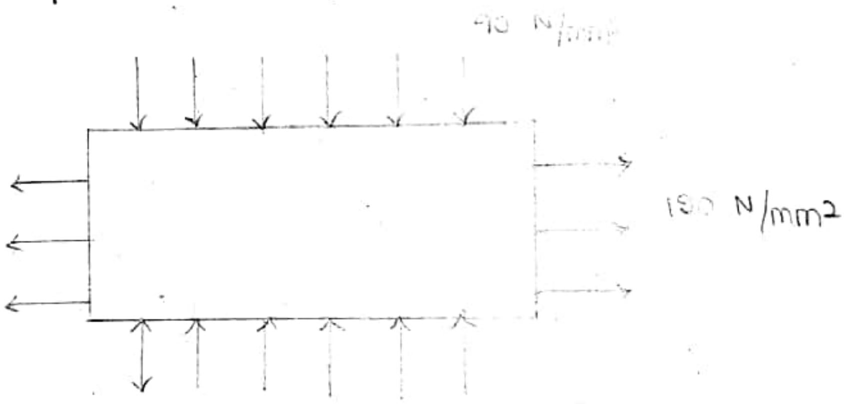
$= 70.4 \ N/mm^2$

* 30 Direct stresses of 120 N/mm² tensile and 90 N/mm² Compressive exist on two perpendicular planes in a body. They are also accompanied by shear stress on the planes. The greatest principal stress due to these is 150 N/mm²

(i) What must be the magnitude of the shear stress on the planes.?

(ii) What will be the maximum shear stress at the point?

Sol



Given: $\sigma_1 = 120 \ N/mm^2$

$\sigma_2 = -90 \ N/mm^2$

$\tau = ?$

Major principal stress = 150 N/mm²

(i) Shear stress = ?

$$\left(\frac{\sigma_1+\sigma_2}{2}\right) + \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2+\tau^2} = 150$$

31 $\left(\frac{120-90}{2}\right) + \sqrt{\left(\frac{120+90}{2}\right)^2 + \tau^2} = 150$

$\Rightarrow 15 + \sqrt{11025 + \tau^2} = 150$

$\sqrt{11025 + \tau^2} = 135$

$11025 + \tau^2 = 18225$

$\tau^2 = 7200$

$\tau = 84.85 \ N/mm^2$

(ii) Maximum shear stress = ?

$= \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$

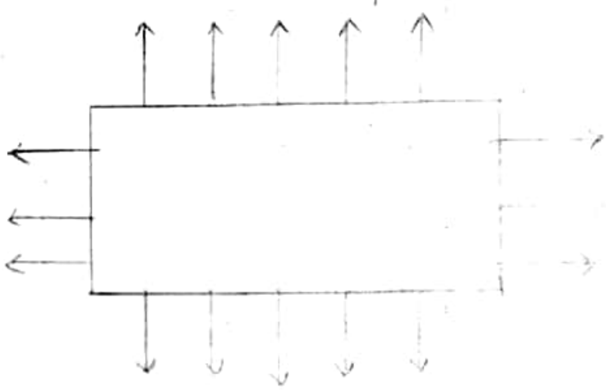
$= \sqrt{\left(\frac{120+90}{2}\right)^2 + (84.85)^2}$

$= \sqrt{11025 + 7199.52}$

$= 134.99$

$= 135 \ N/mm^2$

* 32 The normal stress in two mutually perpendicular directions are 600 N/mm2 and 300 N/mm2. both tensile. The complimentary shear stress is 450 N/mm2. Find the normal and tangential stresses on the two planes which are equally inclined to the planes carrying the normal stresses mention the above.



Sol

Given: $\sigma_1 = 600$ N/mm^2

$\sigma_2 = 300$ N/mm^2

$\tau = 450$ N/mm^2.

$\theta = 45°$.

Normal stress, $\sigma_n = \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \left(\frac{\sigma_1 - \sigma_2}{2}\right)\cos 2\theta$

$+ \tau \sin 2\theta$.

$\sigma_n = \left(\frac{600+300}{2}\right) + \left(\frac{600-300}{2}\right)\cos 2(45°)$

$+ 450 \sin 2(45°)$

$$\sigma_n = 450 + 150 \cos 90^\circ + 450 \sin 90^\circ$$

$$= 450 + 450$$

$$\sigma_n = 900 \ N/mm^2$$

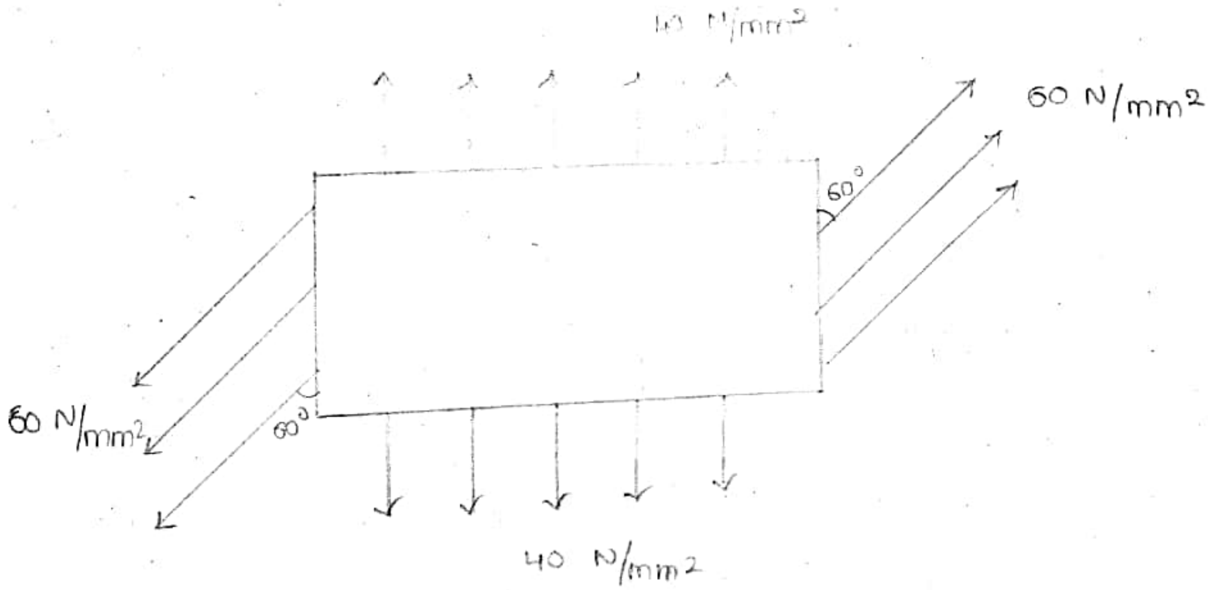
Tangential stress, $\sigma_t = \left(\frac{\sigma_1 - \sigma_2}{2}\right) \sin 2\theta - \tau \cos 2\theta$.

$$\sigma_t = \left(\frac{600 - 300}{2}\right) \sin 2(45^\circ) - 450 \cdot \cos 2(45^\circ).$$

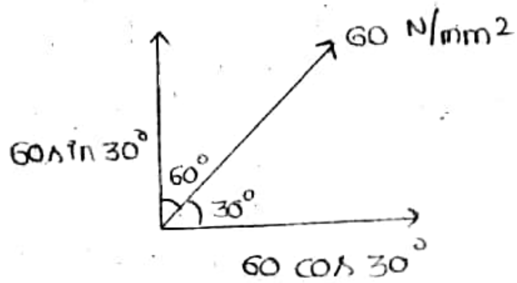
$$= 150 \sin 90^\circ - 450 \cos 90^\circ.$$

$$\sigma_t = 150 \ N/mm^2$$

* A point in strained material is subjected to the stresses as shown in figure. Evaluate their principal stresses and their directions.



The stress on the face in x-direction is not normal. This stress can be resolved in two components as shown in figure.



Normal stress in x-direction $= 60 \cos 30°$

$\sigma_1 = 52$ N/mm2

$\sigma_2 = 40$ N/mm2

$\tau = 60 \sin 30°$

$= 30$ N/mm2.

(i) Maximum principal stress

$$= \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \left(\frac{52 + 40}{2}\right) + \sqrt{\left(\frac{52 - 40}{2}\right)^2 + 30^2}$$

$$= 46 + \sqrt{36 + 900}$$

$$= 46 + \sqrt{936}$$

$$= 46 + 30.59$$

$$= 76.59 \text{ N/mm}^2$$

Minor principal stress

$$= -\sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2+\tau^2} + \left(\frac{\sigma_1+\sigma_2}{2}\right)$$

$$= -\sqrt{\left(\frac{52-40}{2}\right)^2+30^2} + \left(\frac{52+40}{2}\right)$$

$$= -30.59 + 46$$

$$= 15.41 \text{ N/mm}^2$$

Direction, $\tan 2\theta = \dfrac{2\tau}{\sigma_1-\sigma_2}$

$$= \frac{2\times 30}{52-40}$$

$$= 5$$

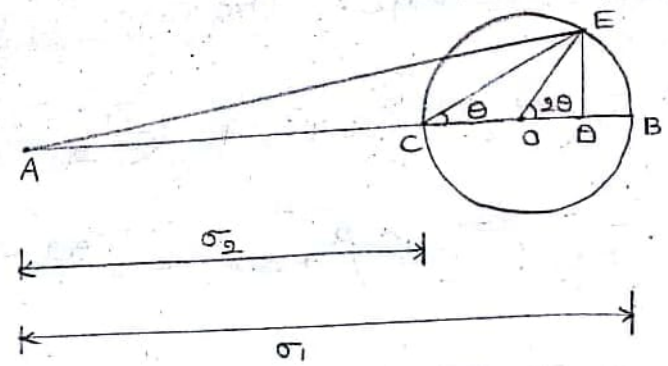
$$2\theta = 78.69$$

$$\theta = 39.3^\circ$$

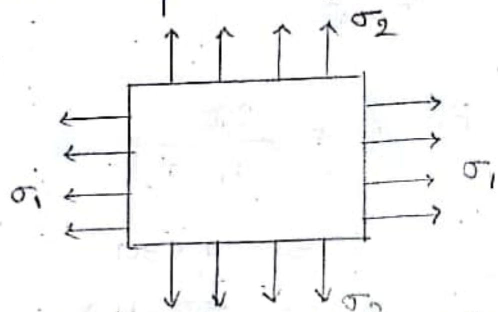
Graphical methods

Mohr's circle:

Mohr's circle when a body is subjected to two mutually perpendicular principal tensile stresses of unequal intensities:



tensile stresses of unequal intensities.



It is required to find the resultant stress.

Mohr's circle procedure:

* Take any point A and draw any horizontal line through A.
* Take $AB = \sigma_1$ and $AC = \sigma_2$ towards right from A to some suitable scale. With BC as diameter describe a circle.
* Let O is the centre of the circle.
* Now through O draw a line OE making an angle of 2θ with OB.

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* From E, draw ED perpendicular on AB.

Join AE.

* The resultant stress on plane equal to AE

normal stress on oblique plane = AD

tangential stress on oblique plane = ED.

<u>Proof:</u>

Radius of Mohr's circle $= \frac{\sigma_1 - \sigma_2}{2}$

$$CO, OB, OE = \frac{\sigma_1 - \sigma_2}{2}$$

$$\sigma_n = AD = AO + OD$$

$$AO = AC + CO$$

$$= \sigma_2 + \frac{\sigma_1 - \sigma_2}{2}$$

$$= \frac{2\sigma_2 + \sigma_1 - \sigma_2}{2}$$

$$= \frac{\sigma_1 + \sigma_2}{2}$$

$$OD = OE \cos 2\theta$$

$$= \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta$$

$$\sigma_n = AO + OD$$

$$= \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \left(\frac{\sigma_1 - \sigma_2}{2}\right) \cos 2\theta$$

$$\sigma_t = EB = OE \sin 2\theta$$

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta$$

11/12. * The tensile stresses at a point across two mutually perpendicular planes are 120 N/mm² and 60 N/mm². Determine normal, tangental and resultant stresses on a plane inclined at 30° to the axis of minor stress by using mohr's circle method.

Sol **Analytical method:**

$\sigma_1 = 120 \; N/mm^2$

$\sigma_2 = 60 \; N/mm^2$

$\theta = 30°$

$$\sigma_n = \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \left(\frac{\sigma_1 - \sigma_2}{2}\right)\cos 2\theta$$

$$= \left(\frac{120+60}{2}\right) + \left(\frac{120-60}{2}\right)\cos 2(30°)$$

$$= 90 + 30 \cos 60°$$

$$= 105 \; N/mm^2$$

$$\sigma_t = \left(\frac{\sigma_1 - \sigma_2}{2}\right)\sin 2\theta$$

$$= \left(\frac{120-60}{2}\right)\sin 2(30°)$$

$$= 30 \sin 60° = 25.98 \simeq 26 \; N/mm^2$$

$$\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2} = \sqrt{105^2 + 26^2}$$

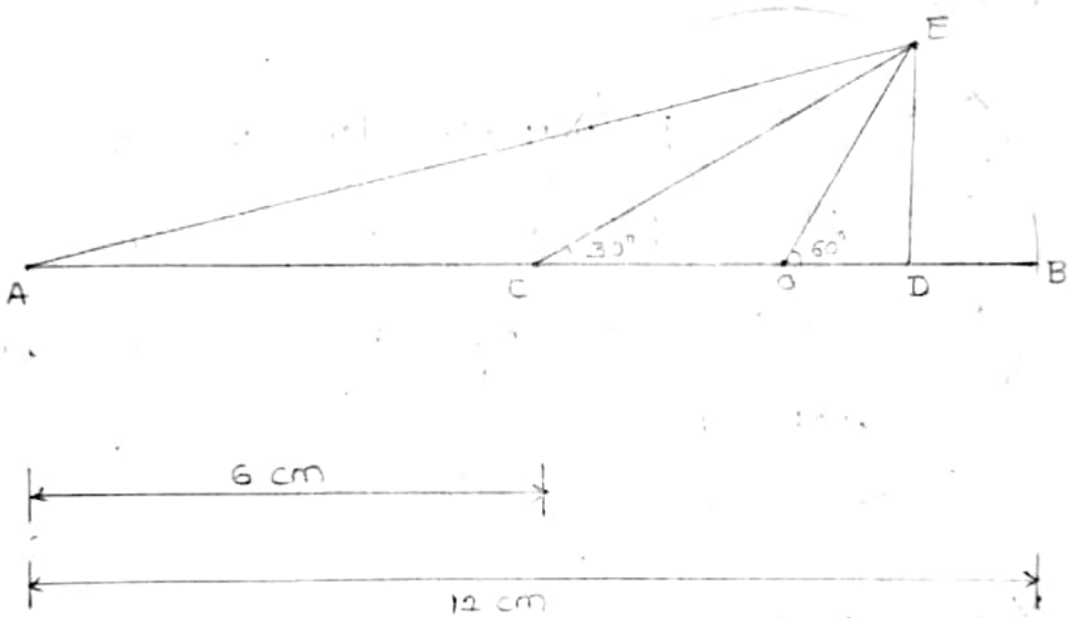
$$\sigma_R = 108.17 \text{ N/mm}^2$$

Graphical method:

Scale $\quad 1 \text{ cm} = 10 \text{ N/mm}^2$

$$\sigma_1 = \frac{120}{10} = 12 \text{ cm}$$

$$\sigma_2 = \frac{60}{10} = 6 \text{ cm}$$



Length $AD = 10.5$ cm

Normal stress, $\sigma_n = 10.5 \times 10 = 105 \text{ N/mm}^2$

Length $ED = 2.6$ cm

Tangential stress, $\sigma_t = 2.6 \times 10 = 26 \text{ N/mm}^2$

Length $AE = 10.8$

Resultant stress, $\sigma_R = 10.8 \times 10 = 108 \text{ N/mm}^2$

40 Mohr's circle when a body is subjected to two mutually perpendicular principal stresses which are unequal and unlike.

* Consider a rectangular body subjected to two mutually perpendicular principal stresses which are unequal and one of them is tensile and other is compressive.

* Let.

σ_1 = major principal tensile stress

σ_2 = minor principal compressive stress.

θ = angle made by the oblique plane with the axis of minor principal stress.

Mohr's circle procedure:

Take any point A.

Draw a horizontal line through A. on both sides of A as shown in figure..

Take $AB = \sigma_1$ towards right of A and $AC = \sigma_2$ towards left of A.

Bisect BC at O.

With O as centre and radius equal to CO (OB), draw a circle.

Through O draw a line OE making an angle 2θ.

From E, draw ED perpendicular to AB.

Join AE and CE.

Then

length AD = normal stress on olique section

length ED = tangential stress

length AE = Resultant stress

The stresses at a point in a bar are 200 N/mm² (tensile) and 100 N/mm² (compressive). Determine the resultant stress in magnitude and direction on the plane inclined at 60° to the axis of the major stress.

<u>Analytical method:</u>

$\sigma_1 = 200 \ N/mm^2$

$\sigma_2 = -100 \ N/mm^2$

$\theta = 90 - 60 = 30°$

$$\sigma_n = \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \left(\frac{\sigma_1 - \sigma_2}{2}\right)\cos 2\theta$$

$$= \left(\frac{200 - 100}{2}\right) + \left(\frac{200 + 100}{2}\right)\cos 2(30^\circ)$$

$$= 50 + 150 \cos 60^\circ$$

$$= 125 \text{ N/mm}^2$$

$$\sigma_t = \left(\frac{\sigma_1 - \sigma_2}{2}\right)\sin 2\theta$$

$$= \left(\frac{200 + 100}{2}\right)\sin 2(30^\circ)$$

$$= 150 \sin 60^\circ$$

$$= 129.9 \text{ N/mm}^2 \simeq \cancel{180}\ 130 \text{ N/mm}^2$$

$$\sigma_R = \sqrt{125^2 + 130^2} = 180.34 \text{ N/mm}^2$$

Obliquity, $\tan\theta = \dfrac{\sigma_t}{\sigma_n} = \dfrac{130}{125} = 1.04$

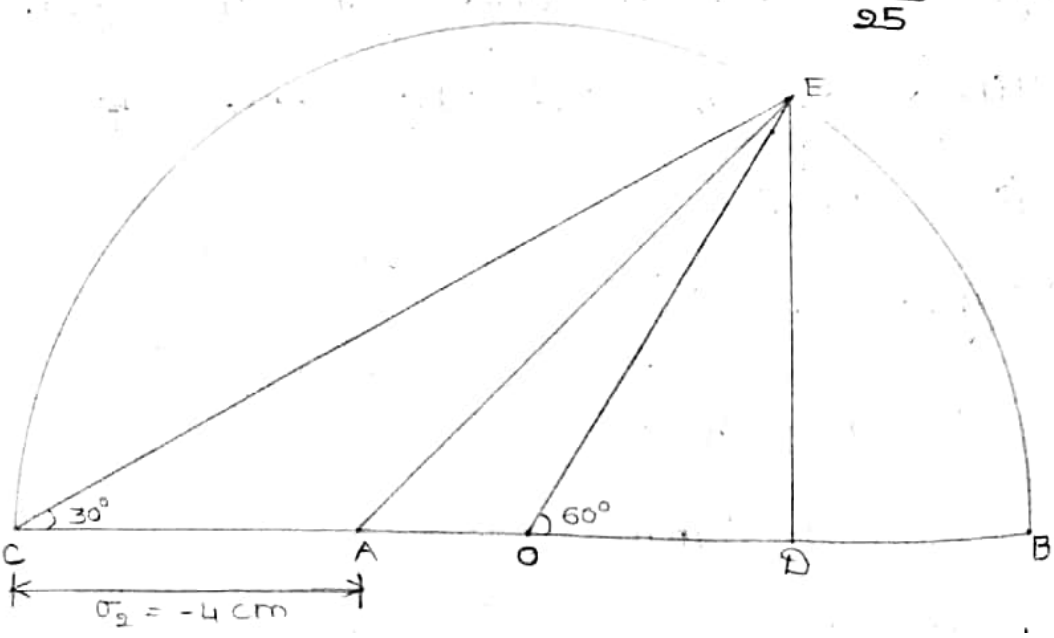
$$\theta = 46.12^\circ$$

Graphical method:

Scale: $1 \text{ cm} = 25 \text{ N/mm}^2$

$$\sigma_1 = \frac{200}{25} = 8 \text{ cm}$$

$$\sigma_2 = \frac{-100}{25} = -4 \text{ cm}$$



Length AB = 5 cm

Normal stress, $\sigma_n = 5 \times 25 = 125 \ N/mm^2$

Length EB = 5.2 cm

Shear stress, $\sigma_t = 5.2 \times 25 = 130 \ N/mm^2$

Length AE = 7.2 cm

Resultant stress, $\sigma_R = 7.2 \times 25 = 180 \ N/mm^2$

Mohr's circle when a body is subjected to two mutually perpendicular stresses accompanied by a simple shear stress:

* When a rectangular body is subjected to two mutually perpendicular principal tensile stresses of unequal intensities accompanied by a simple shear stress.

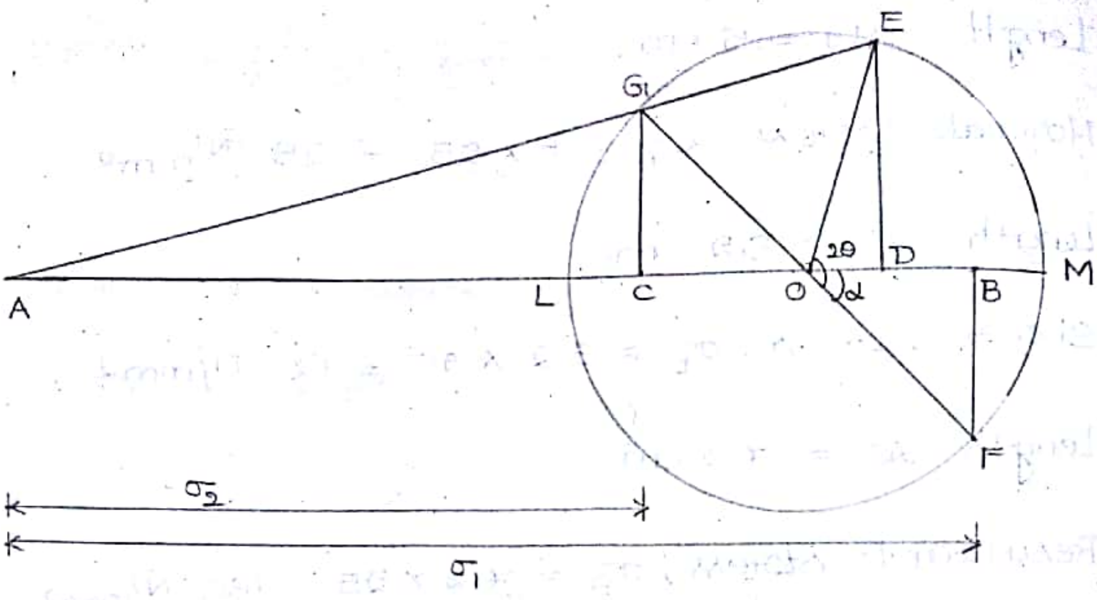
* Let

σ_1 = Major tensile stress.

σ_2 = Minor tensile stress.

τ = shear stress.

θ = angle made by the oblique plane.



Mohr's circle procedure:

Take any point A

Draw a horizontal line through A.

Take $AB = \sigma_1$ and $AC = \sigma_2$ towards right of A.

Draw perpendicular at B and C and cut off BF and CG equal to shear stress.

Bisect BC at O.

Now with O centre and radius equal to OG (OF), draw a circle.

Through O draw a line OE making an angle 2θ with OF.

From E draw ED perpendicular to AB

Join AE.

Then length, AE = resultant stress

AD = normal stress

ED = shear stress.

Proof:

$\sigma_n = AD = AO + OD$

$$AO = AC + CO$$

$$= \sigma_2 + \frac{\sigma_1 - \sigma_2}{2}$$

$$= \frac{2\sigma_2 + \sigma_1 - \sigma_2}{2}$$

$$= \frac{\sigma_1 + \sigma_2}{2}$$

$$OD = OE \cos(2\theta - \alpha)$$

$$= OF(\cos 2\theta \cos\alpha + \sin 2\theta . \sin\alpha)$$

$$= OF \cos\alpha . \cos 2\theta + OF \sin\alpha . \sin 2\theta$$

$$= OB \cos 2\theta + BF \sin 2\theta$$

$$= \left(\frac{\sigma_1 - \sigma_2}{2}\right)\cos 2\theta + \tau \sin 2\theta.$$

$\sigma_n = AD = AO + OD$

$$\sigma_n = \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \left(\frac{\sigma_1 - \sigma_2}{2}\right)\cos 2\theta + \tau \sin 2\theta$$

$$\sigma_t = ED = OE \sin(2\theta - \alpha)$$

$$= OF(\sin 2\theta . \cos\alpha - \cos 2\theta \sin\alpha)$$

$$= OF \cos\alpha . \sin 2\theta - OF \sin\alpha . \cos 2\theta$$

$= OB \sin 2\theta - BF \cos 2\theta$

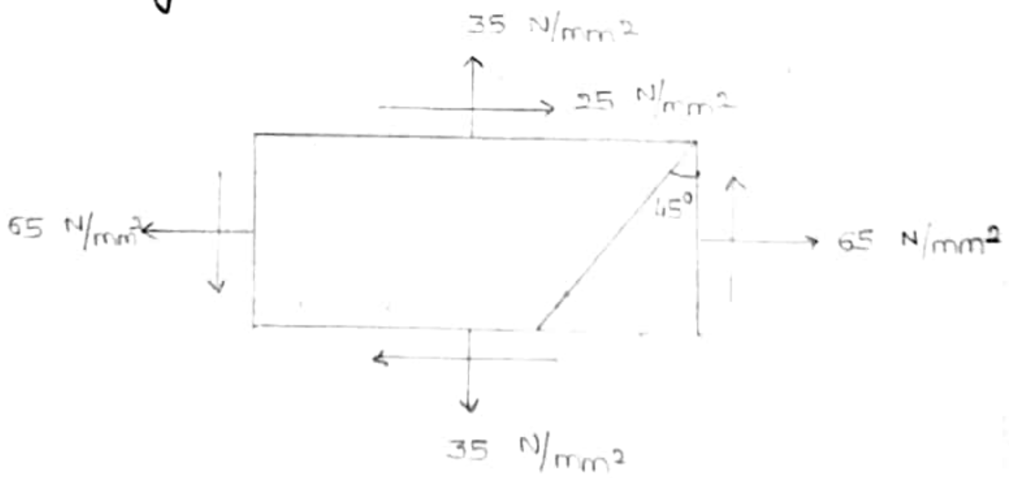
$= \left(\frac{\sigma_1 - \sigma_2}{2}\right) \sin 2\theta - \tau \cos 2\theta$

Maximum principal stress, = AM

Minimum principal stress = AL

Maximum shear stress = OF

A point in a strained material is subjected to stresses shown in figure. Using mohr circle method determine the normal, tangental and resultant stresses across the oblique plane. Check the answers with analytical method.



Given. $\sigma_1 = 65 \ N/mm^2$

$\sigma_2 = 35 \ N/mm^2$

$\tau = 25 \ N/mm^2$

$\theta = 45^\circ$

Analytical method:

$$\sigma_n = \left(\frac{\sigma_1+\sigma_2}{2}\right) + \left(\frac{\sigma_1-\sigma_2}{2}\right)\cos 2\theta + \tau \sin 2\theta$$

$$= \left(\frac{65+35}{2}\right) + \left(\frac{65-35}{2}\right)\cos 90 + 25 \sin 90$$

$$= 50 + 0 + 25$$

$$= 75 \ N/mm^2$$

$$\sigma_t = \left(\frac{\sigma_1-\sigma_2}{2}\right)\sin 2\theta - \tau \cos 2\theta$$

$$= \left(\frac{65-35}{2}\right)\sin 90 - 25 \cos 90$$

$$= 15 \ N/mm^2$$

$$\sigma_R = \sqrt{75^2+15^2}$$

$$= 76.48 \ N/mm^2$$

Graphical method

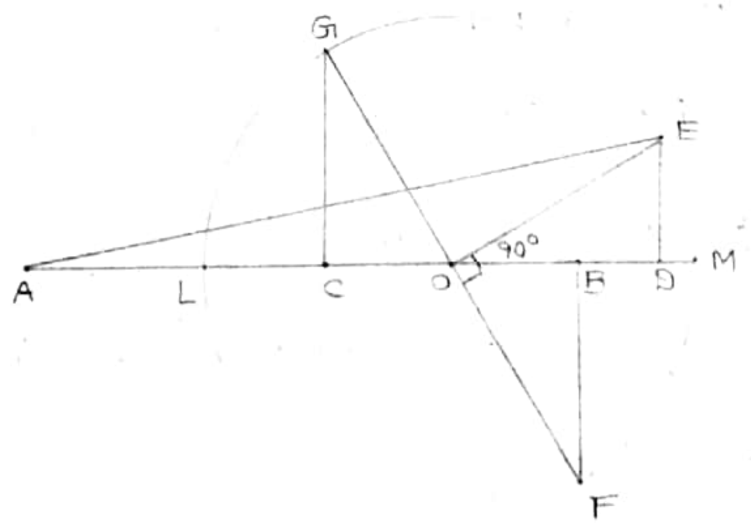
Scale: 1 cm = 10 N/mm^2

$\sigma_1 = \frac{65}{10} = 6.5$ cm

$\tau = \frac{25}{10} = 2.5$ cm

$\sigma_2 = \frac{35}{10} = 3.5$ cm

$\theta = 45°$



Length of AD = 7.5 cm

Normal stress, $\sigma_n = 7.5 \times 10 = 75 \ N/mm^2$

Length of ED = 1.5 cm

Tangential stress = 1.5 × 10 = 15 N/mm²

Length of AE = 7.6 cm

Resultant stress = 7.6 × 10 = 76 N/mm²

Max. principal stress, $(\sigma_n)_{max} = \left(\frac{\sigma_1+\sigma_2}{2}\right) + \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2+\tau^2}$

$$\sigma_{n_{max}} = \left(\frac{65+35}{2}\right) + \sqrt{\left(\frac{65-35}{2}\right)^2+25^2}$$

$$= 79.15 \text{ N/mm}^2$$

Min. principal stress, $(\sigma_n)_{min} = \left(\frac{\sigma_1+\sigma_2}{2}\right) - \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2+\tau^2}$

$$\sigma_{n_{min}} = \left(\frac{65+35}{2}\right) - \sqrt{\left(\frac{65-35}{2}\right)^2+25^2}$$

$$= 20.84 \text{ N/mm}^2$$

Max. shear stress, $(\sigma_t)_{max} = \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2+\tau^2}$

$$(\sigma_t)_{max} = \sqrt{\left(\frac{65-35}{2}\right)^2+25^2}$$

$$= 29.15 \text{ N/mm}^2$$

Length of AM = 7.9 cm

Max. principal stress = 7.9 × 10 = 79 N/mm²

Length of AL = 2.1 cm

Min. principal stress = 2.1 × 10 = 21 N/mm²

Length of OF = 2.9 cm

Max. shear stress = 2.9 × 10 = 29 N/mm²

Theories of failures

* When some external load is applied on the body the stresses and strains are produced in the body.
* These stresses are directly proportional to the strain within the elastic limit.
* This means when the load is removed, the body will return to its original state.
* There is no permanent deformation in the body.
* According to the important theories, the failure takes place when a certain limiting value is reached. by one of the following:

1. The maximum principal stress.
2. The maximum principal strain
3. The maximum shear stress
4. The maximum strain energy.
5. The maximum shear strain energy.

* According to this theory, the failure of a material will occur when the maximum principal tensile stress in this complex system reaches the value of maximum limit stress at the elastic limit. in simple tension or the minimum principal stress reaches the value of maximum stress at the elastic limit in simple compression.

* Let σ_1, σ_2 and σ_3 principal stresses at a point in 3 perpendicular directions (σ_1, σ_2 tensile, σ_3 compressive)

$\sigma_t^* =$ tensile stress at elastic limit in simple tension.

$\sigma_c^* =$ compressive stress at elastic limit in simple compression.

* Then according to this theory, the failure takes place when

$\sigma_1 \geq \sigma_t^*$ (In tension)

$\sigma_3 \geq \sigma_c^*$ (In compression)

51 This theory is also known as Rankine's theory.

* If the maximum principal stress (σ_1) is the design criterion, then the maximum principal stress must not exceed the principal stress (σ_t) for the given material.

$$\sigma_1 = \sigma_t$$

$$\sigma_t = \frac{\sigma_t^*}{FOS}$$

<u>Maximum principal strain theory:</u>

* This theory is also known as saintvenane theory.
* According to this theory, the failure will occur in a material when the max. principal strain reaches the strain due to yield stress in simple tension or when the min. principle strain (max. Compressive strain) reaches the strain

due to yield stress in simple compression.

* Principal strain in the direction of

σ_1, $\varepsilon_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E}$

$$= \frac{1}{E}(\sigma_1 - \mu\sigma_2 - \mu\sigma_3)$$

$$= \frac{1}{E}(\sigma_1 - \mu(\sigma_2 + \sigma_3))$$

$$\therefore \frac{1}{E}(\sigma_1 - \mu(\sigma_2+\sigma_3)) \geq \frac{\sigma_t^*}{E}$$

$$\Rightarrow \sigma_1 - \mu(\sigma_2+\sigma_3) \geq \sigma_t^*$$

* Principal strain in the direction of σ_3,

$$\varepsilon_3 = \frac{\sigma_3}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

$$= \frac{1}{E}(\sigma_3 - \mu(\sigma_1 + \sigma_2))$$

$$\therefore \frac{1}{E}(\sigma_3 - \mu(\sigma_1+\sigma_2)) \geq \frac{\sigma_c^*}{E}$$

$$\Rightarrow \sigma_3 - \mu(\sigma_1+\sigma_2) \geq \sigma_c^*$$

The principal stresses at a point in a elastic material are 100 N/mm² (tensile), 80 N/mm² (tensile) and 50 N/mm² (compressive). If the stress at the elastic limit in simple tension is 200 N/mm², 53 determine whether the failure of material will occur according to max. principal stress theory if not determine the factor of safety.

Sol Given:

$$\sigma_1 = 100 \ N/mm^2$$

$$\sigma_2 = 80 \ N/mm^2$$

$$\sigma_3 = -50 \ N/mm^2$$

$$\sigma_t^* = 800 \ N/mm^2$$

Maximum principal tensile stress is σ_1

$$\sigma_1 = 100 \ N/mm^2$$

As σ_1 , σ_t^*

then the failure will not occur.

$$\sigma_t = \frac{\sigma_t^*}{FOS}$$

$$\Rightarrow FOS = \frac{\sigma_t^*}{\sigma_t}$$

$$= \frac{800}{100}$$

$$FOS = 2.$$

* 54 The principal stresses at a point in an elastic material are 200 N/mm² (tensile), 100 N/mm² (t) and 50 N/mm² (compressive). If the stress at the elastic limit in simple tension is 200 N/mm², determine whether the failure of material will occur, according to max. principal strain theory. Take Poisson's ratio 0.3.

Sol Given: $\sigma_1 = 200 \ N/mm^2$

$\sigma_2 = 100 \ N/mm^2$

$\sigma_3 = -50 \ N/mm^2$

$\sigma_t^* = 200 \ N/mm^2$

$\mu = 0.3$

① Max. principal stress theory:

$\sigma_1 = 200 \ N/mm^2$

$\sigma_t^* = 200 \ N/mm^2$

$\sigma_1 \geq \sigma_t^*$

The material will fail according to max. principal stress theory.

② Max. principal strain theory:

Strain in the direction of max. principal stress, $\varepsilon_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} - \mu \frac{\sigma_3}{E}$

$$= \frac{1}{E}\left(\sigma_1 - \mu(\sigma_2 + \sigma_3)\right)$$

$$= \frac{1}{E}\left(200 - 0.3(100 - 50)\right)$$

$$= \frac{1}{E}\left(200 - \cancel{15}\ 15\right)$$

$$= \frac{185}{E}$$

Max strain in simple tension $= \frac{\sigma_t^*}{E}$

$$= \frac{200}{E}$$

$$\sigma_1 - \mu(\sigma_2 + \sigma_3) \geq \sigma_t^*.$$

$$185 \ngeq 200$$

The material will not fail according to maximum principal strain theory.

Determine the diameter of bolt which is subjected to an axial pull of 9 KN. together with a transverse shear force of 4.5 KN using ① max. principal stress theory

ⓘ max. principal strain theory.

Given the elastic limit in simple tension is 225 N/mm². Factor of safety 3, Poisson's ratio is 0.3.

Let the diameter of the bolt = d.

Axial pull, $P = 9$ KN.

Shear force, $F = 4.5$ KN.

$$\sigma_t^* = 225 \ N/mm^2$$

$$FOS = 3$$

Poisson's ratio = 0.3

$$\sigma_t = \frac{\sigma_t^*}{FOS} = \frac{225}{3} = 75 \ N/mm^2$$

Max. tensile stress, $\sigma_t = 75 \ N/mm^2$

Axial stress, $\sigma_x = \frac{P}{A} = \frac{9 \times 10^3}{\frac{\pi}{4} \times d^2}$

$$\Rightarrow \sigma_x = \frac{11459.15}{d^2}$$

Shear stress, $\tau = \frac{F}{A} = \frac{4.5 \times 10^3}{\frac{\pi}{4} \times d^2}$

$$\tau = \frac{5729.57}{d^2}.$$

Max. principal stress,

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$\sigma_1 = \frac{11459.15}{2d^2} \pm \sqrt{\left(\frac{11459.15}{2d^2}\right)^2 + \left(\frac{5729.57}{d^2}\right)^2}$$

$$= \frac{5729.57}{d^2} \pm \frac{1}{d^2}\sqrt{(5729.57)^2 + (5729.57)^2}$$

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$$= \frac{1}{d^2}\left(5729.57 \pm 8102.83\right)$$

$$\sigma_1 = \frac{13832.4}{d^2}$$

$$\sigma_2 = \frac{-2373.26}{d^2}$$

Max. principal stress theory

$$\sigma_1 = \sigma_t$$

$$\frac{13832.4}{d^2} = 75$$

$$\Rightarrow \quad d^2 = \frac{13832.4}{75}$$

$$d = 13.58 \text{ mm}$$

Max. principal strain theory

$$\sigma_1 - \mu(\sigma_2 + \sigma_3) = \sigma_t$$

$$\Rightarrow \frac{13832.4}{d^2} - 0.3\left(\frac{-2373.26}{d^2} - 0\right) = 75$$

$$\Rightarrow \frac{13832.4}{d^2} + \frac{711.97}{d^2} = 75$$

$$\Rightarrow \frac{14544.37}{d^2} = 75$$

$$d^2 = \frac{14544.37}{75}$$

$$d = 13.92 \text{ mm}$$

Maximum shear stress theory:

* This theory is due to guest and tresca and therefore known as guest theory.
* According to this theory, the failure of the material will occur when the max. principal shear stress in the material reaches max. shear stress in simple tension at the elastic limit.
* The max. shear stress in the material is equal to the half the distance difference between maximum principal and minimum principal stresses.
* If σ_1, σ_2 and σ_3 are principal stresses at a point in a material for which σ_t^* is the principal stress in simple tension at elastic limit.

$$\text{Max. shear stress} = \frac{\sigma_1 - \sigma_3}{2}$$

In case of simple tension at the elastic limit in simple tension the principal stresses are σ_t^*, 0 and 0.

∴ Max. shear stress in simple tension at elastic limit $= \frac{\sigma_t^* - 0}{2}$

$$= \frac{\sigma_t^*}{2}.$$

$$\frac{\sigma_1 - \sigma_3}{2} \geq \frac{\sigma_t^*}{2}$$

$$\Rightarrow \quad \sigma_1 - \sigma_3 \geq \sigma_t^*$$

The principal stresses at a point in a elastic material are 100 N/mm² (tensile), 80 N/mm² (tensile) and 50 N/mm² (Compressive) If the stress at the elastic limit in simple tension is 200 N/mm². Determine whether the failure of material will occur according to max. shear stress theory, if not determine FOS.

Given: $\sigma_1 = 100 \text{ N/mm}^2$

$\sigma_2 = 80 \text{ N/mm}^2$

$\sigma_3 = -50 \text{ N/mm}^2$

$\sigma_t^* = 200 \text{ N/mm}^2.$

Max. shear stress $= \frac{\sigma_1 - \sigma_3}{2}$

$$= \frac{100+50}{2} = 75 \text{ N/mm}^2$$

Max. shear strength $= \frac{\sigma_t^*}{2} = \frac{200}{2} = 100 \ N/mm^2$.

The material will not fail according to max. shear stress theory.

Maximum strain Energy theory:

* This theory is due to Haigh and is known as Haigh's theory.
* According to this theory, the failure of the material occurs when the total strain energy per unit volume in the material reaches the strain energy per unit volume of the material at the elastic limit in simple tension.
* This strain energy in a body equal to

~~half~~ $= \frac{1}{2} \times P \times \delta l$.

$= \frac{1}{2} \times \sigma \times A \times \varepsilon \times l$

$= \frac{1}{2} \times \sigma \times \varepsilon \times V$

Strain energy per unit volume.

$u = \frac{1}{2} \times \sigma \times \varepsilon$.

$$u = \frac{1}{2} \times \sigma \times \varepsilon$$

$$= \frac{1}{2} \times \sigma_1 \times \varepsilon_1 + \frac{1}{2} \times \sigma_2 \times \varepsilon_2 + \frac{1}{2} \times \sigma_3 \times \varepsilon_2$$

$$= \frac{1}{2} \times \sigma_1 \times \left(\frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} - \mu \frac{\sigma_3}{E} \right) +$$

$$\frac{1}{2} \times \sigma_2 \times \left(\frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E} - \mu \frac{\sigma_3}{E} \right) +$$

$$\frac{1}{2} \times \sigma_3 \times \left(\frac{\sigma_3}{E} - \mu \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} \right)$$

$$= \frac{1}{2E} \left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu \left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 \right) \right)$$

This strain energy per unit volume corresponding to stress at elastic limit in simple tension.

$$u = \frac{1}{2} \times \sigma_t^* \times \varepsilon_t^*$$

$$= \frac{1}{2} \times \sigma_t^* \times \frac{\sigma_t^*}{E}$$

$$u = \frac{(\sigma_t^*)^2}{2E}$$

$$\frac{1}{2E} \left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu \left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 \right) \right) \geq \frac{\sigma_t^{*2}}{2E}$$

$$\Rightarrow \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu \left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 \right) \geq \sigma_t^{*2}$$

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Maximum shear strain energy theory:

* This theory is due to Mises and Henky and is known as Mises & Henky theory.
* This theory is also called energy distortion theory.
* According to this theory, the failure of a material occurs when the total shear strain energy per unit volume in the stressed material reaches a value equal to the shear strain energy per unit volume at the elastic limit in simple tension.
* The total shear strain energy per unit volume due to principal stresses σ_1, σ_2 and σ_3 in a stressed material

$$= \frac{1}{12C}\left\{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right\}$$

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* Hence at the elastic limit in the simple tension test, the principal stresses are σ_t^*, 0 and 0.

$\therefore$ The shear strain energy per unit volume $= \frac{1}{12C}\left\{(\sigma_t^* - 0)^2 + (0-0)^2 + (0-\sigma_t^*)^2\right\}$

$$= \frac{1}{12C}\left(2\sigma_t^{*2}\right)$$

$$\frac{1}{12C}\left\{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right\} \geq \frac{1}{12C}\left(2\sigma_t^{*2}\right)$$

$$(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2 \geq 2\sigma_t^{*2}$$

* The principal stresses at a point in a elastic material are 100 N/mm2 (tensile), 80 N/mm2 (tensile) and 50 N/mm2 (compressive). If the stress at the elastic limit in simple tension is 200 N/mm2. Determine whether the failure of material will occur according to maximum ~~shear~~ strain energy theory, if not determine FOS.

Sol <u>Given:</u>

$\sigma_1 = 100$ N/mm^2

$\sigma_2 = 80$ N/mm^2

$\sigma_3 = -50$ N/mm^2

$\sigma_t^* = 200$ N/mm^2

Max. strain energy theory:

Max strain energy per unit volume.

$$= \frac{1}{2E}\left\{\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu\left(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1\right)\right\}$$

$$= \frac{1}{2E}\left\{100^2+80^2+50^2-2\times0.3\left((100\times80)+(80\times(-50))\right.\right.$$
$$\left.\left.+(-50\times100)\right)\right\}$$

$$= \frac{1}{2E}\left\{18900-0.6(-1000)\right\}$$

$$= \frac{1}{2E}\times19500$$

$$u = \frac{19500}{2E} \longrightarrow (1)$$

Max. strain energy per unit volume at the elastic limit $= \frac{1}{2E}\left(\sigma_t^{*2}\right)$

$$= \frac{1}{2E}(200)^2$$

$$= \frac{40000}{2E} \longrightarrow (2)$$

If we compare eqn (1) & (2), the total strain energy per unit volume is less than max. strain energy per unit volume at the elastic limit.

∴ The failure will not occur according to maximum strain energy theory.

Max. shear strain energy theory:

Max. shear strain energy. ~~theor~~ per unit volume $= \frac{1}{12C}\left\{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right\}$

$$= \frac{1}{12C}\left\{(100-80)^2+(80+50)^2+(-50-100)^2\right\}$$

$$= \frac{1}{12C} \times 39800 \longrightarrow (1)$$

Max. shear strain energy per unit volume at the elastic limit $= \frac{1}{12C}(2\sigma_t^{*2})$

$$= \frac{1}{12C}(2\times 200^2)$$

$$= \frac{80000}{12C} \longrightarrow (2)$$

If we compare eqn (1) & (2),

the total shear strain energy per unit volume is less than max. shear strain energy per unit volume at the elastic limit.

∴ The failure will not occur according to maximum shear strain energy theor

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UNIT-II

(1) TORSION OF CIRCULAR SHAFTS AND SPRINGS

*→ Topics :-
→ Theory of pure torsion
→ Derivation of torsion equations $\left[\frac{T}{J} = \frac{q}{r} = \frac{N}{L}\right]$.
→ Assumptions made in theory of pure torsion
→ Torsional moment of resistance.
→ Polar section modulus
→ Power transmitted by shafts
→ Combined bending & torsion & trust
→ Design of shafts according to theory of failure.

* Springs :- Deflection of close & open closed helical springs under axial pull & axial couple.
Springs in series & parallel
Carriage (or) leaf springs

→ The member which is used to observe the energy (move upto downward) is called "Spring".

→ The member which is used to subject one place to another place is called "Shaft" (transmit the power)

* Assumptions made in theory of torsion:-

(1) The material is homogeneous & isotropic

(2) The C/s of shaft is circular, throughout

(3) The material of the section is uniform throughout.

(4) The twist along the shaft is uniform.

(5) The C/s of the shaft which is plain before twist and regains after twist.

(6) All the radii which are straight before twist & remain straight after twist

* Derivation of Torsional equation:-

If any member subjected to torsion S.S are induced.
(Shear stress)

Consider a circular shaft which is fixed at one end and free at another end and subjected torsion "T" at free end.

Because of twisting the end BB' will move in clock-wise direction. Hence, CD will change to CD'; OD will change to OD'

Let,

L = Length of the shaft

D = Dia of the shaft

R = Radius of Shaft

T = Twisting moment applied at section DB.

τ = Shear stress.

C/G/N. = Modulus of rigidity

θ = Angle of twist

ϕ = Shear strain.

Distorsion in shaft = DD'

Shear strain (ϕ) = Distorsion per unit length

$$= \frac{DD'}{L}$$

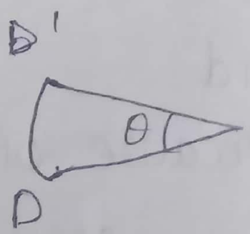
from COD'

$$\Rightarrow \tan\phi = \frac{DD'}{L}$$

[smaller values of ϕ $\tan\phi = \phi$]

from c/s

$$DD' = R\cdot\theta$$



W.K.T $\phi = \frac{DD'}{L}$

$$\phi = \frac{R\theta}{L}$$ but $DD' = R\theta$.

Modulus of rigidity = $\frac{\text{Shear stress}}{\text{Shear strain}}$

$$C = \tau/\phi$$

$$C = \frac{\tau}{(R\theta/L)}$$

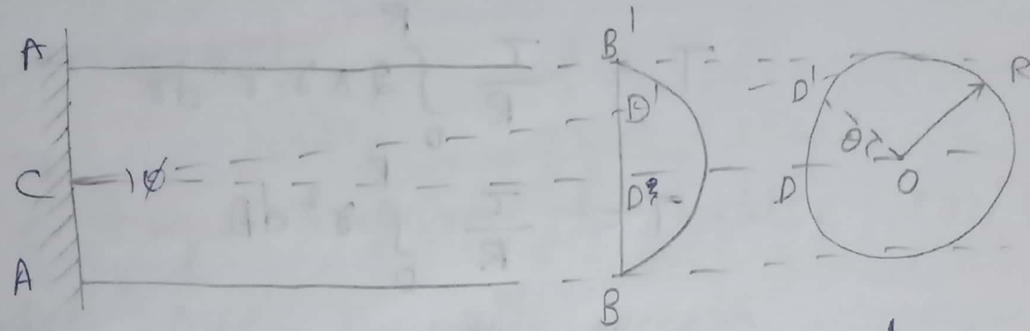
$$C = \frac{\tau L}{R\theta}$$

$$T = \frac{C\theta}{L} \times R$$

$$\boxed{T \propto R}$$

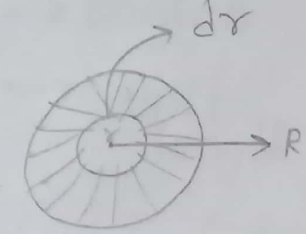
$$\boxed{\frac{\tau}{R} = \frac{C\theta}{L}} \rightarrow (1)$$

T is max at outer fibers



Consider an elemental ring at a distance "r"

thickness - dr.



$\therefore$ S.S in the circular ring

$$\tau' = \frac{\tau}{R} \times r$$

$$\frac{\tau}{R} = \frac{C\theta}{L}$$

$$\frac{\tau'}{r} = \frac{\tau}{R}$$

$R - \tau$

$r - \tau'$

Twisting force in the ring

= (Shear stress) x (Area of Ring)

$= \left(\frac{\tau}{R} \times r\right) \times (2\pi r \times dr)$

$= \frac{\tau}{R} \times 2\pi r^2 \times dr$

The twisting moment at ring (dT)

= Twisting force x r

$$= \frac{\tau}{R} \times 2\pi r^2 \times dr \times r$$

$$= \frac{\tau}{R} \times 2\pi r^3 \times dr$$

Total twisting moment in the shaft will be obtained by integrating to $0 \rightarrow R$

$$\int_0^R dT = \int_0^R \frac{\tau}{R} \times 2\pi r^3 \times dr$$

$$T = \frac{\tau}{R} \int_0^R 2\pi r \cdot r^2 dr$$

$$T = \frac{\tau}{R} \int_0^R r^2 dA$$

$$T = \frac{\tau}{R} J$$

$$\boxed{\frac{T}{J} = \frac{\tau}{R}} \rightarrow (2)$$

$$\boxed{\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{L}}$$

$\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{L}$ is the torsional equation

T = Twisting moment in N-mm

J = polar moment of inertia in N/mm^2

R = Radius (mm)

C = Shear modulus in N/mm^2

θ = Angle of twist in radius

L = length of shaft in mm.

* Strength of the shaft :- Max. torque (or) Max. power the shaft can transmit

* Torsional rigidity :- Product of shear modulus (C) and polar moment of Inertia (CJ)

(or)

The torque required to produce unit twist over a unit length.

$$\frac{T}{J} = \frac{C\theta}{L} \rightarrow T = CJ \times \frac{\theta \rightarrow \text{Radian}}{L \rightarrow \text{unit length}}$$

$$\boxed{T = CJ}$$

* Polar moment of Inertia (J) :-

→ For solid circle -

$$I_{xx} = \frac{\pi D^4}{64}$$

$$I_{yy} = \frac{\pi D^4}{64}$$

$$J = I_{xx} + I_{yy}$$

$$J = \frac{\pi D^4}{64} + \frac{\pi D^4}{64}$$

$$J = \frac{\pi D^4}{32}$$

— for Hollow circle —

D_o = External dia

D_i = Internal dia

$$I_{xx} = I_{yy} = \frac{\pi (D_o^4 - D_i^4)}{64}$$

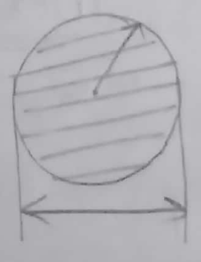
$$J = \frac{\pi (D_o^4 - D_i^4)}{64} + \frac{\pi (D_o^4 - D_i^4)}{64}$$

$$J = \frac{\pi (D_o^4 - D_i^4)}{32}$$

* Polar Modulus (Z_p) :-

$$Z_p = \frac{J}{R}$$

→ For solid.

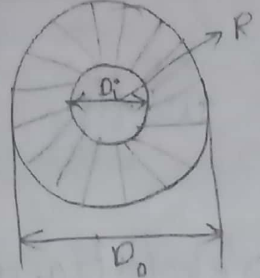


$$Z_p = \frac{J}{R}$$

$$= \frac{\frac{\pi D^4}{32}}{D/2}$$

$$Z_p = \frac{\pi D^3}{16}$$

→ For Hollow



$$Z_p = \frac{J}{R}$$

$$= \frac{\frac{\pi (D_o^4 - D_i^4)}{32}}{D_o/2}$$

$$= \frac{\pi (D_o^4 - D_i^4)}{16 D_o}$$

* **Max torque of solid shaft :-**

$$\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{l}$$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$T = \frac{\tau}{R} \cdot J$$

$$T = \tau \cdot \frac{J}{R} \longrightarrow Z_p$$

⇒ For solid

$$T = \tau \cdot \frac{\pi D^3}{16}$$

⇒ For Hollow :-

$$T = \tau \cdot \frac{\pi (D_o^4 - D_i^4)}{16 D_o}$$

* Power transmitted by shaft :-

$$P = \frac{2\pi NT}{60} \text{ watts}$$

Here,

$N \longrightarrow$ No. of revolutions – r.p.m

T = Torque – N-m

ω – Angular speed – $\frac{2\pi N}{60}$

$$\boxed{P = \omega \times T} \text{ watts}$$

*Q) A solid shaft of 150 mm dia is used to transmit torque. Find max. torque transmitted by shaft. If the max S.S is 45 N/mm².

Sol Given,

dia – 150mm

S.S – 45 N/mm^2

$$T = \tau \times \frac{\pi D^3}{16}$$

$$= 45 \times \frac{\pi \times (150)^3}{16}$$

$$= 29.820 \times 10^6 \text{ N-mm}$$

$$= 29.82 \text{ N-m}.$$

Q) A hollow circular shaft of outer & inner diameter 20 Cm & 10 Cm the shear stress 40mm². Find the maximum torque

Sol Given, $D_o = 20$ Cm

$D_i = 10$

$\tau = 40$ N/mm²

$T = ?$

$$T = \tau \cdot \frac{\pi (D_o^4 - D_i^4)}{16 \cdot D_o}$$

$$= \frac{40 \times \pi [(200)^4 - (100)^4]}{16 (200)}$$

$$= 58.9 \times 10^6 \text{ N/mm}^2$$

Q) A hollow shaft of ext. diameter 120 mm transmit 300 kW power at 200 rpm. Determine internal dia if max stress is limited to 60 N/mm²

Sol Given,

$P = 300$ kW $= 300 \times 10^3$ W

$N = 200$ rpm

$\tau = 60$ N/mm²

$D_o = 120$ mm

$$P = \frac{2\pi NT}{60} \Rightarrow T = \frac{60 \times P}{2\pi N}$$

$$T = \frac{60 \times 300 \times 10^3}{2 \times \pi \times 200}$$

$$T = 14323.944 N-m$$

$$= 14.323 \times 10^3 N-m$$

$$T = 14.323 \times 10^6 N-mm$$

$$T = \tau \times Z_p$$

$$T = \tau \times \frac{\pi (D_o^4 - D_i^4)}{16 \times D_o}$$

$$16 D_o T = \tau \times \pi (D_o^4 - D_i^4)$$

$$D_o^4 - D_i^4 = \frac{16 D_o T}{\tau \times \pi}$$

$$D_i^4 = D_o^4 - \frac{16 D_o T}{\tau \times \pi}$$

$$D_i^4 = (120)^4 - \frac{16 \times 120 \times 14.323 \times 10^6}{60 \times \pi}$$

$$D_i^4 = (120)^4 - 14,58,92,880$$

$$D_i = 88.54 mm$$

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Q) Find the maximum stress in a solid shaft of 15 cms, when the shaft transmits 150 kW power & 180 rpm

Sol Given,

$D = 15\text{ cms} = \cancel{0.15\text{ m}}\ 150\text{ mm}$

$P = 150\text{ kW} = 150 \times 10^3\text{ W}.$

$N = 180\text{ rpm}$

$$P = \frac{2\pi N T}{60}$$

$$\therefore T = \frac{60 \times P}{2\pi N}.$$

$$= \frac{60 \times 150 \times 10^3}{2 \times \pi \times 180}$$

$$= 7957.747\text{ N-m}$$

$$T = 7.9 \times 10^6\text{ N mm}$$

$$T = \tau \times \frac{\pi D^3}{16}$$

$$\tau = \frac{T \times 16}{\pi D^3}$$

$$= \frac{7.9 \times 10^6 \times 16}{\pi \times 150}$$

$=$

Q) A hallow shaft transmits 300 kW power at 80 rpm. If the shear stress 60 N/mm² & internal diameter 0.6 times outer diameter. find the D_o & D_i if maximum torque is equal to 1.4 times mean torque

Sol

$P = 300\ kW = 300 \times 10^3\ W$

$N = 80\ rpm.$

$\tau = 60\ N/mm^2$

$D_i = 0.6 \times D_o.$

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{60 \times P}{2\pi N} = \frac{60 \times 300 \times 10^3}{2 \times \pi \times 80}$$

$= 35.80 \times 10^3\ N\text{-}m$

$= 35.80 \times 10^6\ N\text{-}mm.$

or $[T_{max} = 1.4\, T_{mean} = 1.4 \times 35.80 \times 10^6 = 50.12 \times 10^6]$

$T = \tau \times Z_p$

$$35.80 \times 10^6 = 60 \times \frac{\pi}{16}\left[\frac{D_o^4 - D_i^4}{D_o}\right]$$

$$= 60 \times \frac{\pi}{16}\left[\frac{D_o^4 - (0.6 D_o)^4}{D_o}\right]$$

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$$= 60 \times \frac{\pi}{16}\left[\frac{D_0^4 - 0.1296 D_0^4}{D_0}\right]$$

$$= 60 \times \frac{\pi}{16}\left[\frac{0.8704 D_0^{\cancel{4}3}}{\cancel{D_0}}\right]$$

$$D_0^3 = \frac{35.80 \times 10^6 \times 16 \times 1.4}{60 \times \pi \times 0.8704}$$

$$D_0 = 169.7\,mm = 170\,mm.$$

$$D_i = 0.6 \times D_0 = \underline{102}\ mm.$$

8) Determine dia of solid shaft which will transmit 90 kW at 160 rpm & also determine length of the shaft if angle of twist not exceed 1° over entire length, shear stress limited to 60 N/mm². Take shear modulus 8×10^4 N/mm²

Sol Given,

$P = 90\ kW = 90 \times 10^3\ W$

$\tau = 60\ N/mm^2$

$C = 8 \times 10^4\ N/mm^2$

$N = 160\ rpm$

$\theta = 1^\circ$

$D\ \&\ L = ?$

$$P = \frac{2\pi N T}{60}$$

$$T = \frac{90 \times 10^3 \times 60}{2\pi \times 160}$$

$$T = 5.37 \times 10^3\ N\text{-}m$$

$$T = 5.37 \times 10^6\ N\text{-}mm$$

$$T = \tau \times Z_p$$

$$T = \tau \times \frac{\pi D^3}{16}$$

$$D^3 = \frac{16T}{\tau\pi}$$

$$D^3 = \frac{16 \times 5.37 \times 10^6}{60 \times \pi}$$

$$D = 76.95\ mm.$$

$$\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{L}$$

$$\frac{\tau}{R} = \frac{C\theta}{L}$$

$$L = \frac{RC\theta}{\tau}$$

$$L = \frac{38.47 \times 8\times10^4 \times \left(\frac{\pi}{180}\right)\times 1^\circ}{60}$$

$$L = 894mm$$

(or)

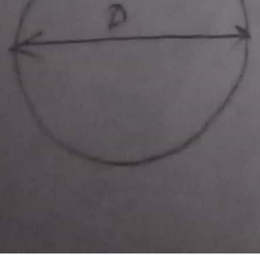
$$\frac{T}{J} = \frac{C\theta}{L}$$

$$L = \frac{C\theta \cdot J}{T}$$

$$L = 894.8mm$$

Q) Two shafts of same material of same length & same torque. If the 1st shaft is solid & 2nd hollow circular section whose internal diameter 2/3 of outer dia. Compare weights of the shaft when subjected to same shear stress

Sol Let, For solid shaft.



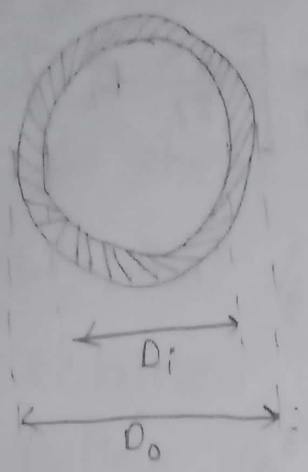
D_s = Diameter

L_s = Length

τ_s = shear stress

γ_s = wt density.

For hollow shaft



D_i = inner diameter

D_o = Outer diameter

$T_h =$

$\tau_h =$

$L_h =$

$\gamma_h =$

$D_i = \frac{2}{3} D_o$

For solid shaft,

$$T_s = \tau_s \times Z_{P_s}$$

$$T = \tau_s \times \frac{\pi \times D_s^3}{16}$$

For hollow shaft

$$T_h = \tau_h \times Z_{P_h}$$

$$= \tau_h \times \frac{\pi}{16} \times \left[\frac{D_o^4 - D_i^4}{D_o} \right]$$

$$= \tau_h \times \frac{\pi}{16} \left[\frac{D_o^4 - \left(\frac{2}{3} D_o\right)^4}{D_o} \right]$$

$$= \tau_h \times \frac{\pi}{16} \left[\frac{D_o^4 - \frac{16}{81} D_o^4}{D_o} \right]$$

$$T_h = \tau_h \times \frac{\pi}{16} \left[\frac{81 D_o^4 - 16 D_o^4}{81 \times D_o} \right]$$

$$T_h = \tau_h \times \frac{\pi}{16} \left[\frac{65 D_o^{\cancel{4}\,3}}{81 \cancel{D_o}} \right]$$

We know, $T_s = T_h$

$$\tau_s \times \frac{\cancel{\pi} \times D_s^3}{\cancel{16}} = \tau_h \times \frac{\cancel{\pi}}{\cancel{16}} \left[\frac{65}{81} D_o^3 \right]$$

$$\tau_s = \tau_h$$

$$D_s^3 = \frac{65}{81} D_o^3$$

$$D_s = \left[\frac{65}{81} \right]^{1/3} D_o$$

$$D_s = 0.929 D_o$$

$$W_s = \gamma_s \times V_s = \gamma_s \times A_s \times L_s = \gamma_s \times \frac{\pi \times D_s^2}{4} \times L_s$$

$$W_h = \gamma_h \times V_h = V_h \times A_h \times L_h = \gamma_h \times \frac{\pi (D_o^2 - D_i^2)}{4} \times L_h$$

$$\frac{W_s}{W_h} = \frac{\cancel{\gamma_s} \times \frac{\cancel{\pi} \times D_s^2}{\cancel{4}} \times \cancel{L_s}}{\cancel{\gamma_h} \times \frac{\cancel{\pi} (D_o^2 - D_i^2)}{\cancel{4}} \times \cancel{L_h}}$$

$$= \frac{D_s^2}{(D_o^2 - \underset{(2/3 D_o)}{D_i^2})} = \frac{(0.929 D_o)^2}{(D_o^2 - 0.66^2 \times D_o^2)}$$

$$= \frac{0.8630 \cancel{D_o^2}}{0.5644 \cancel{D_o^2}} = 1.52$$

$$W_s = 1.52 \times W_h$$

Q) Determine dia of solid shaft which will transmit 300kW at 250 rpm if shear stress $30 N/mm^2$ & angle of twist $1°$. Length - 2m, take $C = 1 \times 10^5 N/mm^2$

Sol

$P = 300 kW = 300 \times 10^3 W$

$N = 250$ rpm

$\tau = 30 N/mm^2$

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{60P}{2\pi N}$$

$$T = \frac{60 \times 300 \times 10^3}{2\pi \times 250}$$

$T = 11.459 \times 10^3$ N-m

$T = 11.459 \times 10^6$ N-mm

$T = \tau \cdot Z_p$

$$T = \tau \cdot \frac{\pi D^3}{16}$$

$$D^3 = \frac{16T}{\tau \cdot \pi}$$

$$D^3 = \frac{16 \times 11.459 \times 10^6}{30 \times \pi}$$

$D = 124.83$

$D \simeq 125$ mm

W.K.T

$$\frac{T}{J} = \frac{C\theta}{l}$$

$$\frac{T}{\frac{\pi D^4}{32}} = \frac{C\theta}{l}$$

$$\frac{32T}{\pi D^4} = \frac{C\theta}{l}$$

$$\frac{l}{C\theta} = \frac{\pi D^4}{32T}$$

$$D^4 = \frac{32lT}{\pi C\theta}$$

$$D^4 = \frac{32 \times 2000 \times 11.459 \times 10^{\cancel{8}}}{\pi \times 1\cancel{0}^{8} \times \left(\frac{\pi}{180^\circ}\right)}$$

$$D = 107.62 \text{mm}$$

The preferred diameter of the shaft is maximum of above two values

$\therefore$ Diameter (D) = 125mm

Q) A hollow shaft of diameter ratio $\frac{3}{8}$ (i.e $\frac{D_i}{D_o}$) is to transmit 375 KW power at 100 rpm. The max. torque is 20% greater than mean. The shear stress is 60 N/mm² Angle of twist 2°, length of shaft 4m. Take $C = 0.85 \times 10^5$ N/mm²

Sol

$P = 375\,kW = 375 \times 10^3\,W$

$N = 100\ rpm$

$\tau = 60\ N/mm^2$

$$P = \frac{2\pi N T}{60}$$

$$T = \frac{60P}{2\pi N}$$

$$T = \frac{60 \times 375 \times 10^3}{2 \times \pi \times 100}$$

$$T = 35.809 \times 10^6\ N\text{-}mm$$

$$T_{max} = 1.2 \times 35.809 \times 10^6$$

$$\left(1 + \frac{20}{100}\right)$$

$$T_{max} = 42.9708 \times 10^6$$

$$T = \tau \times Z_p$$

$$T = \frac{\tau \times \pi \times (D_o^4 - D_i^4)}{16 D_o} \quad \tfrac{3}{8} D_o$$

$$16T = \frac{\tau \times \pi (D_o^4 - D_i^4)}{D_o}$$

$$\frac{16T}{\tau \times \pi} = \frac{D_o^4 - \left(\frac{3}{8} D_o\right)^4}{D_o}$$

$$36.47 \times 10^5 = \frac{D_o^4 - 0.0197 D_o^4}{D_o}$$

$$36.47 \times 10^5 = 0.9803 D_o^3$$

$$D_o = 154.95$$

$$\simeq 155\,mm$$

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(or)

$$\frac{T}{J} = \frac{C\theta}{l}.$$

$$\frac{T}{\left(\frac{\pi D^4}{32}\right)} = \frac{C\theta}{l}.$$

$$\frac{T \times 32}{\pi D^4} = \frac{C\theta}{l}.$$

$$\frac{\pi D^4}{T \times 32} = \frac{l}{C\theta}.$$

$$D^4 = \frac{32 \cdot l T}{\pi C\theta}$$

$$D^4 = \frac{32 \times 4000 \times 42.9708 \times 10^6}{\pi \times 0.85 \times 10^5 \times 2 \times \frac{\pi}{180}}$$

$$D_o = 155.86 \text{ mm}.$$

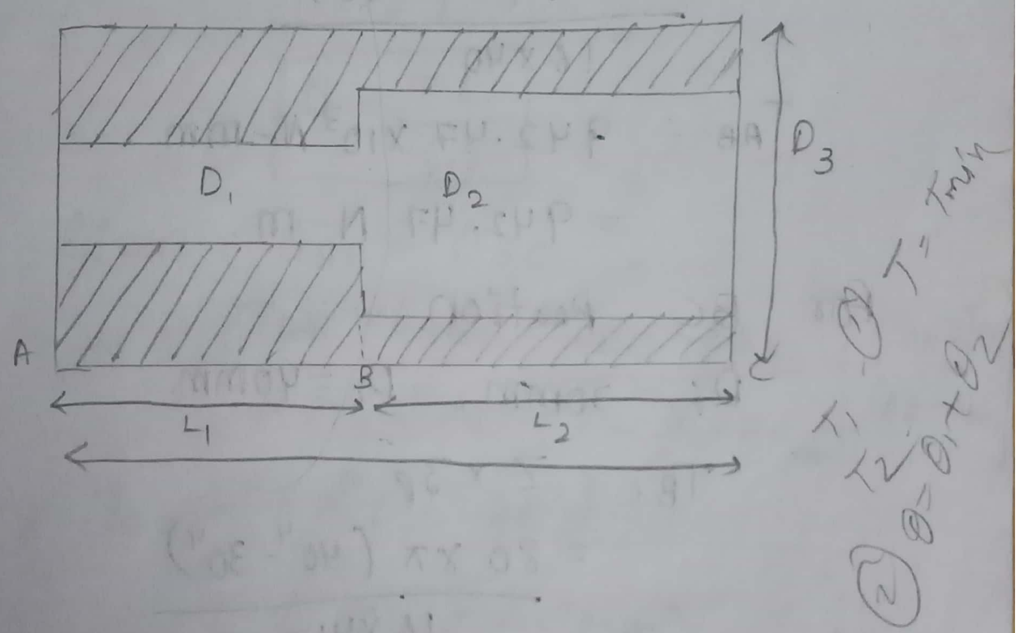
W.K.T $D_i = \frac{3}{8} D_o$

$$D_i = \frac{3}{8} \times 155.86$$

$$D_i = 58.44 \text{mm}$$

* <u>Varying Shafts</u> :- A shaft which is having different cross section over certain length of the shaft are called varying shafts.

For this type of shafts the torque is equal to the minimum torque of shaft portion and the angle of twist is the sum of twists of two portions.

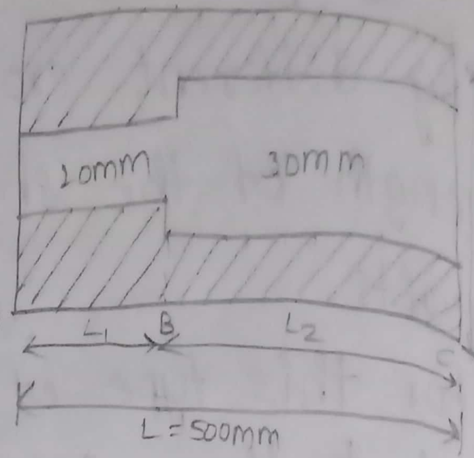


Q) A shaft of ABC 500mm length & 40mm external dia for a part of its length AB & 20mm dia & for the reamaining length BC to 30 mm dia. Shear stress $80 \, N/mm^2$. Find P at 200 rpm. If the angle of twist is same for AB & BC find lengths of AB & BC portions

Sol

$$P = \frac{2\pi N T}{60}$$

For AB portion



$D_i = 20mm, \ D_o = 40mm$

$\tau = 80 N/mm^2$

$T_{AB} = \tau \times Z_p$

$$= \frac{80 \times \pi (D_o^4 - D_i^4)}{16 \times D_o}$$

$$= \frac{80 \times \pi (40^4 - 20^4)}{16 \times 40}$$

$T_{AB} = 942.47 \times 10^3 \ N\text{-}mm.$

$= 942.47 \ N\text{-}m.$

For BC Portion

$D_i = 30mm, \quad D_o = 40mm.$

$T_{BC} = \tau \times Z_p$

$$= \frac{80 \times \pi (40^4 - 30^4)}{16 \times 40}$$

$T_{BC} = 687.2 \times 10^3 \ N\text{-}mm$

$= \cancel{687} \ 687.2 \ N\text{-}m.$

for varying shafts.

$T =$ min of T_{AB} & T_{BC}

$\therefore T = 687.2 \ N\text{-}m$

Power $P = \dfrac{2\pi N T}{60} \text{ W}$ (T in N-m)

$= \dfrac{2 \times \pi \times 200 \times 687.2}{60}$

$= 14.39 \times 10^3 \text{ W}.$

$\therefore P = 14.39 \text{ kW}$

$\dfrac{T}{J} = \dfrac{\tau}{R} = \dfrac{C\theta}{l}.$

$\dfrac{T}{J} = \dfrac{C\theta}{L}$

$\boxed{\theta = \dfrac{TL}{CJ}}$

Given $\theta_{AB} = \theta_{BC}.$

$\dfrac{T_{AB} \times L_{AB}}{C_{AB} \times J_{AB}} = \dfrac{T_{BC} \times L_{BC}}{C_{BC} \times J_{BC}}$ $\left[\because T_{AB} = T_{BC}, \; C_{AB} = C_{BC}\right]$

$J_{AB} = \dfrac{\pi}{32}(40^4 - 20^4)$

$= 235.62 \times 10^3 \text{ mm}^4$

$J_{BC} = \dfrac{\pi}{32}(40^4 - 30^4)$

$= 171.805 \times 10^3 \text{ mm}^4.$

$\therefore L_{AB} = \dfrac{J_{AB}}{J_{BC}} \times L_{BC}$

$L_{AB} = 1.37 \times L_{BC}$

$L_{AB} = 1.37 \times [500 - L_{AB}]$

$\therefore L_{AB} = 289 \text{ mm}$

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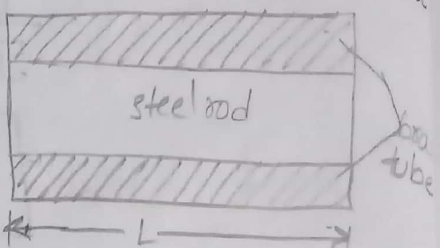
$L_{BC} = 500 - 289$

$= 211\,mm$

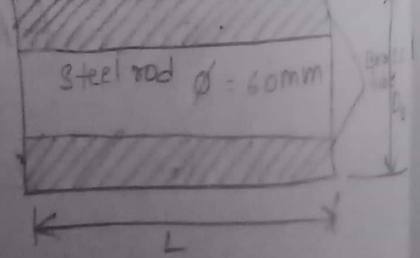
* Composite shafts:- If the shaft is made by using 2 or more materials are called composite shafts. For this type of shafts the torque is sum of torques of two materials. And the angle of twist is equal for the two materials.



$T_{steel},\ T_{brass}$

$$T = T_s + T_b$$
$$\theta_s = \theta_B$$

Q) A composite shaft consists of steel rod 60 mm dia is surrounded by a closely fitting brass tube. Find the outer dia when a torque of 1000 N-m is applied to composite shaft. It will be shared equally b/w the materials. Take $C_s = 8.4 \times 10^4 N/mm^2$, $C_b = 4.2 \times 10^4\ N/mm^2$. Length of shaft = 4m.



Sol

$T_s,\ T_b$

$T = T_s + T_b$

$\theta_s = \theta_b$

$T = 1000\ N\text{-}m$

$T = 1000 \times 10^3\ N\text{-}mm$

$\because$ it is equally shared

$\therefore T_s = T_b = T/2 = \frac{1000 \times 10^3}{2}$

$= 500 \times 10^3$ N-mm.

For composite sections. $\theta_s = \theta_b$.

$$\frac{T_s L_s}{C_s J_s} = \frac{T_b L_b}{C_b J_b}$$

we know, $T_s = T_b$

$L_s = L_b$

we have $C_s J_s = C_b J_b$.

$$J_s = \frac{\pi d_s^4}{32} = \frac{\pi (60)^4}{32} = 12.72 \times 10^5 \text{ mm}^4$$

$$J_b = \frac{\pi (D_o^4 - D_i^4)}{32} = \frac{\pi (D_o^4 - 60^4)}{32} =$$

$[D_s = D_i]$

$\therefore C_s J_s = C_b J_b$.

$$8.4 \times 10^4 \times \frac{\cancel{\pi} \times 60^4}{\cancel{32}} = 4.2 \times 10^4 \times \frac{\cancel{\pi} (D_o^4 - 60^4)}{\cancel{32}}$$

$$D_o^4 - 60^4 = \frac{8.4 \times \cancel{10^4} \times 60^4}{4.2 \times \cancel{10^4}}$$

$$D_o^4 = 2 \times 60^4 + 60^4$$

$D_o = 78.96 \simeq 79$ mm.

$T = \tau \times Z_p$

$\tau = T/Z_p$.

$$\tau_s = T/Z_{p_s} = \frac{500 \times 10^3}{\frac{\pi \times d_s^3}{16}}$$

$$= \frac{500 \times 10^3 \times 16}{\pi \times 60^3}$$

$$= 11.79 \ N/mm^2$$

$$\tau_b = \frac{T}{Z_{Pb}} = \frac{500 \times 10^3}{\frac{\pi (D_o^4 - D_i^4)}{16 D_o}}$$

$$= \frac{500 \times 10^3 \times 16 \times 79}{\pi (79^4 - 60^4)}$$

$$= 7.74 \ N/mm^2$$

Common angle of twist ; $\theta_s = \theta_b$.

$$\theta_s = \frac{T_s L_s}{C_s J_s} = \frac{500 \times 10^3 \times 4 \times 10^3}{8.4 \times 10^4 \times 12.72 \times 10^5}$$

$$\theta_s = 0.01871 \times \frac{180}{\pi}$$

$$= 1^\circ 4' 19'' \simeq 1^\circ 4' 7''$$

$$\theta_b = \frac{T_b L_b}{C_s J_b}$$

$$J_b = \frac{\pi (79^4 - 60^4)}{32} = 25.51 \times 10^5 \ mm^4$$

$$\therefore \theta_b = \frac{500 \times 10^3 \times 4 \times 10^3}{4.2 \times 10^4 \times 25.51 \times 10^5}$$

$$\theta_b = 0.01866 \times \frac{180}{\pi}$$

$$= 1^\circ 4' 1''$$

* Shafts subjected to combined Bending and Torsion :-

Major princip

(1) Solid Shafts :-

→ Major principle stress,

$$\sigma_1 = \frac{16}{\pi D^3}\left[M + \sqrt{M^2 + T^2}\right]$$

→ Minor principle stress

$$\sigma_2 = \frac{16}{\pi D^3}\left[M - \sqrt{M^2 + T^2}\right]$$

→ Max shear stress; $\tau_{max} = \frac{16}{\pi D^3}\left[\sqrt{M^2 + T^2}\right]$

→ Principal plane ; $\tan 2\theta = \frac{T}{M}$

Q) A solid shaft diameter 80mm subjected to twist in moment 8 Mega Newtons (MN) mm and bending moment 5 MN-mm Determine major, Minor principal stresses. Max. shear stress & Position of principal plane.

Sol

$D = 80mm.$

$T = 8\ MN\text{-}mm = 8 \times 10^6\ N\text{-}mm$

$M = 5\ MN\text{-}mm = 5 \times 10^6\ N\text{-}mm$

∴ Major principal stress

$$\sigma_1 = \frac{16}{\pi D^3}\left[M + \sqrt{M^2 + T^2}\right]$$

$$= \frac{16}{\pi \times 80^3}\left[5 \times 10^6 + \sqrt{(8^2 + 5^2)10^{12}}\right]$$

$\sigma_1 = 143.37 \ N/mm^2$

Minor principal stress

$$\sigma_2 = \frac{16}{\pi D^3}\left[M - \sqrt{M^2 + T^2}\right]$$

$$= \frac{16}{\pi (80)^3}\left[5 \times 10^6 - \sqrt{(5^2+8^2)10^{12}}\right]$$

$$= -44. \ N/mm^2$$

Max. shear stress

$$\tau_{max} = \frac{16}{\pi (D^3)}\left[\sqrt{M^2+T^2}\right]$$

$$= \frac{16}{\pi (80)^3}\left[\sqrt{(5^2+8^2)\,10^{12}}\right]$$

$$= 93.84 \ N/mm^2$$

Principal plane

$$\tan 2\theta = \frac{T}{M}$$

$$\tan 2\theta = \frac{8 \times 10^6 \ N\text{-}mm}{5 \times 10^6 \ N\text{-}mm}$$

$$\theta = 28^\circ 59'$$

(2) Hallow circular shafts :-

→ Major principal stress

$$\sigma_1 = \frac{16 \times D_0}{\pi (D_0^4 - D_i^4)}\left[M + \sqrt{M^2 + T^2}\right]$$

→ Minor principal stress

$$\sigma_2 = \frac{16 \times D_0}{\pi (D_0^4 - D_i^4)}\left[M - \sqrt{M^2 + T^2}\right]$$

→ Max shear stress,

$$\tau_{max} = \frac{16 D_o}{\pi (D_o^4 - D_i^4)} \left(\sqrt{M^2 + T^2}\right)$$

→ Principal plane

$$\tan 2\theta = \frac{T}{M}.$$

$$\theta = \frac{\tan^{-1}\left(\frac{T}{M}\right)}{2}$$

Q) In a hollow shaft subjected to twisting moment 4×10^6 N-mm and bending moment 3×10^6 N-mm & maximum shear stress 80 N/mm². De-termine outer & internal diameter & major & minor principal stresses and position of the principal plane if outer dia is twice internal dia.

Sol Given,

$$T = 4 \times 10^6 \text{ N-mm}$$

$$M = 3 \times 10^6 \text{ N-mm}$$

$$\tau_{max} = 80 \text{ N/mm}^2$$

$$D_o = 2D_i \Rightarrow D_i = \frac{D_o}{2}.$$

W.K.T,

$$\tau_{max} = \frac{16 \times D_o}{\pi (D_o^4 - D_i^4)} \left[\sqrt{M^2 + T^2}\right]$$

$$= \frac{16 \times D_o}{\pi \left(D_o^4 - \left(\frac{D_o}{2}\right)^4\right)} \left(\sqrt{(3 \times 10^6)^2 + (4 \times 10^6)^2}\right)$$

$$= \frac{16 D_o}{\pi [D_o^4 - 0.0625 D_o^4]} \left[5 \times 10^6\right]$$

$$= \frac{16 \times D_0}{\pi \times [0.9375 D_0^{4/3}]} \times [5 \times 10^6]$$

$$\tau_{max} = \frac{16}{\pi \times 0.9375 D_0^3} \times [5 \times 10^6]$$

$$\frac{80 \times \pi \times 0.9375}{5 \times 10^6 \times 16} = \frac{1}{D_0^3}$$

$$D_0^3 = \frac{5 \times 10^6 \times 16}{80 \times \pi \times 0.9375}$$

$$D_0 = 69.78 \text{ mm}$$

$$\simeq 70 \text{ mm}.$$

$$D_i = 35 \text{ mm}.$$

→ Major principal stress

$$\sigma_1 = \frac{16 \times D_0}{\pi (D_0^4 - D_i^4)} \left[M + \sqrt{M^2 + T^2}\right]$$

$$= \frac{16 \times 70}{\pi (70^4 - 35^4)} \left[3 \times 10^6 + \sqrt{(3^2 + 4^2) 10^{12}}\right]$$

$$= 126.70 \text{ N/mm}^2$$

→ Minor principal stress

$$\sigma_2 = \frac{16 \times D_0}{\pi (D_0^4 - D_i^4)} \left[M - \sqrt{M^2 + T^2}\right]$$

$$= \frac{16 \times 70}{\pi (70^4 - 35^4)} \left[3 \times 10^6 - 5 \times 10^6\right]$$

$\sigma_2 = -30.29\ N/mm^2$

→ Max shear stress

$$\tau_{max} = \frac{16 D_0}{\pi(D_0^4 - D_i^4)} \left[\sqrt{M^2+T^2}\right]$$

$$= \frac{16 \times 70}{\pi(70^4 - 35^4)} \left[\cancel{8}\ 5 \times 10^6\right]$$

$$= 80\ N/mm^2$$

→ Principal plane

$$\tan 2\theta = \frac{T}{M} \Rightarrow$$

$$\Rightarrow \theta = 26^\circ 33'$$

* **Strain Energy stored by shafts :- (U)**

Solid Shaft

$$U = \frac{\tau^2}{4C} \times \text{volume}$$

$$= \frac{\tau^2}{4C} \times \frac{\pi d^2}{4} \times l$$

hollow shaft

$$U = \frac{\tau^2}{4CD_0^2}(D_0^2 + D_i^2) \times \text{volume}$$

$$= \frac{\tau^2}{4CD_0^2}(D_0^2 + D_i^2) \times \left(\frac{\pi(D_0^2 - D_i^2)}{4}\right) \times l$$

Q) Determine max. strain energy stored in a solid shaft of 10 cm dia. Length 1.25 m subjected to 50 N/mm2 shear stress. Take $C = 8 \times 10^4\ N/mm^2$

Sol given,

$d = 10\ cm = 100\ mm$

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$l = 1.25 \times 10^3$ mm.

$\tau = 50 \, N/mm^2$.

$C = 8 \times 10^4 \, N/mm^2$

$$U = \frac{\tau^2}{4C} \times \frac{\pi d^2}{4} \times l$$

$$= \frac{(50)^2 \times \pi (100)^2 \times 1.25 \times 10^3}{4 \times 4 \times \cancel{\epsilon}\, 8 \times 10^4}$$

$= 76699.03$ N-mm.

Q) A hollow shaft 40 cm outer dia, 20 cm inner dia, having length 5 m subjected to $\tau = 50 \, N/mm^2$. Take $C = 8 \times 10^4 \, N/mm^2$. Determine U

Sol

$$U = \frac{\tau^2}{4 C D_o^2} (D_o^2 + D_i^2) \times \frac{\pi (D_o^2 - D_i^2)}{4} \times l.$$

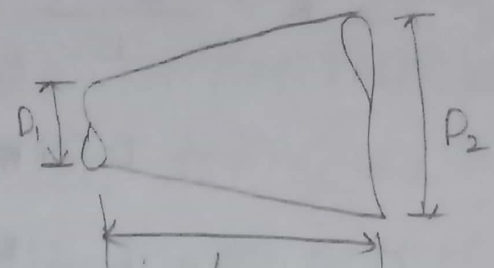
$$= \frac{(50)^2 \times (400^2 + 200^2) \times \pi (400^2 - 200^2) \times 5 \times 10^3}{4 \times 8 \times 10^4 \times (40)^2 \times 4}$$

$= 4.6 \times 10^6$ N-mm

(GATE) not in syllabus

* Tapering Shaft :-

angle of twist



$$\theta = \frac{32T}{\pi C} \times \frac{1}{3K}\left\{\frac{1}{D_1^3} - \frac{1}{D_2^3}\right\}$$

where $K = \dfrac{D_2 - D_1}{L}$

$$\therefore \quad T = \tau \times Z_p = \frac{\tau \times \pi \times D_1^3}{16}$$

Q) Determine angle of twist and shear stress developed in a shaft which tapers uniformly from 160mm to 240mm having length 2m subjected to torque of 48 KN-m. Take $C = 80\ GN/m^2$

Sol Given,

$D_1 = 160mm$

$D_2 = 240mm$

$L = 2m = 2\times10^3\ mm$

$T = 48KN\text{-}m = 48\times10^6\ N\text{-}mm$

$$\therefore \quad \theta = \frac{32T}{\pi C} \times \frac{1}{3K}\left\{\frac{1}{D_1^3} - \frac{1}{D_2^3}\right\}$$

$C = 80\ GN/m^2$

$= 80\times10^9\ N / (10^3)^2\ mm^2$

$C = 80\times10^3\ N/mm^2$

$$k = \frac{D_2 - D_1}{L} = \frac{240-160}{2000}$$

$$= 0.04$$

$$\theta = \frac{32 \times 48 \times 10^3}{\pi \times 80 \times 10^3} \times \frac{1}{3 \times 0.04} \left\{ \frac{1}{(160)^3} - \frac{1}{(240)^3} \right\}$$

$$\theta = 0.0087 \text{ radians}$$

$$= 0.0087 \times \frac{180}{\pi} = 0^\circ 30' 5''$$

$$T = \tau \times z_p$$

$$\tau = \frac{T}{z_p}$$

$$\tau = \frac{48 \times 10^6}{\frac{\pi \times D_1^3}{16}}$$

$$\tau = \frac{48 \times 16 \times 10^6}{\pi \times D_1^3} \rightarrow (160)^3$$

$$\tau = 59.683 \text{ N/mm}^2$$

* **SPRINGS** :- Springs are elastic bodies having extension ~~or~~ or compression or twisting abilities when subjected to external force and they can be regain to their original shape after removing the loads. It is also termed as ~~reg~~. Resilient member.

→ **Classification of Springs**

SPRINGS (Resilient/elastic)

- Helical spring
  - Tension spring (closed-coil spring)
  - Compression spring (open-coiled spring)
  - Torsional spring
  - Spiral spring.
- leaf spring/Laminated spring/ Carriage spring/ Built-up spring

* **Helical Spring** :- It is made of wire coil in the form of Helix. Generally in circular, Square or rectangular in cross section

i, **Tension spring or closed coil spring** :- If the slope of the helix of the coil is so small that bending effect can be neglected or plane of helix is

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perpendicular to axis of coil. Then this type of helical spring is known as close coil helical spring. Due to axial load the section of wire is subjected to pure torsion & direct shear.

Let W - axial load

D - Mean dia of coil

d - dia of wire

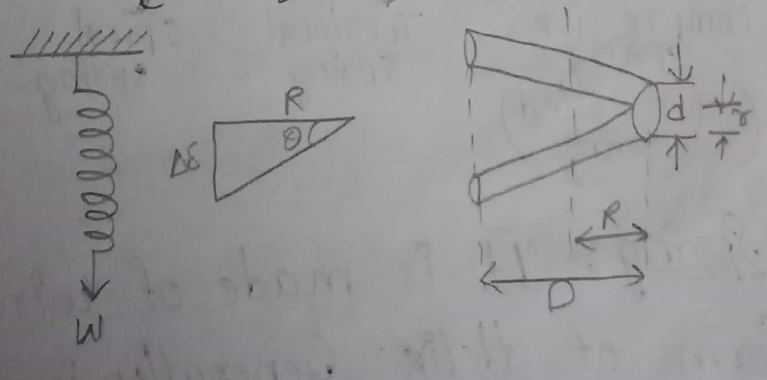
δ - deflection due to load

C - Shear modulus

θ - Angular deformation

n - No. of coils

τ - shear stress



Consider a cutting plane AA; Length of spring $L = 2\pi n R$; The moment produced $T = WR$; The deflection in the coil $\Delta\delta = R\theta$.

from the torsional equation

$$\underbrace{\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{L}}$$

$$\frac{T}{J} = \frac{C\theta}{L}$$

$$\Rightarrow \theta = \frac{TL}{CJ}$$

$\therefore$ Total deflection $\delta = \int R\,d\theta$

$$= R\theta$$

$$= R \cdot \frac{TL}{CJ}$$

$$\delta = \frac{R \times WR \times 2\pi n R}{C \times \frac{\pi d^4}{32}}$$

$$\boxed{\delta = \frac{64 WR^3 n}{Cd^4}}$$

* <u>Stiffness</u> of <u>spring</u> :- It is the ratio of load per deflection.

$$s = \frac{W}{\delta}$$

$$= \frac{W}{\frac{64 WR^3 n}{Cd^4}}$$

$$\boxed{s = \frac{Cd^4}{64 R^3 n}}$$

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Q) A closely coiled helical spring is to carry a load of 500N. The mean dia of coil ~~wire~~ is 10 times dia of the coil wire. Calculate diameters if max. shear stress is $80 N/mm^2$ & if stiffness is 20 N/mm. Find the no. of coils & $C = 8.6 \times 10^4 N/mm^2$

Sol Given,

$W = 500 N$

$D = 10d$

$\tau = 80 N/mm^2$

$S = 20 N/mm$

$C = 8.6 \times 10^4 N/mm^2$

$T = WR \quad = W \times \frac{D}{2}$

$T = \tau \times Z_p$

$= \tau \times \frac{\pi d^3}{16}$

$\tau = T/Z_p = \frac{W \times D/2}{\pi d^3/16}$

$\tau = \frac{W \times 10\cancel{d} \times \cancel{16}\,8}{\pi d^{\cancel{3}\,2} \times \cancel{2}}$

$\tau = \frac{W \times 10 \times 8}{\pi \times d^2}$

$d^2 = \frac{500 \times 10 \times 8}{\pi \times 80}$

$d = \cancel{12.619} \; 12.614 \text{ mm}$

$D = 10 \times d = 126.14 \text{ mm}$

$S = \frac{W}{\delta}$

$\delta = \frac{W}{S} = \frac{500}{20}$

$\delta = 25 \text{mm}.$

$\delta = \frac{64 WR^3 n}{Cd^4}$

$25 = \frac{64 \times 500 \times \left(\frac{126.7}{2}\right)^3 \times n}{8.6 \times 10^4 \times 12.6^4}$

$\therefore n = 6.7 \simeq 7$ coils.

Q) A closely coiled helical spring of steel wire 10mm dia having 10 complete turns with mean dia of coil 12cm subjected to axial load 200N. Determine deflection, shear stress & stiffness of spring. Take $C = 8 \times 10^4 \text{ N/mm}^2$.

Sol Given,

$d = 10\text{mm}.$

$n = 10$

$D = 12 \text{ cm} = 120\text{mm} ; R = 60\text{mm}$

$W = 200\text{N}.$

$\delta = \frac{64 WnR^3}{Cd^4}$

$\delta = \frac{64 \times 200 \times 10 \times (60)^3}{8 \times 10^4 \times (10)^4}$

$\delta = 34.56\,mm.$

$$\tau = T/Z_P$$

$$= \frac{W \times D/2}{\pi d^3/16}$$

$$\tau = \frac{200 \times 60 \times 16}{\pi \times (10)^3}$$

$$\tau = 61.11\,N/mm^2$$

$$S = \frac{W}{\delta} = \frac{200}{34.56}$$

$$= 5.78\,N/mm.$$

Q) The stiffness of a closed coil helical spring is 1.5 N/mm under 60 N load. If the shear stress 125 N/mm² & the length of the spring is 5cm. Find dia of coil wire & dia of the coil & no. of coils. Take $C = 4.5 \times 10^4\,N/mm^2$.

Sol

$S = 1.5\,N/mm.$

$W = 60\,N.$

$\tau = 125\,N/mm^2$

$L = nd = 5\,cm = 50\,mm$ (solid length)

$C = 4.5 \times 10^4\,N/mm^2$

$$S = \frac{Cd^4}{64R^3n} \rightarrow (1)$$

$$\tau = T/Z_p = \frac{16WR}{\pi d^3} \rightarrow ②$$

$$① \Rightarrow 1.5 = \frac{4.5 \times 10^4 \times d^4}{64 \times R^3 \times \frac{50}{d}}$$

$$R^3 = \frac{4.5 \times 10^4 \times d^5}{64 \times 1.5 \times 50}$$

$$R^3 = 9.375 \times d^5$$

$$R = 2.10 \times d^{5/3}$$

$$② \Rightarrow \tau = T/Z_p = \frac{16WR}{\pi d^3}$$

$$125 = \frac{16 \times 60 \times R}{\pi \times d^3}$$

$$125 = \frac{16 \times 60 \times 2.10 \times d^{5/3}}{\pi \times d^3}$$

$$125 = \frac{16 \times 60 \times 2.10 \times d^{4/3}}{\pi}$$

$$d^{4/3} = \frac{125\pi}{16 \times 60 \times 2.10}$$

$$d^{4/3} = 0.194$$

$$d = \cancel{2.085} \approx 3.4 \text{ mm}$$

$$R = 2.1 \times (3.4)^{5/3}$$

$$= 16.2 \text{ mm}$$

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$D = 16.2 \times 2 = 32.4 mm$

$n = \frac{50}{d} = \frac{50}{3.4} = 14.6 \simeq 15 no's$

Q) A closed coil helical spring is made of 6mm diameter wire. The coil dia-meter is 80mm. No. of coils is 10. The max. stress in the spring is not to exceed 180 MPa. Determine

(a) The proof load ; (b) Extension of spring

Take $C = 80 G.Pa.$

Sol

$\tau = 180 \times 10^6 \ N/m^2$

$= 180 \ N/mm^2.$

$C = 80 \ G.Pa = 80 \times 10^9 \ N/m^2$

$= 80 \times 10^9 \ N/(10^3)^2 mm^2$

$= 80 \times 10^9 \times 10^{-6} \ N/mm^2$

$= 80 \times 10^3 \ N/mm^2.$

Dia. of coil $D = 80mm.$

$R = \frac{D}{2} = \frac{80}{2} = 40mm$

Dia. of wire $d = 6mm.$

$\tau = 180 \ N/mm^2$

$C = 8 \times 10^3 \ N/mm^2$

$n = 10.$

$W, \delta = ?$

Shear stress, $\tau = T/Z_P$

$$= \frac{WR}{\frac{\pi d^3}{16}} = \frac{16WR}{\pi d^3}$$

$$180 = \frac{16 \times W \times 40}{\pi \times 6^3}$$

$$W = 190.8\ N.$$

Deflection, $\delta = \frac{64 WR^3 n}{C d^4}$

$$= \frac{64 \times 190.8 \times 40^3 \times 10}{80 \times 10^3 \times 6^4}$$

$$\delta = 75.37\ mm.$$

Q) A closed coil helical spring subjected to axial load 200N having 10 coils of wire diameter 18mm, made with coil diameter 20cm. Find max. deflection, max. shear stress & strain energy stored in the spring. Take $C = 80 G.N/m^2$

Sol Given, $W = 200N$.

$n = 10$.

$d = 18mm$; $D = 20cm$

$= 200mm$

$\Rightarrow R = 100mm$

$C = 80 \times 10^3 N/mm^2$

$$\delta = \frac{64 WR^3 n}{C d^4}$$

$$\delta = \frac{64 \times 200 \times (100)^3 \times 10}{80 \times 10^3 \times (18)^4}$$

$\delta = \cancel{15.2415\ 7}\ \delta = 15.24\ mm.$

Max shear stress

$$\Rightarrow \quad \tau = \frac{T}{Z_P}$$

$$= \frac{WR}{\frac{\pi d^3}{16}} = \frac{16WR}{\pi d^3}$$

$$\tau = \frac{16 \times 200 \times 100}{\pi \times (18)^3}$$

$$\tau = 17.46\ N/mm^2$$

Strain Energy $= \frac{1}{2} W.\delta$

$= \frac{1}{2} \times 200 \times (15.24)$

$= 100 \times (15.24)$

$= 1524\ N\text{-}mm.$

Q) A closed coil helical spring has mean diameter 80mm has spring constant 100 kN/m. It has 10 coils. Maximum shear stress 200M. What is the diameter of coil wire & what is the max. load the spring can carry. Take C or G or N = 80 G.Pa.

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Sol given, D = 80mm

spring constant = Stiffness

$s = 100 kN/m$

$s = 100 N/mm$

$n = 10$

$\tau = 200 MPa$

$= 200 \times 10^6 N/m^2$

$= 200 N/mm^2$

$C = 80 \times 10^3 N/mm^2$

$$\cancel{\tau = \frac{T}{Z_p} = \frac{16WR}{\pi d^3}}$$

$$\cancel{d^3 = \frac{16WR}{\pi \tau}}$$

$$s = \frac{Cd^4}{64 nR^3}$$

$$100 = \frac{80 \times 10^3 \times d^4}{64 \times 10 \times (40)^3}$$

$$d^4 = \frac{100 \times 64 \times (40)^3 \times 10}{80 \times 10^3}$$

$d = 15mm$

$$\tau = \frac{T}{Z_p} = \frac{16WR}{\pi d^3}$$

$\cancel{200 = 16 \times W}$

$$W = \frac{\tau \times \pi \times d^3}{16 \times (40)} = 3313 N$$

$= 3.3 kN$

Q. A closely coiled helical spring has stiffness of 1 kN/m with a maximum load 50 N and max shear stress of 150 N/mm² 45 mm. find the soled length of spring, mean diameter coil the wire diameter radius and no of coils take C = 40 GPa

Sol: Given

$W = 50N$

$S = 1 kN/m$

$= \frac{1 \times 10^3 N}{1000 mm}$

$= 1 N/mm$

$nd = 45 mm$

$n = \frac{45}{d}$

$C = 40 \times 10^3 N/mm^2$

$\tau = 150 N/mm^2$

$S = \frac{Cd^4}{64R^3 n}$

$\tau = 3p \times \tau$

$\tau = T/3p$

$= \frac{16WR}{\pi d^3}$

$S = \frac{40 \times 10^3 \times d^5}{64 \times R^3 \times 45}$

$R^3 = 13.88 \times d^5$

$R = (13.88)^{1/3} \times d^{5/3}$

$R = 2.40 \times d^{5/3}$

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$$\tau = \frac{16WR}{\pi d^3}$$

$$\tau = \frac{16 \times 50 \times 2.40 \times d^{5/3}}{\pi \times d^3}$$

$$150 = \frac{16 \times 50 \times 2.40 \times d^{-4/3}}{\pi}$$

$$d^{-4/3} = \frac{150 \times \pi}{16 \times 50 \times 2.40}$$

$$\boxed{d = 2.86}$$

$$R = 2.40 \times (2.86)^{5/3}$$

$$\boxed{R = 13.82}$$

$$n = \frac{45}{d} = \frac{45}{2.86} = 15.73 \cong 16$$

$$\boxed{n = 16}$$

Q. A closed coil Helical Spring has to extend by 120mm under an axial force of 1200 N. If the mean coil radius = 40mm and max Shear Stress = 300Mpa. Find the wire dia, no of coils, and length of the Spring take C = 80 Gpa.

Sol Given, $\tau = 300$ M.Pa.

$\delta = 120$mm $\quad = 300 \times 10^6 N/m^2$

$= 300 N/mm^2$

$d = ?$ $\quad W = 1200N$

$n = ?$ $\quad C = 80 \times 10^3 N/mm^2$

$L = ?$ $\quad R = 40mm$

$$\Delta = \frac{64WR^3n}{Cd^4}$$

$$n = \frac{80 \times 10^3}{64 \times 1200 \times 40^3} \times d^4$$

$$n = 1.953 \times 10^{-3} \times (9.34)^4$$

$$n = 14.83 \cong 15 \quad \boxed{n = 15}$$

$$\tau = \frac{16WR}{\pi d^3}$$

$$\tau = \frac{16 \times 1200 \times 40}{\pi \times d^3}$$

$$\tau \times \pi \times d^3 = 16 \times 1200 \times 40$$

$$d^3 = \frac{16 \times 1200 \times 40}{\tau \times \pi}$$

$$d^3 = \frac{16 \times 1200 \times 40}{300 \times \pi}$$

$$d = 9.34 mm.$$

$$\boxed{d = 9.34 mm}$$

$$L = 2\pi nR$$

$$= 2 \times \pi \times n \times R$$

$$= 2 \times \pi \times 15 \times 40$$

$$L = 3.7 m.$$

8) A closed coil helical spring has to absorb 100 N-m energy when compressed to 10cm the coil diameter is 10 times the wire diameter. The no. of coils = 12. find the diameter of the wire. mean Radius

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and maximum shear stress. Take C = 80 Gpa

Sol Given, $C = 80 \times 10^3 \, N/mm^2$

$n = 12$

$D = 10d \; ; \; d = \frac{D}{10}$

$d = ? \; ; \; R = ? \; ; \; \tau = ?$

$U = 100 \, N\text{-}m$

$= 100 \times 10^3 \, N\text{-}mm$

$\delta = 10 \, cm = 100 \, mm$

$U = \frac{1}{2} \delta W$

$W = \frac{2U}{\delta}$

$= \frac{2 \times 100 \times 10^3}{100}$

$= 2 \times 10^3 \, N$

$= 2 \, kN$

$R = \frac{D}{2}$

$= \frac{5\cancel{10}d}{\cancel{2}}$

$\Delta = \frac{64 W R^3 n}{C d^4}$

$100 = \frac{64 \times 2000 \times R^3}{80 \times 10^3 \times (d^4)}$

$100 = \frac{64 \times 2000 \times (5d)^3}{80 \times 10^3 \times d^4}$

$100 \times 80 \times 10^3 \times d^{\cancel{4}} = 64 \times 2000 \times 5\cancel{d^3}$

$d = \frac{64 \times 2000 \times 5^3}{100 \times 80 \times 10^3}$

$\boxed{d = 24 \, mm}$

$D = 10 \times d$

$= 10 \times 24$

$\boxed{D = 240mm}$

$R = \frac{D}{2}$

$= \frac{240}{2} = 120mm.$

$\boxed{R = 120mm}$

$\tau = \frac{16WR}{\pi d^3}$

$= \frac{16 \times 2000 \times 120}{\pi \times (24)^3}$

$= 88.41 N/mm^2$

Q). Design a closely coiled helical spring which will deflect 100mm under a load of 600N. The radius of the coil 6 times the wire diameter the maximum stress not to exceed 8 MPa take C = 80 GPa.

Sol:

$\delta = 100mm$

$W = 600N.$

$R = 6d \Rightarrow d = \frac{R}{6}$

$C = 80 \times 10^3 N/mm^2$

$\tau = 80 N/mm^2$

$$\tau = \frac{16WR}{\pi d^3}$$

$$d^3 = \frac{16WR}{\tau \times \pi}$$

$$d^2 = \frac{16 \times 600 \times 6\cancel{d}}{80 \times \pi}$$

$$\boxed{d = 15.13\,mm}$$

$$R = 6 \times d$$

$$= 6 \times 15.13$$

$$\boxed{R = 90.82}$$

$$\Delta = \frac{64WR^3 n}{Cd^4}$$

$$100 = \frac{64 \times 600\,(90.82)^3 \times n}{80 \times 10^3 \times (15.13)^4}$$

$$n = 14.61 \approx 15$$

$n = 15$ coils

$$R = \frac{D}{2}$$

$$D = 2 \times 90.82$$

$$D = 181.66\,mm$$

Q. A closed coil helical spring has a stiffness 10 N/mm. Its length when fully compressed with adjacent coils touching each other 400mm. $C = 80$ Gpa.

(a) Determine the wire diameter and mean coil radius if their ratio is 0.02.

(b) If the Gap between any two adjacent coils is 2mm. What maximum load can be applied before the adjacent coil touch.

(c) What is the corresponding maximum shear stress in the spring.

Sol.

$$S = 10 N/mm$$

$$nd = 400mm$$

$$C = 80 \times 10^3 N/mm^2$$

$$d = ?$$

$$\frac{d}{R} = 0.2$$

$$R = \frac{d}{0.02} \quad \frac{d}{0.2} \quad R = 5d$$

$$S = \frac{Cd^4}{64R^3n}$$

$$= \frac{80 \times 10^3 \times d^4}{64\left(\frac{d}{0.02}\right)^3 \times \left(\frac{400}{d}\right)}$$

$$= \frac{80 \times 10^3 \times d^4}{64 \times \frac{d^3}{(0.02)^3} \times \frac{400}{d}}$$

$$= \frac{80 \times 10^3 \times d^4 \times 10^{...}}{64 \times d^2 \times 400}$$

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$$10 = \frac{80\times10^3 \times d^2 \times 8\times10^{-6}\left(\frac{1}{0.02}\right)^3}{64\times400}$$

$$10\times64\times400 = 80\times10^3\times d^2\times8\times10^{-6}\left(\frac{1}{0.02}\right)^3$$

$$d^2 = \frac{10\times64\times400\times\left(\frac{1}{0.02}\right)^3}{80\times10^3\times8\times10^{-6}}$$

$$d = 632.4\,mm$$

(b) Given $\frac{d}{R} = 0.02$

$$\frac{632.4}{R} = 0.02$$

$$R = \frac{632.4}{0.02}$$

$$\boxed{R = 31.6\,m}$$

Stiffness $S = \frac{Cd^4}{64R^3n}$

$$= \frac{80\times10^3\times d^4}{64\times(5d)^3\times\frac{400}{d}}$$

$d = 20mm$

$R = 5d$

$= 5\times20$

$= 100mm$

$n = \frac{400}{d}$

$n = \frac{400}{20}$

$n = 20$

Gap between no of coils = 20mm

Total deflection = $n\times2$

= 40mm

deflection $\Delta = \frac{gd^4}{64\hat{W}R^3 n} = \frac{64WR^3 n}{Cd^4}$

$= \frac{64 \times W \times 100^3 \times 20}{80 \times 10^3 \times (20)^4}$

$W = \underline{\underline{400N}}$.

(c) $\tau = \frac{16WR}{\pi d^3}$

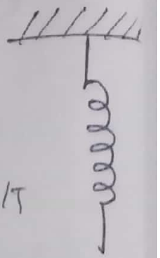
$= \frac{16 \times 400 \times 100}{\pi \times (20)^3}$

$= \underline{\underline{25.46 N/mm^2}}$

* Closed coiled helical spring subjected to axial couple :-

$l = 2\pi nR$.

angle of twist, $\phi = \frac{128 nMR}{Ed^4}$



Strain Energy, $U = \frac{M^2 l}{2EI} = \frac{1}{2} M \phi$ M/T

stress $= \frac{32M}{\pi d^3}$

$I = \frac{\pi d^4}{64}$

Q) A closely coiled helical spring subjected to an axial moment of 16 N-m. If the spring has 15 no's of coils with wire dia 10mm & Mean coil radius 60mm. Find strain Energy stored & maximum bending stress and twist if $E = 200$ GPa

Sol $M = 16\ N\text{-}m = 16 \times 10^3\ N\text{-}mm$

$n = 15$

$d = 10mm$

$R = 10\ cm = 100mm$

$E = 200 \times 10^3\ N/mm^2$

⇒ Angle of twist $(\phi) = \dfrac{128\, nMR}{E d^4}$

$$= \frac{128 (15)(16 \times 10^3) \times 100}{200 \times 10^3 \times (10^4)}$$

$\phi = 1.536$ Radians $= 1.5\left(\frac{180}{\pi}\right) = 88^\circ$

⇒ Strain Energy.

$$U = \frac{M^2 L}{2EI} = \frac{1}{2} \phi M$$

$$U = \frac{M^2 L}{2EI} = \frac{(16 \times 10^3)^2 \times (150)}{2 \times 200 \times 10^3 \times \frac{\pi}{64} (10)^4}$$

$l = 2\pi n R$

$= 2\pi \times 15 \times 100$

$= 9424.7mm$

$$= \frac{(16 \times 10^3)^2 \times (9424.7)}{2 \times 200 \times 10^3 \times \frac{\pi}{64} (10)^4}$$

$U = 12287.8\ N\text{-}mm$

$= 12.2\ kN\text{-}mm$

$U = \frac{1}{2} M\phi$

$= \frac{1}{2} \times 16 \times 10^3 \times 1.536$

$= 12288 = 12.28$

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* Open coil subjected to axial loads :-

$$\phi = \frac{64 WR^3 n \sin\alpha}{d^4}\left[\frac{1}{C} - \frac{2}{E}\right]$$

(or)

$$= \frac{64 MR^2 n \sin\alpha}{d^4}\left[\frac{1}{C} - \frac{2}{E}\right]$$

$$\Delta = \frac{64 WR^3 n \sec\alpha}{d^4}\left[\frac{2\sin^2\alpha}{E} + \frac{\cos^2\alpha}{C}\right]$$

$$U = \frac{W^2R^2 l}{2}\left[\frac{\sin^2\alpha}{EI} + \frac{\cos^2\alpha}{CJ}\right]$$

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$$\sigma_b = \frac{32 WR \sin\alpha}{\pi d^3}$$

$$\tau = \frac{16 WR \cos\alpha}{\pi d^3}.$$

$$\sigma_{1,2} = \frac{\sigma_b}{2} \pm \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$2 \tan\alpha = \frac{\sigma_b}{\tau}$$

Q) In an open coil spring of 10 coils subjected to bending stress 120 N/mm^2 & twisting 150 N/mm^2; When the spring is loaded axially. Assume mean radius of coil 5 times wire dia. Find maximum permeassible axial load & the dia of wire for maximum deflection 20mm. Take $E = 200\,GPa$ & $C = 80\,GPa$.

Sol Given,

$n = 10$ coils.

$\sigma_b = 120\ N/mm^2$

$R = 5d$

$E = 200\ G.Pa = 200 \times 10^3\ N/mm^2$

$C = 80\ G.Pa = 80 \times 10^3\ N/mm^2$.

$\tau = 150\ N/mm^2$

$$2\tan\alpha = \frac{\sigma_b}{\tau}$$

$$2\tan\alpha = \frac{120}{150}$$

$$\alpha = \tan^{-1} 0.4$$
$$= 21.80 \text{ rad.}$$

$$\Delta = \frac{64 W R^3 n \sec\alpha}{d^4}\left[\frac{2\sin^2\alpha}{E} + \frac{\cos^2\alpha}{C}\right]$$

$$20 = \frac{64 \times W \times (5d)^3 \times 10 \times \sec 21.80}{d^4}\left[\frac{2\sin^2 21.8}{200 \times 10^3} + \frac{\cos^2 21.8}{80 \times 10^3}\right]$$

$$20 = \frac{86{,}161.78\, W}{d}\left[(1.379 \times 10^{-6}) + 1.0776 \times 10^{-5}\right]$$

$$20 = \frac{1.046\, W}{d}$$

$$W = \frac{20}{1.046}\, d = 19.09\, d$$

$$\boxed{W = 19.09\, d} \rightarrow (1)$$

$$\sigma_b = \frac{32 W R \sin\alpha}{\pi d^3}$$

$$120 = \frac{32 \times W \times 5\cancel{d} \times \sin 21.80}{\pi d^{\cancel{3}\,2}}$$

$$120 = \frac{59.41 \times W}{\pi d^2}$$

$$\frac{W}{d^2} = \frac{120 \times \pi}{59.41}$$

$$\boxed{\frac{W}{d^2} = 6.34} \rightarrow (2)$$

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① ÷ ②

$$\frac{w}{d} \times \frac{d^2}{w} = \frac{19.09}{6.34}$$

$$d = 3.008$$

$$\therefore \sigma_b = \frac{32 WR \sin\alpha}{\pi d^3}$$

$$120 = \frac{32 \times W \times (5 \times 3) \times \sin 21.8}{\pi \times 3^3}$$

$$W = 57.10 \text{ N}$$

(or)

$$\tau = \frac{16 WR \cos\alpha}{\pi d^3}$$

$$150 = \frac{16 \times W \times (5 \times 3) \cos 21.8}{\pi \times 3^3}$$

$$W = 57.09 \text{ N}$$

Q) An open coil helical spring having 10 complete turns is made of 16mm dia steel rod having mean coil radius 50mm and α = angle of helix $\alpha = 28°$. Find deflection under 300N. Calculate direct shear stress & principle shear stress and maximum shear stress. Take $E = 200$ GPa, $C = 80$ GPa.

Sol

$E = 200\ G.Pa$
$= 200 \times 10^3\ N/mm^2$

$C = 80\ G.Pa$
$= 80 \times 10^3\ N/mm^2$

$n = 10$
$R = 50mm$
$d = 16mm$
$\alpha = 28^\circ$
$W = 300N$

$$\Delta = \frac{64 WR^3 n \sec\alpha}{d^4}\left[\frac{2\sin^2\alpha}{E} + \frac{\cos^2\alpha}{C}\right]$$

$$\Delta = \frac{64 \times 300 (50)^3 (10) \sec 28^\circ}{(16)^4}\left[\frac{2\sin^2 28^\circ}{200\times10^3} + \frac{\cos^2 28}{80\times10^3}\right]$$

$$= 414759.54\left[2.204\times10^{-6} + 1.073\times10^{-5}\right]$$

$\Delta = 8.958.$

Deflection shear stress

$$\sigma_b = \frac{32 WR \sin\alpha}{\pi d^3}$$

$$= \frac{32 \times 300 \times (50) \times \sin 28^\circ}{\pi \times (16)^3}$$

$= 17.51$

Principal shear stress

$$\sigma_{1,2} = \frac{\sigma_b}{2} \pm \sqrt{\frac{(\sigma_b)^2}{2} + \tau^2}$$

$$\sigma_1 = \frac{17.51}{2} \pm \sqrt{\frac{(17.51)^2}{2}}$$

$$\sigma_1 = \frac{\sigma_b}{2} + \sqrt{\frac{(\sigma_b)^2}{2} + \tau^2}$$

$$\sigma_2 = \frac{\sigma_b}{2} - \sqrt{\frac{(\sigma_b)^2}{2} + \tau^2}$$

$$\tau = \frac{16 WR \cos\alpha}{\pi d^3} = \frac{16 \times 300 \times 50 \times \cos 28^\circ}{\pi \times (16)^3}$$

$$\tau = 16.46 \ N/mm^2$$

Q) In an open coil helical spring having angle of helix 38°. If the inclination of the coil is ignored. Calculate % by which the axial extension is under estimated. E = 200 GPa, C = 80 GPa

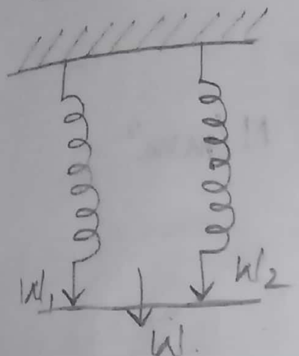
Sol Given,

$\alpha = 38^\circ$

$E = 200 \ GPa = 200 \times 10^3 \ N/mm^2$

* **Compound Springs** :- ~~Comb~~ If the springs are cast with two or more numbers is called Compound spring.

→ **Case ①** :- **Springs in parallel**



For this type of case the load carried by entire spring system is equal to the sum of the loads carried by two springs.

$$\therefore W = W_1 + W_2.$$

→ The deflection of the entire spring system is equal to deflections in each spring.

$$\Delta = \Delta_1 \text{ (or) } \Delta_2$$

→ The stiffness of the entire spring system is equal to sum of the stiffnesses carried by the each spring.

$$K = K_1 + K_2.$$

Q) Two close coiled helical springs are compressed b/w the 2 ||el plates by a load of 1000N. The spring have diame-

-ter of 10mm and radius of coils 50mm & 75 mm. Each spring has 10 coils & of same length. If the smaller spring is placed inside the larger one find deflection & stress in each spring. Take

$C = 40 GPa$

Sol Given,

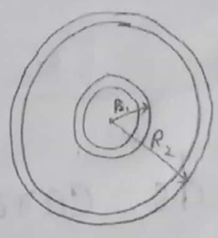
$C = 40 GPa = 40 \times 10^3 N/mm^2$

$n = 10$

$d = 10mm.$

$R_1 = 50mm$

$R_2 = 75mm.$



$W = 1000N$

$$\Delta_{50} = \Delta_{75}$$

$$\frac{\cancel{64} W_1 R_1^3 \cancel{n}}{\cancel{C} \cancel{d_1^4}} = \frac{\cancel{64} W_2 R_2^3 \cancel{n}}{\cancel{C} \cancel{d_2^4}}$$

$$W_1 R_1^3 = W_2 R_2^3$$

$$W_1 (50)^3 = W_2 (75)^3$$

$$W_1 = W_2 \times \frac{75^3}{50^3}$$

$$W_1 = 3.37 W_2$$

We know, $W = W_1 + W_2$

$$1000 = 3.37 W_2 + W_2$$

$$W_2 = \frac{1000}{3.37} = 228.57N$$

$$W_1 = W - W_2 = 1000 - 228.57$$

$$W_1 = 771.42\,N.$$

$$\Delta = \Delta_1 = \Delta_2$$

$$\Delta_1 = \frac{64 W_1 R_1^3 n}{C d_1^4} \; ; \; \Delta_2 = \frac{64 W_2 R_2^3 n}{C d_2^4}$$

$$\Delta_1 = \frac{64 \times 771.42 \times 50^3 \times 10}{40 \times 10^3 \times 10^4}$$

$$= 154.28$$

$$\Delta_2 = \frac{64 \times 228.57 \times 75^3 \times 10}{40 \times 10^3 \times 10^4}$$

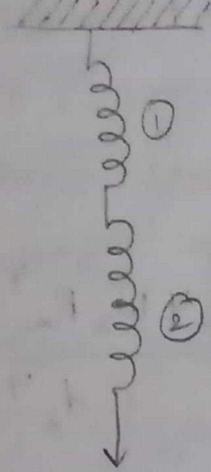
$$= 154.28.$$

$$\tau_1 = \frac{16 W_1 R_1}{\pi d_1^3} = \frac{16 \times \overset{771.42}{\cancel{1000}} \times 50}{\pi \times (10)^3} = \cancel{254.64}\ 196.44\,N/mm^2$$

$$\tau_2 = \frac{16 W_2 R_2}{\pi d_2^3} = \frac{16 \times \overset{228.57}{\cancel{1000}} \times 75}{\pi \times (10)^3} = \cancel{381.97}\ 87.30\,N/mm^2$$

→ **Case-II :- Springs in series :** For these type of springs the load carried by entire spring system is equal to load carried by each spring.

The deflection of the spring system is equal to the sum of the deflections of each spring.



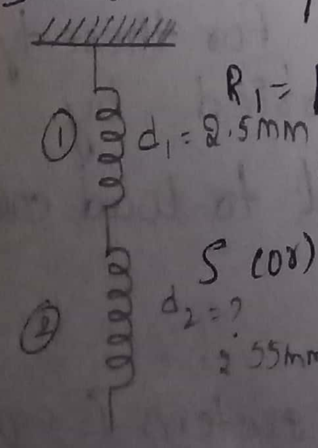
$$W = W_1 = W_2$$

$$\Delta = \Delta_1 + \Delta_2$$

$$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}$$

Q) A composite spring has 2 closed coiled helical springs connected in series. Each spring has 10 coils at a mean radius of 15mm. Find the dia of one spring if the other is 2.5mm. The stiffness for the entire spring is 750 N/m. Calculate the greatest load that the spring can be carried as composite spring & corresponding deflection if shear stress not to exceed 200 N/mm². Take C = 80 G.Pa.

Sol Given,



$n = 10.$

$R_1 = R_2 = 15mm.$

$d_1 = 2.5mm.$

S (or) $K = 750\ N/m = 0.750\ N/mm.$

$\tau = 200 N/mm^2$

$C = 80 G.Pa = 80 \times 10^3 N/mm^2$

$$\frac{1}{S} = \frac{1}{S_1} + \frac{1}{S_2}$$

$$S_1 = \frac{W_1}{\Delta_1} = \frac{W}{\frac{64WR_1^3n}{Cd_1^4}} = \frac{Cd_1^4}{64R_1^3n}$$

$$S_1 = \frac{80\times10^3 \times (2.5)^4}{64\times(15)^3\times10}$$

$$= 1.44 \text{ N/mm}$$

$$S_2 = \frac{W_2}{\Delta_2} = \frac{W}{\frac{64WR_2^3n}{Cd_2^4}} = \frac{Cd_2^4}{64R_2^3n}$$

$$S_2 = 0.037\, d_2^4 \text{ N/mm}$$

We know, $\frac{1}{S} = \frac{1}{S_1} + \frac{1}{S_2}$

$$\frac{1}{0.75} = \frac{1}{1.44} + \frac{1}{0.037\, d_2^4}$$

$$0.638 = \frac{1}{0.037 \cdot d_2^4}$$

$$d_2^4 = \frac{0.638}{0.037}$$

$$d_2 = 2.55 \text{ mm.}$$

∴ Shear stress, $\tau = \frac{16WR}{\pi d^3}$

$$W = \frac{200\times10^3\times\pi\times2.5^3}{16\times15}$$

$$W = 40.90 \text{N.}$$

$$S = \frac{W}{\Delta} \Rightarrow \Delta = \frac{W}{S} = \frac{40.90}{0.75}$$

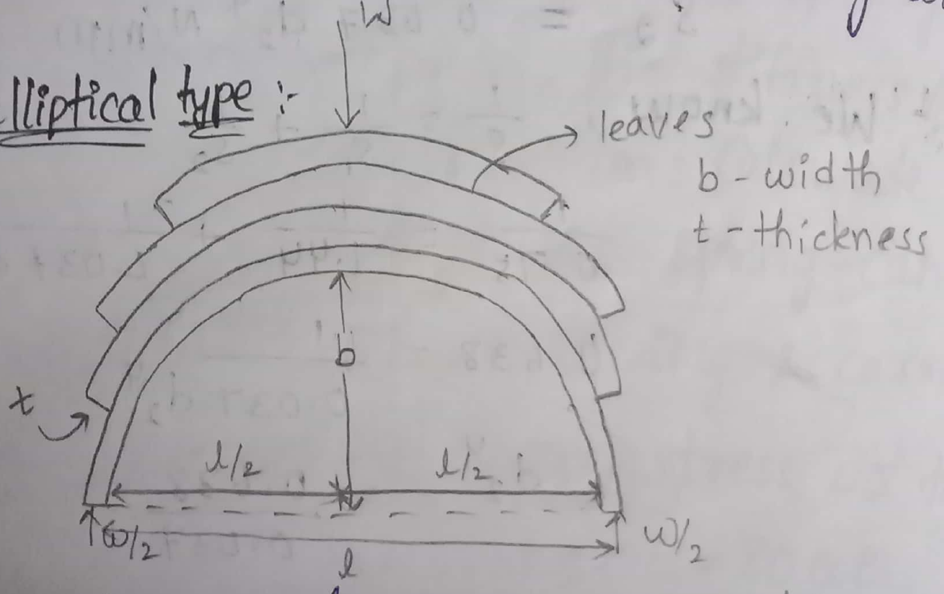
$$\Delta = 54.50 \text{mm}$$

* Leaf Springs:

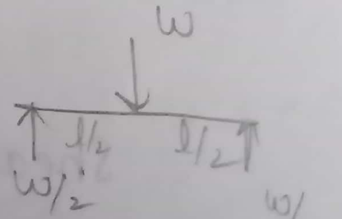
(Built-up (or) Carraige (or) Laminated :-)

→ These are made of no. of leaves of equal width & thickness but varying length. The no. of leaves decreases towards the end of the spring. The spring is designed such that the maximum bending stress is same at all sections. These are used in cars, lorries, Automobiles & railway wagons.

① semi-elliptical type :-



$$M = \frac{Wl}{4}$$



$$M = \frac{W}{2} \times \frac{l}{2} = \frac{Wl}{4}$$

$$\Delta = \frac{3Wl^3}{8nbt^3E}$$

$$s = \frac{W}{\Delta} = \frac{8Enbt^3}{3l^3}$$

$$\sigma = \frac{3Wl}{2nbt^2}$$

$$U = \frac{1}{2} W \Delta = \frac{3W^2 l^3}{16 n b t^3 E}$$

Q) A laminated steel spring simply supported at its ends & centrally loaded with a span of 0.8 m is required to carry a load 8 kN with deflection not to exceed 50 mm & stress not to exceed 400 N/mm². Determine, b, t & n. Take b = 12t. E = 200 G.Pa.

Sol Given, $E = 200 \times 10^3$ N/mm²

$l = 0.8\,m = 800\,mm$

$W = 8\,kN = 8 \times 10^3\,N$

$\Delta = 50\,mm$

$\sigma = 400\,N/mm^2$

$b = 12 \cdot t$

W.K.T $\Delta = \dfrac{3Wl^3}{8nbt^3E}$

$$50 = \frac{3 \times 8 \times 10^3 \times 800^3}{8n \times 12t \times t^3 \times 200 \times 10^3}$$

$$8 \times n \times 12 \times 200 \times 10^3 \times t^4 = \frac{3 \times 8 \times 10^3 \times 800^3}{50}$$

$$nt^4 = \cancel{0.02}\ 12800 \rightarrow (1)$$

$$\sigma = \frac{3Wl}{2nbt^2}$$

$$400 = \frac{3 \times 8 \times 10^3 \times 800}{2 \times n \times 12t \times t^2}$$

$nt^3 = 2000 \rightarrow (2)$

$(1) \div (2) \Rightarrow \frac{nt^4}{nt^3} = \frac{12800}{2000}$

$t = 6.4$

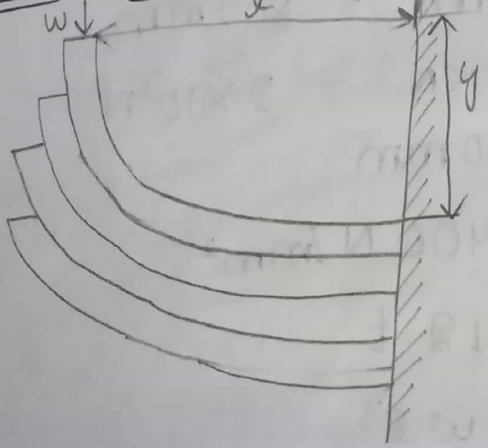
$b = 12 \times 6.4$

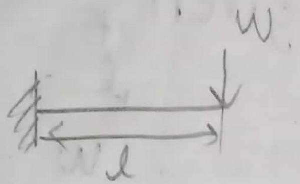
$b = 76.8$

$(1) \Rightarrow \quad n = \frac{12,800}{t^4} = \frac{12,800}{(6.4)^4}$

$n = 7.6 \simeq 8$ No's.

(2) quarter-elliptic type :-





$M = Wl.$

$$\Delta = \frac{6Wl^3}{nbt^3E}$$

$$S = \frac{W}{\Delta} = \frac{nbt^3E}{6l^3}$$

$$\sigma = \frac{6Wl}{nb^2}$$

$$U = \frac{1}{2}W\Delta = \frac{3W^2l^3}{nbt^3E}$$

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Q) A laminated spring of quarter elliptic type 0.6 m long is to provide a static 80mm deflection under an end load of 2000N Width 60mm, thickness 5mm, max. Find no. of leaves required & max stress if E = 200 GPa.

Sol:

$$\Delta = \frac{6Wl^3}{nbt^3E}$$

given,

$l = 0.6m = 600mm$

$\Delta = 80mm$

$W = 2000N$

$b = 60mm$

$t = 5mm.$

$E = 200 \times 10^3 \, N/mm^2$

$$n = \frac{6Wl^3}{\Delta bt^3E}$$

$$= \frac{6 \times 2000 \times (600)^3}{80 \times 60 \times (5)^3 \times 200 \times 10^3}$$

$n = 21.6$

$\simeq 22.$

$$\sigma = \frac{6Wl}{nb^2} = \frac{6 \times 2000 \times 600}{22 \times (60)^2}$$

$\sigma = 90.90. \, N/mm^2$

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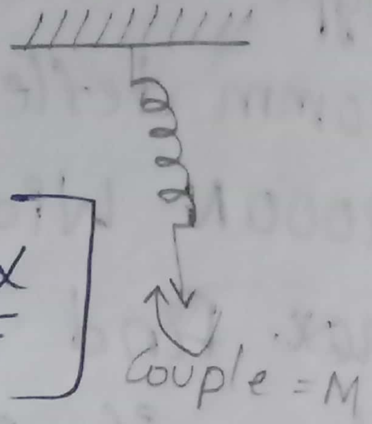
* Open coil helical spring subjected to axial couple :-



$$l = 2\pi n R \cos\alpha$$

$$\phi = T l \left[\frac{\cos^2\alpha}{EI} + \frac{\sin^2\alpha}{CJ} \right]$$

$$\phi = \frac{64 W R^3 n \sec\alpha}{d^4} \left[\frac{2\cos^2\alpha}{E} + \frac{\sin^2\alpha}{C} \right]$$

$$\sigma_b = \frac{32 T \cos\alpha}{\pi d^3}$$

$$\tau = \frac{16 T \sin\alpha}{\pi d^3}$$

$$U = \frac{T^2 l}{2} \left[\frac{\cos^2\alpha}{EI} + \frac{\sin^2\alpha}{CJ} \right]$$

UNIT - III

COLUMNS & STRUTS

* **Column** :- Column is a vertical member subjected to compressive loads which is rigidly fixed at its ends
  Ex:- A pillar b/w floor and roof.

* **Strut** :- Strut is a member which subjected to compressive loads provided in a truss.

* **Types of columns** :-

(1) Short column. - $l/D < 8$ & $\lambda < 32$.
(2) Medium column. - $l/D = 8$ to 30 & $\lambda = 32$ to 120
(3) Long column. - $l/D > 30$ & $\lambda > 120$.

Slenderness ratio $(\lambda) = \frac{l}{k}$

Least radius of gyration $k = \sqrt{\frac{I}{A}}$.

$I \rightarrow$ M.I.

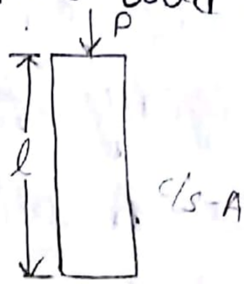
A — c/s area.

I_{xx} & I_{yy}

$I =$ Min of I_{xx} & I_{yy}.

Failure of Columns :-

1) **Failure of short column** :- Consider a column of length "l" & uniform cross sectional area A subjected to a compressive load of P.



The stress induced in column,

$$\sigma = \frac{P}{A}.$$

→ If P increases the stress will also increase and the column will fail at certain point of load.

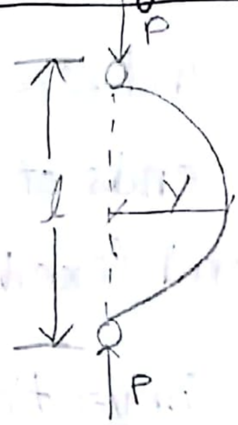
→ The point at which the column fails is failure point and the corresponding load is called Failure load or crushing load. And the corresponding stress is called crushing stress.

$$\therefore \sigma_c = \frac{P_c}{A}.$$

→ If $\sigma > \sigma_c$ the column will fail.

The short column fails by crushing when $\sigma > \sigma_c$ $\therefore$ for safety the σ is always less than σ_c.

(2) **Failure of long column:-** A long column of uniform cross section "A" & length "l" subjected to load P. Long column mainly fails by buckling/crippling load. The load at which ~~the~~ bending long column fails is known as buckling load/crippling/critical load.



direct stress, $\sigma_d = P/A$

$\therefore$ bending stress, $\sigma_b = \pm \frac{P \times Y}{Z}$

$\therefore$ Max. stress, $\sigma = \sigma_d + \sigma_b = \frac{P}{A} + \frac{Py}{Z}$

Min. stress, $\sigma = \sigma_d - \sigma_b = \frac{P}{A} - \frac{Py}{Z}$.

* **Euler's long column theory:-**

→ **Assumptions:-**

(1) The column initially perfect, straight and the load is applied axially

(2) Cross section of the column is uniform throughout.

(3) Column material is perfectly elastic, Homogeneous, isotropic & obeys Hooke's law.

(4) Length of the column is very large compared to its lateral dimension.

Direct stress is very small as compared
to buckling stress

The column will fail by buckling alone.

Self weight of the column is negligeble.

* End conditions for long Columns :-

1) Both ends of columns are hinged (pinned)

2) One end fixed, other end free

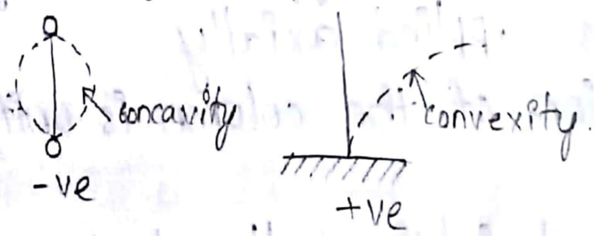
3) Both ends of column are fixed

4) One end fixed other end hinged

* Sign Conventions :-

(1) A moment which will bend the column wi-th its convexity towards its initial centre is taken as positive.

(2) A moment which will bend the column with its concavity towards its initial position is taken as negative.

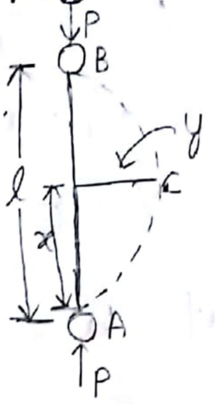


(1) Load Carrying capacity of long column with both ends hinged :- Consider a long column of length "l" having uniform

cross section throughout & subjected to buckling load "P."

→ Because of load application the column will bend to a curved portion ACB

→ Consider a point at a distance "x" from A & the corresponding deflection y.



∴ Moment at x = $-Py$.

But we know the moment at any distance $= EI\frac{d^2y}{dx^2} = 0$

$$\therefore EI\frac{d^2y}{dx^2} = -Py$$

$$EI\frac{d^2y}{dx^2} + Py = 0.$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}\cdot y = 0 \rightarrow (1)$$

$$y = C_1 \cos\sqrt{\frac{P}{EI}}\cdot x + C_2 \sin\sqrt{\frac{P}{EI}}\, x.$$

$$\left[\begin{array}{c} \frac{d^2y}{dx^2} + \alpha^2 y = 0. \\ y = C_1 \cos\alpha x + C_2 \sin\alpha x. \\ \alpha^2 = P/EI \\ \alpha = \sqrt{P/EI} \end{array}\right]$$

The hinged support will have 0 deflection

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Boundary condition.

$x = 0 \, ; \, Y = 0$

$$0 = C_1 \cos\sqrt{\frac{P}{EI}} \cdot 0 + C_2 \sin\sqrt{\frac{P}{EI}} \times 0$$

$$0 = C_1 \times 1 + 0$$

$$C_1 = 0$$

$$Y = 0 + C_2 \sin\sqrt{\frac{P}{EI}}\, x$$

at $x = l \; ; \; Y = 0$

$$0 = C_2 \sin\sqrt{\frac{P}{EI}} \times l$$

$C_2 = 0$ (or) $\sin l\sqrt{\frac{P}{EI}} = 0$

$C_2 \neq 0$ because If $C_2 = 0$ entire bending stress is zero.

Hence $\boxed{C_2 \neq 0}$

$$\therefore \sin l \cdot \sqrt{\frac{P}{EI}} = 0.$$

$$\sin l\sqrt{\frac{P}{EI}} = \sin 0, \sin \pi, \sin 2\pi, \sin 3\pi \ldots$$

Least value, $l \cdot \sqrt{\frac{P}{EI}} = \pi$

$$\boxed{P = \frac{\pi^2 EI}{l^2}}$$

where P = Load carrying capacity of Column

E = Young's modulus of column material.

I = Moment of Inertia of column section

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l = actual length of the column.

∴ Load carrying capacity of column ~~each~~ with both ends hinged is

$$\boxed{P = \frac{\pi^2 EI}{l^2}}$$

$$\frac{\frac{N}{mm^2} \times mm^4}{mm^2}$$

8) A column 3m long 5cm in diameter is used as a column with both ends hinged. Take $E = 2\times10^5\ N/mm^2$. Determine the crippling load carried by the column.

Sol Given,

length l = 3m = 3000mm.

diameter = 5cm = 50mm.

$E = 2\times10^5\ N/mm^2$.

$$I = \frac{\pi d^4}{64} = \frac{\pi (50)^4}{64}$$

$$I = 306,796.15\ mm^4$$

$$\therefore P = \frac{\pi^2 \times 2\times10^5 \times 306,796.15}{(3000)^2}$$

$$P = 67287.92\ N$$

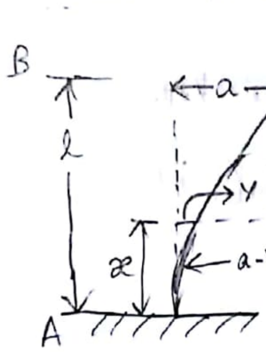
$$= 67.28792\times10^3\ N$$

$$= 67.28\ kN$$

* Load carrying capacity of long column with.

→ One end fixed other end free:- Consider a long column with one end fixed & other end free subjected to load "P" at a distance 'A':

Consider a point at a distance "x" with corresponding deflection "y" the moment at x.



∴ The moment @ x = $P\times(a-y)$

$$\therefore M = EI\cdot\frac{d^2y}{dx^2}$$

$$EI\cdot\frac{d^2y}{dx^2} = P(a-y)$$

$$EI\cdot\frac{d^2y}{dx^2} = Pa - Py$$

$$\Rightarrow \frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{P}{EI}\cdot a$$

$$\boxed{\begin{gathered}\frac{d^2y}{dx^2} + \alpha^2 y = \alpha^2 a \\ \alpha^2 = P/EI \Rightarrow \alpha = \sqrt{P/EI} \\ y = C_1\cos\alpha x + C_2\sin\alpha x + \frac{\cancel{\alpha^2} a}{\cancel{\alpha^2}}\end{gathered}}$$

∴ deflection

$$y = C_1\cos\sqrt{\frac{P}{EI}}\cdot x + C_2\sin\sqrt{\frac{P}{EI}}\,x + a$$

slope $\Rightarrow \frac{dy}{dx} = C_1\left(-\sin x\sqrt{\frac{P}{EI}}\cdot\sqrt{\frac{P}{EI}}\right) + C_2\cdot\cos x$

$\sqrt{\frac{P}{EI}}\cdot\sqrt{\frac{P}{EI}} + 0$

for fixed end, Boundary conditions $y=0$; $\frac{dy}{dx}=0$

① $x=0$; $y=0$.

$$0 = C_1\cos\sqrt{\frac{P}{EI}}\cdot 0 + C_2\sin\sqrt{\frac{P}{EI}}\cdot 0 + a$$

$$= C_1\times 1 + C_2\times 0 + a$$

$$\boxed{C_1 = -a}$$

② $x=0$; $\frac{dy}{dx}=0$

$$\therefore 0 = \left(-C_1\underbrace{\sin 0}_{0}\cdot\sqrt{\frac{P}{EI}}\cdot\sqrt{\frac{P}{EI}}\right) + C_2\cos 0\sqrt{\frac{P}{EI}}$$

$$\sqrt{\frac{P}{EI}} + 0$$

$$0 = C_2\times 1\times\sqrt{\frac{P}{EI}}$$

$C_2 = 0$ (or) $\sqrt{\frac{P}{EI}} = 0$

$\therefore C_2 = 0$;

$\therefore$ Final $y = -a\cos\sqrt{\frac{P}{EI}}\cdot x + a$

B.C ; if $x=l$, $y=a$.

$$\cancel{a} = -a\cos\sqrt{\frac{P}{EI}}\cdot l + \cancel{a}$$

$$a\cos\sqrt{\frac{P}{EI}}\cdot l = 0$$

if $a=0$ (or) $\cos l\cdot\sqrt{\frac{P}{EI}} = 0$

$$\cos l\cdot\sqrt{\frac{P}{EI}} = 0$$

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$$\cos l\sqrt{\frac{P}{EI}} = \cos\frac{\pi}{2}, \cos\frac{3\pi}{2}, \cos\frac{5\pi}{2}$$

Taking, least value

$$\cos l\sqrt{\frac{P}{EI}} = \cos\frac{\pi}{2}$$

$$l\sqrt{\frac{P}{EI}} = \frac{\pi}{2} \quad (\text{S.O.B.S})$$

$$\boxed{P = \frac{\pi^2 EI}{4l^2}}$$

8) A column 3m long 5cm in diameter is used as a column with one end fixed & other end free. Take $E = 2\times10^5\ N/mm^2$. Determine the crippling load carried by the column.

Sol Given,

$$l = 3000\ mm$$

$$d = 50\ mm$$

$$E = 2\times10^5\ N/mm^2$$

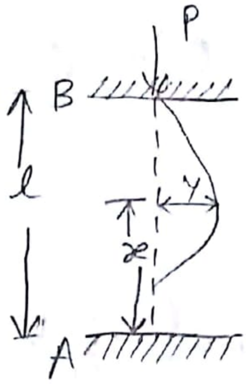
$$I = \frac{\pi d^4}{64} = \frac{\pi(50)^4}{64} = 306.79\times10^3$$

$$P = \frac{\pi^2 EI}{4l^2}$$

$$= 16.82\ kN$$

→ **Both ends fixed** :- Consider a long column with both ends fixed subjected to compressive load P having uniform cross section through out. Consider a point at a distance x from A with corresponding deflection y.

∴ M_0 is fixed end moment @ fixed supports.



∴ Moment @ $x = M_0 - Py$

$$\therefore EI \cdot \frac{d^2y}{dx^2} = M_0 - Py$$

$$EI \cdot \frac{d^2y}{dx^2} + Py = M_0$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} \cdot y = \frac{M_0}{EI}$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} \cdot y = \frac{P}{P} \times \frac{M_0}{EI}$$

deflection ; $y = C_1 \cos\sqrt{\frac{P}{EI}} \cdot x + C_2 \sin\sqrt{\frac{P}{EI}} \cdot x + \frac{M_0}{P}$.

∴ Slope ; $\frac{dy}{dx} = C_1 \left[-\sin\sqrt{\frac{P}{EI}} \cdot x \sqrt{\frac{P}{EI}}\right] + C_2 \cos . x\sqrt{\frac{P}{EI}} \sqrt{\frac{P}{EI}} + 0$.

B.C ∋ $x = 0$; $\frac{dy}{dx} = 0$

Slope $0 = C_1 \left(-\sin 0 \sqrt{\frac{P}{EI}} \sqrt{\frac{P}{EI}}\right) + C_2 \cos 0 \sqrt{\frac{P}{EI}} \cdot \sqrt{\frac{P}{EI}}$

$C_2 \times 1 \times \frac{P}{EI} = 0$

$C_2 = 0$ (or) $\frac{P}{EI} = 0$

Put $x = 0$, $y = 0$

$\Rightarrow 0 = C_1 \cos 0 \sqrt{\frac{P}{EI}} + C_2 \sin 0 \sqrt{\frac{P}{EI}} + \frac{M_0}{P}$

$0 = C_1 \times 1 \times \sqrt{\frac{P}{EI}} + \frac{M_0}{P}$

$C_1 = -\frac{M_0}{P}$

Final, $y = -\frac{M_0}{P} \cos \sqrt{\frac{P}{EI}}\, x + \frac{M_0}{P}$.

$x = l$, $y = 0$

$0 = -\frac{M_0}{P} \cos l \cdot \sqrt{\frac{P}{EI}} + \frac{M_0}{P}$.

$\frac{M_0}{P} \cos l \cdot \sqrt{\frac{P}{EI}} = \frac{M_0}{P}$

$\cos l \sqrt{\frac{P}{EI}} = \frac{\cancel{M_0}}{\cancel{P}} \times \frac{\cancel{P}}{\cancel{M_0}}$

$\cos l \cdot \sqrt{\frac{P}{EI}} = 1$

$= \cos 0, \cos 2\pi, \cos 4\pi, \cos 6\pi, \ldots$

Least value

$\cos l \cdot \sqrt{\frac{P}{EI}} = \cos 2\pi$

$l \cdot \sqrt{\frac{P}{EI}} = 2\pi$ (S.O.B.S)

$$\boxed{P = \frac{4\pi^2 EI}{l^2}}$$

//

Q7 A column 3m long 5cm in diameter is used as a column with both ends fixed. Take $E = 2 \times 10^5 \, N/mm^2$. Determine the crippling load ~~carrying~~ carried by the column

Sol. Given,

$l = 3000 \, mm$

$d = 50 \, mm.$

$E = 2 \times 10^5 \, N/mm^2$

$$I = \frac{\pi d^2}{64} = 306.79 \times 10^3$$

$$P = \frac{4\pi^2 EI}{4l^2}$$

$=$

Q8) A column of timber section 15cm × 20 cm is 6m long with both ends fixed. If $E = 17.5 \, kN/mm^2$. Determine

i, Crippling load (P)

ii, Safe load with factor of safety as 3.

Sol Given,

$E = 17.5 \, kN/mm^2$ $\nearrow 17.5 \times 10^3 \, N/mm^2$

$l = 6 \, m = 6000 \, mm.$

$$I_{xx} = \frac{bd^3}{12} = \frac{(150)(200)^3}{12} = 10^8 \, mm^4$$

$$P = \frac{4\pi^2 EI}{L}$$

$$= \frac{4\pi^2 \times 17.5 \times 10^8}{(6000)^2}$$

$$= 1919.08 \; kN/mm^2$$

$$I_{YY} = \frac{DB^3}{12} = \frac{200 \times 150^3}{12} = 56.25 \times 10^6 \; mm^4$$

The column will fail about minimum moment of inertia of axis.

$\therefore I = I_{YY}$

i, $\therefore P = \frac{4\pi^2 E \cdot I_{min}}{l^2}$

$$= \frac{4\pi^2 \times 17.5 \times 10^3 \times 56.25 \times 10^6}{6000^2}$$

$$= 1079.4 \; kN$$

ii, Safe load / Working load.

$\therefore S.L = \frac{P}{F.S} = \frac{1079.4 \times 10^3}{3} = 359.82 \; kN$

Q) A hallow mild steel column 6 m long 4 Cm internal dia & 5mm thick is used as a column with both ends hinged. Find the i, Crippling load

ii, Safe load with factor of safety 3.

Take $E = 2 \times 10^5\ N/mm^2$.

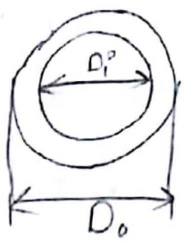
Sol Given, $E = 2 \times 10^5\ N/mm^2$

$l = 6m = 6000\,mm.$

$D_i \Rightarrow 4\ cms \Rightarrow 40mm$

$D_o \Rightarrow ?$

$t = 5mm$



$D_o = D_i + 2t$

$= 40 + 2(5)$

$\boxed{D_o = 50mm}$

$I_{min} \Rightarrow I_{xx} = I_{yy} \Rightarrow \frac{\pi}{64}(D_o^4 - D_i^4)$

$= \frac{\pi}{64}\left[50^4 - 40^4\right]$

$= 181.13 \times 10^3\ mm^4$

Both ends are hinged.

$\Rightarrow \frac{\pi^2 E I}{l^2} \Rightarrow \frac{\pi^2 \times (2\times10^5) \times (181.13\times10^3)}{(6000)^2}.$

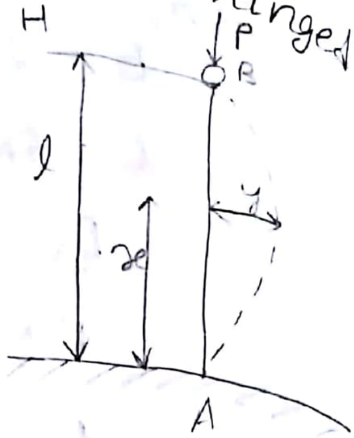
$P = 9.93kN.$

Safe load $= \frac{P}{FOS} = \frac{9.93}{3}$

$P_{SF} = 3.31\ kN$

→ Load carrying capacity of long column with one end fixed & other end hinged

Consider a long column with one end fixed and other end hinged of length "l" subjected to "P" of uniform cross section through out



Let M_0 be the fixed end moment at fixed support.

H is the horizontal reaction at hinged end.

Consider a point at distance "x" from "A" and corresponding deflection "y"

$$\text{Moment at "}x\text{"} = H(l-x) - P.y$$

$$EI\frac{d^2y}{dx^2} = H(l-x) - Py$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{H(l-x)}{EI} \times \frac{P}{P}$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}\cdot y = \frac{P}{EI}\cdot\frac{H}{P}(l-x)$$

Compare with $\frac{d^2y}{dx^2} + \alpha^2 y = \alpha^2 \cdot a$

$$\therefore \alpha^2 = \frac{P}{EI}$$

$$\alpha = \sqrt{\frac{P}{EI}}$$

$$a = \frac{H}{P}(l-x)$$

Solution for differential eqn is

$$y = C_1 \cos\alpha x + C_2 \sin\alpha x + a$$

$$\therefore y = C_1 \cos\sqrt{\frac{P}{EI}}\cdot x + C_2 \sin\sqrt{\frac{P}{EI}}\cdot x + \frac{H}{P}(l-x)$$

$$\therefore \theta = \frac{dy}{dx} = -C_1 \sin\sqrt{\frac{P}{EI}}\cdot x\cdot\sqrt{\frac{P}{EI}} + C_2 \cos\sqrt{\frac{P}{EI}}x \sqrt{\frac{P}{EI}} - \frac{H}{P}$$

Boundary conditions at "A"

$$x = 0 \Rightarrow y = 0$$

$$0 = C_1(1) + C_2(0) + \frac{H}{P}(l-0)$$

$$\Rightarrow C_1 = -\frac{Hl}{P}$$

$$x = 0;\ \frac{dy}{dx} = 0$$

$$0 = C_1(0) + C_2\sqrt{\frac{P}{EI}} - \frac{H}{P}$$

$$C_2 = \frac{H}{P}\sqrt{\frac{EI}{P}}$$

Boundary condition at "B"

$$\therefore x = l;\ \Rightarrow y = 0$$

$$0 = -\frac{Hl}{P}\cos\sqrt{\frac{P}{EI}}\cdot l + \frac{H}{P}\sqrt{\frac{EI}{P}}\sin\sqrt{\frac{P}{EI}}\cdot l + \cancel{\frac{Hl}{P}} - \cancel{\frac{Hl}{P}}$$

$$0 = -l\cos\sqrt{\frac{P}{EI}}\,l + \sqrt{\frac{EI}{P}}\sin\sqrt{\frac{P}{EI}}\cdot l$$

$$\sqrt{\frac{EI}{P}}\sin l\sqrt{\frac{P}{EI}} = l\cos l\sqrt{\frac{P}{EI}}$$

$$\tan l\sqrt{\frac{P}{EI}} = l\sqrt{\frac{P}{EI}}$$

$$l\cdot\sqrt{\frac{P}{EI}} = 4.5^2$$

$$P = \frac{4.5^2 \times EI}{l^2}$$

$$P = \frac{2\pi^2 EI}{l^2}$$

Q) A column 3m long, 5 cm dia is used as a column with both ends fixed $E = 2\times10^5$ N/mm². Determine the crippling load carried by column.

Sol

$l = 3m = 3000$ mm

$d = 50$ mm

$E = 2\times10^5$ N/mm²

$$I = \frac{\pi d^4}{64} = 306796.16$$

$$P = \frac{2\pi^2 EI}{l^2}$$

$$= \frac{2\pi^2 \times 2\times10^5 \times 306796.16}{(3000)^2}$$

$P = 134.57$ kN

* <u>Eulers crippling load</u> :- Euler's ~~given~~ gave the load carrying capacity of a long column

$$P = \frac{\pi^2 EI}{le^2}$$ for any end conditions.

le = effective length (or) Equivalent length

The condition for le & l are given in the table below

End condition	$P \to l$	$P \to l_e$	l_e & l relation
(1) Both ends hinged	$\frac{\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{l_e^2}$	$l_e = l$; x
(2) One end fixed other end free	$\frac{\pi^2 EI}{4l^2}$	$\frac{\pi^2 EI}{l_e^2}$	$l_e = 2l$, $\frac{x}{4}$
(3) Both ends fixed	$\frac{4\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{l_e^2}$	$l_e = \frac{l}{2}$; $4x$
(4) One end fixed other end hinged	$\frac{2\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{l_e^2}$	$l_e = \frac{l}{\sqrt{2}}$; $2x$

* Effective length :- The length of an equivalent column of same material and cross-section with both ends hinged having the value of crippling load is called effective length.

Q) Determine the crippling load for I-Section $40 \times 20 \times 1$ cm . 5m. long with , ends fixed Take $E = 2.1 \times 10^5$ N/mm^2.

Q) Calculate the crippling load for T- column flange with 10cm, depth 8cm both flange & web 10cm thickness with 3m long is built at both ends take $E = 2 \times 10^5$ N/mm^2.

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* **Rankine- Gordan Formula:-** The load carrying Capacity of any type of column can be determined by using Rankine- Gordan formula. is given by

$$\frac{1}{P_R} = \frac{1}{P_C} + \frac{1}{P_E}$$

$$\frac{1}{P_R} = \frac{P_C + P_E}{P_C P_E}$$

$$P_R = \frac{P_C P_E}{P_C + P_E}.$$

Where P_R = Rankine's crippling load

P_C - crushing load $\Rightarrow$ $P_C = \sigma_C \times A$

P_E = Euler's crippling load

$$P_E = \frac{\pi^2 E I}{l_e^2}$$

$$P_R = \frac{(P_C P_E) \times \frac{1}{P_E}}{(P_C + P_E) \times \frac{1}{P_E}}$$

$$P_R = \frac{P_C}{1 + \frac{P_C}{P_E}}$$

$$P_R = \frac{\sigma_C \times A}{1 + \frac{\sigma_C A l_e^2}{\pi^2 E I}}$$

Radius of gyration $K = \sqrt{\frac{I}{A}}$

$\Rightarrow I = AK^2$

$$P_R = \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c \cdot l_e^2 \cdot A}{\pi^2 E A K^2}}$$

$$P_R = \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c}{\pi^2 E}\left(\frac{l_e}{K}\right)^2}$$

Rankine's constant

$$a = \frac{\sigma_c}{\pi^2 E}$$

Q) Determine the Rankine's crippling load in the external & internal dia of column are 5cm & 4cm respectively with a length of 3m both ends fixed. Take $\sigma_c = 550\ N/mm^2$ & Rankine constant $a = \frac{1}{1600}$

Sol given, $D_o = 5cm = 50mm$

$D_i = 4cm = 40mm$

$l = 3m = 3000mm$

$l_e = \frac{l}{2} = \frac{3000}{2} = 1500$

$\sigma_e = 550\ N/mm^2$

$a = \frac{1}{1600}$

$$A = \frac{\pi(D_o^2 - D_i^2)}{4}$$

$$= \frac{\pi(50^2 - 40^2)}{4}$$

$A = 706.86 \, mm^2$

$$I = I_{xx} = I_{yy} = \frac{\pi (D_o^4 - D_i^4)}{64}$$

$$I = \frac{\pi (50^4 - 40^4)}{64}$$

$$I = 1.81 \times 10^5 \, mm^4$$

$$K = \sqrt{\frac{I}{A}} = 16$$

$$P_R = \frac{\sigma_c A}{1 + a \left(\frac{l_e}{k}\right)^2}$$

$$= \frac{550 \times 706.86}{1 + \frac{1}{1600}\left(\frac{1500}{16}\right)^2}$$

$$= \frac{550 \times 706.86}{1 + 5.493}$$

$$P_R = 59.875 \, kN$$

Q) the external & internal dia. of hollow column are 5 cm & 4 cm with a compressive load 240 kN when 2 m of column has tested with both ends fixed the load at failure was 158 kN

i, Find Rankine's Constant

ii, What will be the failure load if it is used as 3 m long one end fixed other end hinged.

Given, $D_o = 5\,cm = 50\,mm$

$D_i = 4\,cm = 40\,mm$

$l = 2000\,mm$

$P_c = 240\,kN = 240 \times 10^3\,N$

$P_R = 158\,kN = 158 \times 10^3\,N$

$$A = \frac{\pi(D_o^2 - D_i^2)}{4} = 706.86\,mm^2$$

$$I = \frac{\pi(D_o^4 - D_i^4)}{64} = 1.18 \times 10^5\,mm^4$$

$$\sigma_c = \frac{P_c}{A} = \frac{240 \times 10^3}{706.86} \qquad K = \sqrt{\frac{I}{A}} = 16$$

$$\sigma_c = 339.53\,N/mm^2$$

$$l_e = \frac{l}{2} = \frac{2000}{2} = 1000\,mm$$

$$P_R = \frac{\sigma_c \cdot A}{1 + a\left(\frac{l_e}{k}\right)^2}$$

$$158 \times 10^3 = \frac{339.53 \times 706.86}{1 + a\left(\frac{1000}{16}\right)^2}$$

$$0.658 = \frac{1}{1 + a \times 3906.25}$$

$$1 + a \times 3906.5 = \frac{1}{0.658}$$

$$a = 1.33 \times 10^{-4}$$

$$a = \frac{1}{7518.79} \rightarrow (1)$$

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One end fixed other end hinged.

$$l = 3000 m$$

$$l_e = \frac{l}{\sqrt{2}} = 2121.32 mm.$$

$$P_R = \frac{\sigma_c A}{1 + a\left(\frac{l_e}{k}\right)^2}$$

$$= \frac{339.53 \times 706.36}{1 + \frac{1}{7518.79}\left(\frac{2121.32}{16}\right)^2}$$

$$= \frac{339.53 \times 706.36}{1 + 2.338}$$

$$P_R = 71.899 kN$$

* **Limitations of Euler's Rankine's formula:**

Euler's & Rankine's formula gives only approximate values of crippling load due to following :-

(1) The pin joints not practically friction less

(2) The end supports never perfectly rigid.

(3) The euler's formula neglected the effect of direct compression.

(4) This is not suitable for slenderless ratio less 80 types of columns.

(5) the load is not exactly applied axially.

(6) Columns are never perfectly straight & uniform in section

(7) Materials aren't homogeneous & isotropic

* Straight line formula :-

$$P = \sigma_c \cdot A - n\left[\frac{l_e}{k}\right] \times A$$

2 Here σ_c = crushing stress

n = a constant depends on material of constant

A = Area

P = load

l_e = effective length

k = radius of gyration

$$P = \left\{\sigma_c - n\left[\frac{l_e}{k}\right]\right\} \times A$$

$$\frac{P}{A} = \sigma_c - n\left[\frac{l_e}{k}\right]$$

l_A is stress corresponding to Area A

* Johnson parabolic formula :-

$$P = \sigma_c A - \gamma\left[\frac{l_e}{k}\right] A$$

Here γ = Johnson constant

$$\gamma = \frac{\sigma_c}{4\pi^2 E^2}$$

* Proff perry's & roberton formula :-

Proff perry formula.

$$\sigma = \frac{1}{2}\left[f_y + \sigma_c(1+\theta) - \sqrt{f_y + \sigma_c(1+\theta)^2 - 4 f_y \sigma_c} \right]$$

$$\theta = \frac{W \cdot C}{K^2}$$

Robertson's formula.

$$\theta = 0.003\lambda.$$

λ = Slenderness ratio $= \frac{l}{k}$.

Here

f_y = yield stress of elastic limit

σ_e = Average tension corresponding to euler's load

W = Amplitude of initial geometrical imperfection.

C = distance of centroid section to the most stressed fibre

k = radius of gyration

* Indian Standard code formula (IS code) :-

The direct stress in compression of gross area of section of an axially loaded column shall not exceed the values of σ_c

The values of σ_c are calculated as follows

$$\sigma_c = \sigma_c' \times \frac{\sigma_y / m}{1 + 0.2 \sec\left[\frac{l_e}{k}\sqrt{\frac{MP_c'}{4E}}\right]}$$

here,

$$\frac{l_e}{k} = 0 \text{ to } 160$$

$$\sigma_c = \sigma_c' \times \left[1.2 \times \frac{l_e}{800k}\right]$$

here l_e = 160 & above.

here σ_c = allowable axial compressive stress obtained from table.

σ_c' = value obtained from IS formula

σ_y = Yeild stress.

m = factor of safety

$\frac{l_e}{k}$ = slenderness Ratio

E = Young's Modulus.

l_e/k	σ_c (N/mm^2)
250	16.6
300	19.9
350	7.6

Table:-

l_e/k	σ_c N/mm^2
0	1.25
10	124.6
20	123.9
30	122.4
40	120.3
50	117.2
60	113.0
70	107.5
80	100.7
90	92.8
100	84.0
110	75.3
120	67.1
130	59.7
140	53.1
150	47.4
160	42.3
170	37.3
180	33.6
190	30.0
200	27.0
210	24.3
220	21.9
230	10.9
240	18.1

Q) Determine the safe load by IS code for hollow column which external dia 4cm & internal dia 3 cm. When the length of column is 2.5cm with both ends hinged & other conditions.

Sol Given

i, Both ends hinged

$D_0 = 4\ cm = 40mm$

$D_i = 3\ cm = 30mm$

$l = 2.5\ cm = 2500\ mm$

$l_e = l = 2500mm$

$$A = \frac{\pi(D_0^2 - D_i^2)}{4} = \frac{\pi(40^2 - 30^2)}{4}$$

$A = 549.78\ mm^2$

$$I = \frac{\pi(D_0^4 - D_i^4)}{64} = \frac{\pi(40^4 - 30^4)}{4}$$

$I = 85902.92\ mm^4$

$$k = \sqrt{\frac{I}{A}} \Rightarrow k = 12.5$$

$$\frac{l_e}{k} = \frac{2500}{12.5} = 200$$

From IS code table

For $\frac{l_c}{k} = 200$

$\Rightarrow \sigma_c = 27$

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$$\text{Safe load} = \frac{\sigma_c}{A} = \frac{27}{549.78}$$

$$\text{Safe load} = 0.049\,N$$

(2) One end fixed, other end free

$D_o = 40mm$

$D_i = 30\,mm$

$l_e = 2l = 5000$

$A = 549.78\,mm^2$

$I = 85902.92\ mm^4$

$k = 12.5$

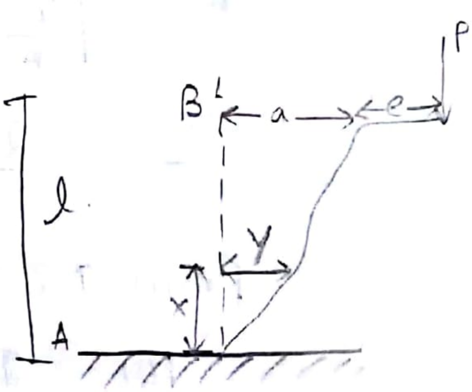
$\frac{l_e}{k} = 400$

1

$\Rightarrow \sigma_c =$

* Eccentricity loaded column :- Consider a column AB of length L which is fixed at one end & free at another end Subjected to compressive load P with an eccentricity of "e".

If a is the deflection at free end Consider a section at a distance x from "A" with corresponding deflection y.



Moment at $x = P_x (a+e-y)$

$$M = \frac{d^2y}{dx^2} EI$$

$$EI \cdot \frac{d^2y}{dx^2} = P\cancel{x}(a+e-y)$$

$$= P\cancel{x}(a+e) - P \cdot y$$

$$EI \cdot \frac{d^2y}{dx^2} + P \cdot y = P(a+e)$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} y = \frac{P}{EI}(a+e)$$

$$\frac{d^2y}{dx^2} + \alpha^2 y = \alpha^2 a \qquad (2)$$

$$\alpha^2 = \frac{P}{EI}$$

$$\alpha = \sqrt{\frac{P}{EI}}$$

$$y = C_1 \cos[\alpha \cdot x] + C_2 \sin[\alpha \cdot x] + \frac{\cancel{\alpha^2} a}{\cancel{\alpha^2}}$$

$\therefore$ Solution.

$$y = C_1 \cos\left[x\sqrt{\frac{P}{EI}}\right] + C_2 \sin\left[x\sqrt{\frac{P}{EI}}\right] + (a+e)$$

$$\frac{dy}{dx} = -C_1 \sin\left[x\sqrt{\frac{P}{EI}}\right] \cdot \sqrt{\frac{P}{EI}} + C_2 \cos\left[x\sqrt{\frac{P}{EI}}\right] \cdot \sqrt{\frac{P}{EI}} + 0.$$

Boundary conditions at A;

$x = 0$; $y = 0$

$$0 = C_1 \cos\left[0\sqrt{\frac{P}{EI}}\right] + C_2 \sin\left[0 \cdot \sqrt{\frac{P}{EI}}\right] + (a+e)$$

$$= C_1 \times 1 + C_2 \times 0 + (a+e)$$

$$e_1 = -(a+e)$$

$$x = 0 ; \quad \frac{dy}{dx} = 0$$

$$0 = -c_1 \sin\left[0\sqrt{\frac{P}{EI}}\right]\sqrt{\frac{P}{EI}} + c_2 \cos 0\left[0\cdot\sqrt{\frac{P}{EI}}\right]\cdot\sqrt{\frac{P}{EI}}$$

$$= -c_1 \times 0 + c_2 \times 1 \sqrt{\frac{P}{EI}}$$

$$0 = c_2\sqrt{\frac{P}{EI}}$$

If $c_2 = 0$ (or) $\sqrt{\frac{P}{EI}} = 0$

hence $c_2 = 0$

find $y = c_1 \cos\left[x\sqrt{\frac{P}{EI}}\right] + 0 + (a+e)$

$$y = -(a+e)\cdot\cos\left[x\sqrt{\frac{P}{EI}}\right] + (a+e)$$

at B $\quad x = L ; \quad y = a \qquad$ 3

$$a = -(a+e)\cos\left[l\sqrt{\frac{P}{EI}}\right] + (a+e)$$

$$(a+e)\cos\left[l\sqrt{\frac{P}{EI}}\right] = \cancel{a} + e - \cancel{a}$$

$$a+e = \frac{e}{\cos\left[l\sqrt{\frac{P}{EI}}\right]}$$

Max moment at $B = P \times (a+e)$

$$M_{max} = P \times e \sec\left[l\sqrt{\frac{P}{EI}}\right]$$

$$\sigma_{max} = \sigma_d + \sigma_b$$

$$= \frac{P}{A} + \frac{M}{Z}$$

$$= \frac{P}{A} + \frac{M \times y}{I}$$

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1) A circular column subjected to a load of 120 kN with an ecentricity 2.5mm. Outer dia 60mm, inner dia 50mm length 2.1m with both ends hinged take E = 200 gega N/m². Determine the max stress

Sol

$P = 120\ kN = 120 \times 10^3\ N$

$e = 2.5\ mm$

$D_o = 60\ mm;\quad D_i = 50\ mm$

$L = 2.1\ m = 210\ mm$

$E = 200\ G\ N/m^2$ (4)

$= 200 \times 10^3\ N/mm^2$

$$\sigma_{max} = \frac{P}{A} + \frac{P e \cdot \sec\left[\frac{l_e}{2}\sqrt{\frac{P}{EI}}\right]}{Z}$$

$A = \pi(D_o^2 - D_i^2)$

$= \frac{\pi(60^2 - 50^2)}{4}$

$= 863.93\ mm^2$

$I = \frac{\pi}{64}\left[D_o^4 - D_i^4\right]$

$= \frac{\pi}{64}\left[60^4 - 50^4\right]$

$= 3.29 \times 10^5\ mm^4$

$Z = \frac{I}{Y} = \frac{3.29 \times 10^5}{60/2}$

$= 10.97 \times 10^3\ mm^3$

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le for both ends hinged

$$\left[\frac{le}{2} \times \sqrt{\frac{P}{EI}}\right] = \left[\frac{2100}{2} \times \sqrt{\frac{120 \times 10^3}{200 \times 10^3 \times 3.29 \times 10^5}}\right]$$

$$= 1050 \sqrt{\frac{120000}{6.58 \times 10^{10}}}$$

$$= 1050 \times 1.35 \times 10^{-3}$$

$$= 1.417$$

$$= \sec\left[\frac{le}{2} \times \sqrt{\frac{P}{EI}} \text{ rad}\right]$$

$$= \sec\left[1.417 \times \frac{180}{\pi}\right]$$

$$= 6.56.$$

(5)

$$\sigma_{max} = \frac{120 \times 10^3}{863.93} + \frac{120 \times 10^3 \times 2.5 \times 6.56}{10.97 \times 10^3}$$

$$= 138.90 + 179.39$$

$$= 318.29 \text{ N/mm}^2.$$

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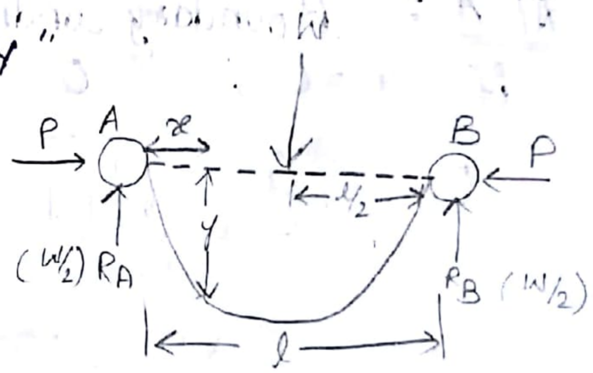
Struts (beam columns):- Columns carry axial compressive loads. If the column subjected to a transverse load then that column is called beam column (6)

1) Strut subjected to axial compressive load and a transverse load "W" acting at centre with both ends hinged:- Consider a column AB of length "l" subjected to axial compressive load "P" and it transfers load "W" acting at centre. Consider a section at a distance "x" from A & the corresponding deflection "y"



∴ Moment at x

$$= -Py - \frac{Wx}{2}$$

$$M = EI\frac{d^2y}{dx^2}$$

$$EI\frac{d^2y}{dx^2} = -Py - \frac{W}{2}x$$

$$EI\frac{d^2y}{dx^2} + Py = -\frac{W}{2}x$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = -\frac{W}{2}\cdot x\cdot\frac{1}{EI}$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = -\frac{W}{2}x\cdot\frac{1}{EI}\cdot\frac{P}{P}$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{P}{EI}\times\frac{-Wx}{2P}$$

* **Struts (beam columns):-**

$$\frac{d^2y}{dx^2} + \alpha^2 y = \alpha^2 a$$

$$\alpha^2 = P/EI$$

$$\alpha = \sqrt{P/EI}$$

$$y = C_1 \cos(\alpha x) + C_2 \sin \alpha x + \frac{\cancel{\alpha^2} a}{\cancel{\alpha^2}}$$

$$\Delta = y = C_1 \cos\left[\sqrt{P/EI}\, x\right] + C_2 \sin\left[\sqrt{P/EI}\, x\right] - \frac{Wx}{2P}$$

$$\theta = \frac{dy}{dx} = -C_1 \sin\left[x \sqrt{P/EI}\right]\sqrt{\frac{P}{EI}}$$

$$+ C_2 \cos\left[x \cdot \sqrt{P/EI}\right]\sqrt{\frac{P}{EI}} - \frac{W}{2P}$$

At A :- Boundary condition-1

if $x = 0$; $y = 0$ (i)

$$0 = C_1 \underbrace{\cos\left[0 \sqrt{\frac{P}{EI}}\right]}_{1} + C_2 \underbrace{\sin\left[0 \cdot \sqrt{\frac{P}{EI}}\right]}_{0} - \frac{W \times 0}{2P}$$

$$= C_1 \times 1 + C_2 \times 0 - 0$$

$$\boxed{C_1 = 0}$$

Boundary condition-2

if $x = l/2$; $\frac{dy}{dx} = 0$

$$0 = -0 \times \sin\left[\frac{l}{2} \sqrt{\frac{P}{EI}}\right]\sqrt{\frac{P}{EI}} + C_2 \cos\left[\frac{l}{2} \sqrt{\frac{P}{EI}}\right] \times \sqrt{\frac{P}{EI}} - \frac{W}{2P}$$

$$= 0 + C_2 \cos\left[\frac{l}{2} \sqrt{\frac{P}{EI}}\right]\sqrt{\frac{P}{EI}} - \frac{W}{2P}$$

$$C_2 = \frac{W}{2P} \times \sqrt{\frac{EI}{P}} \times \frac{1}{\cos\left[\frac{l}{2} \cdot \sqrt{\frac{P}{EI}}\right]}$$

$$Y = \frac{W}{2P} \times \sqrt{\frac{EI}{P}} \cdot \frac{1}{\cos\left[\frac{l}{2}\sqrt{\frac{P}{EI}}\right]} \sin\left[x\sqrt{\frac{P}{EI}}\right] - \frac{Wx}{2P}$$

Boundary condition-3.

if $x = l/2$; $Y = Y_{max}$.

$$Y_{max} = \frac{W}{2P} \times \sqrt{\frac{EI}{P}} \times \frac{1}{\cos\left[\frac{l}{2}\cdot\sqrt{\frac{P}{EI}}\right]} \sin\left[\frac{l}{2}\sqrt{\frac{P}{EI}}\right] - \frac{W\, l/2}{2P}$$

$$\boxed{Y_{max} = \frac{W}{2P} \times \sqrt{\frac{EI}{P}} \times \tan\left[\frac{l}{2}\cdot\sqrt{\frac{P}{EI}}\right] - \frac{Wl}{4P}}$$

Max Moment -

$$M = -PY - \frac{W}{2}\cdot x$$

$$M = -\left[PY + \frac{W}{2}\cdot x\right] \quad (8)$$

Boundary condition -4.

If $x = l/2$; $Y = Y_{max}$; $M = M_{max}$

$$M_{max} = -\left[P \times Y_{max} + \frac{W}{2}\cdot x\right]$$

$$= -\left[P \times \left\{\frac{W}{2P} \times \sqrt{\frac{EI}{P}} \times \tan\left[\frac{l}{2}\sqrt{\frac{P}{EI}}\right] - \frac{Wl}{4P}\right\} + \frac{W}{2} \times \frac{l}{2}\right]$$

$$= -\left[\cancel{P} \times \left\{\frac{W}{2\cancel{P}} \times \sqrt{\frac{EI}{P}} \times \tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) - \frac{Wl}{4\cancel{P}} + \frac{Wl}{4}\right]$$

$$= -\left[\frac{W}{2} \times \sqrt{\frac{EI}{P}} \times \tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) - \cancel{\frac{Wl}{4}} + \cancel{\frac{Wl}{4}}\right]$$

$$M_{max} = -\left[\frac{W}{2} \times \sqrt{\frac{EI}{P}} \times \tan\left[\frac{l}{2}\sqrt{\frac{P}{EI}}\right]\right]$$

(-ve hogging moment)

$$\boxed{M_{max} = \frac{W}{2} \times \sqrt{\frac{EI}{P}} \times \tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)}$$

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$$\sigma_{max} = \sigma_d + \sigma_b$$
$$= \frac{P}{A} + \frac{M}{Z}$$

***

$$\boxed{\sigma_{max} = \frac{P}{A} + \frac{M \cdot y}{I}} = \frac{P}{A} + \frac{My}{Ak^2}$$

Q) Determine max. stress induced in a strut of length 1.2m & diameter 30mm. The strut is hinged at both ends subjected to an axial compressive load 20kN & a transverse point load 1.8kN at centre. Take $E = 208\ GN/m^2$ (9)

sol Given, length = 1.2m = 1200mm
diameter = 30mm.
load P = 20kN. = 20×10^3N.
W = 1.8 kN = 1.8×10^3

$$A = \frac{\pi d^2}{4} = \frac{\pi(30)^2}{4} = 706.85\ mm^2$$

$$I = \frac{\pi d^4}{64} = \frac{\pi(30)^4}{64} = 39.76\times10^3\ mm^4$$

$$y = \frac{d}{2} = 15mm.$$

$$\tan\left[\frac{l}{2}\sqrt{\frac{P}{EI}}\right] = \tan\left(\frac{1200}{2} \times \sqrt{\frac{20\times10^3}{208\times10^3\times39.76\times10^3}}\right)$$

$$= \tan[0.93\ rad]$$
$$= \tan\left[0.93 \times \frac{180}{\pi}\right].$$
$$= \tan[53.46°]$$

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$= 1.34$

$$M_{max} = \frac{1.8 \times 10^3}{2} \times \sqrt{\frac{208 \times 10^3 \times 39.76 \times 10^3}{20 \times 10^3}} \times 1.34$$

$$M_{max} = 77.55 \times 10^4 \text{ N-mm}$$

$$\sigma_{max} = \frac{P}{A} + \frac{MY}{I}$$

$$\frac{P}{A} = \frac{20 \times 10^3}{706.85} = 28.29$$

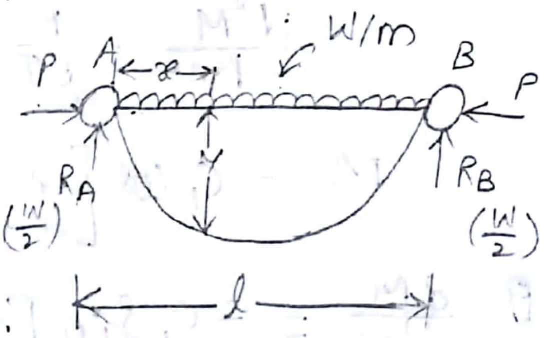
$$\frac{MY}{I} = \frac{77.55 \times 10^4 \times 15}{39.76 \times 10^3}$$

$$= 292.56$$

10

$$\sigma_{max} = 320.8 \text{ N/mm}^2$$

(2) **A Strut subjected to axial compressive load P & transverse load W/m over the entire span:-**



Consider a strut AB of length "l" subjected to Axial compressive load "P" & a transverse load W/m over the entire span. with both ends hinged.

Consider a section at a distance "x" from A & the corresponding deflection y

$$\text{Moment at } x = -PY - \frac{wl}{2} \cdot x + Wx \cdot \frac{x}{2}$$

$$M = -PY - \frac{wlx}{2} + \frac{Wx^2}{2}$$

$$\boxed{C_1 = \frac{-W \cdot EI}{P}}$$

B.C - (2)

$x = l/2$; $\frac{dM}{dx} = 0$

$$0 = -\left[\frac{-W EI}{P}\right] \sin\left[\frac{l}{2} \cdot \sqrt{\frac{P}{EI}}\right] \sqrt{\frac{P}{EI}} + C_2 \cos\left[\frac{l}{2}\sqrt{\frac{P}{EI}}\right] \times \sqrt{\frac{P}{EI}}.$$

$$= \frac{-\frac{W EI}{P} \cdot \sin\left[\frac{l}{2}\sqrt{\frac{P}{EI}}\right] \cdot \cancel{\sqrt{\frac{P}{EI}}}}{\cos\left[\frac{l}{2} \cdot \sqrt{\frac{P}{EI}}\right] \cancel{\sqrt{\frac{P}{EI}}}} = C_2$$

$$C_2 = -\frac{W \cdot EI}{P} \cdot \tan\left[\frac{l}{2} \cdot \sqrt{\frac{P}{EI}}\right] \quad (12)$$

Find M,

$$M = -\frac{W EI}{P} \cdot \cos\left[x \cdot \sqrt{\frac{P}{EI}}\right] - \frac{W \cdot EI}{P} \tan\left[\frac{l}{2} \cdot \sqrt{\frac{P}{EI}}\right] \cdot \sin\left[x \cdot \sqrt{\frac{P}{EI}}\right] + \frac{W EI}{P}.$$

$$M = -\frac{W EI}{P} \left\{ \cos\left[x\sqrt{\frac{P}{EI}}\right] + \tan\left[\frac{l}{2} \cdot \sqrt{\frac{P}{EI}}\right] \cdot \sin\left[x\sqrt{\frac{P}{EI}}\right] - 1\right]$$

B.C - (3)

$x = l/2$; $M = M_{max}$.

$$M_{max} = -\frac{W \cdot EI}{P}\left[\cos\left[l/2 \sqrt{\frac{P}{EI}}\right] + \frac{\sin\left[l/2 \sqrt{\frac{P}{EI}}\right]}{\cos\left[\frac{l}{2}\sqrt{\frac{P}{EI}}\right]} \sin\left[\frac{l}{2}\sqrt{\frac{P}{EI}}\right] - 1\right]$$

$$= -\frac{W \cdot EI}{P}\left[\frac{\cos^2\left[\frac{l}{2}\cdot\sqrt{P/EI}\right]+\sin^2\left[\frac{l}{2}\sqrt{P/EI}\right]}{\cos\left[\frac{l}{2}\cdot\sqrt{P/EI}\right]}\right.$$

$$M_{max} = -\frac{W \cdot EI}{P}\left[\frac{1}{\cos\left[\frac{l}{2}\cdot\sqrt{\frac{P}{EI}}\right]}-1\right]$$

[-ve sign indicates hogging]

$$\boxed{M_{max} = \frac{W \cdot EI}{P}\cdot\left[\frac{1}{\cos\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)}-1\right]}$$

$$M = -PY - \frac{Wl}{2}\cdot x + \frac{Wx^2}{2} \qquad (13)$$

$x = l/2$; $Y = Y_{max}$; $M = M_{max}$.

$$M_{max} = P \cdot Y_{max} - \frac{Wl}{2}\cdot\frac{l}{2} + \frac{W(l/2)^2}{2}$$

$$= -P \cdot Y_{max} - \frac{Wl^2}{4} + \frac{Wl^2}{8}$$

$$M_{max} = -PY_{max} - \frac{Wl^2}{8}$$

$$\frac{W \cdot EI}{P} \times \left[\frac{1}{\cos\left(\frac{l}{2}\cdot\sqrt{\frac{P}{EI}}\right)}-1\right] = -PY_{max} - \frac{Wl^2}{8}$$

$$Y_{max} = \left[\frac{W \cdot EI}{P}\left[\frac{1}{\cos\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)}-1\right]+\frac{Wl^2}{8}\right]\times\frac{1}{P}$$

$$\boxed{Y_{max} = \frac{W \cdot EI}{P^2}\left[\frac{1}{\cos\left[\frac{l}{2}\cdot\sqrt{P/EI}\right]}-1\right]-\frac{Wl^2}{8P}}$$

$$\sigma_{max} = \sigma_d + \sigma_b$$

$$= P/A + \frac{MY}{I}$$

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UNIT-4

Direct & Bending Stresses

30/1/18

(1) Combined direct and bending stress :-

* Consider the case of a column subjected to a compressive load 'P' acting along the axis of the column.
* This load will cause only direct stresses.

∴ Direct stress, $\sigma = \frac{P}{A}$

* Now consider the case of a column subjected by a compressive load P whose line of action at a distance e from the axis of the column.
* The eccentric load will cause direct and bending stress.

Direct stress, $\sigma = \frac{P}{A}$

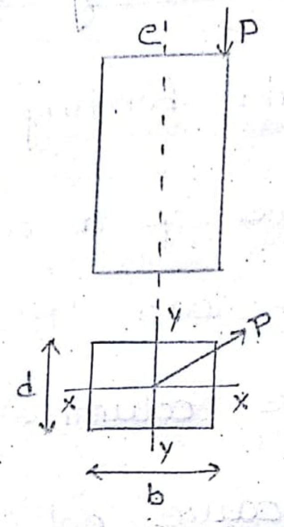
Bending stress, $\sigma = \frac{My}{I}$

Resultant stress when a column of rectangular section subjected to an eccentric load :

* A column of rectangular section

subjected to an eccentric load as

(2) shown in figure.



* Let the load is eccentric with respect to y-y axis.

Let P = Eccentric load on column.

e = Eccentricity of the load.

b = width of column.

d = Depth of column.

σ_0 = Direct stress

σ_b = Bending stress.

$\therefore$ Area of column, $A = b \times d$

M = Moment due to eccentric load

$M = P.e.$

$\therefore$ Direct stress, $\sigma_0 = \frac{P}{A} = \frac{P}{bd}.$

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∴ Bending stress, $\frac{M}{I} = \frac{\sigma_b}{y}$

(3) $$\sigma_b = \pm \frac{My}{I}$$

$$\sigma_b = \pm \frac{Pe.y}{\left(\frac{b^3 d}{12}\right)}$$

$I = \frac{db^3}{12}$ about y-y axis.

$$\sigma_b = \pm P.e. \frac{b}{2} . \frac{12}{b^3 d}$$

$$\sigma_b = \pm \frac{6Pe}{b^2 d}.$$

∴ Total stress, $\sigma = \sigma_0 + \sigma_b$.

$$\sigma = \frac{P}{bd} \pm \frac{6Pe}{b^2 d}$$

$$= \frac{P}{bd}\left(1 \pm \frac{6e}{b}\right)$$

$$= \frac{P}{A}\left(1 \pm \frac{6e}{b}\right)$$

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{6e}{b}\right).$$

$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right).$$

1/12/18

* A rectangular column of width 200 mm and depth 150 mm carries a load of 240 kN at an eccentricity of 100 mm. Determine the max & min stresses. (4)

Sol Given: Load, $P = 240$ KN

Width, $b = 200$ mm

Depth, $d = 150$ mm

Eccentricity, $e = 100$ mm

Area, $A = b \times d = 200 \times 150 = 30000 \text{ mm}^2$.

Max. stress, $\sigma_{max} = \frac{P}{A}\left(1 + \frac{6e}{d}\right)$

$$= \frac{240 \times 10^3}{30000}\left(1 + \frac{6 \times 100}{200}\right)$$

$$= 32 \text{ N/mm}^2$$

Min stress, $\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{d}\right)$

$$= \frac{240 \times 10^3}{30000}\left(1 - \frac{6 \times 100}{200}\right)$$

$$= -16 \text{ N/mm}^2$$

* If in the above problem the min. stress on the section is given zero then find the eccentricity of the point load of 240 KN acting on the rectangular section. Also calculate the corresponding

max. stress on the section.

Sol (5) Given: Load, $P = 240\ KN = 240 \times 10^3\ N$.

Width, $b = 200\ mm$

Depth, $d = 150\ mm$

$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right) = 0.$$

$$\frac{240 \times 10^3}{30000}\left(1 - \frac{6e}{200}\right) = 0.$$

$$\frac{6e}{200} = 1$$

$$e = 33.33\ mm$$

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{6e}{b}\right)$$

$$= \frac{240 \times 10^3}{30000}\left(1 + \frac{6 \times 33.33}{200}\right)$$

$$\sigma_{max} = 15.99\ N/mm^2.$$

* In the above problem, the eccentricity is given 50 mm the find max & min stress

Sol $$\sigma_{max} = \frac{P}{A}\left(1 + \frac{6e}{b}\right) = \frac{240 \times 10^3}{30000}\left(1 + \frac{6 \times 50}{200}\right)$$

$$= 20\ N/mm^2$$

$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right)$$

$$= \frac{240 \times 10^3}{30000}\left(1 - \frac{6 \times 50}{200}\right)$$

$$= -4\ N/mm^2$$

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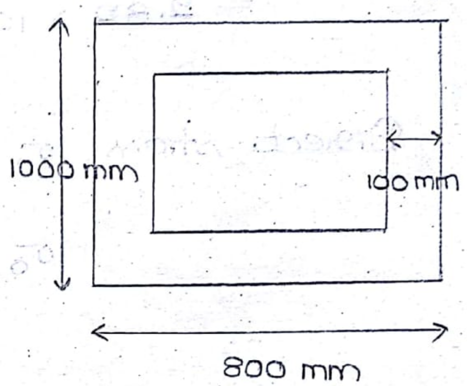
A hollow rectangular column of external depth 1 m and external width 0.8 m and thickness 10 cm. Calculate the max & min stresses in the section of the column. If the vertical load of 200 KN is acting with an eccentricity of 15 cm.

<u>Given:</u>

Load, $P = 200$ KN

$= 200 \times 10^3$ N.



Eccentricity i.e $= 15$ cm

$= 150$ mm

External Width, $B = 800$ mm

Internal width, $b = 800 - 200$

$= 600$ mm.

External depth, $D = 1000$ mm

Internal depth, $d = 1000 - 200$

$= 800$ mm

Area of hollow rectangle, $A = BD - bd$.

$A = (800 \times 1000) - (600 \times 800)$

$= 800000 - 480000$

$= 320000 = 32 \times 10^4 \text{ mm}^2$.

(8)

Moment of inertia, $I_{yy} = \frac{BD^3}{12} - \frac{bd^3}{12}$

$$I = \frac{800 \times 1000^3}{12} - \frac{600 \times 800^3}{12}$$

$$I_{xx} = \frac{B^3D}{12} - \frac{b^3d}{12}$$

$$= \frac{800^3 \times 1000}{12} - \frac{600^3 \times 800}{12}$$

$$= 2.82 \times 10^{10} \text{ mm}^4.$$

Direct stress, $\sigma_0 = \frac{P}{A}$

$$\sigma_0 = \frac{200 \times 10^3}{32 \times 10^4}$$

$$\sigma_0 = 0.625 \text{ N/mm}^2$$

Bending stress, $\sigma_b = \frac{My}{I} = \frac{Pey}{I} = \frac{Pe \cdot \frac{b}{2}}{I}$

$$= \frac{200 \times 10^3 \times 150 \times \frac{800}{2}}{2.82 \times 10^{9}}$$

$$\sigma_b = 0.425 \times \cancel{10^6} \text{ N/mm}^2$$

Max. stress = direct stress + bending stress

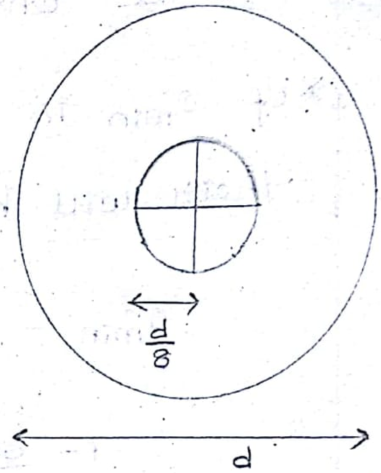
$$\sigma_{max} = 0.625 + 0.425$$

$$\sigma_{max} = 1.05 \text{ N/mm}^2.$$

Min stress = direct stress − bending stress

Middle quarter rule for circular section:

(11) * The cement concrete columns are weak in tension and hence the load must be applied on these columns in such a way that their is no tensile stress anywhere in the section.



* Consider a circular section of diameter 'd' as shown in figure.

* Let this section is subjected to the load which is eccentric to the y-y axis.

* ∴ Min stress, $\sigma_{min} = \sigma_0 - \sigma_b$

$$= \frac{P}{A} - \frac{My}{I}$$

$$= \frac{P}{A} - \frac{Pe \cdot \frac{d}{2}}{\frac{\pi d^4}{64}}$$

$$= \frac{P}{A} - \frac{32\,Pe}{\pi d^3}$$

$$= \frac{P}{A} - \frac{8\,Pe}{\left(\frac{\pi}{4} d^2\right) d}$$

$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{8\,Pe}{d}\right)$$

(12)

* If σ_{min} is negative, then the stresses will be tensile.

* If σ_{min} is zero (or) positive, then their will be no tensile stress.

$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{8e}{d}\right) \geq 0.$$

$$1 - \frac{8e}{d} \geq 0$$

$$1 \geq \frac{8e}{d}$$

$$e \leq \frac{d}{8}$$

* ∴ The eccentricity must be less than or equal to $\frac{d}{8}$.

* Hence the range within which the load can be applied so as not to produce tensile stress and is within the middle quarter of the base.

similarly

* If the load had been eccentric with respect to the x-x axis, the condition that the tensile stress will not occur is when the eccentricity of the load with respect to y-y axis does not

exceed. $\frac{d}{8}$

13 $$e \leq \frac{d}{8}$$

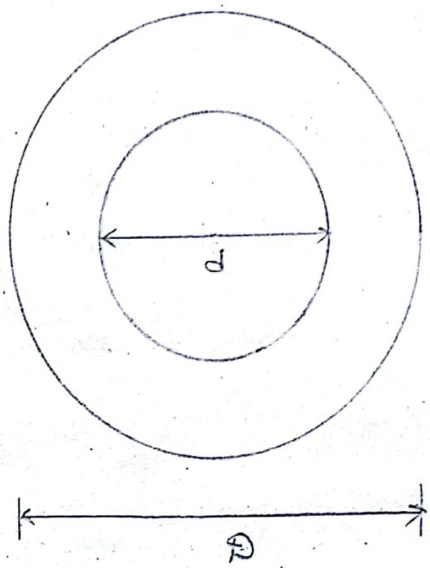
* **Kernel of hollow circular section:**

$$\sigma_{min} \geq 0$$

$$\sigma_0 - \sigma_b \geq 0$$

$$\sigma_b \leq \sigma_0$$

$$\frac{My}{I} \leq \frac{P}{A}$$



$$\frac{Pey}{I} \leq \frac{P}{A}$$

$$e \leq \frac{I}{Ay}$$

$$e \leq \frac{\pi (D^4 - d^4)}{64 \times \frac{\pi}{4}(D^2 - d^2) \times \frac{D}{2}}$$

$$e \leq \frac{(D^4 - d^2)}{8D(D^2 - d^2)}$$

$$e \leq \frac{(D^2 + d^2)\cancel{(D^2 - d^2)}}{8D\cancel{(D^2 - d^2)}}$$

$$e \leq \frac{D^2 + d^2}{8D}$$

* Draw neat sketches of kernel section for the following.

① rectangular section (200 mm x 300 mm)

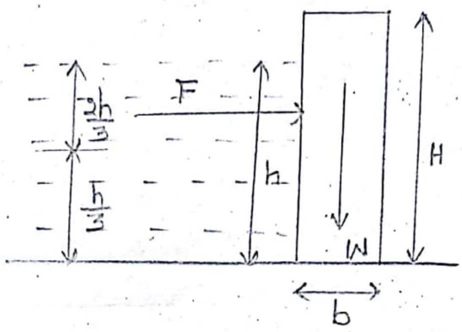
(ii) Hollow circular section.

3/2/18

15 **Dams and retaining walls:**

Types of dams:

① **Rectangular dam**



② ~~Trapezoidal dam~~:

* Let h = height of water. $\quad \omega = 9.81 \text{ kN/m}^3$

H = height of dam

b = width of dam.

F = force exerted by water.

W = weight of dam per meter length

ω = weight density of dam.

* Consider one meter length of the dam
* The forces acting on the dam

① Water pressure.

The force (F) due to water.

$F = \omega A \bar{h}$

$F = \omega (h \times 1) . \frac{h}{2}$

$F = \omega (h \times 1) \frac{h}{2}$

$$\boxed{F = \frac{wh^2}{2}}$$

$\omega = 9.81\ KN/m^3$

16 * The force (F) will be acting horizontally at a height of h/3 above the base as shown in figure.

* (ii) The weight (W) of the dam.

$W = \omega_c \times V$

$= \omega_c \times (b \times H \times 1)$

$$\boxed{W = \omega_c \times b \times H}$$

$\omega_D = \frac{W_D}{V_D}$

$\omega_c = \frac{W_D}{V_D}$

$\omega_c = 25\ KN/m^3$

* The weight (W) of the dam will be acting downwards through the C.G. of the dam.

* The resultant force may be determined by the parallelogram law.

$R = \sqrt{F^2 + W^2}$

and the angle made by the resultant with vertical.

$\tan\theta = \frac{F}{W}$

* The horizontal distance between the line of action of W and the point through which the resultant cuts the base equal to 'x'.

(17)

* Taking moments at N.

$$F \times \frac{h}{3} = W \times x$$

$$\boxed{x = \frac{F}{W} \times \frac{h}{3}}$$

5/2/18 * Distance, between A and N is

$$d = AM + MN$$

$$\boxed{d = \frac{b}{2} + x.}$$

Max. stress, $\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right)$

Min. stress, $\sigma_{min} = \frac{W}{b}\left(1 - \frac{6e}{b}\right)$

Eccentricity, $e = d - \frac{b}{2}$

* A masonry dam of rectangular section is 20 m height and 10 m wide, as water upto a height of 16 m. On its one side. Find.

(i) pressure force due to water.

(ii) Position of centre of pressure.

(iii) The point at which the resultant cuts the base.

(iv) The resultant

Take weight density of masonry = 19.62 KN and of water 9.81 KN/m³

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Sol (18) Given: Height of dam, $H = 20\ m$.

Width of dam, $b = 10\ m$

Heigth of water, $h = 16\ m$

Weight density of masonry, $\omega_m = 19.62\ \frac{KN}{m^3}$

Weight density of water, $\omega = 9.81\ \frac{KN}{m^3}$

(i) Pressure force due to water,

$$F = \frac{\omega h^2}{2} = \frac{9.81 \times 16^2 \times 1}{2}$$

$$F = 1255.68\ KN/m$$

$$F = 1255.68\ KN$$

(ii) Position of centre of pressure from base

$$= \frac{h}{3}$$

$$= \frac{16}{3}$$

$$= 5.33\ m$$

(iii) Let x = horizontal distance between line of action of W and resultant cuts the base.

$$x = \frac{F}{W} \times \frac{h}{3}$$

$$= \frac{1255.68}{\omega_m \times b \times H} \times \frac{16}{3}$$

$$= \frac{1255.68}{19.62 \times 10 \times 20} \times \frac{16}{3}$$

$x = 1.706$ m.

(A) (iv) Resultant, $R = \sqrt{F^2 + W^2}$

$$= \sqrt{(1255.68)^2 + (3924)^2}$$

$$R = 4120.013 \text{ KN.}$$

(v) Max. stress, $\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right)$

$$= \frac{3924}{10}\left(1 + \frac{6\left(d - \frac{b}{2}\right)}{b}\right)$$

$$= 392.4\left(1 + \frac{6(1.706)}{10}\right)$$

$$\sigma_{max.} = 794.06 \text{ KN/m}^2$$

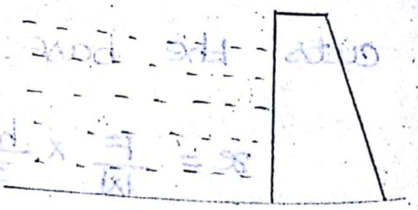
(vi) Min. stress, $\sigma_{min} = \frac{W}{b}\left(1 - \frac{6e}{b}\right)$

$$= \frac{3924}{10}\left(1 - \frac{6(1.706)}{10}\right)$$

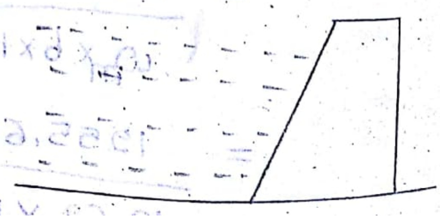
$$= -9.26 \text{ KN/m}^2 \text{ (tension)}$$

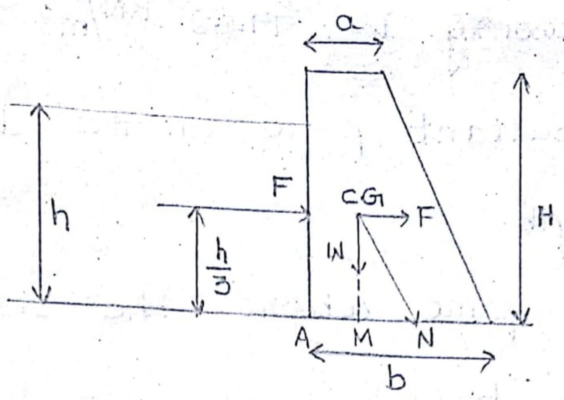
(2) Trapezoidal dam:

(i) Water face vertical.



(ii) Water face inclined.





(i) Force, $F = \frac{wh^2}{2}$.

(ii) Position, $= \frac{h}{3}$

(iii) Weight of dam, $W = w_c \times \frac{H}{2}(a+b) \times 1$

(iv) Centre of pressure, $C.G = AM$.

(v) Resultant, $R = \sqrt{F^2 + W^2}$.

(vi) Distance between A and N

$$d = AM + x$$

(vii) Resultant cuts the base, $x = \frac{F}{W} \cdot \frac{h}{3}$

(viii) Max. stress, $\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right)$

(ix) Min. stress, $\sigma_{min} = \frac{W}{b}\left(1 - \frac{6e}{b}\right)$

(x) Eccentricity, $e = d - \frac{b}{2}$.

* A trapezoidal masonry dam of 18m height The dam is having water upto a depth of 15 m on its vertical side. The top and bottom width of the dam are 4m and 8m respectively. Weight density

of masonry is 19.62 KN/m^3. Determine

(21) (i) Resultant force on the dam per unit length.

(ii) The point where the resultant cuts the base.

(iii) Max. and min. stresses at the base

Sol Given:

Height of the dam, $H = 18$ m

Height of water in dam, $h = 15$ m.

Width of dam at top, $a = 4$ m.

Width of dam at bottom, $b = 8$m

Weight density of masonry, $w_m = 19.62$ KN/m^3

(i) Resultant force, $R = \sqrt{F^2 + W^2}$

$$F = \frac{wh^2}{2} = \frac{9.81 \times 15^2}{2}$$

$$F = 1103.62 \text{ KN}$$

$$W = w_m \times b \times \frac{H}{2}(a+b) \times 1$$

$$= 19.62 \times \frac{18}{2}(4+8) \times 1$$

$$= 2118.96 \text{ KN}$$

$$R = \sqrt{(1103.62)^2 + (2118.96)^2}$$

$$R = 2389.13 \text{ KN}$$

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(ii) Point where the resultant cuts the base.

$$x = \frac{F}{W} \cdot \frac{h}{3}$$

$$x = \frac{1103.62}{2118.96} \times \frac{15}{3}$$

$$x = 2.604 \text{ m}$$

(iii) Max. stress, $\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right)$

$$e = d - \frac{b}{2}.$$

$$d = AM + x.$$

$$d = CG + 2.604.$$

$$AM = C.G = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

$$= \frac{(4 \times 18)\left(\frac{4}{2}\right) + \left(\frac{1}{2} \times 4 \times 18\right)\left(4 + \frac{1}{3} \cdot 4\right)}{(4 \times 18) + \left(\frac{1}{2} \times 4 \times 18\right)}$$

$$= \frac{144 + 192}{72 + 36}$$

$$= 3.11 \text{ m}$$

$$d = 3.11 + 2.604 = 5.714 \text{ m}.$$

$$e = d - \frac{b}{2} = 5.714 - \frac{8}{2}$$

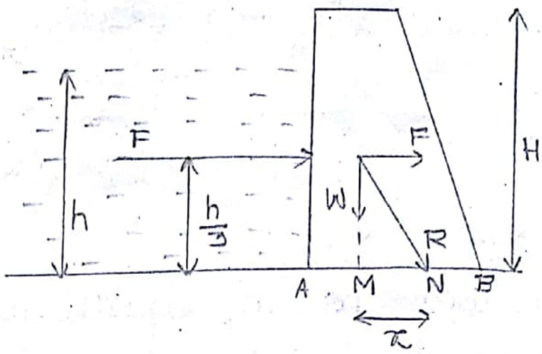
$$e = 1.714$$

$$\sigma_{max} = \frac{2118.96}{8}\left(1 + \frac{6(1.714)}{8}\right)$$

$$\sigma_{max} = 605.36 \text{ kN/m}^2$$

Condition to prevent overturning of the [dam]

(25)



* If the resultant 'R' of the weight of the dam on the horizontal force (F) strikes the base within its width. i.e., the point N lies within the base AB there will be no overturning of the dam.

* Moment due to horizontal force $= F \times \frac{h}{3}$

* Moment due to weight $= W \times MB$

* There will be no overturning about B if MB is greater than MN.

$$MB > MN.$$

Condition to avoid tension in dam:

* The concrete dam is weak in tension. hence the tension should be avoided.

$\therefore$ Minimum stress, $\sigma_{min} = \frac{W}{b}\left(1 - \frac{6e}{b}\right) \geq 0$

$$1 - \frac{6e}{b} \geq 0.$$

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$$\frac{6e}{b} \leq 1$$

$$e \leq \frac{b}{6}$$

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$$d - \frac{b}{2} \leq \frac{b}{6} \Rightarrow d \leq \frac{2b}{3}$$

Condition to avoid excessive compressive stresses :

* The condition to avoid excessive compressive stresses in masonry of the dam is that the P_{max} i.e., max. stress in the masonry should be less than the permissible stress in the masonry.

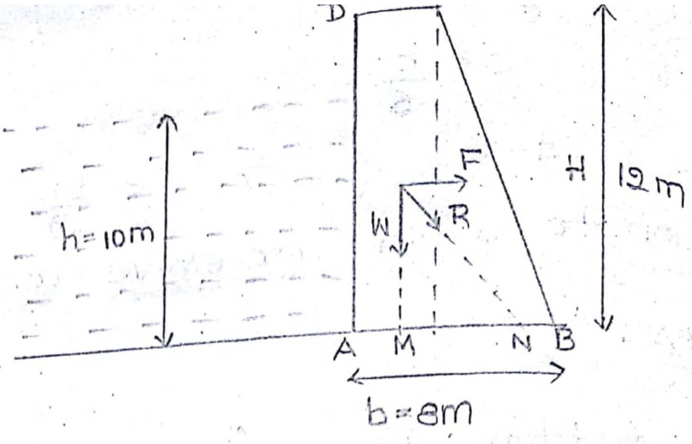
$$\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right) \leq P_{max}.$$

12/2

* A trapezoidal masonry dam having 4m top width, 8 m bottom width and 12 m height is retaining water upto a height of ~~10~~ 12 m. The density of masonry is 2000 kg/m³. and coefficient of friction between dam and soil is 0.55. The allowable compressive stress is 343350 N/m². Check the stability of the dam. $\phi = 30°$

801

27



Given:

Top width, a = 4 m

Bottom width, b = 8 m

Height of dam, H = 12 m

Height of water, h = 10 m.

Density of masonry, ρ = 2000 kg/m³

Weight density, $\gamma_m = \rho g = 2000 \times 9.81$

$\gamma = 19620 \text{ N/m}^3$

$= 19.62 \text{ kN/m}^3$

Weight density of water, $w = 9.81 \text{ kN/m}^3$

(1) Water pressure, $F = \dfrac{wh^2}{2}$

$= \dfrac{9.81 \times 10^2}{2}$

$= 490.5 \text{ kN}$

Position of F $= \dfrac{h}{3}$

$= \dfrac{10}{3}$

$= 3.33 \text{ m}$

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(2) Weight of dam, $W = \gamma \times \frac{H}{2}(a+b)$

$= 19.62 \times \frac{12}{2}(4+8)$

$= 1412.64$ KN.

Position of W = $AM = \frac{a^2+ab+b^2}{3(a+b)}$

$AM = \frac{4^2+(4\times 8)+8^2}{3(4+8)}$

$= \cancel{2.55\ m} = 3.11\ m$

Resultant, $R = \sqrt{F^2+W^2}$

$= \sqrt{(490.5)^2+(1412.64)^2}$

$= 1495.37$ KN

Let x = horizontal distance between line of action of W and the resultant cuts the base.

$MN = x = \frac{F}{W} \times \frac{h}{3}$

$= \frac{490.5}{1412.64} \times \frac{10}{3}$

$MN = 1.15$ m.

$d = AM + MN = 3.55 + 1.15 = \cancel{3.7}\ m$

$= 4.26$ m

Eccentricity, $e = d - \frac{b}{2}$

$= 4.26 - \frac{8}{2}$

$= +0.26$ m

(27)

$$\sigma_{max} = \frac{W}{b}\left(1+\frac{6e}{b}\right)$$

$$= \frac{1412.64}{8}\left(1+\frac{6(0.26)}{8}\right)$$

$$= 211.01 \ KN/m^2.$$

$$\sigma_{min} = \frac{W}{b}\left(1-\frac{6e}{b}\right)$$

$$= \frac{1412.64}{8}\left(1-\frac{6(0.26)}{8}\right)$$

$$= 142.14 \ KN/m^2$$

Check for stability:

(i) Sliding.

Frictional force > water pressure.

Frictional force $= \mu W$

$= 0.55 \times 1412.64$

$= 776.95$ KN

$776.95 > 490.5$

No sliding occurs in the dam.

(ii) Overturning

$MB > MN$

$MN = 1.15$ m

$MB = 8 - AM = 8 - 3.11 = 4.89$ m.

$4.89 > 1.15$

(ii) Overturning doesn't occurs.

(iii) Check for tension

$$d \leq \frac{2b}{3}$$

$$4.26 \leq \frac{2 \times 8}{3}$$

$$4.26 \leq 5.33$$

No tension occurs in the dam.

(iv) Check for compression

$$\sigma_{max} = 211.01 \ KN/m^2$$

$$P_{max} = 343350 \ N/m^2$$

$$= 343.35 \ KN/m^2$$

$$\sigma_{max} \leq P_{max}$$

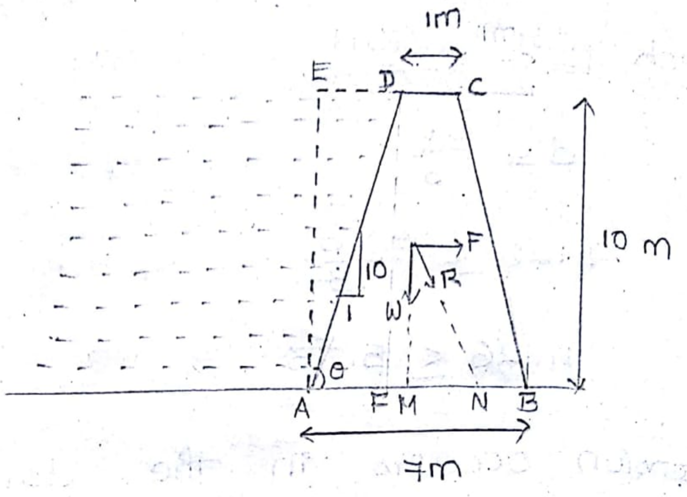
$$211.01 \leq 343.35$$

No. compression occurs in the dam.

* A masonry dam of trapezoidal section is 10 m height it has top width 1m and bottom width 7m. The face exposed to water as a slope of 1H to 10 V. (1H to 10V) Calculate the max. and minimum stresses on the base when the water level coincides with the top of the dam.

(31) Take weight density of masonry as 19.62 KN/m2

Sol



Given:

Top width, $a = 1m$

Bottom width, $b = 7m$

Dam height, $H = 10m$

Height of water, $h = 10m$

Slope $= 1H$ to $10V$.

Weight density of masonry, $\gamma = 19.62\ KN/m^2$

$$\tan\theta = \frac{10}{1}$$

$$\theta = 84.28$$

$$\tan\theta = \frac{10}{AF}$$

$$\Rightarrow \tan 84.28 = \frac{10}{AF}$$

$$AF = 1m$$

① Horizontal water pressure, $F_h = \frac{\omega h^2}{2}$

$$F_h = \frac{9.81 \times 10^2}{2} = 490.5\ KN$$

Position of F_h from base $= \frac{h}{3}$

$= \frac{10}{3}$

$= 3.33$ m

(ii) Vertical water pressure, $F_V = \omega \times V$

$F_V = \omega \times \left(\frac{1}{2} \times 10 \times 1\right) \times 1$

$= 9.81 \times 5$

$F_V = 49.05$ KN.

Position of F_V from AE $= \frac{BE}{3}$

$= \frac{1}{3}$

$= 0.33$ m

(iii) Weight of dam, $W = \gamma \times \frac{H}{2}(a+b) \times 1$

$W = 19.62 \times \frac{10}{2}(1+7) \times 1$

$= 784.8$ KN

$$AM = \bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$$

Here A_1 = Area of (1) triangle

$= \frac{1}{2} \times 1 \times 10$

$= 5\ m^2$

$x_1 = \frac{2b}{3} = \frac{2}{3} \times 1 = \frac{2}{3}$ m

$= 0.67$ m.

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33

A_2 = Area of ② rectangle.

$= 1 \times 10$

$= 10\ m^2$

$x_2 = 1 + \frac{1}{2}$

$= 1.5\ m$

A_3 = Area of ③ triangle.

$= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times (7-2) \times 10$

$= 25\ m^2$

$x_3 = 1 + 1 + \frac{b}{3}$

$= 1 + 1 + \frac{5}{3}$

$= 3.66\ m$

$$\bar{x} = \frac{(5 \times 0.67) + (10 \times 1.5) + (25 \times 3.66)}{5 + 10 + 25}$$

AM $\bar{x} = 2.745\ m.$

Let x = horizontal distance between line of action of W and resultant cuts the base.

$F_h \times \frac{h}{3} = W \times MN + F_y \times GN.$

$490.5 \times \frac{10}{3} = 784.8 \times MN + 49.05(AN - AG)$

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$1635.~~36~~ = 784.8 \times MN + 49.05(AM + MN - 0.33)$

$1635 = 784.8 \times MN + 49.05(2.745 + MN - 0.33)$

~~$1635 = 784.8 MN + 118.45 MN$~~

$903.~~25~~ MN = ~~1635~~$

~~$MN = 1.81 m.$~~

$1635 = 784.8 MN + 49.05(2.415 + MN)$

$1635 = 784.8 MN + 118.45 + 49.05 MN$

$833.85 MN = 1516.55$

$MN = 1.81 m$

$d = AM + MN = 2.745 + 1.81 = 4.55 m.$

Eccentricity, $e = d - \frac{b}{2} = 4.55 - \frac{7}{2} = 1.05.$

$\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right)$

$= \frac{784.8}{7}\left(1 + \frac{6(1.05)}{7}\right)$

$= 213.01 \ KN/m^2$

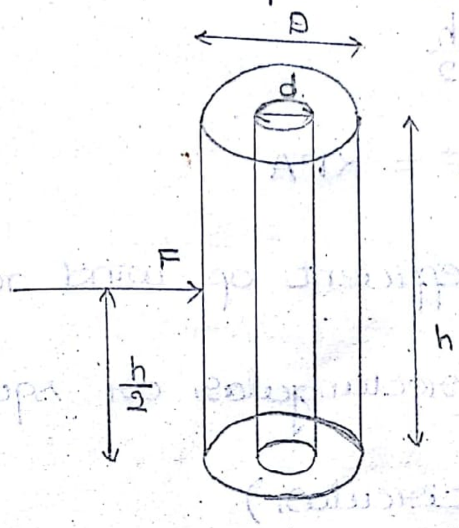
$\sigma_{min} = \frac{W}{b}\left(1 - \frac{6e}{b}\right)$

$= \frac{784.8}{7}\left(1 - \frac{6(1.05)}{7}\right)$

$= 11.21 \ KN/m^2$

Chimneys

(42) Chimneys are tall structures subjected to horizontal wind pressure



$$M = F \times \frac{h}{2}$$

The base of the chimney are subjected to B.M due to horizontal wind force.

This B.M at the base produces bending stresses.

The base of the chimney is also subjected to direct stress due to self-weight of the chimney.

Hence at the base, bending stress and direct stress are acting.

Direct stress, $\sigma_0 = \frac{W}{A}$

$$= \frac{\text{self weight of chimney}}{\text{c/s Area of chimney}}$$

(43)

* Bending stress, $\sigma_b = \frac{My}{I}$

$\sigma_b = \frac{M}{Z}$ $\boxed{\because \frac{I}{y} = Z}$

Here $M = F \times \frac{h}{2}$

* Wind force, $F = KPA$

Here K = co-efficient of wind resistance.

K = 1 (rectangular or square)

$K = \frac{2}{3}$ (circular)

P = intensity of wind pressure.

A = projected area of the surface

A = D × h (circular chimney)

= b × h (rectangular or square)

* ∴ Maximum pressure $= \sigma_0 + \sigma_b$

Minimum pressure $= \sigma_0 - \sigma_b$.

* Determine the max & min stresses at the base of a hollow circular chimney of height 20 m. and external dia 4m and internal dia 2m. The chimney is subjected to a horizontal wind pressure of intensity 1 KN/m².

The specific weight of the material of chimney is 22 KN/m².

(4)

Given:

Height of chimney, $h = 20\ m$

External dia, $D = 4\ m$

Internal dia, $d = 2\ m$.

Intensity of pressure, $p = 1\ KN/m^2$.

Specific weight, $\gamma = 22\ KN/m^2$.

$\sigma_{max} = ?$

$\sigma_{min} = ?$

Self weight of chimney $= \gamma V$. $\quad \gamma = \frac{W}{V}$

$W = \gamma . A . h$

$= \cancel{22 \times \frac{\pi}{4}}$

$\therefore$ Direct stress, $\sigma_0 = \frac{W}{A} = \frac{\gamma A h}{A}$

$= \gamma h$

$= 22 \times 20$

$= 440\ KN/m^2$

$\therefore$ Bending stress, $\sigma_b = \frac{M}{Z}$

$M = F \times \frac{h}{2}$

$F = KpA = \frac{2}{3} \times 1 \times (4 \times 20)$

$= 53.33$

45

$$M = 53.33 \times \frac{20}{2}$$

$$= 533.3 \text{ KN-m}$$

$$Z = \frac{I}{y}$$

$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$= \frac{\pi}{64}(4^4 - 2^4)$$

$$= 11.78 \text{ m}^4$$

$$y = \frac{D}{2} = \frac{4}{2} = 2$$

$$Z = \frac{11.78}{2} = 5.89$$

$$\sigma_b = \frac{M}{Z} = \frac{533.3}{5.89}$$

$$\sigma_b = 90.54 \text{ KN/m}^2$$

Maximum stress, $\sigma_{max} = \sigma_0 + \sigma_b$

$$= 440 + 90.54$$

$$= 530.54 \text{ KN/m}^2$$

Minimum stress, $\sigma_{min} = \sigma_0 - \sigma_b$

$$= 440 - 90.54$$

$$= 349.46 \text{ KN/m}^2$$

UNIT-V

UNSYMMETRICAL BENDING

* The plane of loading or that of bending doesnot lie in a plane that coincide the principal axis of cross section then the bending is called unsymmetrical bending.
* Thus in unsymmetrical bending the direction of neutral axis is not perpendicular to plane of bending.
* **Reasons for unsymmetrical bending :-**

→ The section is symmetrical (rectangle, circle or symmetrical I- section) but the load line is inclined to both the principal axis

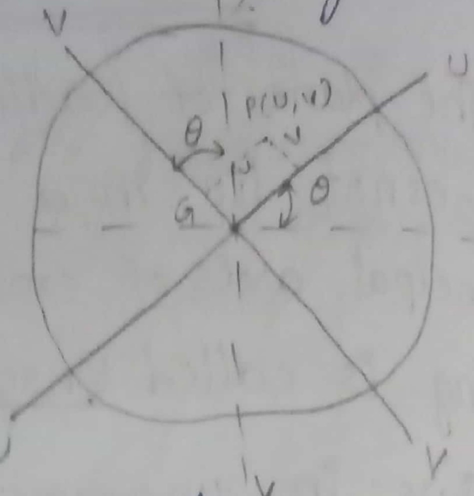
→ Section is itself unsymmetrical (angle sections or channel sections) but the load line is acting along any centroidal axis.

* **Stresses due to unsymmetrical bending :-**

Consider a beam cross section under an action of bending moment M acting in a plane Y-Y

→ Let G is centroid of the section X-X & Y-Y are the two co-ordinate axis passing through C.G.

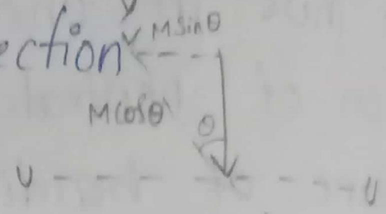
→ U-U & V-V are principle axis inclined at an angle θ to X-X & Y-Y respectively.



→ The moment M can be resolved along U-U and V-V.

→ M_U = Moment in U-U direction = $M\sin\theta$

→ M_V = Moment in V-V direction = $M\cos\theta$



→ The resultant bending stress at a point P(u, v)

$$\boxed{\sigma_b = \frac{M_U u}{I_{VV}} + \frac{M_V v}{I_{UU}}}$$

where I_{VV} & I_{UU}

$$\sigma_b = \frac{(M\sin\theta)u}{I_{VV}} + \frac{(M\cos\theta)v}{I_{UU}}$$

$$= M\left[\frac{\sin\theta \times u}{I_{VV}} + \frac{\cos\theta \times v}{I_{UU}}\right]$$

→ The neutral axis is present at where the stress is zero.

<u>N.A position</u> :- $\sigma_b = 0$

$$M\left[\frac{\sin\theta \times u}{I_{VV}} + \frac{\cos\theta \times v}{I_{UU}}\right] = 0.$$

$$\boxed{v = \cancel{\frac{I_{UU}}{I_{VV}}}\left[\frac{I_{UU}}{I_{VV}} \times \tan\theta\right] u}$$

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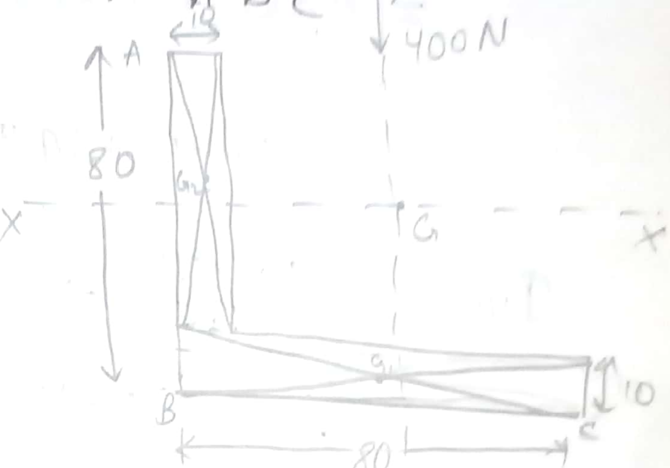
→ This is an equation of a straight line passing through CG & inclined at an angle α with UU

$$\boxed{\tan\alpha = -\left[\frac{I_{UU}}{I_{VV}}\tan\theta\right]}$$

Q) An angle section 80x80x10 mm as shown in figure is used as simply supported beam over a span of 2.4m carries a load of 400N along the line YG. Determine stresses at points A B C



Sol

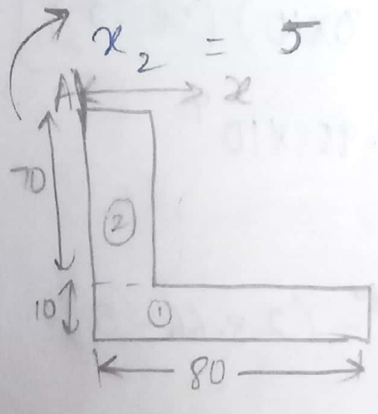
$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1+A_2}$$

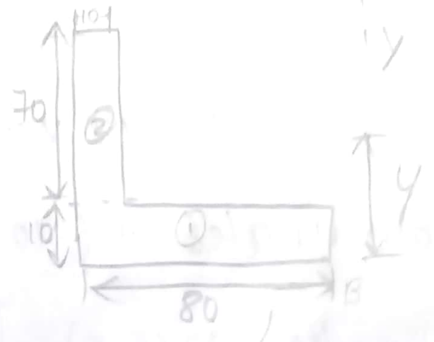
$A_1 = 80\times10$

$x_1 = 40$

$A_2 = 70\times10$

$x_2 = 5$





$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1+A_2}$$

$A_1 = 80\times10$ | $A_2 = 70\times10$

$y_1 = 5$ | $y_2 = 10+\frac{70}{2}$

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$$\bar{x} = \frac{(80\times10)(40) + (70\times10)(5)}{(80\times10) + (70\times10)}$$

$\bar{x} = 23.66$

$$\bar{y} = \frac{(80\times10)(5) + (70\times10)(45)}{(80\times10)+(70\times10)}$$

$\bar{y} = 23.66$

$$I_{xx} = \left[\frac{80\times10^3}{12} + (80\times10)(\bar{y}-y_1)^2\right] + \left[\frac{10\times70^3}{12} + (70\times10)(\bar{y}-y_2)^2\right]$$

$$= \left[\frac{80\times10^3}{12} + (80\times10)(23.66-5)^2\right] + \left[\frac{10\times70^3}{12} + (70\times10)(23.66-45)^2\right]$$

$= 28.52\times10^4 + 60.46\times10^4 = 88.98\times10^4$

$$I_{yy} = \left[\frac{10\times80^3}{12} + (10\times80)(\bar{x}-x_1)^2\right] + \left[\frac{70\times10^3}{12} + (70\times10)(\bar{x}-x_2)^2\right]$$

$= 64.02\times10^4 + 24.95\times10^4$

$= 88.97\times10^4$

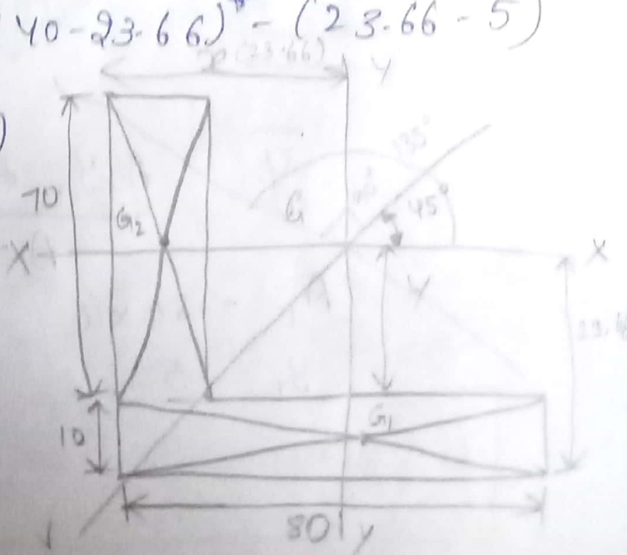
G_1 - coordinates $+(40-23.66)$ - $(23.66-5)$

G_2 - coordinates $-(23.66-5)$

$+(45-23.66)$

$G_1 = 16.34, -18.66$

$G_2 = +21.33, -18.66$



Product of inertia. $I_{xy} = A_1 G_1 + A_2 G_2$

$$= (80\times10)(+16.33)\times(-18.66) + (70\times10)\times(-18.66)\times(+21.33)$$

$$= 522386.7 \text{ mm}^4$$

If θ is angle at which principal axes makes with xx through G.

$$\tan 2\theta = \frac{2\times I_{xy}}{I_{xx} - I_{yy}}$$

$$\tan 2\theta = \infty = 90^\circ$$

$$\theta = 45^\circ.$$

∴ Other principal plane makes an angle $45^\circ + 90^\circ = 135^\circ$ with x-axis

<u>Principal moments</u>:-

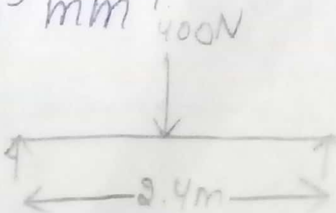
$$I_{uu} = \frac{I_{xx}+I_{yy}}{2} + \frac{I_{xx}-I_{yy}}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \frac{\cancel{2}\times 88.98\times10^4}{\cancel{2}} - 522386.7\times \sin(90^\circ)$$

$$= \cancel{56.74\times10^4}\ 1422186.7 \text{ mm}^4$$

W.K.T, $\boxed{I_{uu} + I_{vv} = I_{xx} + I_{yy}}$

$$I_{vv} = 3.67\times10^5 \text{ mm}^4$$



Moment on section

$$M = \frac{Wl}{4} = \frac{400\times2.4\times10^3}{4}$$

$$M = 240 \times 10^3 \ N\text{-}mm$$

Moment in UU, $M_U = M \sin\theta = 240 \times 10^3 \times \sin 45^\circ$

$= 169705.62$

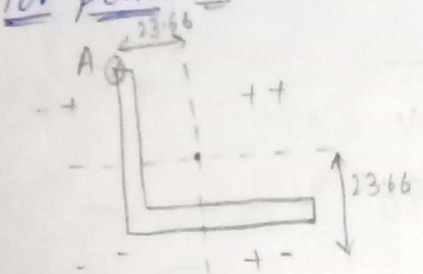
$= 169.7 \times 10^3 \ N\text{-}mm$

Moment in VV, $M_V = M \cos\theta = 240 \times 10^3 \times \cos 45^\circ$

$= 169.7 \times 10^3 \ N\text{-}mm$

$$\sigma_b = \frac{M_U \, u}{I_{VV}} + \frac{M_V \cdot v}{I_{UU}}$$

<u>for point A</u>:



$x = -23.66$

$y = +(80 - 23.66) = 56.34$

$u = x \cos\theta + y \sin\theta$

$= -23.66 \cos 45^\circ + 56.34 \sin 45^\circ$

$= 23.108$

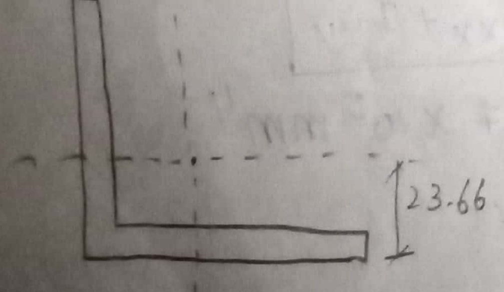
$v = y \cos\theta - x \sin\theta$

$= 56.34 \cos 45^\circ + 23.66 \cos 45^\circ = 56.56$

$$\sigma_b = \frac{169.7 \times 10^3 \times 23.10}{3.67 \times 10^5} + \frac{169.7 \times 10^3 \times 56.56}{1422186.7}$$

$= 17.42 \ N/mm^2$

<u>Point B</u>:

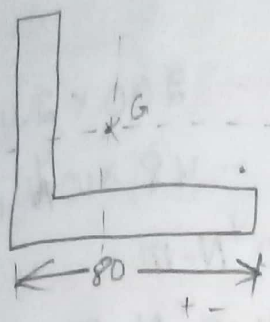


$x = -23.66$

$y = -23.66$

$\therefore \sigma_b = -15.47 \ N/mm^2$

Point C :-

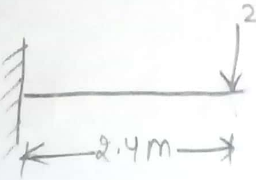


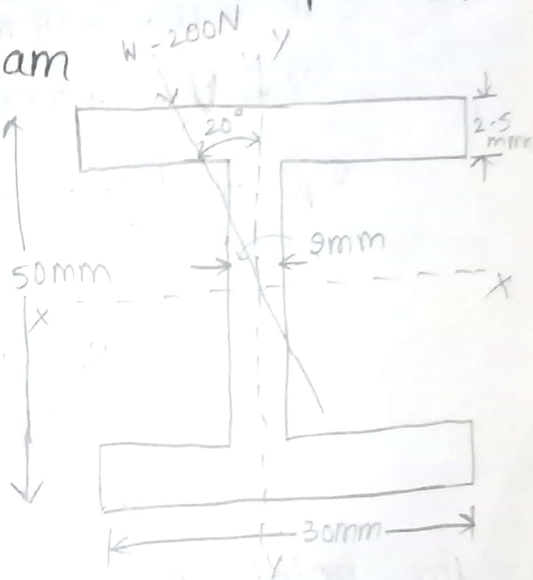
$x = +(80 - 23.6)$

$Y = -23.66$

$\sigma_b = 3.646 \ N/mm^2$

Q) Determine stresses on an I-section as shown in figure subjected to a load of 200 N at the free end over a span of 2.4 m of cantilever beam



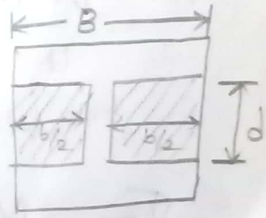


Sol

For symmetrical I-section

$\bar{x} = \frac{30}{2} = 15mm$

$\bar{y} = \frac{50}{2} = 25mm$



$I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12}$

$= \frac{30 \times 50^3}{12} - \frac{28 \times (50-(2.5 \times 2))^3}{12}$

$= 99875 \ mm^4$

I_{yy} = I_{yy} of web about C.G + 2 x I_{yy} of flange about C.G

$= \frac{45 \times 2^3}{12} + \left[2 \times \frac{2.5 \times 30^3}{12}\right]$

$= 30 + 11250$

$= 11280 \ mm^4$

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For symmetrical sections, $I_{vv} = I_{xx}$ &

$I_{uu} = I_{yy}$.

Moment on section, $M = w \times l = 200 \times 2.4 \times 10^3$

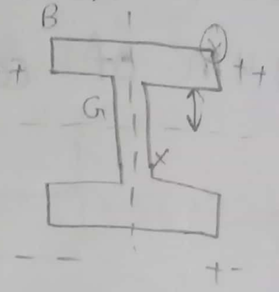
$= 48 \times 10^4$ N-mm

$\therefore M_u = M \sin\theta = 16.4163 \times 10^4$ N-m

$M_v = M \cos\theta = 45.1052 \times 10^4$ N-m

For symmetrical sections,

$u = x$ & $v = y$. for A



$x = u = +15$ mm

$y = v = +25$ mm

$$\sigma_A = \frac{M_u \cdot u}{I_{vv}} + \frac{M_v \cdot v}{I_{uu}}$$

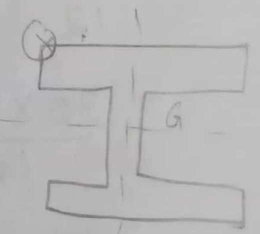
$$= \frac{16.4163 \times 10^4 \times 15}{11280} + \frac{45.10 \times 25 \times 10^4}{99875}$$

$= 218.21 + 112.89 = 331.10$ N/mm^2

For point B.

$u = x = -15$ mm

$v = y = +25$ mm



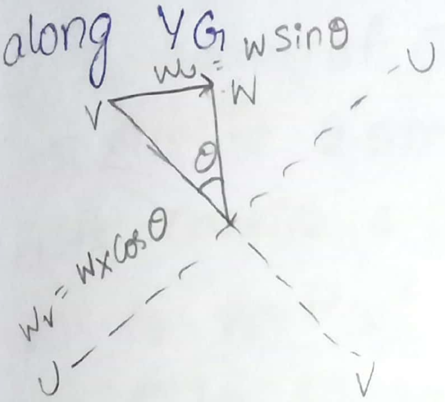
$$\sigma_B = \frac{M_u \cdot u}{I_{vv}} + \frac{M_v \cdot v}{I_{uu}}$$

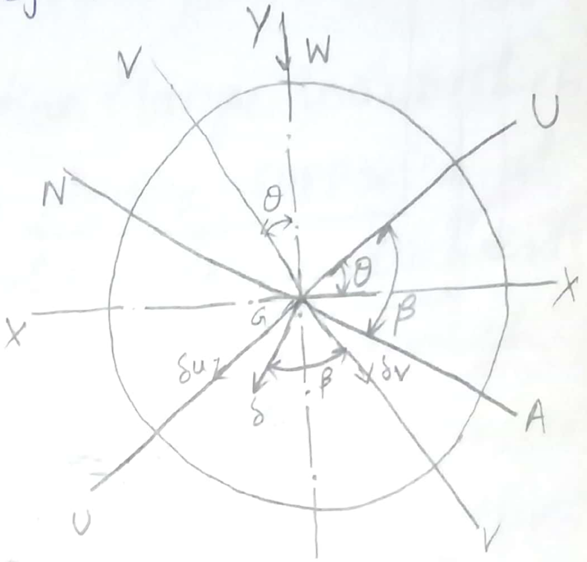
$$= \frac{16.4 \times (-15) \times 10^4}{11280} + \frac{45.10 \times 25 \times 10^4}{99878}$$

$= -218.21 + 12.801$

$= -105.32$ N/mm^2

Deflection of beams due to unsymmetrical bending :- Consider a beam of cross section. Let XX & YY are co-ordinate axis. UU & VV principal axis making an angle θ with XX & YY. Let G is centroid of section. This section subjected to a load of W along YG





Resolving W along UU & VV axes.

$$W_u = W \times \sin\theta$$

$$W_v = W \times \cos\theta.$$

Let δ_u - deflection due to W_u along UU.

δ_v - deflection due to W_v along VV.

Deflection for any beam $= \delta = \dfrac{kWl^3}{EI}$

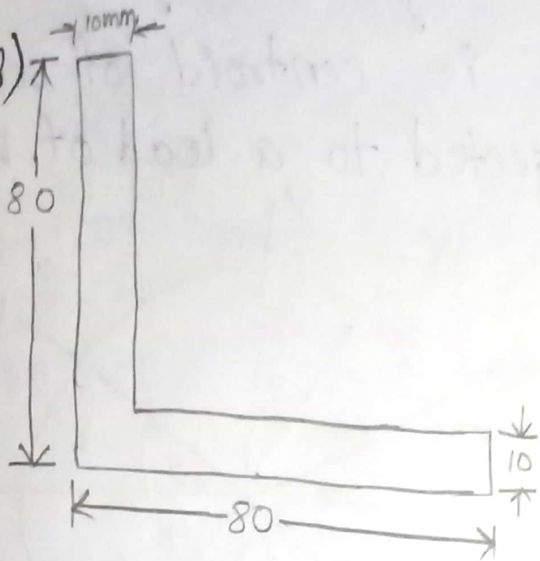
$$\therefore \delta_U = \frac{k \times (W_u \times \sin\theta) l^3}{EI_{VV}}$$

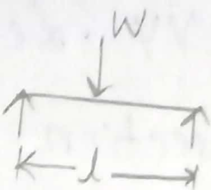
$$\delta_V = \frac{k \times [W_v \cdot \cos\theta] l^3}{E \cdot I_{UU}}$$

Resultant deflection, $\delta = \sqrt{(\delta_U)^2 + (\delta_V)^2}$

$$\boxed{\delta = \frac{kwl^3}{E}\sqrt{\frac{\sin^2\theta}{I^2_{uu}} + \frac{\cos^2\theta}{I^2_{vv}}}}$$

& direction $\beta = \frac{I_{uu}}{I_{vv}} \times \tan\theta$.

Q) 



$$\delta = \frac{wl^3}{48EI}$$

$$K = \frac{1}{48}$$

$W = 400N$, $l = 2.4 \times 10^3 mm$

$I_{uu} = 14.1246 \times 10^5 mm^4$

$I_{vv} = 3.67 \times 10^5 mm^4$

$\theta = 45^0$.

$E = 200 GN/m^2 = 200 \times 10^3 N/mm^2$

$W_u = W \times \sin\theta = 282.84$

$W_v = W \times \cos\theta = 282.84$

$$\delta_u = \frac{K(W_u \sin\theta)}{E \times I_{vv}}$$

$$= \frac{\frac{1}{48}(282.84)(2.4 \times 10^3)}{(200 \times 10^3) \times (3.57 \times 10^5)}$$

$= 0.019$.

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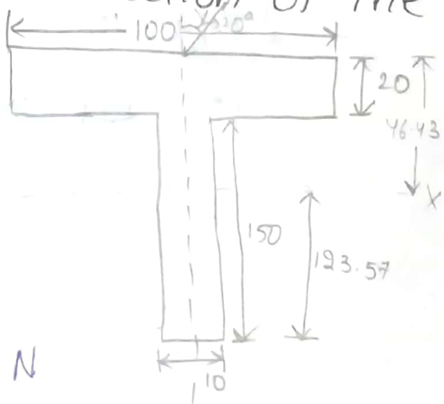
$$\delta = \frac{Kwl^3}{E}\sqrt{\frac{\sin^2\theta}{I^2_{UU}} + \frac{\cos^2\theta}{I^2_{VV}}}$$

$$= \frac{\left(\frac{1}{48}\right)(400)(2.4\times10^3)^3}{200\times10^3}\sqrt{\frac{\sin^2 45}{(14.12\times10^5)^2} + \frac{\cos^2 45}{(3.67\times10^5)^2}}$$

$$= 576000\sqrt{2.50\times10^{-13} + 3.762\times10^{-12}}$$

$$\delta = 1.14\,mm$$

Q) A beam of T-section flange 100x20 mm web 150x10 mm is 2.5m long simply supported at ends carries a load of 3.2kN inclined at 20° to vertical axis & passing through CG. Take E = 200 GN/m². Determine stresses at flange corners and deflection of the section.



Sol
$E = 200\,GN/m^2$
$= 200\times10^3\,N/mm^2$
$\theta = 20°$
$W = 3.2\,kN = 3.2\times10^3\,N$

for symmetrical section, (YY)

$$\bar{x} = \frac{100}{2} = 50$$

$$\bar{Y} = \frac{A_1Y_1 + A_2Y_2}{A_1 + A_2}$$

$$= \frac{(100\times20)(10) + (150\times10)(95)}{(100\times20) + (150\times10)}$$

$A_1 = 100\times20$
$A_2 = 150\times10$
$Y_1 = \frac{20}{2}$
$Y_2 = 20 + \frac{150}{2}$

$\bar{y} = 46.42\ mm$

I_{yy} = I_{yy} of web along C.G + I_{yy} of flange about CG

$$= \frac{150 \times 10^3}{12} + \frac{20 \times 100^3}{12}$$

$$= 12500 + 16.66 \times 10^5$$

$$= 16.78 \times 10^5\ mm^4$$

I_{xx} = (I_{xx} of flange + $A\bar{h}_1^2$) + (I_{xx} of web + $A\bar{h}_2^2$)

$$= \left[\frac{100 \times 20^3}{12} + (100 \times 20)(\underset{46.42}{\bar{y}} - \underset{10}{y_1})^2\right]$$

$$+ \left[\frac{10 \times 150^3}{12} + (10 \times 150)(\underset{46.42}{\bar{y}} - \underset{95}{y_2})^2\right]$$

$$= 27.19 \times 10^5 + 63.52 \times 10^5$$

$$= 90.71\ \times 10^5\ mm^4$$

6. Analysis of pin-jointed plane frames

(1) A structure made up of several members riveted or welded together is known as a frame.

*. If the frame is composed of such members which are just sufficient to keep the frame in equilibrium when the frame is supporting an external load, then the frame is known as perfect frame.

Types of frames:

(1) Perfect frame.

(2) Imperfect frame.

*. A frame in which no. of members are more than the required members then the frame is known as imperfect frame.

Analysis of frame:

(2) A frame is analyzed in the following methods.

(1) Method of joints

(2) Method of sections

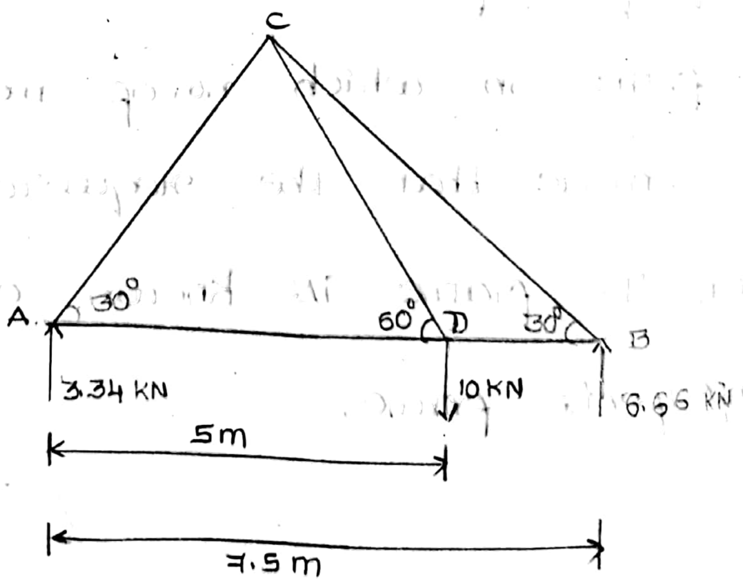
(3) Graphical method

Method of joints:

* For a perfect frame, the no. of joints and no. of members are given by.

$$n = 2j - 3$$

* A truss of span 7.5 m carries a point load of 10 KN at joint D as shown in fig. Find the reactions and forces in the members of the truss.



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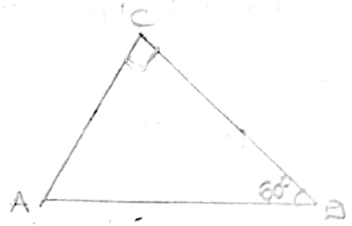
Let F_{AD}, F_{AC}, F_{BD}, F_{DC} & F_{BC} be the forces in the members of the truss i.e., AD, AC, BD, DC & BC respectively

<u>Reactions</u>

Let R_A and R_B be the support reactions at A and B respectively.

$$\sin 60^\circ = \frac{AC}{AD}$$



$$AC = 5 \sin 60^\circ$$

$$AC = 4.33 \text{ m}$$

$\Sigma V = 0 \Rightarrow R_A + R_B = 10 \text{ KN}$

$\Sigma M_A = 0 \Rightarrow (-R_B \times 7.5) + (10 \times 5) = 0$

$$7.5 R_B = 50$$

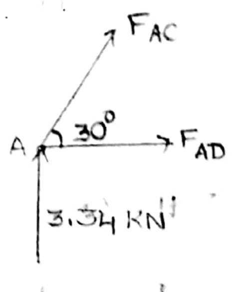
$$R_B = 6.66 \text{ KN}$$

$$R_A = 10 - 6.66$$

$$R_A = 3.34 \text{ KN.}$$

<u>Joint-A</u>

$\Sigma V = 0$



$\Rightarrow F_{AC} \sin 30^\circ + 3.34 = 0$

$$F_{AC} \sin 30^\circ = -3.34$$

$$F_{AC} = -6.68 \text{ KN}$$

$\Sigma H = 0$

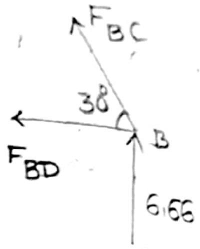
(4) $F_{AD} \cos 30^\circ + F_{AD} = 0$

$-6.68 \cos 30^\circ + F_{AD} = 0$

$F_{AD} = 5.78$ KN

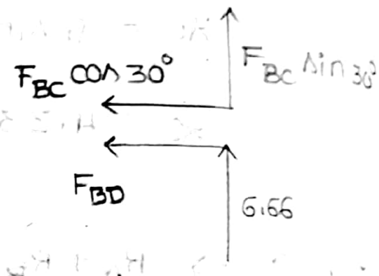
Joint - B

$\Sigma V = 0$



$\Rightarrow 6.66 + F_{BC} \sin 30^\circ = 0$

$F_{BC} \sin 30^\circ = -6.66$



$F_{BC} = -13.32$ KN

$\Sigma H = 0$

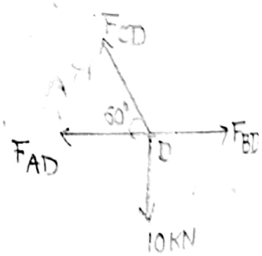
$\Rightarrow -F_{BD} - F_{BC} \cos 30^\circ = 0$

$F_{BD} = -(-13.32 \cos 30^\circ)$

$F_{BD} = 11.53$ KN

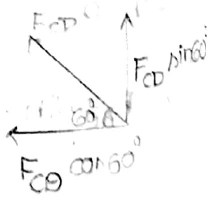
Joint - D

$\Sigma V = 0$



$\Rightarrow -10 + F_{CD} \sin 60^\circ = 0$

$F_{CD} \sin 60^\circ = +10$



$F_{CD} = +11.54$ KN

$\Sigma H = 0$

$\Rightarrow F_{BD} - F_{AD} - F_{CD} \cos 60^\circ = 0$

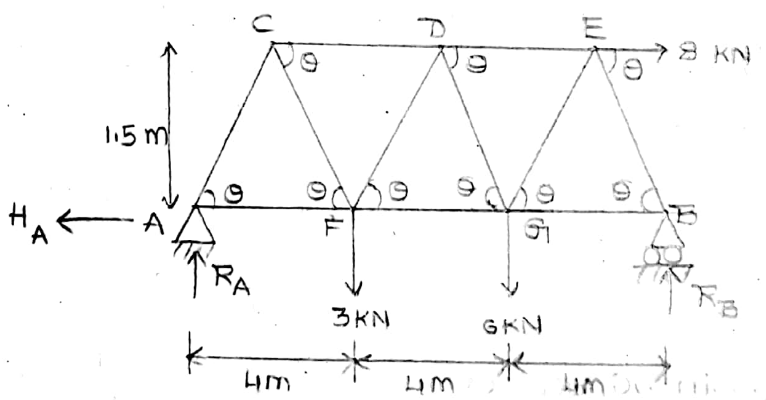
$\Rightarrow F_{BD} - 5.78 - 11.54 \cos 60^\circ = 0$

$F_{BD} = 11.55$

Member	Force in the member	Nature of force
AD	5.78 KN	Tensile
AC	6.68 KN	Compression
BD	11.53 KN	Tensile
BC	13.32 KN	Compression
CD	11.54 KN	Tensile

9/2

* Determine the forces in the members of the truss as shown in figure.



Sol **Reactions:**

Let R_A, H_A & R_B be the support reactions at A and B.

$\Sigma V = 0$

$\Rightarrow R_A + R_B = 3 + 6$

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$R_A + R_B = 9$ KN

6) $\Sigma H = 0$

$\Rightarrow H_A = 8$ KN

$\Sigma M_A = 0$

$\Rightarrow (-R_B \times 12) + (6 \times 8) + (3 \times 4) + (8 \times 1.5) = 0$

$12 R_B = 72$

$R_B = 6$ KN

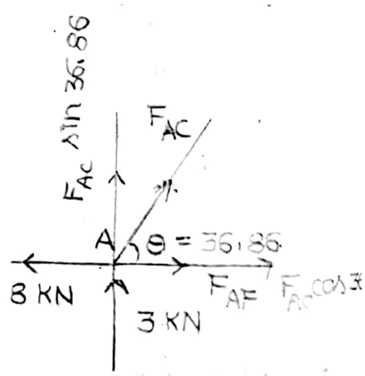
$R_A = 9 - 6$

$R_A = 3$ KN

Methods of joints

Let F_{AC}, F_{AF}, F_{DF}, F_{CD}, F_{DG}, F_{DE}, F_{CF}, F_{GE}, F_{EB}, F_{FG}, F_{GB} be the forces in the members respectively

Joint-A



$\Sigma V = 0$

$\Rightarrow 3 + F_{AC} \sin 36.86 = 0$

$F_{AC} \sin 36.86 = -3$

$$\boxed{F_{AC} = -5.001 \text{ KN}}$$

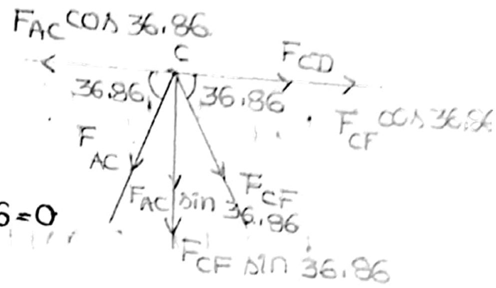
$\Sigma H = 0$

$\Rightarrow -8 + F_{AF} + F_{AC} \cos 36.86 = 0$

$F_{AF} = 8 - (-5.001 \times \cos 36.86)$

$$\boxed{F_{AF} = 12.001 \text{ KN}}$$

Joint - C



$\Sigma V = 0$

$\Rightarrow -F_{AC}\sin 36.86 - F_{CF}\sin 36.86 = 0$

$\Rightarrow F_{AC} = -F_{CF}$

$F_{CF} = -(-5.001) = 5.001 \text{ KN}$

$F_{CF} = 5.001 \text{ KN}$

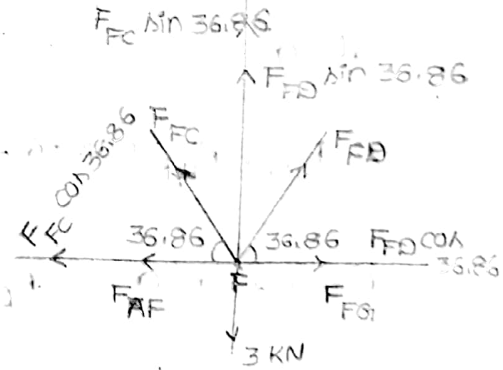
$\Sigma H = 0$

$\Rightarrow -F_{AC}\cos 36.86 + F_{CD} + F_{CF}\cos 36.86 = 0$

$-(-5.001)\cos 36.86 + F_{CD} + (5.001 \times \cos 36.86) = 0$

$$\boxed{F_{CD} = -8.002 \text{ KN}}$$

Joint - F



$\Sigma V = 0$

$\Rightarrow F_{FC}\sin 36.86$

$+ F_{FB}\sin 36.86 = 3 \text{ KN}$

$\Rightarrow (5.001 \sin 36.86) + F_{FB}\sin 36.86 = 3$

$F_{FB}\sin 36.86 = 9.11 \times 10^{-5}$

$F_{FD} = 1.51 \times 10^{-4} = 0.00015$

8

$$\boxed{F_{FD} = 0}$$

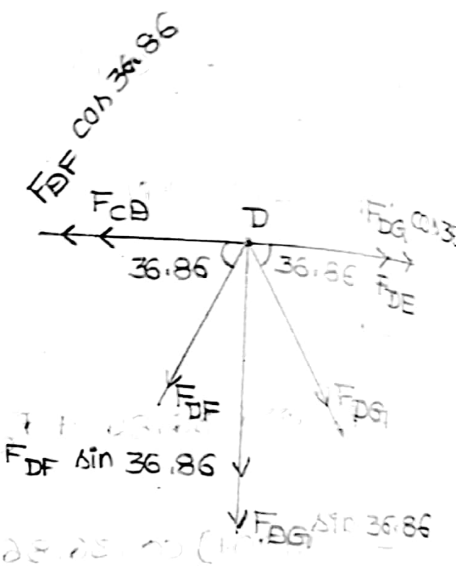
$\Sigma H = 0$

$\Rightarrow F_{AF} + F_{FC} \cos 36.86 = F_{FG} + F_{FD} \cos 36.86$

$\Rightarrow 12.001 + (5.001) \cos 36.86 = F_{FG} + 0$

$$\boxed{F_{FG} = 16 \text{ KN}}$$

<u>Joint-D</u>



$\Sigma V = 0$

$\Rightarrow F_{DF} \sin 36.86$

$+ F_{DG} \sin 36.86 = 0$

$F_{DF} = - F_{DG}$

$$\boxed{F_{DG} = 0}$$

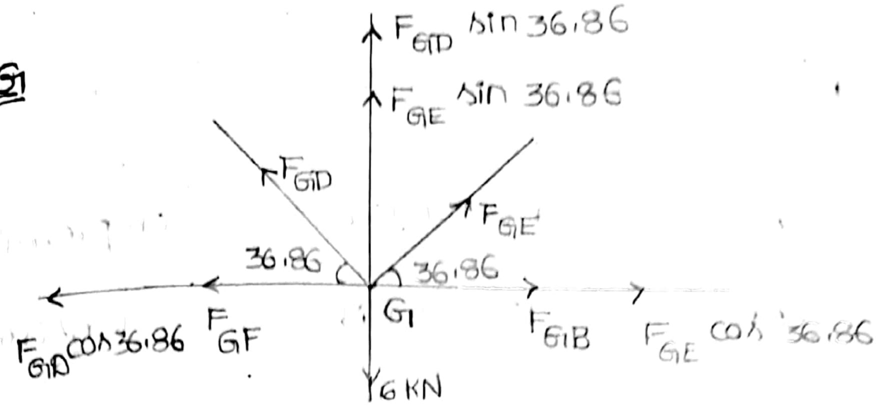
$\Sigma H = 0$

$\Rightarrow F_{CD} + F_{DF} \cos 36.86 = F_{DE} + F_{DG} \cos 36.86$

$(-8.002) + 0 = F_{DE} + 0$

$$\boxed{F_{DE} = -8.002 \text{ KN}}$$

Joint - G



$\Sigma H = 0$

$$\Rightarrow F_{GF} + F_{GD}\cos 36.86 = F_{GB} + F_{GE}\cos 36.86$$

$$16 + 0 = F_{GB} + F_{GE}\cos 36.86$$

$$F_{GB} = 16 - F_{GE}\cos 36.86$$

$$= 16 - 10\cos 36.86$$

$$\boxed{F_{GB} = 8\ KN}$$

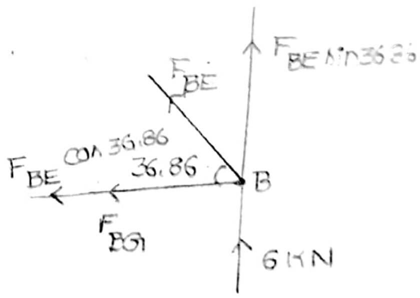
$\Sigma V = 0$

$$\Rightarrow F_{GD}\sin 36.86 + F_{GE}\sin 36.86 = 6$$

$$0 + F_{GE}\sin 36.86 = 6$$

$$\boxed{F_{GE} = 10\ KN}$$

Joint - B



$\Sigma V = 0$

$$\Rightarrow F_{BE}\sin 36.86 + 6 = 0$$

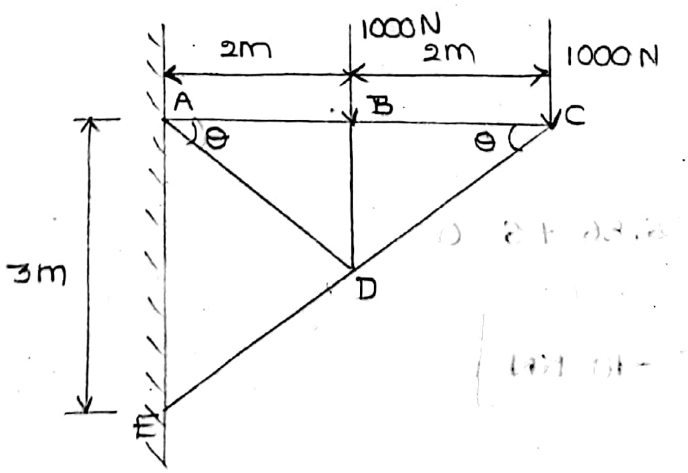
$$\boxed{F_{BE} = -10\ KN}$$

Scanned by CamScanner

(10)

Member	Force in member	Nature of force
AC	+5.001 KN	Compression
AF	12.001 KN	Tensile
CF	5.001 KN	Tensile
CD	8.002 KN	Compression
DF	0	—
FG	16 KN	Tensile
DG	0	—
DE	8.002 KN	Compression
GE	10 KN	Tensile
BE	10 KN	Compression
GB	8 KN	Tensile

* Determine the forces in the members of the truss as shown in figure.



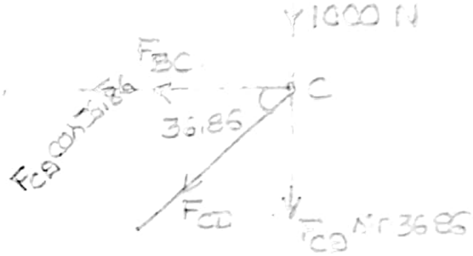
sol, Reactions

11) Let F_{AB}, F_{BC}, F_{CD}, F_{DE}, F_{BD}, F_{AD} be the forces in the members AB, BC, CD, DE, BD, AD respectively.

$$\tan\theta = \frac{3}{4}$$

$$\theta = 36.86^\circ$$

Joint-C:



$\Sigma V = 0$

$\Rightarrow F_{CD}\sin 36.86 + 1000 = 0$

$$F_{CD} = \frac{-1000}{\sin 36.86}$$

$$F_{CD} = -1667.05\ N$$

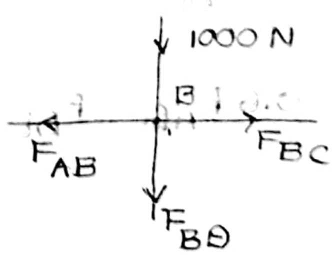
$\Sigma H = 0$

$\Rightarrow F_{BC} + F_{CD}\cos 36.86 = 0$

$$F_{BC} = -(-1667.05)\cos 36.86$$

$$F_{BC} = 1333.8\ N$$

Joint-B



$\Sigma V = 0$

$\Rightarrow F_{BD} + 1000\,N = 0$

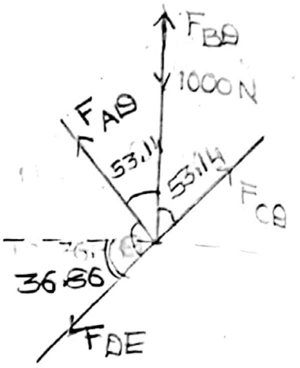
$$\boxed{F_{BD} = -1000\ N}$$

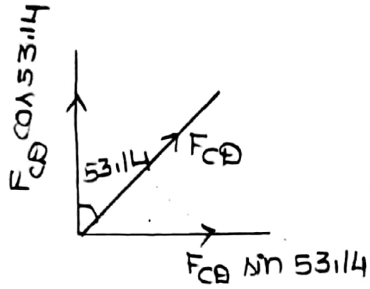
12

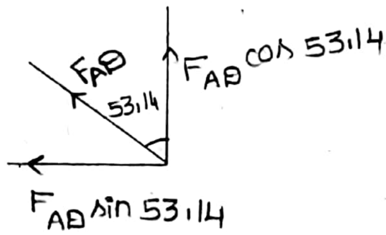
$$F_{BC} = F_{AB}$$

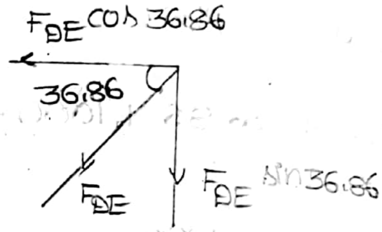
$$\boxed{F_{AB} = 1333.8\ N}$$

Joint-D









$$\Sigma V = 0$$

$$\Rightarrow F_{CD} \cos 53.14 + F_{AD} \cos 53.14 = F_{DE} \sin \overset{36.86}{\cancel{53.14}} + 1000$$

$$\Rightarrow (-1667.05 \cos 53.14) + F_{AD} \cos 53.14 = F_{DE} \sin \overset{36.86}{\cancel{53.14}} + 1000$$

$$0.6 F_{AD} - F_{DE} \sin \overset{36.86}{\cancel{53.14}} = 1000 + 1667.05 \cos 53.14$$

$$0.6 F_{AD} - F_{DE} \sin \overset{36.86}{\cancel{53.14}} = 1999.9\ N \longrightarrow (1)$$

$$0.6 F_{AD} - F_{DE} (0.6) = 1999.9 N \longrightarrow (1)$$

Scanned by CamScanner

$\Sigma H = 0$

(13) $F_{CD} \sin 53.14 = F_{AD} \sin 53.14 + F_{DE} \cos \cancel{53.14}$ 36.86

$(1667.05 \sin 53.14) = F_{AD}(0.8) + F_{DE}(\cancel{0.59})$ 0.8

$0.8 F_{AD} + 0.8 F_{DE} = -1333.8\text{ N} \longrightarrow (2)$

solve (1) & (2) eqns.

$F_{AD} = \cancel{107.17\text{ N}} = 832.95\text{ N}$

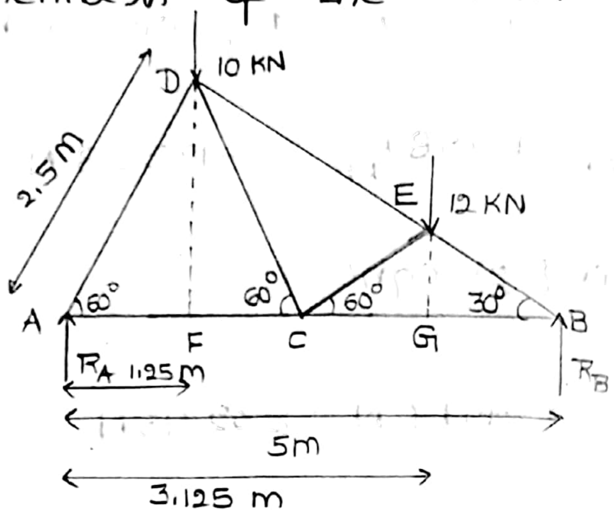
$F_{DE} = \cancel{-2365.9\text{ N}} = -2500.2\text{ N}$

Member	Force in the member	Nature of force.
AB	1333.8 N	Tensile
BC	1333.8 N	Tensile
CD	1667.05 N	Compressive
BD	1000 N	~~Tensile~~ Compressive
DE	2500.2 N	Compressive
AD	832.95 N	Tensile

20/2

* A truss of span 5 m is loaded as shown

(14) in figure. Find the reactions and forces in the members of the truss.



Sol Let R_A & R_B be the reactions at A and B respectively.

$$R_A + R_B = 10 + 12$$

$$R_A + R_B = 22 \text{ KN}$$

ΔABD, $\sin 30° = \dfrac{AD}{AB}$

$$AD = 5 \times \sin 30°$$

$$AD = 2.5 \text{ m}$$

ΔADF, $\cos 60° = \dfrac{AF}{AD}$

$$AF = 2.5 \cos 60°$$

$$AF = 1.25 \text{ m}$$

ΔCEB, $\sin 30° = \dfrac{CE}{BC}$

$$CE = 2.5 \times \sin 30^\circ$$

$$CE = 1.25\ m$$

15 $\Delta\ CEG,\quad \cos 60^\circ = \dfrac{CG}{CE}$

$$CG = 1.25 \cos 60^\circ$$

$$CG = 0.625\ m.$$

$\Sigma M_A = 0$

$$\Rightarrow (+R_B \times 5) = (10 \times 1.25) + (12 \times 3.125)$$

$$5R_B = 50$$

$$R_B = 10\ KN$$

$$R_A = 22 - 10$$

$$R_A = 12\ KN$$

__Method of joints__

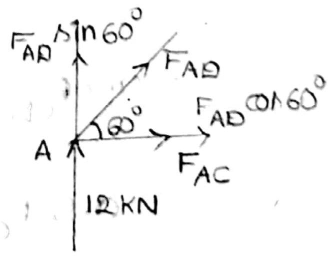
Let F_{AD}, F_{AC}, F_{CD}, F_{CE}, F_{DE},

F_{BC}, F_{BE} be the forces in the members

AD, AC, CD, CE, DE, BC, BE respectively.

__Joint-A__

$\Sigma V = 0$



$$\Rightarrow F_{AD} \sin 60^\circ + 12 = 0$$

$$\boxed{F_{AD} = -13.85\ KN}$$

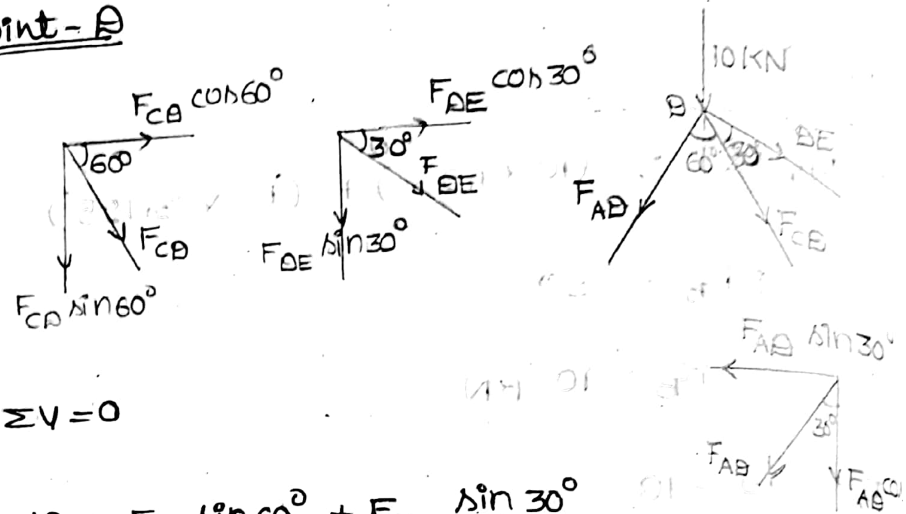
$\Sigma H = 0$

16 $\Rightarrow$ $F_{AB} \cos 60^\circ + F_{AC} = 0$

$-13.85 \cos 60^\circ + F_{AC} = 0$

$$\boxed{F_{AC} = 6.925 \text{ KN}}$$

Joint - B



$\Sigma V = 0$

$\Rightarrow$ $10 + F_{CB} \sin 60^\circ + F_{BE} \sin 30^\circ$

$+ F_{AB} \cos 30^\circ = 0$

$10 + F_{CB} \sin 60^\circ + F_{BE} \sin 30^\circ + (-13.85 \cos 30^\circ) =$

$F_{CB} \sin 60^\circ + F_{BE} \sin 30^\circ = 1.99$

$0.866 F_{CB} + 0.5 F_{BE} = 1.99 \longrightarrow (1)$

$\Sigma H = 0$

$\Rightarrow$ $F_{CB} \cos 60^\circ + F_{BE} \cos 30^\circ = F_{AB} \sin 30^\circ$

$0.5 F_{CB} + 0.866 F_{BE} = (-13.85 \sin 30^\circ)$

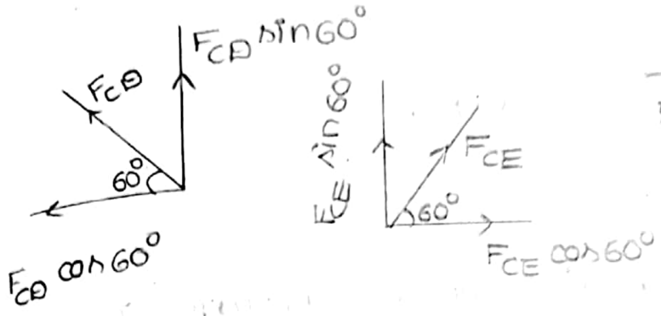
$0.5 F_{CB} + 0.866 F_{BE} = -6.925 \longrightarrow (2)$

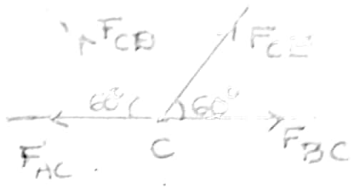
Solve (1) & (2) eqns

$F_{CD} = 10.37\ \text{KN}$

$F_{DE} = -13.98\ \text{KN}.$

(7) Joint-C





$\Sigma V = 0$

$\Rightarrow F_{CD}\sin 60^\circ + F_{CE}\sin 60^\circ = 0$

$10.37 \sin 60^\circ + F_{CE}\sin 60^\circ = 0$

$F_{CE}\sin 60^\circ = -8.98$

$F_{CE} = -10.37\ \text{KN}$

$\Sigma H = 0$

$\Rightarrow F_{AC} + F_{CD}\cos 60^\circ = F_{BC} + F_{CE}\cos 60^\circ$

$6.925 + 10.37\cos 60^\circ = F_{BC} + (-10.37\cos 60^\circ)$

$F_{BC} = 6.925 + 10.37\cos 60^\circ + 10.37\cos 60^\circ$

$F_{BC} = 17.29\ \text{KN}$

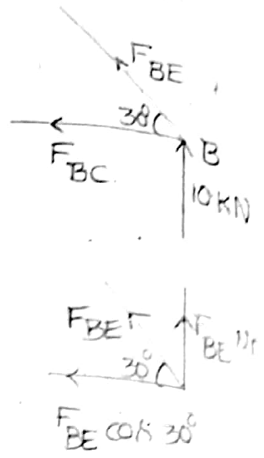
Joint-B

(18) $\Sigma V = 0$

$\Rightarrow 10 + F_{BE} \sin 30^\circ = 0$

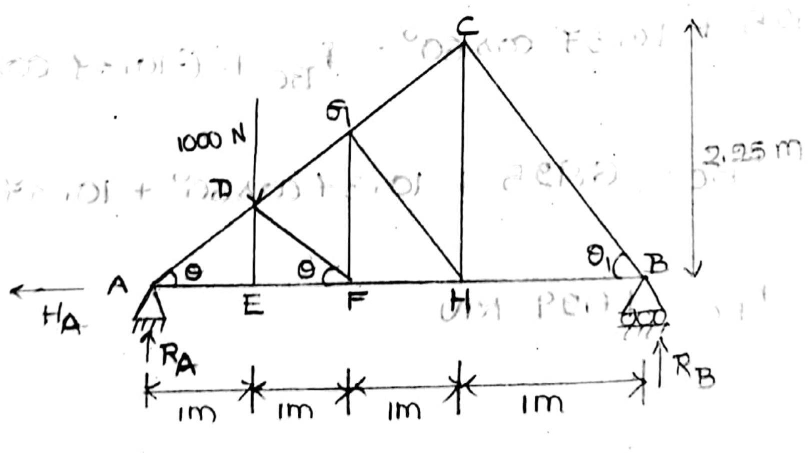
$F_{BE} \sin 30^\circ = -10$

$F_{BE} = -20$ KN



Member	Force in the member	Nature of force
AD	13.85	Compression
AC	6.925	Tensile.
CD	10.37	Tensile
DE	13.98	Compression
CE	10.37	Compression
BC	17.29	Tensile
BE	20	Compression

* Determine the forces in the members of the truss as shown in figure.



Sol Let R_A, H_A & R_B be the support reactions at A and B respectively.

Reactions:

$\Sigma H_A = 0$

$\Sigma V = 0$

$\Rightarrow R_A + R_B = 1000\,N$

$\Sigma M_A = 0$

$\Rightarrow R_B \times 4 = 1000 \times 1$

$R_B = 250\,N$

$R_A = 1000 - 250$

$R_A = 750\,N$

Method of joints

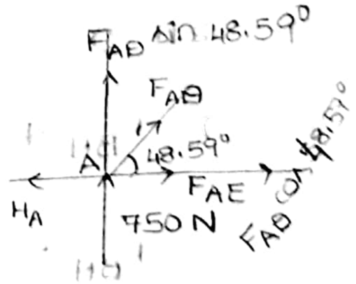
Let F_{AE}, F_{AD}, F_{DE}, F_{DG}, F_{DF}, F_{CG}, F_{GH}, F_{FH}, F_{BC}, F_{CH}, F_{BH} are the forces in the members AE, AD, DE, DG, DF, CG, GH, FH, BC, CH, BH respectively.

From Δ ACH, $\tan\theta = \frac{CH}{AH}$

$\tan\theta = \frac{2.25}{3}$

$\theta = \cancel{48.59^\circ}$ 36.86°

Joint-A



$\Sigma V = 0$

$\Rightarrow 750 + F_{AD}\sin \cancel{48.59^\circ}\ 36.86^\circ = 0$

$F_{AB} = \cancel{-1000\ N}$

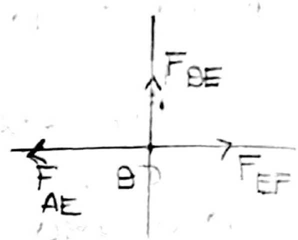
$-1250.28\ N$

(20) $\Sigma H = 0$

$\Rightarrow F_{AE} + F_{AB} \cos \cancel{48.59}\ 36.86^\circ = 0$

$F_{AE} = \cancel{661.44\ N}\ 1000.85\ N$

Joint - E



$\Sigma V = 0$

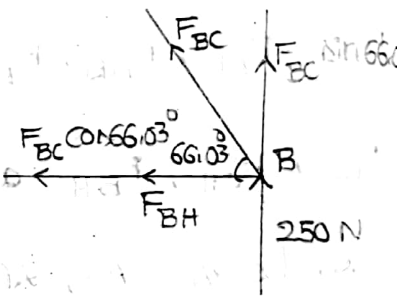
$\Rightarrow F_{DE} = 0$

$\Sigma H = 0$

$\Rightarrow F_{AE} = F_{EF}$

$F_{EF} = \cancel{661.44\ N}.\ 1000.85\ N.$

Joint - B



$\Delta\ BCH,\ \tan\theta_1 = \frac{CH}{BH}$

$= \frac{2.25}{1}$

$\theta_1 = 66.03^\circ.$

$\Sigma V = 0$

$\Rightarrow 250 + F_{BC} \sin 66.03 = 0$

$F_{BC} = -273.59\ N$

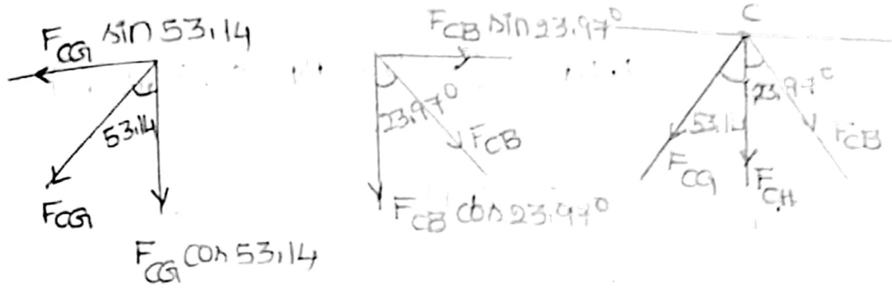
$\Sigma H = 0$

$\Rightarrow F_{BH} + F_{BC} \cos 66.03 = 0$

$F_{BH} = 273.59 . \cos 66.03$

$F_{BH} = 111.14$ N

Joint-C



$\Sigma V = 0$

$\Rightarrow F_{CG}\cos 53.14 + F_{BC}\cos 23.97° + F_{CH} = 0$

$F_{CG}\cos 53.14 - 273.59\cos 23.97 + F_{CH} = 0$

$0.599\, F_{CG} + F_{CH} = 249.99 \longrightarrow (1)$

$\Sigma H = 0$

$\Rightarrow F_{CG}\sin 53.14 = F_{BC}\sin 23.97°$

$F_{CG}\sin 53.14 = -273.59\sin 23.97°$

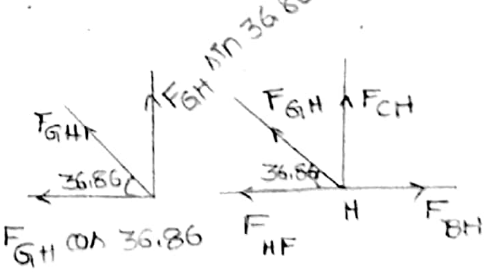
$F_{CG} = -138.91$ N

from (1) $\quad 0.599(-138.91) + F_{CH} = 249.99$

$F_{CH} = 333.19$ N

Joint-H

$\Sigma V = 0$



$\Rightarrow F_{CH} + F_{GH}\sin 36.86.$

$F_{GH} = -555.44$ N

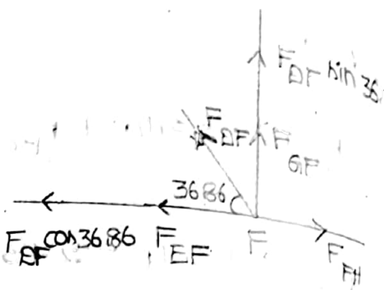
$\Sigma H = 0$

22 $\Rightarrow F_{FH} + F_{GH} \cos 36.86 = F_{BH}$

$F_{HF} = 111.14 - (-555.44 \cos 36.86)$

$F_{FH} = 555.54 \text{ N}$

<u>Joint - F</u>



$\Sigma V = 0$

$\Rightarrow F_{GF} + F_{DF} \sin 36.86 = 0$

$\cancel{F_{DF} = 925.95 \text{ N}}$

$\Sigma H = 0$

$\Rightarrow F_{EF} + F_{DF} \cos 36.86 = F_{FH}$

$\Rightarrow F_{EF} + F_{DF} \cos 36.86 = 555.54$

$1000.05 + F_{DF} \cos 36.86 = 555.54$

$F_{DF} = -555.56 \text{ N}$

$F_{GF} + (-555.56) \sin 36.86 = 0$

$F_{GF} = 333.25 \text{ N}$

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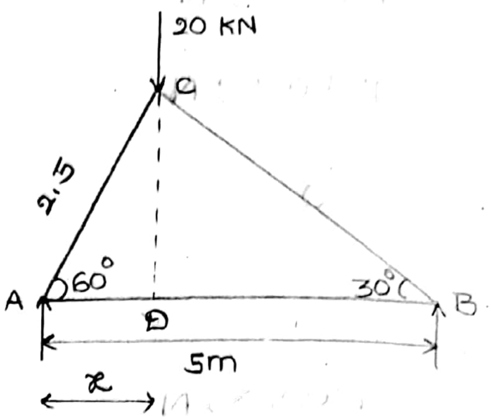
Member	Force in the member	Nature of the force.
AE	1000.35 N	Tensile
AD	1250.28 N	Compressive.
DE	0	—
DF		
EF	1000.35 N	Tensile
DG		
GH		
GC	138.91	Compressive
CH	333.19	Tensile
HB	111.14 ~~273.59~~	Tensile
BC	273.59	Compressive

Methods of sections:

* In this method, a section-line is passed through members in which forces are to be determined.
* The section-line should be drawn in such a way that it does not cut more than three members in which the forces are unknown.

* Find the forces in the members AB & AC as shown in figure.

(24)



Sol Reactions:

$R_A + R_B = 20\ KN$

from ΔACB, $\sin 30° = \frac{AC}{AB}$

$AC = 5 \sin 30°$

$AC = 2.5\ m$

from ΔACD, $\cos 60° = \frac{AD}{AC} = \frac{x}{2.5}$

$x = 2.5 \cos 60°$

$x = 1.25\ m$

$\Sigma M_A = 0$

$\Rightarrow (R_B \times 5) = 20 \times 1.25$

$R_B = 5\ KN$

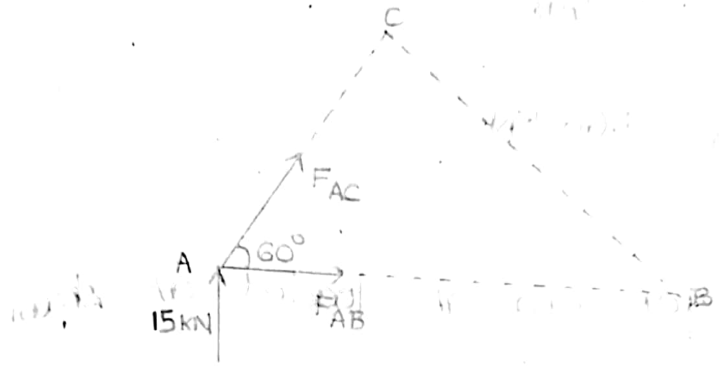
$R_A = 20 - 5$

$R_A = 15\ KN$

Method of sections:

Let F_{AB}, F_{AC}, F_{BC} be the forces in the members AB, AC, BC respectively.

Take a section cutting the members AB and AC as shown in figure.



$\Sigma M_C = 0$

$\Rightarrow (15 \times 1.25) - (F_{AB} \times 2.165) = 0$

$F_{AB} = 8.66$ KN

Δ ACD

$\sin 60° = \frac{CD}{AC}$

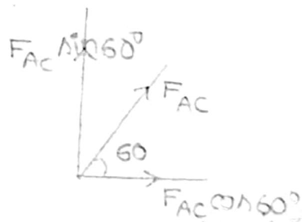
$CD = 2.5 \sin 60°$

$y = 2.165$

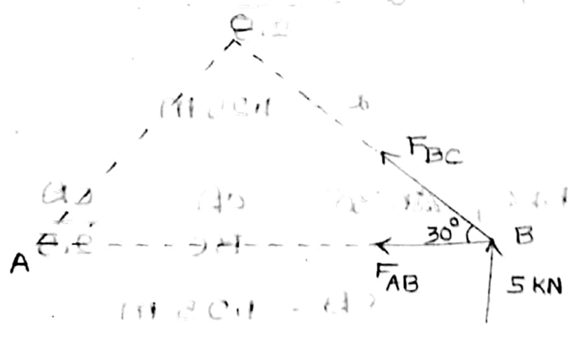
$\Sigma M_B = 0$

$\Rightarrow 15 \times 5 + F_{AC} \sin 60° \times 5 = 0$

$F_{AC} = -17.32$ KN



Take another section cutting the members BC and AB.



$\Sigma M_A = 0$

26 $\Rightarrow$ $+(5 \times 5) - (F_{BC} \sin 30^\circ \times 5) = 0$

$F_{BC} = -10$ KN

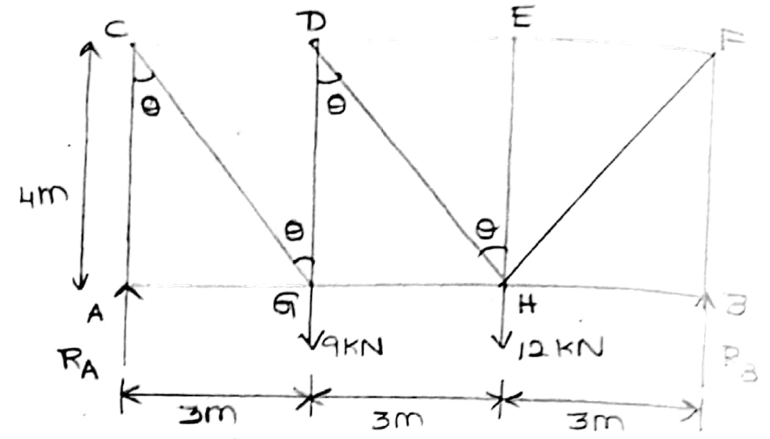
$\Sigma M_C = 0$

$\Rightarrow$ $5 \times (5 - 1.25) - (F_{AB} \times 2.165) = 0$

$F_{AB} = 8.66$ KN

* A truss of span 9m is loaded as shown in fig. Find the reactions and forces in the members.

(28)



Sol Reactions:

Let R_A and R_B be the reactions at A and B respectively.

$\Sigma V = 0$

$$R_A + R_B = 9 + 12$$

$$R_A + R_B = 21 \text{ KN}$$

$\Sigma M_A = 0$

$$\Rightarrow (-R_B \times 9) + (12 \times 6) + (9 \times 3) = 0$$

$$R_B = 11 \text{ KN}$$

$$R_A = 21 - 11 = 10 \text{ KN}$$

Method of sections:

Let $F_{AC}, F_{AG}, F_{CG}, F_{GD}, F_{GH}, F_{CD}, F_{DH}, F_{EH}, F_{DE}, F_{HF}, F_{EF}, F_{BF}, F_{BH}$ be the forces in the members AC, AG, CG, GD, GH, CD, DH, EH, DE, HF, EF, BF, BH respectively

Take a section ①-① cutting the members AC and AG as shown in fig.

$\Sigma M_G = 0$

$\Rightarrow (R_A \times 3) + (F_{AC} \times 3) = 0$

$3R_A + 3(10) + 3F_{AC} = 0$

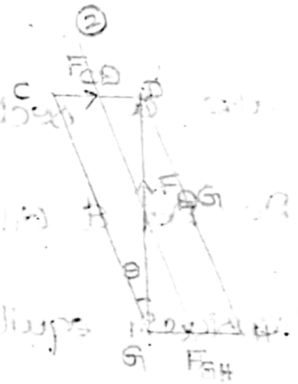
$F_{AC} = -10$ KN

$\Sigma M_C = 0$

$\Rightarrow F_{AG} \times 4 = 0$

$F_{AG} = 0$

Take a section ②-② cutting the members CD, DG, GH as shown in fig.



$\Sigma M_G = 0$

$\Rightarrow F_{CD} \times 4 + R_A \times 3 = 0$

$F_{CD} \times 4 + 30 = 0$

$F_{CD} = -7.5$ KN

$\Sigma M_D = 0$

$\Rightarrow -F_{GH} \times 4 + 10 \times 3 = 0$

$F_{GH} = 7.5$ KN

$\Sigma M_C = 0$

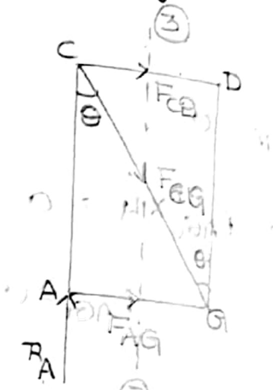
$\Rightarrow -F_{DG} \times 3 - F_{GH} \times 4 + (9 \times 3) = 0$

$$-3F_{DG} + 30 + 27 = 0$$

$$3F_{DG} = -3$$

$$F_{DG} = -1 \text{ KN}$$

Take a section ③-③ cutting the members CD, CG and AG as shown in fig.



$\Sigma M_A = 0$

$$\theta \times 4$$

$$\Rightarrow F_{CD} \times 4 + F_{CG} \sin(\cancel{90-\theta}) \times 4 = 0$$

$$(-7.5 \times 4) + F_{CG} \sin 36.86 \times 4 = 0$$

$$2.399 F_{CG} = 30$$

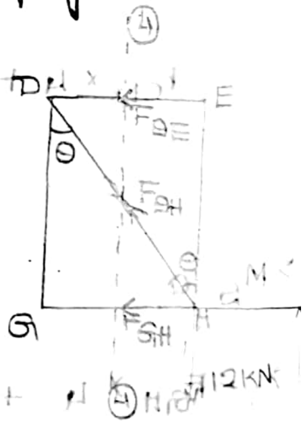
$$F_{CG} = 12.5 \text{ KN}$$

$\tan\theta = \frac{3}{4}$

$\theta = 36.86$

Take a section ④-④ cutting the members DE, DG & GH as shown in fig.

Consider equilibrium of right of the section.



$\Sigma M_H = 0$

$$\Rightarrow -R_B \times 3 - F_{DE} \times 4 = 0$$

$$-33 - F_{DE} \times 4 = 0$$

$$F_{DE} = -8.25 \text{ KN}$$

$\Sigma M_E = 0$

$\Rightarrow (-R_B \times 3) + F_{GH} \times 4 + F_{DH} \sin\theta \times 4 = 0$

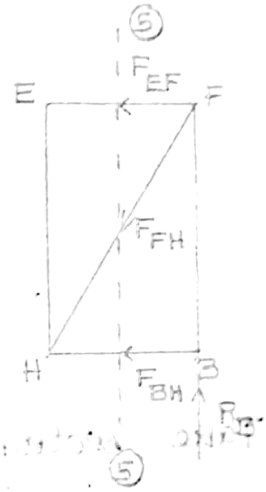
$-33 + 7.5 \times 4 + F_{DH} \sin 36.86 \times 4 = 0$

$2.399\, F_{DH} = 3$

$F_{DH} = 1.25 \text{ KN}$

Taking section ⑤-⑤ cutting EF, HF, & BH as shown in fig.

Consider equilibrium of right of the section.



$\Sigma M_H = 0$

$\Rightarrow -R_B \times 3 - F_{EF} \times 4 = 0$

$-33 - F_{EF} \times 4 = 0$

$F_{EF} = -8.25 \text{ KN}$

$\Sigma M_F = 0$

$\Rightarrow F_{BH} \times 4 = 0$

$F_{BH} = 0$

$\Sigma M_B = 0$

$\Rightarrow -F_{EF} \times 4 - F_{FH} \sin\theta \times 4 = 0$

$-(-8.25 \times 4) - F_{FH} \sin 36.86 \times 4 = 0$

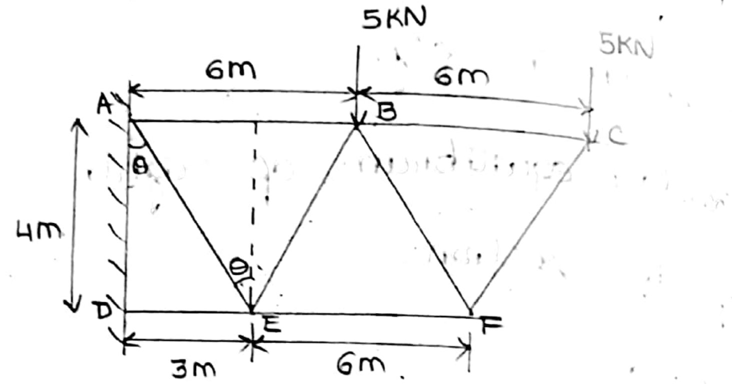
$2.399\, F_{FH} = 33$

$$F_{FH} = +13.75 \text{ KN}$$

By observing the structure $F_{EH} = 0$, F_{BF}

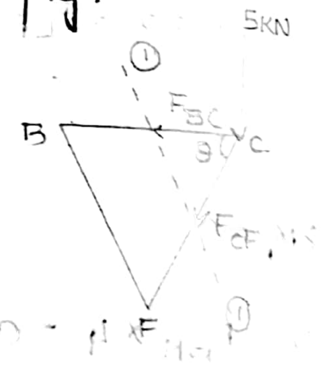
$F_{BF} = -11$ KN.

A cantilever truss is loaded as shown in the fig. Find the forces in the members.



Sol Take section ①-① cutting the members BC and CF as shown in fig.

Consider equilibrium of right of section.



$\Sigma M_F = 0$

$\Rightarrow$ ~~F_F~~ $(5 \times 3) - F_{BC} \times 4 = 0$

$$F_{BC} = 3.75 \text{ KN}.$$

$\Sigma M_B = 0$

$\Rightarrow (5 \times 6) + (F_{CF} \sin\theta \times 6) = 0$ $\qquad \tan\theta = \frac{4}{3}$

$\theta = 53.13$

$30 + F_{CF} \times \sin 53.13 \times 6 = 0$

$4.799\, F_{CF} = -30$

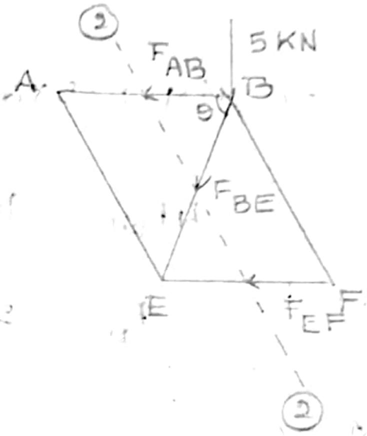
$F_{CF} = -6.25$ KN.

Take a section (2)-(2) cutting the members.

~~BC, BF & EF as~~

AB, BE & EF as shown in the fig.

Consider equilibrium of right of the section.



$\Sigma M_B = 0$

$\Rightarrow +F_{EF} \times 4 + (5 \times 6) = 0$

$F_{EF} = -7.5$ KN

$\Sigma M_E = 0$

$\Rightarrow (5 \times 9) + (5 \times 3) - F_{AB} \times 4 = 0$

$45 + 15 - 4 F_{AB} = 0$

$F_{AB} = 15$ KN

$\Sigma M_A = 0$

$\Rightarrow (5 \times 12) + (5 \times 6) + F_{EF} \times 4 + F_{BE} \sin\theta \times 6 = 0$

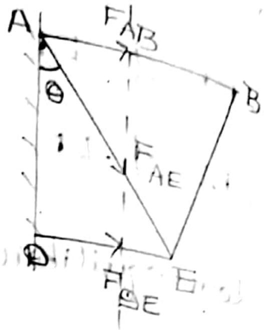
$\Rightarrow 60 + 30 + 4(7.5) + F_{BE} \sin 53.13 \times 6 = 0$

$4.799\, F_{BE} = -60$

$F_{BE} = -12.5$ KN

(34) Take a section ③-③ cutting the members AB, AE & DE as shown in fig.

Consider equilibrium of left of the section.



$\Sigma M_A = 0$

$\Rightarrow \quad -F_{DE} \times 4 + (5 \times 6) + (5 \times 12) = 0$

$4 F_{DE} = 90$

$F_{DE} = 22.5$ kN

$\Sigma M_D = 0$

$\Rightarrow \quad F_{AB} \times 4 + F_{AE} \sin\theta \times 4 + (5 \times 6) + (5 \times 12) = 0$

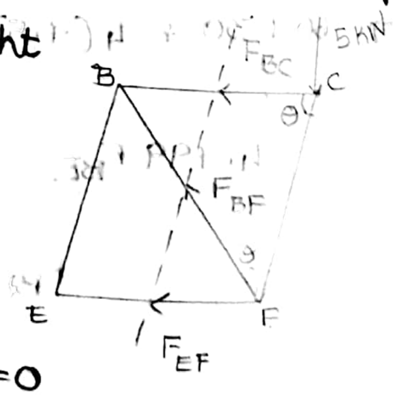
$15 \times 4 + F_{AE} (\sin 36.86) \times 4 + 30 + 60 = 0$

$2.399 \; F_{AE} = 90 - 60$

$F_{AE} = 12.5$ kN.

Take a section ④-④ cutting the members BC, BF & EF as shown in fig.

Consider equilibrium of right of the section.



$\Sigma M_C = 0$

$\Rightarrow \quad F_{EF} \times 4 + F_{BF} \sin\theta \times 4 = 0$

$(7.5 \times 4) + F_{BF} \sin 36.86 \times 4 = 0$

$2.399 \, F_{BF} = 30$

$F_{BF} = 12.5 \text{ KN}$