1. Uniform flow in Open channels

Syllabus:

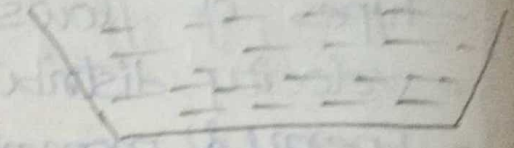
- Types of channels ✓
- Types of flows ✓
- Velocity distribution ✓
- Energy & momentum correction factors
- Chezy's & Manning's formulae for uniform flow ✓
- Most economical sections ✓
- Critical flow: specific energy ✓
  - Critical depth
  - computation of critical depth.

Open channel flow:

It is defined as the top surface of the channel is exposed to Atmospheric pressure.

Ex: Rivers, canals, Lakes etc.

The Action of gravitational force and by maintaining Bed slope of the channel the flow will be takes place in open channels.



Types of flows:

1. steady flow:

The flow in open channel is said to be steady if the fluid characteristics such as velocity, depth of flow do not changes with respect to time at any section.

2. Unsteady flow:

The flow in open channel is said to be unsteady if the fluid characteristics such as velocity, depth of flow changes with respect to time at any section.

3. Uniform flow:

The fluid characteristics does not change with space (or) length of the channel flow at any section is called uniform flow.

4. Non-uniform flow:

The fluid characteristics changes with space (or) length of the channel flow at any section is called Non-uniform flow.

It is of two types: 1. Gradually Varied flow
2. Rapid Varied flow

Gradually Varied flow:

The depth of the flow varies along with the space (or) length of channel is called Gradually varied flow.

Rapid Varied flow:

The depth of the flow varies suddenly With the space (or) length of channel is called Rapid Varied flow.

5. Laminar flow:

The flow which is parallel with respect to Adjacent Layer.

Re-Number < 500 : Laminar flow
Re-Number > 2000 : Turbulent flow
Re-Number between 500 to 2000 : Transistion flow

Froude Number:

It is the square root of ratio Inertia force to the Gravitational force is called froude number.

It is denoted by "F_r".

$$F_r = \sqrt{\frac{\text{Inertia force}}{\text{Gravitational force}}}$$

$$F_r = \frac{V}{\sqrt{gD}}$$

Where; V = Velocity of flow
g = Acceleration due to gravity
D = Hydraulic depth

Classification based upon fradde Number:

Froude number < 1 = subcritical

$F_r < 1$

Froude number = 1 = critical

$F_r = 1$

Froude number > 1 = super critical

$F_r > 1$

Regims of flow:

Laminar:

subcritical; Re < 500; $F_r < 1$
critical; Re < 500; $F_r = 1$
super critical; Re < 500; $F_r > 1$

Turbulent:

subcritical; Re > 2000; $F_r < 1$
critical; Re > 2000; $F_r = 1$
super critical; Re > 2000; $F_r > 1$

Transistion:

subcritical; Re < 500 > 2000; $F_r < 1$
critical; Re < 500 > 2000; $F_r = 1$
super critical; Re < 500 > 2000; $F_r > 1$

Types of channels:

(i) Natural channel (ii) Artificial channel

(i) Natural channel:

Channel which do not have proper geometrical cross section and slopes, bed slopes are called Natural channel.

Ex: Rivers, Lakes.

(ii) Artificial channel:

These are man made constructions and have proper shape, size & slopes called Artificial channel.

Manning's velocity equation:

$$V = \frac{1}{n} \cdot R^{2/3} S^{1/2}$$

Where; n = manning's constant

'n' varies from 0.002 to 0.0

According to chezy's $C\sqrt{RS} = \frac{1}{n} \cdot R^{2/3} S^{1/2}$

$$C R^{1/2} S^{1/2} = \frac{1}{n} \cdot R^{2/3} S^{1/2}$$

$$C = \frac{1}{n} R^{1/6}$$

Kutter's equation:

$$C = \frac{23 + 1/N + \frac{0.00155}{S}}{1 + \left(23 + \frac{0.00155}{S}\right)\frac{N}{\sqrt{R}}}$$

Bazin's constant:

$$C = \frac{157.6}{1.81 + \frac{K}{\sqrt{R}}}$$

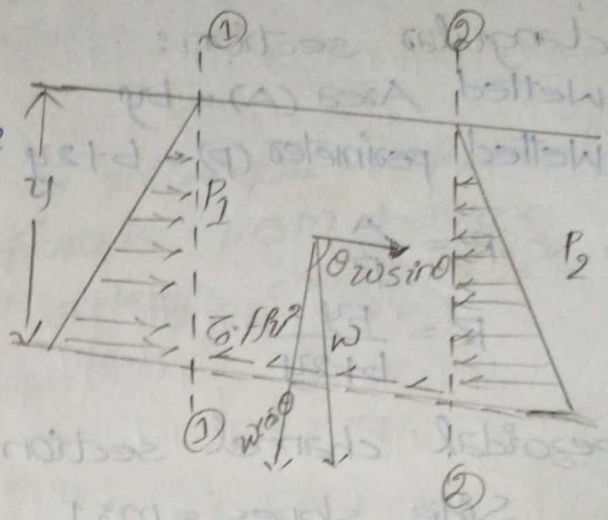
Where; K = Bazin's constant

Derivation for Chezy's formula:

Consider an open channel flow section as shown in figure. The various forces acting on the portion we considered are

(i) pressure force (P_1 & P_2) which are equal in Magnitude and acts in opposite direction.

(ii) Weight component ; ($W\sin\theta$, $W\cos\theta$) the frictional force (F)



In steady uniform flow, $\Sigma F = 0$

$$P_1 - P_2 + W\sin\theta - F = 0$$

$$P_1 - P_2 + w\forall \sin\theta - F = 0$$

$$P_1 - P_2 + w \times A \times L \sin\theta - F = 0$$

$$0 + w \times A \times L \sin\theta - \tau_0 \times P \times L = 0$$

$$w \times A \times L \sin\theta = \tau_0 \times P \times L$$

$$wA\sin\theta = \frac{f\rho V^2}{8} \times P$$

$$V^2 = \frac{8wA\sin\theta}{f\rho P} \quad \left\{ \because \frac{A}{P} = R \right\}$$

$$V^2 = \frac{8w}{f\rho} \cdot R\sin\theta$$

$$V = \sqrt{\frac{8w}{f\rho} \cdot R \cdot \sin\theta}$$

$$\boxed{V = C\sqrt{RS}}$$

$$\left\{ \begin{array}{l} w = \frac{W}{\forall} \\ W = w\forall \\ \because \forall = A \times L \end{array} \right\}$$

$$\left\{ \begin{array}{l} \sin\theta = S \\ \frac{8w}{f\rho} = C \end{array} \right\}$$

Where; C = Chezy's constant

R = Hydraulic mean radius = $\frac{\text{Wetted Area}}{\text{Wetted perimeter}} = \frac{A}{P}$

S = Bed slope

V = velocity of fluid.

Channel Sections:

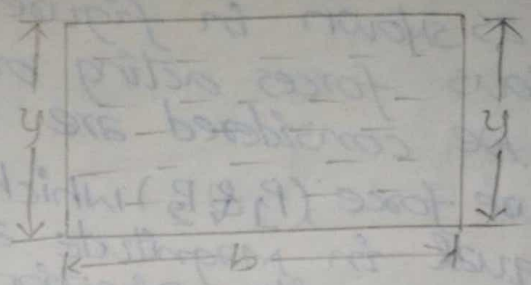
(i) Rectangular section:

Wetted Area $(A) = by$

Wetted perimeter $(P) = b + 2y$

$$R = \frac{A}{P}$$

$$R = \frac{by}{b+2y}$$



(ii) Trapezoidal channel section:

Side slopes $= m:1$

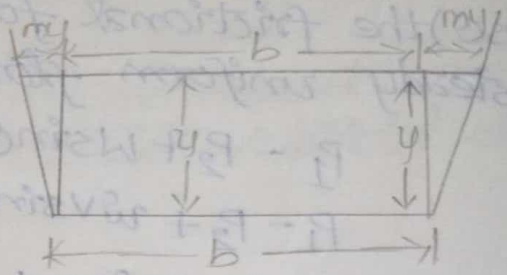
Top width $= b + 2my$

Wetted Area $(A) = \frac{1}{2}(a+b)h$

$$= \frac{1}{2}(b+b+2my)y$$

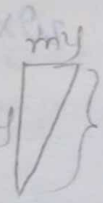
$$= \frac{(2b+2my)y}{2}$$

$$= (b+my)y$$



Wetted perimeter $(P) = b + 2y\sqrt{1+m^2}$

$$\therefore R = \frac{A}{P} = \frac{(b+my)y}{b+2y\sqrt{1+m^2}}$$



$$= \sqrt{y^2+m^2y^2}$$

$$= y\sqrt{1+m^2}$$

Hydraulic mean radius of Trapezoidal channel section is $R = \frac{(b+my)y}{b+2y\sqrt{1+m^2}}$

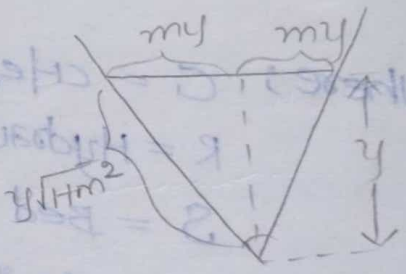
(iii) Triangular channel section:

Side slopes $= m:1$

Top width $= 2my$

Wetted Area $(A) = \frac{1}{2} \times 2my \times y$

$$= my^2$$



Wetted perimeter $(P) = 2y\sqrt{1+m^2}$

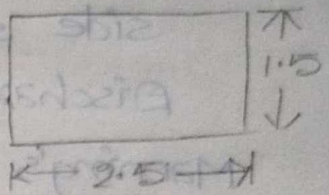
$$\therefore R = \frac{A}{P} = \frac{my^2}{2y\sqrt{1+m^2}} = \frac{my}{2\sqrt{1+m^2}}$$

Hydraulic depth (d) = $\frac{A}{T}$; $\frac{\text{Wetted Area}}{\text{Top width}}$

section factor (Z) = $A\sqrt{d}$.

1. Find the discharge through a rectangular channel having 2.5m wide & depth of flow is 1.5m having bed slope 1 in 2000 . Consider cHezy's constant C = 55

Sol: Given : Rectangular channel section



Width (b) = 2.5m

Depth (y) = 1.5m

bed slope (S) = 1/2000

cHezy's constant (c) = 55

We know; Discharge Q = AXV

Velocity (V) = $C\sqrt{RS}$

Where; R = $\frac{\text{Area/Wetted Area}}{\text{Wetted perimeter}} = \frac{A}{P}$

Wetted Area (A) by = $2.5 \times 1.5 = 3.75 m^2$

Wetted perimeter (p) = $b + 2y = 2.5 + 2(1.5) = 5.5m$

$$R = \frac{A}{P} = \frac{3.75}{5.5} = 0.681$$

$$V = C\sqrt{RS}$$

$$= 55\sqrt{0.681 \times \frac{1}{2000}}$$

$$V = 1.014 m/sec$$

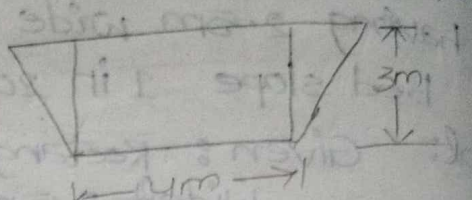
$$\rightarrow Q = AXV$$

$$= 3.75 \times 1.014$$

$$Q = 3.802 m^3/sec$$

(2) Find the bed slope of the Trapezoidal channel section having bottom width 4m, Depth of slope 3m and side slope is 2:3 & Discharge is $20 m^3/sec$. Consider manning's constant (n) = 0.03

sol: Given:



channel width (b) = 4m

channel depth (y) = 3m

side slope = 2:3

Discharge (Q) = $20 m^3/sec$

Manning's constant (n) = 0.03

We know; Discharge (Q) = AXV

Where; Manning's velocity (V) = $\frac{1}{n} R^{2/3} \cdot S^{1/2}$

Where; Hydraulic Mean radius of Trapezoidal section (R) = $\frac{\text{Wetted Area}}{\text{Wetted perimeter}}$

Wetted Area (A) = $(b+my)y$

$= (4+2\times3\times3)3$

$= 18 m^2$

Wetted perimeter (P) = $b+2y\sqrt{1+m^2}$

$= 4+2(3)\sqrt{1+(2\times3)^2}$

$= 4+6\sqrt{1+4}$

$= 4+6\times6.082$

$= 11.211 m$

$\Rightarrow R = \frac{A}{P} = \frac{18}{11.211} = 1.605$

$\Rightarrow Q = AXV$

$20 = 18 \times \frac{1}{n} R^{2/3} S^{1/2}$

$20 = 18 \times \frac{1}{0.03} (1.605)^{2/3} \cdot S^{1/2}$

$20 = \frac{18 \times 1.3708}{0.03} S^{1/2}$

$S^{1/2} = 0.024$

$S = 0.591 \times 10^{-4}$

$\therefore$ Bed slope $S = \frac{1}{0.591 \times 10^{-4}}$

$S = \frac{1}{1691}$

(3) A varying wide channel has to carry silt free water at a depth of 2m. The permisable shear stress on the bed is $2.5 N/m^2$. Find the bed slope.

Sol: Given:

Depth of channel = 2m

permisable shear stress $= 2.5 N/m^2$

Condition: varying wide channel

$\therefore R = y$

$R = 2m$

$S = ?$

$\Rightarrow$ permisable shear stress $\tau = \rho RS$

$2.5 = 9.810 \times 2 \times S$

$S = \frac{2.5}{(9810 \times 2)}$ (1.27×10^{-4})

$S = 0.0001$

(4) Find the discharge throught a rectangular channel 14m wide having depth of water 3m and bed slope is 1 in 15000. N = 0.03 in Kutter's formula.

Sol: Given:

Wide of channel (b) = 14m

Depth of channel (y) = 3m

Bed slope (S) $= \frac{1}{15000}$

$N = 0.03$

By kutter's formula; $C = \frac{23 + \frac{1}{N} + \frac{0.00155}{S}}{1 + (23 + \frac{0.00155}{S})\frac{N}{\sqrt{R}}}$

Discharge $(Q) = AV$

$= A \times C \sqrt{RS}$

Wetted Area (A) = by = 14 x 3 = 42

Wetted perimeter (p) = (b + 2y) = 14 + 2x3 = 20

$$R = \frac{A}{P} = \frac{42}{20} = 2.1$$

$$\Rightarrow Q = AV$$

$$= 14 \times 3 \times \left(\frac{23 + \frac{1}{0.03} + \frac{0.00155}{1/1500}}{1 + \left(23 + \frac{0.00155}{1/1500}\right) \cdot \left(\frac{0.03}{\sqrt{2.1}}\right)} \right) (2.1)^{1/2} \times \left(\frac{1}{1500}\right)^{1/2}$$

$$= 42 \times \left(\frac{23 + 33.33 + 0.775}{1 + (23 + 0.775)(0.0207)} \right) 1.449 \times 0.025$$

$$= 42 \times \frac{57.105}{1.492} \times 1.449 \times 0.025$$

$$Q = 60.43 \, m^3/sec$$

5) A sewer pipe laid at a slope of 0.0001 flowing half fulled conveys a discharge of 800 lit/sec. Find the size of the pipe. Consider Manning's N = 0.015

Sol: Given:

Slope (S) = 0.0001.

Discharge (Q) = 800 lit/sec = $0.8 \, m^3/sec$

Mannings constant (N) = 0.015

$$\text{Discharge } (Q) = A \times V$$

$$= A \times \frac{1}{n} R^{2/3} S^{1/2}$$

Wetted Area of circle (A) = $\frac{\pi r^2}{2}$ =

Wetted perimeter of circle (p) = $\frac{2\pi r}{2}$

$$Q = \frac{\pi r^2}{2} \times \frac{1}{0.015} \times \left(\frac{r}{2}\right)^{2/3} \cdot (0.0001)^{1/2}$$

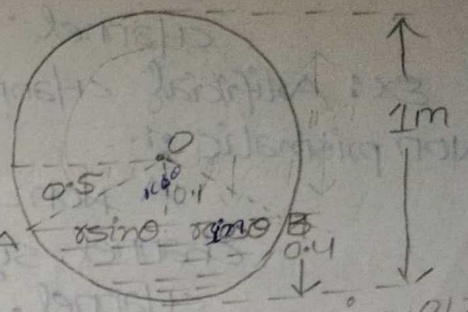
$$0.8 = \frac{(0.0001)^{1/2}}{0.0015} \cdot \frac{\pi r^2}{2} \times \left(\frac{r}{2}\right)^{2/3}$$

$$\frac{0.8 \times 0.0015 \times 2 \times (2)^{2/3}}{\pi \times (0.0001)^{1/2}} = r^{2 + \frac{2}{3}}$$

$$0.121 = r^{\frac{8}{3}}$$

$$r = 1.075 m$$

6) A drinage pipe of 1m diameter conveys water at a depth of 0.4m The pipe is laid at a slope of 1 in 1000. Find discharge; then manning's N = 0.015.



Sol: Given:

Diameter (d) = 1m

Depth (y) = 0.4m

slope (S) = 1/1000

N = 0.015

Radius (r) = 0.5m

Wetted Area (A) = Area of ABOA - Area of AOB

$= r^2\theta - \frac{1}{2} \times 2r\sin\theta \times r\cos\theta$

$(A) = r^2\left(\theta - \frac{\sin 2\theta}{2}\right)$

Wetted perimeter (P) = $2r\theta$

Hydraulic mean radius (R) = $\frac{A}{P}$

$= \frac{r^2\left(\theta - \frac{\sin 2\theta}{2}\right)}{2r\theta}$

$= \frac{r\left(\theta - \frac{\sin 2\theta}{2}\right)}{2\theta}$

$A = (0.5)^2\left(78^\circ 27' \times 0.0174^\circ - \frac{\sin 2(78^\circ 27')}{2}\right)$

$= 0.25(1^\circ 21' - 0.196)$

$A = 0.2885$

$A \simeq 0.29 m^2$

$P = 2r\theta$

$= 2 \times 0.5 \times (78^\circ 27' \times 0.0174^\circ)$

$P = 1.365m$

$\Rightarrow R = \frac{0.29}{1.365}$

$R = 0.212m$

Velocity of flow by manning's formula $(V) = \frac{1}{n} R^{2/3} \times S^{1/2}$

$= \frac{1}{0.015} (0.212)^{2/3} (1/1000)^{1/2}$

$V = 0.749 m/sec$

Discharge (Q) = $A \times V = 0.29 \times 0.749 = 0.217 m^3/sec$

Define prismatic & nonprismatic channel:

Prismatic: A prismatic channel is which has same or definite size & constant slope is called as prismatic channel.

Ex: Artificial channel constructed for Irrigation purpose.

Non prismatic:

A Non-prismatic channel is which has different channel, size & slope is called as nonprismatic channel.

Ex: Lakes.

**Hydraulically economical (or) efficient channel:

Def: A channel is said to be most economical or more efficient, it gives maximum discharge for a given cross sectional Area, given bed slope & roughness coefficient (or) Rugosity coefficient.

We know that; Discharge (Q) = AV [continuity equation]

$Q = AC\sqrt{RS}$ [∵ Chezy's velocity equation]

As per the condition defination to maximum condition for discharge, area, slope & roughness coefficient are constant

∴ Q is directly propotional to Area, perimeter

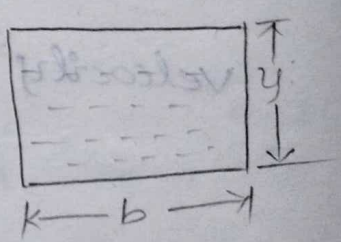
$$Q = \frac{A}{P}$$

$$Q \propto A \;\&\; Q \propto \frac{1}{P}$$

For obtaining maximum discharge wetted perimeter should be minimum.

Derive an expression for economical for Rectangular channel:

Consider a rectangular channel having with 'b' & depth 'y' as shown in figure.



We know; For rectangular channel

Wetted Area (A) = $by \Rightarrow b = \frac{A}{y}$

Wetted perimeter (P) = $b + 2y$

$= \frac{A}{y} + 2y$

For maximum Discharge wetted perimeter should be minimum

$$\frac{dP}{dy} = 0$$

$$\frac{d}{dy}\left(\frac{A}{y} + 2y\right) = 0$$

$$-\frac{A}{y^2} + 2 = 0$$

$$A = 2y^2$$

$$\rightarrow by = 2y^2 \quad [\because A = by$$

$$\boxed{b = 2y}$$

$$\boxed{y = b/2.}$$

Hydraulic Mean radius $(R) = \frac{A}{P} = \frac{by}{b+2y}$

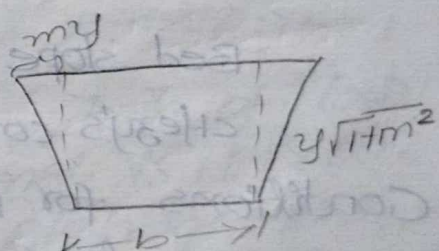
$$= \frac{2y \times y}{2y+2y} = \frac{2y^2}{4y} = \frac{y}{2}$$

$$\boxed{R = \frac{y}{2}}$$

∴ The condition for economical rectangular channel is $R = y/2$.

Expression for most economical Trapezoidal section:

Consider a Trapezoidal channel having width 'b' & depth 'y' & slope m:1 as shown in figure.



We know; For Trapezoidal section

Wetted Area $(A) = (b+my)y \Rightarrow b = \frac{A}{y} - my$

Wetted perimeter $(p) = b + 2y\sqrt{1+m^2}$

$$P = \frac{A}{y} - my + b + 2y\sqrt{1+m^2}$$

For maximum Discharge; $\frac{dP}{dy} = 0$

$$\frac{d}{dy}\left(\frac{A}{y} - my + b + 2y\sqrt{1+m^2}\right) = 0$$

$$-\frac{A}{y^2} - m + 2\sqrt{1+m^2} = 0$$

$$-\frac{(b+my)y}{y^2} - m + 2\sqrt{1+m^2} = 0 \quad [\because A = (b+my)y$$

$$-\frac{(b+my)}{y} - m + 2\sqrt{1+m^2} = 0$$

$$-b - 2my + 2y\sqrt{1+m^2} = 0$$

$$\boxed{b + 2my = 2y\sqrt{1+m^2}}$$

Hydraulic Mean Radius $(R) = \dfrac{(b+my)y}{b+2y\sqrt{1+m^2}}$

$$= \frac{(b+my)y}{b+b+2my}$$

$$= \frac{(b+my)y}{(b+my)2}$$

$$\boxed{R = y/2}$$

(1) A rectangular channel carries water at the rate of 400 lit/sec, when the bed slope is 1 in 2000. Find the most economical dimensions of the channel. If C = 15

Sol: Given:

Discharge $(Q) = 400$ lit/sec $= 0.4\ m^3/sec$

Bed slope $(S) = \frac{1}{2000}$

Chezy's constant $(C) = 15$

Conditions for most economical rectangular channel: $b = 2y$

$$R = y/2$$

Discharge $(Q) = A \times V$

$$= A \times C\sqrt{RS}$$

$$0.4 = by \times C\sqrt{\frac{y}{2}S}$$

$$0.4 = 2y \times y \times C\sqrt{\frac{y}{2}S}$$

$$0.4 = 2y^2 \times C\sqrt{\frac{y}{2}S}$$

$$0.4 = 2y^2 \times 15 \times \sqrt{\frac{y}{2} \times \frac{1}{2000}}$$

$$0.4 = 2y^2 \times 15 \times \sqrt{y^2} \times 0.0158$$

$$0.4 = 0.474 y^4$$

$$y = 0.934 m$$

$$b = 2 \times 0.934 = 1.868$$

(2) The rectangular channel of width 4m having bed slope of 1 in 500. Find the maximum discharge through the channel. $C = 15$

Sol: Given:

Width $(b) = 4m$

Bed slope $(s) = \frac{1}{500}$

$C = 15$

$\Rightarrow b = 2y$

$$y = \frac{b}{2} = \frac{4}{2} = 2m$$

$$\Rightarrow R = \frac{y}{2} = \frac{2}{2} = 1m$$

Discharge $(Q) = A \times V$

$$= 4 \times 2 \times C\sqrt{RS}$$

$$= 8 \times 15 \sqrt{1 \times \frac{1}{500}}$$

$$Q = 5.36 m^3/sec.$$

(3) A rectangular channel 4m wide has depth of flow 1.5m. The slope of bed of the channel is 1 in 1000. $C = 55$. It is desired to increase the discharge to a maximum by changing the dimensions of the sections for constant area of cross-section, slope of the bed & roughness of bed. Find new dimensions, increase in discharge of the channel.

Sol: Given:

width $(b) = 4m$

depth $(y) = 1.5m$

slope $(s) = 1/1000$; $C = 55$

∴ Discharge is not maximum:

(i) Discharge (Q) $= AV$

$$Q = by \times C\sqrt{RS}$$

$$R = \frac{A}{P} = \frac{by}{b+2y} = \frac{4\times1.5}{4+2\times1.5} = 0.857m$$

$$Q = 4\times1.5\times55\sqrt{0.857\cdot\frac{1}{1000}}$$

$$Q = 9.66 m^3/sec.$$

(ii) Discharge is maximum

Discharge (Q') $= AV$

$$= b'y' \times C\sqrt{RS}$$

$$= 2y'y' \times C\sqrt{\frac{y'}{2}S}$$

$$= 2\times y'^2 \times C\sqrt{\frac{y'}{2}S}$$

$$= 2\times(1.73)^2 \times 55\sqrt{\frac{1.73}{2}\times\frac{1}{1000}}$$

$$Q' = 9.711 m^3/sec.$$

$\because A = b'y'$ $\quad b = \frac{y'}{2}$

$G = 2y'y'$ $\quad b = 2y$

$y' = 1.732$ $\quad R = y/2$

Increase in Discharge $= Q' - Q = 9.711 - 9.66$

$= 0.051 m^3/sec.$

(4) A Trapezoidal channel section has side slope one Horizontal, two vertical 1:2 & slope of bed is 1 in 1500. Area of cross section $40m^2$. Find the Dimensions of the section if it is most economial. Determine 'Q' of most economical section. consider Chezy's rugosity coefficient (C) = 150.

Sol: Given: side slope (m) = 1/2

Bed slope (S) $= \frac{1}{1500}$

Area (A) $= 40m^2$

Chezy's constant (C) = 150

Conditions for most economical Trapezoidal section are $b + 2my = 2y\sqrt{1+m^2}$

$$R = y/2$$

Wetted Area (A) $= (b+my)y$

Wetted perimeter (P) $= b + 2y\sqrt{1+m^2}$

Discharge (Q) $= AXV$

$= AXC\sqrt{RS}$

$A = 40$

$(b+my)y = 40$

$(b+\frac{1}{2}y)y = 40$

$(b+0.5y)y = 40$. ——①

→ condition: $b+2my = 2y\sqrt{1+m^2}$

$b+2\cdot\frac{1}{2}y = 2y\sqrt{1+(1/2)^2}$

$b+y = 2.236y$

$b = 2.236y - y$

$b = 1.236y$

put 'b' in ①

①⇒ $(1.236y+0.5y)y = 40$

$1.736y^2 = 40$

$y^2 = 40/1.736$

$\boxed{y = 4.80m}$

⇒ $\boxed{b = 5.93m}$

Top width $= b+2my$

$= 5.93+2\times\frac{1}{2}\times4.8$

$= 10.73m$

Discharge (Q) $= AV$

$Q = 40XC\sqrt{RS}$

Wetted Area (A) $= 40$

Wetted perimeter (p) $= b+2y\sqrt{1+m^2}$

$= 5.89+2\times4.76\sqrt{1+(0.5)^2}$

$P = 15.41$

$R = \frac{A}{P} = 2.595$

→ $Q = 40\times1.5\sqrt{2.595\times1/1500}$

$Q = 80.00m^3/sec$

(5) A trapezoidal channel has side slope 3 Horizontal to 4 vertical ; ie 3:4 ; slope (s) = $\frac{1}{2000}$. Determine the optimum dimensions of the channel if it is so carry water 0.5 m^3/sec ; C = 80.

Sol: Given :

side slope (m) = 3:4

slope (s) = $\frac{1}{2000}$

carry water (Q) = 0.5 m^3/sec

C = 80

Conditions : $R = y/2$

$b + 2my = 2y\sqrt{1+m^2}$

Wetted Area (A) = $(b+my)y$

Wetted perimeter (p) = $b + 2y\sqrt{1+m^2}$

$\Rightarrow b + 2my = 2y\sqrt{1+m^2}$

$b + 2(\frac{3}{4})y = 2y\sqrt{1+(\frac{3}{4})^2}$

$b + 1.5y = 2.5y$

$b = 2.5y - 1.5y$

$\boxed{b = y}$

$\Rightarrow$ Area (A) = $(by + my)y$

$= (by + \frac{3}{4}y)y$

$= byy + \frac{3}{4}y^2$

$= byy + 0.75y^2$

$= byxy + 0.75y^2$

$A = 1.75y^2$

$1.75b^2$

$(p) = b + 2y\sqrt{1+m^2}$

$= b + 2b\sqrt{1+(\frac{3}{4})^2}$

$= b + 2.5b$

$P = 3.5b$

Discharge (Q) = AV

$0.5 = 1.75y^2 \times C\sqrt{RS}$

$= 1.75y^2 \times 80\sqrt{\frac{y}{2} \times \frac{1}{2000}}$

$0.5 = 2.21y^2 \times \sqrt{y}$

$y^2 \times y^{1/2} = 0.226$

$y^2 = 0.566m$

$\boxed{y = 0.56m}$

$\boxed{b = 0.56m}$

Top width $= b + 2my$

$= 0.56 + 2\left(\frac{3}{4}\right)(0.56)$

$= 1.4m$

(6) A Trapezoidal channel with side slope of 1:1 has to be designed to convey $10m^3/sec$ at a velocity of 2m/sec, so that the Amount of concrete lining for the bed & the side slopes is minimum. Calculate the Area of lining required for 1m length of channel.

Sol: Given:

Side slope (m) = 1:1

Velocity (v) = 2m/sec.

Discharge (Q) = $10m^3/sec.$

length (l) = 1m

Conditions for Trapezoidal channel: $R = y/2$,

$b + 2my = 2y\sqrt{1+m^2}$

Area of lining = perimeter x length

$= b + 2y\sqrt{1+m^2} \times 1$

$= b + 2y\sqrt{1+m^2}$ ——— (1)

Discharge (Q) = A x V

$10 = A \times 2$

$\boxed{A = 5m^2}$

Area (A) $= (b + my)y$

$5 = (b + 1y)y$

$5 = by + y^2$ ——— (2)

condition:

$b + 2my = 2y\sqrt{1+m^2}$

$b + 2y = 2y \times 1.414$

$b + 2y = 2.828y$

$b = 0.828y$

Put 'b' in ② = $5 = by + y^2$

$5 = (0.828)y + y^2$

$5 = 1.828y^2$

$y^2 = 2.735$

$y = 1.65m$ $\Rightarrow$ $b = 0.828 \times 1.65$

$b = 1.36m$

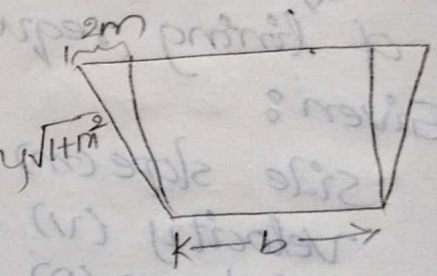
① Area $= b + 2y\sqrt{1+m^2}$

$= 1.36 + 2 \times 1.65\sqrt{1+1}$

Area of lining $= 6.02 m^2$

Derive the condition for best side slope in case of most economical Trapezoidal channel section.

For Trapezoidal channel section;



Area $(A) = (b+my)y$

perimeter $(p) = b + 2y\sqrt{1+m^2}$

$\Rightarrow A = (b+my)y$

$\frac{A}{y} = b + my$

$b = \frac{A}{y} - my.$

put 'b' in perimeter

$\Rightarrow P = \frac{A}{y} - my + 2y\sqrt{1+m^2}$

most economical channel;

$\frac{dP}{dm} = 0$

$\frac{d}{dm}\left(\frac{A}{y} - my + 2y\sqrt{1+m^2}\right) = 0$

$-y + \frac{2y}{2\sqrt{1+m^2}}(2m) = 0$

$-y\sqrt{1+m^2} + 2my = 0$

$2my = y\sqrt{1+m^2}$

$2m = \sqrt{1+m^2}$

squaring on both sides

$4m^2 = 1 + m^2$

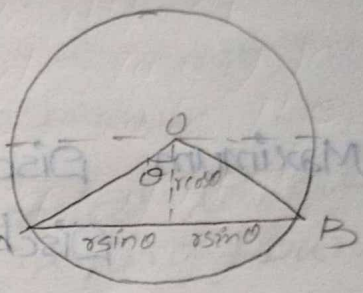
$$3m^2 = 1$$

$$m = \pm\sqrt{1/3}$$

Derive the condition for most economical circular channel section:

In the circular channel section as the depth of flow varies due to the convergence of boundaries towards top. so in this case for estimating the most economical conditions we can derive the expressions for maximum velocity, Maximum discharge criteria.

(i) Maximum velocity;



We know that;

Velocity of flow (V) = $C\sqrt{RS}$ (∵ Chezy's)

$\Rightarrow V \propto \frac{A}{P}$

Wetted Area (A) = $r^2(\theta - \frac{\sin 2\theta}{2})$ (∵ problem-6)

Wetted perimeter (P) = $2r\theta$.

Hydraulic mean radius (R) = $\frac{A}{P} = \frac{r(\theta - \sin\frac{2\theta}{2})}{2\theta}$.

In the circular channel section as the depth of flow channel section, Wetted Area & Wetted perimeter changes for obtaining maximum velocity.

Differentiating "R" with respect to 'θ'.

$$\frac{d}{d\theta}\left(\frac{A}{P}\right) = 0$$

$$P \cdot \frac{dA}{d\theta} - A \cdot \frac{dP}{d\theta} = 0$$

$$2r\theta \cdot \frac{d}{d\theta}\left(r^2(\theta - \frac{\sin 2\theta}{2})\right) - \left(r^2\theta - \frac{r^2\sin 2\theta}{2}\right) \cdot \frac{d}{d\theta}(2r\theta) = 0$$

$$2r\theta \cdot \left(r^2 - \frac{r^2\cos 2\theta}{2} \cdot 2\right) - \left[r^2\theta - \frac{r^2\sin 2\theta}{2} \cdot (2r\right] = 0$$

$$r^3(2\theta - 2\cos 2\theta) - r^3(2\theta - \sin 2\theta) = 0$$

$$r^3(2\theta - 2\cos 2\theta - 2\theta + \sin 2\theta) = 0$$

$$-2\theta\cos 2\theta + \sin 2\theta = 0$$

$$\tan 2\theta = 2\theta.$$

$\theta = 127.8°$.

from calci
$\tan 2\theta = 2\theta$
$\theta = 2.246$ rad
$1° = 0.0174$
$1° = 57°9'$
$\theta = 127.8$

Depth of flow $(y) = r(1-\cos\theta)$

$= r(1-\cos 127°8')$

$\boxed{y = 0.89r}$

Hydraulic Mean radius $(R) = \dfrac{r(\theta - \sin\frac{2\theta}{2})}{2\theta}$

$= \dfrac{r(2.246 - \frac{\sin(2\times 127.8')}{2})}{2\times 2.246}$

$R = r(2.409)$

$\boxed{R = 0.606r}$

(iii) Maximum Discharge:

Discharge $(Q) = A \times V$

$= A \times C\sqrt{RS}$

$Q \propto \dfrac{A^3}{P}$

$\dfrac{d}{d\theta}\left(\dfrac{A^3}{P}\right) = 0$

$P \cdot \dfrac{d}{d\theta}(A^3) - A^3 \dfrac{dP}{d\theta} = 0$

$P \cdot (3A^2)\dfrac{dA}{d\theta} - A^3 \cdot \dfrac{dP}{d\theta} = 0$

$A^2\left(3P \cdot \dfrac{dA}{d\theta} - A \cdot \dfrac{dP}{d\theta}\right) = 0$

$3(2r\theta) \cdot \dfrac{d}{d\theta}\left(r^2\left(\theta - \dfrac{\sin 2\theta}{2}\right)\right) - A \cdot \dfrac{d}{d\theta}(2r\theta) = 0$

$6r\theta \cdot \dfrac{d}{d\theta}\left(r^2 - r^2\dfrac{\cos 2\theta}{2}\cdot 2\right) - A\cdot(2r) = 0$

$6r\theta \cdot \dfrac{d}{d\theta}(r^2 - r^2\cos 2\theta) - r^2\left(\theta - \dfrac{\sin 2\theta}{2}\right)2r = 0$

$r^3(6\theta - 6\cos 2\theta) - r^3(2\theta - \sin 2\theta) = 0$

$6\theta - 6\theta\cos 2\theta - 2\theta + \sin 2\theta = 0$

$4\theta - 6\theta\cos 2\theta + \sin 2\theta = 0$

$\theta = 154°$.

(9) Depth of flow $(y) = r(1-\cos\theta)$

$= r(1-\cos 154^\circ)$

$\boxed{y = 1.899r}$

Hydraulic Mean radius $(R) = \dfrac{r\left(\theta - \frac{\sin 2\theta}{2}\right)}{2\theta}$

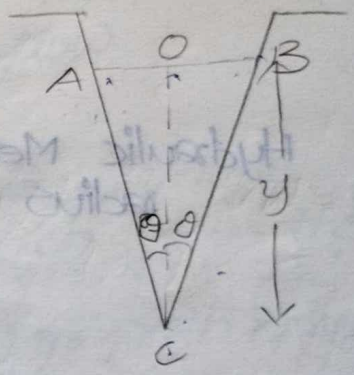
$= \dfrac{r\left(154^\circ \times 0.0174^\circ - \frac{\sin 154^\circ}{2}\right)}{2 \times (154^\circ \times 0.0174^\circ)}$

$\boxed{R = 0.481r}$

$\Rightarrow R = 0.5r$

Derive the condition for most economical triangular channel section: A triangular channel of depth of flow.

For a most economical triangular channel section;

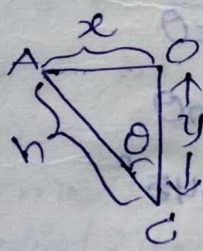


Wetted Area $(A) = \frac{1}{2} AB \times BC$

$= \frac{1}{2}(AO+OB) \times BC$

$= \frac{1}{2}(AO+OB)\,y$

From $\triangle le$ AOC;



$\tan\theta = \frac{x}{y}$

$x = y\tan\theta$

$AO = y\tan\theta$

$AO+OB = 2y\tan\theta$

$= \frac{1}{2} \times 2y\tan\theta \times y$

Wetted Area $(A) = y^2\tan\theta \Rightarrow y = \sqrt{\frac{A}{\tan\theta}}$

Wetted perimeter $(P) = AC + BC$.

$P = 2y\sec\theta$

$= 2 \times \sqrt{\frac{A}{\tan\theta}} \times \sec\theta$

From $\triangle le$ AOC;

$\cos\theta = \frac{y}{h}$

$y = h\cos\theta$

$h = y\sec\theta$

$$\frac{dP}{d\theta} = 0$$

$$\frac{d}{d\theta}\left(2\sqrt{\frac{A}{\tan\theta}} \cdot \sec^2\theta\right) = 0$$

$$2\sqrt{A} \cdot \frac{d}{d\theta}\left(\frac{1}{\sqrt{\sin\theta \cdot \cos\theta}}\right) = 0$$

$$\frac{d}{d\theta}\left(\sqrt{\operatorname{cosec}\theta} \cdot \sqrt{\sec\theta}\right) = 0$$

$$\sqrt{\operatorname{cosec}\theta} \cdot \frac{\sec\theta \cdot \tan\theta}{2\sqrt{\sec\theta}} - \sqrt{\sec\theta} \cdot \frac{\sqrt{\operatorname{cosec}\theta}\cot\theta}{2\sqrt{\operatorname{cosec}\theta}} = 0$$

$\because \tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sec^2\theta = \frac{1}{\cos^2\theta}$

$\because \frac{1}{\sqrt{\sin\theta}} = \sqrt{\operatorname{cosec}\theta}$

$\frac{1}{\sqrt{\cos\theta}} = \sqrt{\sec\theta}$

$$\frac{\sqrt{\operatorname{cosec}\theta} \cdot \sqrt{\sec\theta}}{2} \cdot \tan\theta - \frac{\sqrt{\sec\theta} \cdot \sqrt{\operatorname{cosec}\theta}}{2} \cdot \cot\theta = 0$$

$$\frac{\sqrt{\operatorname{cosec}\theta} \cdot \sqrt{\sec\theta}}{2}(\tan\theta - \cot\theta) = 0$$

$$\tan\theta = \cot\theta$$

$$\theta = 45^\circ$$

Hydraulic Mean radius $(R) = \frac{A}{P}$

$$= \frac{y^2 \tan\theta}{2y\sec\theta}$$

$$= \frac{y\tan\theta}{2\sec\theta}$$

$$= \frac{y}{2} \cdot \sin\theta$$

$$R = \frac{y}{2}\sin 45^\circ$$

$$R = \frac{y}{2\sqrt{2}}$$

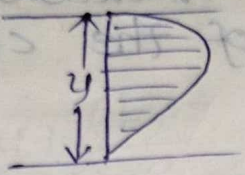
$$\boxed{R = \frac{y}{2.82}}$$

Write a note on velocity distribution in open channels:

Velocity distribution in open channels is

→ non uniform due to the free surface is present in open channels & frictional resistance.

→ The maximum velocity across the channel is just below the top surface. i.e $V_{max} = 0.05$ to $0.15y$



$V_{max} = 0.05 - 0.15y$

→ The velocity distribution depends upon the shape of channel section, Roughness coefficient, Rogosity coefficient, presence of beds / curves.

→ Mean velocity (or) Average velocity in open channels is estimated by ~~two~~ (point) methods.

(i) one point method: In this method the velocity is at 0.6 to the depth of flow

$$V_{avg} = V_{0.6y}$$

(ii) TWO point method: In this method the Average velocity by the following formula, which gives the Accurate results

$$V = \frac{V_{0.2y} + V_{0.8y}}{2}$$; Where; y = Depth of flow

→ Velocity distribution in open channels is measured by current meters, pitot tube.

→ For the measurement of velocity two correction factors to be applied. That are

(i) Kinetic energy correction factor

$$\alpha = \frac{\int V^3 dA}{}$$

It's Limits varies from 1.03 to 1.36

(ii) momentum correction factor

$$\beta = \frac{\int V^2 . dA}{V^2 A}$$

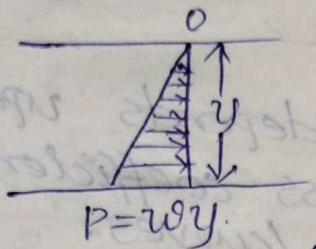
It's Limits varies from 1.01 to 1.13

pressure distribution in open channels:

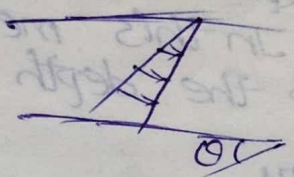
In static fluid pressure can be estimated by

$$\text{pressure } (p) = \rho g h$$
$$= \rho g y$$
$$p = w y$$

In case of open channels; pressure distribution is as follows; i.e o at the top & it increases with depth of flow and maximum at the bottom of the channel.

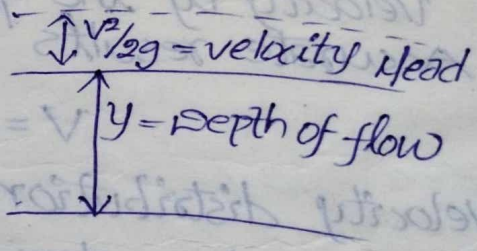


Whenever the channel bed makes an Angle 'θ' with the datum line the pressure can be estimated by $P = w y^2 \cos\theta$



Specific energy:

The total energy per unit weight of fluid with Horizontal Datum $= y + \frac{v^2}{2g} + z$



If we consider the channel bottom as datum, the total energy per unit weight of fluid gives the Specific weight of energy.

$$V = y + \frac{v^2}{2g}$$

Where; y = potential energy of flow

$v^2/2g$ = Kinetic energy of flow

$$\Rightarrow E = E_p + E_k$$

$$E = y + \frac{V^2}{2g}$$

from continuity equation; $V = Q/A$.

In case of rectangular channel section; $A = by$.

$$\Rightarrow V = \frac{Q}{by}$$

$$V = \frac{q}{y} \quad \left[\because \frac{Q}{b} = q = \text{Discharge for unit width}\right]$$

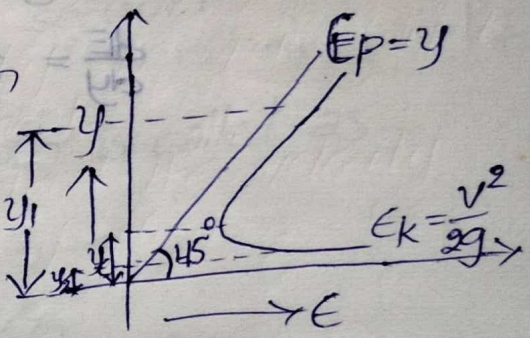
$$E = y + \frac{(q/y)^2}{2g}$$

$$\Rightarrow \boxed{E = y + \frac{q^2}{2gy^2}}$$

Specific energy curve;

It is a graphical plot between specific energy & depth of flow.

The features of specific energy curve are (i) the curve for potential energy is a straight line which makes an angle 45° with the origin.



Specific energy Curve

(ii) The curve for kinetic energy 'E_k' is assymptitos to the 45° line & represents a parabola.

(iii) By combining two lines gives the specific energy diagram & there it has two depths y_1 & y_2 represented in the diagram.

Where; y_1 = subcritical depth of flow

y_2 = supercritical depth of flow

y_c = critical depth; i.e the depth at which the specific energy is Minimum.

(iv) If the specific energy is same at y_1, y_2 then those depths are called Alternative depths.

Derive the condition for critical depth or derive the general equation for critical flow:

From the specific energy equation;

$$E = y + \frac{V^2}{2g}$$

Discharge $Q = AV$

$$V = \frac{Q}{A}$$

$$E = y + \frac{Q^2}{2gA^2}$$

For critical depth specific energy is minimum

$$\frac{dE}{dy} = 0$$

$$\frac{dE}{dy} = \frac{d}{dy}\left(y + \frac{Q^2}{2gA^2}\right) = 0$$

$$1 + \frac{d}{dy}\left(\frac{Q^2}{2gA^2}\right) = 0$$

$$1 + \frac{Q^2}{2g}\left(\frac{-2}{A^3} \cdot \frac{dA}{dy}\right) = 0$$

$$1 - \frac{Q^2}{gA^3} \cdot \frac{dA}{dy} = 0$$

$$1 - \frac{Q^2}{gA^3} \cdot d = 0$$

$$1 - \frac{Q^2 T}{gA^3} = 0$$

$$\frac{Q^2}{g} = \frac{A^3}{T} \quad (\because \text{General equation})$$

Critical depth for rectangular section:

∴ From the specific energy equation.

$$E = y + \frac{q^2}{2gy^2} \qquad \left[\because Q = AV,\ V = \frac{Q}{A} = \frac{Q}{by} = \frac{q}{y}\right]$$

$$\frac{dE}{dy} = 0$$

$$\frac{d}{dy}\left(y + \frac{q^2}{2gy^2}\right) = 0$$

$$1 + \frac{v^2}{2g}\left(\frac{-2}{y^3}\right) = 0$$

$$1 - \frac{v^2}{gy^3} = 0$$

$$y_c^3 = \frac{v^2}{g}$$

$$\therefore y_c = (v^2/g)^{1/3}$$

$\therefore$ critical depth for rectangular section is $y_c = \left(\frac{q^2}{g}\right)^{1/3}$

Derive the equation for critical velocity in case of rectangular channel section:

We know; For rectangular channel section

$$y_c = (q^2/g)^{1/3}$$

THE velocity of flow at critical depth is called as critical velocity.

We know that; $Q = AV$

$$V_c = \frac{Q}{A_c}$$

Area of the section $A = b \times y_c$

$$V_c = \frac{Q}{b \times y_c}$$

$$V_{c_1} = \frac{q_1}{y_c} \quad \left[\because \frac{Q}{b} = q_1\right]$$

$$y_{c_1} = \frac{q_1}{V_{c_1}}$$

$$\Rightarrow V_{c_1} = \frac{q_1}{(q^2/g)^{1/3}} \quad \left[\because y_c = \left(\frac{q_1^2}{g}\right)^{1/3}\right]$$

$$= \frac{q_1 \times (g)^{1/3}}{(q_1^2)^{1/3}}$$

[$\because$ Apply cube on B.S]

$$V_c^3 = \frac{q^3 \times g}{q^2}$$

$$V_c^3 = \frac{g q^3}{q^2}$$

$$V_c^3 = q^2 g \qquad \left[\because V_c = \frac{q}{y_c}\right]$$

$$V_c^3 = V_c \times y_c g$$

$$V_c^2 = g y_c$$

$$V_c = \sqrt{g y_c}$$

$$\frac{V_c}{\sqrt{g y_c}} = 1 \qquad \left[\because Fr = \frac{V_c}{\sqrt{g y_c}}\right]$$

froude Num

$$\boxed{Fr = 1}$$

Show that for critical flow the froude Number is equal to 1.

Sol: From General equation of specific energy

$$\frac{Q^2}{g} = \frac{A^3}{T}$$

$$\Rightarrow Q = AV$$

$$\frac{A^2 V^2}{g} = \frac{A^3}{T}$$

$$\frac{V^2}{g} = \frac{A}{T}$$

$$V^2 = g \cdot \frac{A}{T}$$

$$V^2 = g y$$

$$V = \sqrt{g y}$$

$$\frac{V}{\sqrt{g y}} = 1$$

$$Fr = 1$$

Derive the minimum specific energy equation in case of rectangular channels:

Energy equation $E = y + \frac{V^2}{2g}$

$Q = AV$ [∵ continuity equation]

$\therefore V = \frac{Q}{A}$

In case of rectangular channels

Wetted Area $(A) = by$

specific energy $\epsilon = y + \frac{V^2}{2g}$

$= y + \frac{Q^2}{A^2 2g}$

$= y + \frac{Q^2}{b^2 y^2 2g}$ $= y + \frac{q^2}{2gy^2}$ $[\because \frac{Q}{b} = q]$

We know; for rectangular channel $y_c = (\frac{q^2}{g})^{1/3}$

$\Rightarrow y_c^3 = \frac{q^2}{g}$

For minimum specific energy $\epsilon_{min} = y_c + \frac{q^2}{2g y_c^2}$

$= y_c + \frac{y_c^3}{2y_c^2}$

$\epsilon_{min} = \frac{3y_c}{2}$

Derive the condition for maximum discharge for rectangular channel for a given specific energy:

Specific energy equation $E = y + \frac{V^2}{2g}$

$V = \frac{Q}{A}$ [∵ continuity equation]

$V = \frac{Q}{by}$

$\Rightarrow E = y + \frac{Q^2}{b^2 y^2 \cdot 2g}$

$= y + \frac{q^2}{2gy^2}$

$$E - y = \frac{Q^2}{2gb^2y^2}$$

$$Q^2 = (E-y)(2gb^2y^2)$$

$$Q = \sqrt{(E-y)(2gb^2y^2)}$$

$$Q = \sqrt{2gb^2(y^2)(E-y)}$$

$$\frac{dQ}{dy} = 0$$

$$\frac{d}{dy}(y^2(E-y)) = 0$$

$$\frac{d}{dy}(Ey^2 - y^3) = 0$$

$$2Ey - 3y^2 = 0$$

$$2E = 3y$$

$$E = \frac{3}{2}y$$

→ The condition for critical flow ; $fr = 1$

$$\Rightarrow \quad fr = \frac{V}{\sqrt{gD}} = 1$$

→ Specific energy is minimum for a given Discharge.

→ Discharge is Maximum for a given specific energy $E = 3/2\,y$.

→ The specific force is Minimum for a given discharge

→ Discharge is Maximum for a given specific force.

1) The specific energy in a rectangular channel is 3NM-N. calculate the critical depth & also calculate the Maximum discharge than one occur if width of the channel is 6m.

Sol: Given data:

$E = 3\ N\text{-}m/N$

$b = 6m$

$y_c = ?$

$Q = ?$

condition for rectangular channels are $y_c = \left(\frac{q^2}{g}\right)^{1/3}$

$E = \frac{2}{3} y_c$

$\Rightarrow E = \frac{2}{3} y_c$

$y_c = \frac{2E}{3} = \frac{2 \times 3}{3} = 2m$

$y_c = \left(\frac{q^2}{9.81}\right)^{1/3}$

$2 = \left(\frac{q^2}{9.81}\right)^{1/3}$

$q = 8.85\ m^2/sec$

$q = Q/b$

$Q = q \times b$

$= 8.85 \times 6$

$Q = 53.1\ m^3/sec$

2) Find the critical depth & critical velocity of water flowing through a rectangular channel of width 5m, discharge $Q = 15\ m^3/sec$.

Sol: Given: $b = 5m$

$Q = 15 m^3/sec$

conditions: $y_c = \left(\frac{q^2}{g}\right)^{1/3}$

$E = 2/3\ y_c$

We know; $q = Q/b = 15/3 = 3 m^2/sec$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$= \left(\frac{3^2}{9.81}\right)^{1/3}$$

$$y_c = 0.97 m$$

$$y_c \simeq 1 m$$

$f_r = 1$

$$\frac{V_c}{\sqrt{g y_c}} = 1$$

$$V_c = \sqrt{g y_c}$$

$$= \sqrt{9.81 \times 1}$$

$$V_c = 3.13$$

∴ critical depth = 1m

critical velocity = 3.13 m/sec

3) The Discharge of water through a rectangular channel of width 8m is $15 m^3/sec$. When the depth of flow of water is 1.2m. calculate.

(i) specific energy of flowing water.

(ii) critical velocity, critical depth.

(iii) value of minimum specific energy.

Given:

$b = 8$

$Q = 15 m^3/sec$

$y = 1.2$

(i) specific energy $(E) = y + \frac{V^2}{2g}$

$$= y + \frac{q^2}{2gy^2}$$

$\Rightarrow q = \frac{Q}{b} = \frac{15}{8} = 1.87 m^2/sec$

$$E = 1.2 + \frac{(1.87)^2}{2 \times 9.81 \times (1.2)^2}$$

$$E = 1.2 + 0.123$$

$$E = 1.324$$

(ii) critical depth; critical velocity:

$$\text{critical depth} = \left(\frac{q^2}{g}\right)^{1/3}$$

$$y_c = \left(\frac{1.87^2}{9.81}\right)^{1/3}$$

$$\boxed{y_c = 0.709m}$$

$$f_r = 1$$

$$\frac{v_c}{\sqrt{g y_c}} = 1$$

$$v_c = \sqrt{g y_c}$$

$$= \sqrt{9.81 \times 0.709}$$

$$\boxed{v_c = 2.63 m/sec}$$

(iii) Minimum specific energy $(E) = \frac{3}{2} y_c$

$$= \frac{3}{2} \times 0.709$$

$$\boxed{E = 1.0635}$$

4) The specific energy for a 5m wide rectangular channel is to be 4N-m/m. If the rate of flow of water through the channel is $20 m^3/sec$. Determine Alternate depth of flow.

Sol: Given: $b = 5m$

$E = 4N\text{-}m/m$

$Q = 20 m^3/sec$

From specific energy equation $E = y + \frac{V^2}{2g}$.

$y_1 = 3.94$
$y_2 = -0.42$
$y_3 = 0.48$

$$V = \frac{Q}{A} = \frac{Q}{by} = \frac{20}{5y} = \frac{4}{y}$$

sq. on B.S

$$\Rightarrow V^2 = \frac{16}{y^2}$$

$$\Rightarrow E = y + \frac{V^2}{2g}$$

$$4 = y + \frac{16}{y^2(2\times 9.81)}$$

$$4 - y + \frac{0.815}{y^2}$$

$$4y^2 = y^3 + 0.815$$

$$y^3 - 4y^2 + 0.815 = 0$$

$$y_1 = 3.94m \; ; \; y_2 = 0.48m$$

5) A rectangular channel has a width of 1.8m & carries a discharge $1.8 m^3/sec$ at a depth of 0.2m. calculate (i) specific energy
(ii) Depths Alternate to existing depth
(iii) Froude Number at Alternate depths.

Given:

$b = 1.8m$
$Q = 1.8 m^3/sec$
$y = 0.2m$

$$A = by = 1.8 \times 0.2 = 0.36$$

$$\Rightarrow Q = AV$$

$$V = \frac{Q}{A}$$

$$= \frac{1.8}{0.36}$$

$$V = 5 m/sec$$

(i) specific energy:

$$E = y + \frac{V^2}{2g}$$

$$= 0.2 + \frac{5^2}{2 \times 9.81}$$

$$E = 1.474$$

(ii) Depths Alternate to existing depth:

$$V = \frac{Q}{A}$$

$$V = \frac{1.8}{1.8y} = \frac{1}{y}$$

$$E = y + \frac{V^2}{2g}$$

$$= y + \frac{V^2}{2g}$$

$$1.45 = y + \frac{(1/y)^2}{2g}$$

$$1.45 = y + \frac{1}{2 \times 9.81} \cdot \frac{1}{y^2}$$

$$1.45 = y + \frac{0.050}{y^2}$$

$$1.45y^2 = y^3 + 0.050$$

$$y^3 - 1.45y^2 + 0.050 = 0$$

$$y_1 = 1.42 \; ; \; y_2 = 0.2$$

(iii) Froude Numbers at Alternate depths:

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}} \qquad V_1 = \frac{1.8}{1.8 \times y_1}$$

$$= \frac{0.704}{\sqrt{9.81 \times 1.42}} \qquad = \frac{1.8}{1.8 \times 1.42}$$

$$V_1 = 0.704 \text{ m/sec}$$

$$Fr_1 = 0.188$$

$\rightarrow$ subcritical

$$Fr_2 = \frac{V_2}{\sqrt{gy_2}}$$

$$= \frac{5}{\sqrt{9.81 \times 0.2}}$$

$$Fr_2 = 3.56$$

$\Rightarrow$ supercritical

$$V_2 = \frac{Q}{A_2}$$

$$= \frac{Q}{b \times y_2}$$

$$= \frac{1.8}{1.8 \times 0.2}$$

$$V_2 = 5 \text{ m/sec}$$

6) If y_1, y_2 are the Alternate depths; show that $y_c^3 = \frac{2(y_1 y_2)^2}{y_1 + y_2}$ and specific energy $\epsilon = \frac{y_1^2 + y_1 y_2 + y_2^2}{y_1 + y_2}$

Sol:

$E_1 = E_2$ for Alternate depths

$$E_1 = y_1 + \frac{V_1^2}{2g} \; ; \; E_2 = y_2 + \frac{V_2^2}{2g}$$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$\Rightarrow V_1 = \frac{Q}{A_1} = \frac{Q}{by_1} = \frac{q}{y_1}$$

$$\Rightarrow V_2 = \frac{q}{y_2}$$

$$y_1 + \frac{(q/y_1)^2}{2g} = y_2 + \frac{(q/y_2)^2}{2g}$$

$$\frac{q^2}{2gy_1^2} - \frac{q^2}{2gy_2^2} = y_2 - y_1$$

$$\frac{q^2}{2g}\left[\frac{1}{y_1^2} - \frac{1}{y_2^2}\right] = y_2 - y_1$$

$$\left[\because \left(\frac{q^2}{g}\right)^{1/3} = y_c\right]$$

$$\frac{y_c^3}{2}\left[\frac{1}{y_1^2} - \frac{1}{y_2^2}\right] = y_2 - y_1$$

$$y_c^3 = \frac{2(y_1 y_2)^2}{y_2 + y_1}$$

$$\Rightarrow E_1 = y_1 + \frac{V_1^2}{2g}$$

$$V_1 = \frac{Q}{A_1} \rightarrow \frac{Q}{by_1} \Rightarrow \frac{q}{y_1}$$

$$V_1 = \frac{q}{y_1}$$

$$E_1 = y_1 + \frac{q^2}{2gy_1^2}$$

$$= y_1 + \frac{y_c^3}{2y_1^2} \quad \left[\because y_c^3 = \frac{q^2}{g}\right]$$

$$= y_1 + \frac{2(y_1 y_1)^2/y_2 + y_1}{2y_1^2}$$

$$= y_1 + \frac{2y_1^2 y_2^2}{2y_1^2(y_2 + y_1)}$$

$$= \frac{y_1(y_2 + y_1) + y_2^2}{y_2 + y_1}$$

$$= \frac{y_1 y_2 + y_1^2 + y_2^2}{y_1 + y_2}$$

$$\therefore E_1 = \frac{y_1^2 + y_2^2 + y_1 y_2}{y_1 + y_2}$$

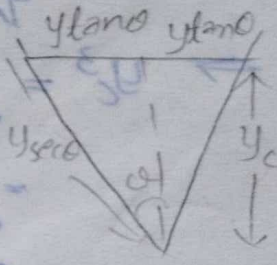
7) For a triangular channel having vertex angle of 120°. calculate critical depth for a discharge of 3m³/sec.

Sol: Given: Angle $\theta = 120^\circ$

Discharge $(Q) = 3m^3/sec$.

From; General equation of specific gravity

$$\frac{Q^2}{g} = \frac{A^3}{T}$$



$$A = \frac{1}{2} y_c \tan\theta \times y_c$$

$$A = y_c^2 \tan\theta.$$

$$\frac{3^2}{9.81} = \frac{(y_c^2 \tan\theta)^3}{2y_c \tan\theta}$$

$$\frac{9}{9.81} = \frac{y_c^6 \tan\theta^3}{2y_c \tan\theta}$$

$$0.917 = y_c^5 \tan\theta^2$$

$$0.917 = y_c^5 \tan 60^2$$

$$\frac{0.917}{3} = y_c^5$$

$$y_c^5 = 0.305$$

$$y_c = 0.951m$$

(8) Calculate y_c corresponding to a discharge of $6m^3$ per second.

(i) Rectangular channel of ~~size~~ width 3m

(ii) In a triangular channel of side slope 1.5 Horizontal to one Vertical.

(iii) In a trapezoidal channel of bottom width 2m & side slope one Horizontal one vertical.

Sol:- (i) Given:

$$Q = 6m^3/sec$$

$$b = 3m$$

From the condition of rectangular channel

$$y_c^3 = (q^2/g)$$

$$q = \frac{Q}{b}$$

$$= \frac{6}{3}$$

$$q = 3$$

$$\Rightarrow y_c^3 = \frac{2^2}{9.81}$$

$$= \frac{4}{9.81}$$

$$= 0.407$$

$$y_c = \sqrt{0.407} = 0.74m$$

(ii) Given: $m:1 = 1:5$

$m = 1.5$

from General specific energy equation;

$$\frac{Q^2}{g} = \frac{A^3}{T}$$

$$\Rightarrow A = \frac{1}{2} \times 2my_c \times y_c$$

$$A = my_c^2$$

~~$= 1.5 \times (0.97)^2$~~

$$A = 1.5y_c^2$$

$$T = 2my_c = 2 \times 1.5y_c = 3y_c$$

~~$= 2 \times 1.5 \times 0.97$~~

$$\Rightarrow \frac{(6)^2}{9.81} = \frac{(1.5y_c^2)^3}{3y_c}$$

$$\frac{36}{9.81} = \frac{(1.5y_c^3)}{3}$$

$$108 = 14.71y_c^3$$

$$y_c^3 = 7.34$$

$$y_c =$$

$$\frac{Q^2}{g} = \frac{A^3}{T}$$

$$\frac{6^2}{9.81} = \frac{(my_c^2)^3}{2my_c}$$

$$\frac{36}{9.81} = \frac{m^3y_c^6}{2my_c}$$

$$\frac{36}{9.81} = \frac{m^2y_c^5}{2}$$

$$36 = 9.81 \times 1.125 y_c^5$$

$$\frac{36}{11.036} = y_c^5$$

$$y_c = 1.26m$$

(iii) $b = 2$; $m = 1$

$$A = (b + my_c)y_c$$

$$T = b + 2my_c$$

$$\frac{Q^2}{g} = \frac{(b+my_c)y_c)^3}{b+2my_c}$$

$$\frac{6^2}{9.81} = \frac{((b+my_c)y_c)^3}{b+2my_c}$$

$$\frac{36}{9.81} = \frac{((b+my_c)y_c)^3}{b+2my_c}$$

$\Rightarrow (b+my_c)y_c = (2+y_c)y_c = 2y_c + y_c^2$

$((b+my_c)y_c)^3 = (y_c^2+2y_c)^3$

$T = b+2my_c = 2+2y_c$

$$\frac{36}{9.81} = \frac{y_c^6 + 2y_c^3}{2+2y_c}$$

$$\frac{36}{9.81} = \frac{y_c^3(y_c^3+2)}{2(1+y_c)}$$

$72(1+y_c) = 9.81 y_c^3 (y_c^3+2)$

$72+72y_c = 9.81y_c^6 + 19.62y_c^3$

$9.81y_c^6 + 19.62y_c^3 - 72y_c - 72 = 0$

$y_{c1} = 0.84.$

Derive the expression $\Delta E = \frac{(y_2-y_1)^3}{4y_1y_2}$.

$\because E = y + \frac{v^2}{2g}$

$\Delta E = E_2 - E_1$

$= y_2 + \frac{v_2^2}{2g} - y_1 - \frac{v_1^2}{2g}$

$= y_2 - y_1 + \frac{1}{2g}(v_2^2 - v_1^2)$

$= y_2 - y_1 + \frac{1}{2g}\left(\frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2}\right)$

$= y_2 - y_1 + \frac{Q^2}{2g}\left(\frac{Q^2}{b^2y_2^2} - \frac{Q^2}{b^2y_1^2}\right)$

$$\Delta E = y_2 - y_1 + \frac{1}{2g}\left(\frac{q^2}{y_2^2} - \frac{q^2}{y_1^2}\right)$$

$$= y_2 - y_1 + \frac{q^2}{2g}\left(\frac{1}{y_2^2} - \frac{1}{y_1^2}\right)$$

$$= y_2 - y_1 + \frac{y_c^3}{2}\left(\frac{y_1^2 - y_2^2}{(y_1 y_2)^2}\right)$$

$\because y_c^3 = \frac{2(y_1 y_2)^2}{y_1 + y_2}$

$$= y_2 - y_1 + \frac{2(y_1 y_2)^2}{2(y_1 + y_2)}\left(\frac{y_1^2 - y_2^2}{(y_1 y_2)^2}\right)$$

$$= y_2 - y_1 + \frac{(y_1^2 - y_2^2)(y_1 y_2)^2}{(y_1 y_2)^2 (y_1 + y_2)}$$

$\frac{2q^2}{g} = y_1 + y_2 \quad (y_1 y_2)$

$$= y_2 - y_1 + \frac{y_1 y_2 (y_1 + y_2)}{4}\left(\frac{1}{y_2^2} - \frac{1}{y_1^2}\right)$$

$\frac{q^2}{g} = \frac{(y_1 + y_2) y_1 y_2}{2}$

$$= (y_1 - y_2) + \frac{y_1 y_2 (y_1 + y_2)}{4}\left(\frac{y_2^2 - y_1^2}{(y_1 y_2)^2}\right)$$

$$\Delta E = \frac{4(y_1 - y_2)}{1} + \frac{(y_1 + y_2)(y_2^2 - y_1^2)}{4 y_1 y_2}$$

$$= (y_1 - y_2) + (y_1 + y_2)(y_1 + y_2)(y_1 - y_2)$$

$$= \frac{4 y_1 y_2 (y_1 - y_2) + y_2^2 y_1 - y_1^3 + y_2^3 - y_1^2 y_2}{4 y_1 y_2}$$

$$= \frac{4 y_1^2 y_2 - 4 y_1 y_2^2 + y_2^2 y_1 - y_1^3 + y_2^3 - y_1^2 y_1}{4 y_1 y_2}$$

$$= \frac{y_2^3 + 3 y_1^2 y_2 + 3 y_1 y_2^2 - y_1^3}{4 y_1 y_2}$$

$$\Delta E = \frac{(y_2 - y_1)^3}{4 y_1 y_2}$$

1. Classify various types of flows.
2. Derive chezy's constant.
3. Derive expressions for most economical channels.
4. velocity, pressure distribution of open channels.
5. problems.
6. specific energy derivations

2. Non Uniform flow in open channels

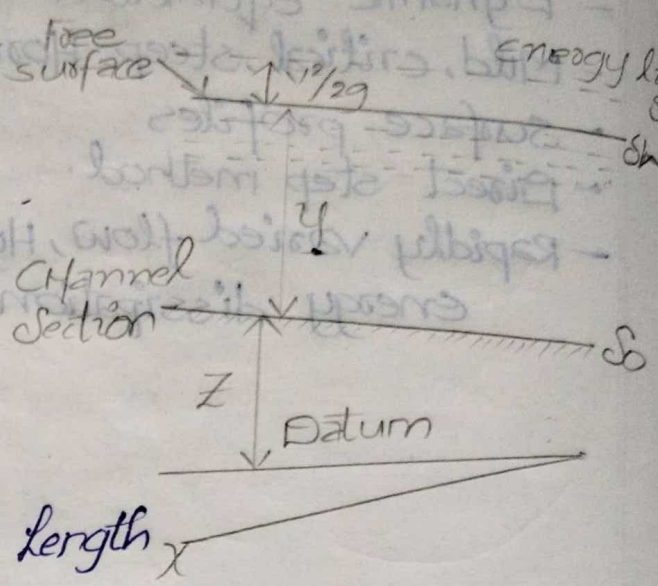
Syllabus:

- Steady Gradually Varied flow.
- Dynamic equation
- Mild, critical, steep, Horizontal & adverse slopes
- surface profiles
- Direct step method.
- Rapidly varied flow, Hydraulic jump, energy dissipation.

Non-uniform flow:

The fluid characteristics varies with respect to depth of channel.

In case of Non-uniform flow in open channels the slope of channel bottom, slope of water surface are different. (i) This is due to because of change in shape & size of the channel. so that slopes are different.



(ii) This is due to obstructions constructed along with the length of channel.

Classification of Non-uniform flow:

(i) Gradually varied flow:

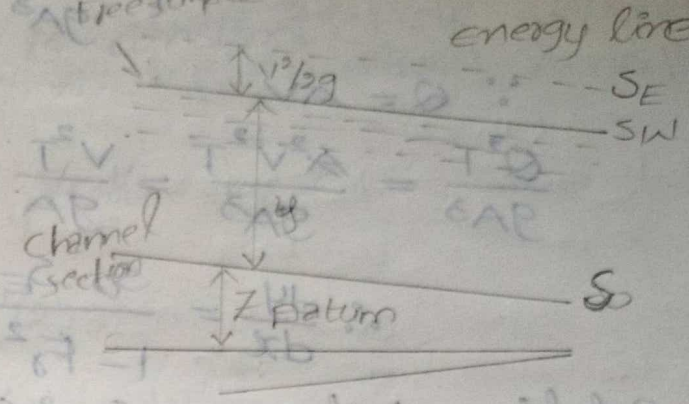
If the depth of flow in open channel varies gradually along the length of channel.

Assumptions of Gradually varied flow:

→ Flow is study.
→ pressure distribution is Hydrostatic.
→ Slope at the bottom is negligible. i.e $\theta = \sin\theta = \tan\theta$.
→ Channel is prismatic.
→ The Kinetic energy correction factor is equal to unity. i.e $\alpha = 1$.
→ the Head loss is same as uniform flow.
→ Chezy's & mannings equations are applicable for gradually varied flow.
→ channel roughness does not depend on depth of flow & is same along the channel.

Derive the General governing equation for gradually Varied flow:

Let us consider a gradually varying flow as shown in figure.



The total Head or total energy per unit weight of liquid measured above the Horizontal Datum.

$$H = Z + y + \frac{V^2}{2g}$$

Take channel bottom as x-axis & differentiate With respect to 'x'.

$$\frac{dH}{dx} = \frac{dZ}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right)$$

$\frac{dH}{dx}$ = slope of energy line 'S_E'.

$\frac{dZ}{dx}$ = slope of channel bed 'S_o'.

H & Z are decreasing in the direction of flow.

∴ The above equation is modified as

$$-S_E = -S_o + \frac{dy}{dx} - \frac{dy}{dx}\left(\frac{V^2}{2g}\right)$$

$$\frac{dy}{dx}\left(\frac{V^2}{2g}\right) = \frac{dy}{dx}\left(\frac{Q^2}{2gA^2}\right)$$

$$= \frac{Q^2}{2g} \cdot \frac{d}{dx}\left(\frac{1}{A^2}\right)$$

$$= \frac{Q^2}{2g} \cdot \frac{-2}{A^3} \frac{dA}{dx}$$

$$= \frac{-Q^2}{gA^3}\left(\frac{dA}{dy} \cdot \frac{dy}{dx}\right) \qquad \left[\because \frac{dA}{dy} = T\right]$$

$$= -\frac{Q^2}{gA^3} \cdot T \cdot \frac{dy}{dx}$$

$$-S_E = -S_o + \frac{dy}{dx} - \frac{dy}{dx}\left(\frac{Q^2 T}{gA^3}\right)$$

$$\frac{dy}{dx}\left(1 - \frac{Q^2 T}{gA^3}\right) = S_o - S_E$$

$$\frac{dy}{dx} = \frac{S_0 - S_E}{1 - \frac{Q^2 T}{gA^3}}$$

$\because Q = AV$

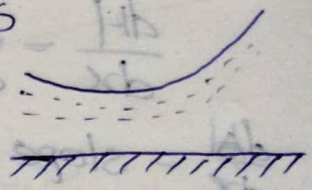
$$\frac{Q^2 T}{gA^3} = \frac{A^2 V^2 T}{gA^3} = \frac{V^2 T}{gA} = \frac{V^2}{gD} = F_r^2$$

$$\therefore \frac{dy}{dx} = \frac{S_0 - S_E}{1 - F_r^2}$$

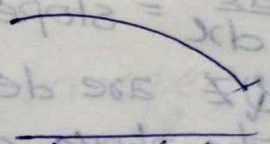
Relation between S_w & S_0 & $\frac{dy}{dx}$: $S_w = S_0 - \frac{dy}{dx}$

(i) If $\frac{dy}{dx} = 0 \longrightarrow S_w = S_0 \longrightarrow$ Uniform flow

(ii) If $\frac{dy}{dx}$ = '+ve'; the water surface will raises
this represents Back water flow
this curve is Back curve



(iii) If $\frac{dy}{dx}$ = '-ve'; the water surface drops
this represents Draw back flow
this curve is Drawdown curve



Derive the general governing equation in case of wide rectangular channel:

In case of wide rectangular channel;

Hydraulic mean radius = Depth of flow

$$R = y$$

We know; $Q = A \times V$

Apply manning's velocity expression in case of uniform flow.

The Depth of flow in uniform flow is termed as normal depth. i.e represented by "y_n".

$$\therefore Q = b y_n \times \frac{1}{n} y_n^{2/3} S_0^{1/2}$$

$$Q = b \cdot \frac{1}{n} y_n^{5/3} \cdot S_0^{1/2} \quad \text{---------- (1)}$$

In case of non uniform flow;

Discharge $Q = b \cdot \frac{1}{n} y^{5/3} S_e^{1/2}$ ——— ②

By Assuming manning's 'n' is constant

∴ From ①&②

$$b \cdot \frac{1}{n} y_n^{5/3} S_0^{1/2} = b \cdot \frac{1}{n} \cdot y^{5/3} S_e^{1/2}$$

$$y_n^{5/3} S_0^{1/2} = y^{5/3} \cdot S_e^{1/2}$$

$$\left(\frac{y_n}{y}\right)^{5/3} = \left(\frac{S_e}{S_0}\right)^{1/2}$$

multiply powers by 2

$$\frac{S_e}{S_0} = \left(\frac{y_n}{y}\right)^{10/3} \longrightarrow S_e = S_0 \left(\frac{y_n}{y}\right)^{10/3}$$

∴ From general equation for gradually varied flow

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_0^2}$$

$$\frac{dy}{dx} = \frac{S_0 - S_0\left(\frac{y_n}{y}\right)^{10/3}}{1 - F_0^2}$$

$$\frac{dy}{dx} = \frac{S_0\left(1 - \left(\frac{y_n}{y}\right)^{10/3}\right)}{1 - \frac{Q^2 T}{gA^3}}$$

$$\therefore \frac{Q^2 T}{gA^3} = \frac{Q^2 T}{g b^3 \cdot y^3} = \frac{Q^2 T}{b^2 \cdot b g y^3} = \frac{q^2 T}{b g y^3} \quad [\because \text{Top width} = b]$$

$$= \frac{q^2}{g y^3} \quad (\because \frac{Q}{b} = q)$$

$$\therefore \frac{Q^2 T}{gA^3} = \frac{y_c^3}{y^3} \quad (\because \frac{q^2}{g} = y_c^3)$$

$$\frac{dy}{dx} = \frac{S_0\left(1 - (y_n/y)^{10/3}\right)}{1 - \frac{y_c^3}{y^3}}$$

Find the slope of water surface in a rectangular channel of width 20m having a depth of flow 5m. The Discharge through channel is $50m^3/sec$. The bed of channel having a slope of 1 in 4000. consider $C_i = 60$.

Sol: Given:

$b = 20m$

$y = 5m$

$Q = 50m^3/sec$

$S_0 = 1/4000$

$C_i = 60$

$\Rightarrow A = b \times y = 20 \times 5 = 100\,m^2$; $P = b + 2y = 20 + 2 \times 5 = 30m$

$$R = \frac{A}{P} = \frac{100}{30} = 3.33m$$

For general governing equation; $V = C\sqrt{RS} \Rightarrow Q = A \cdot C\sqrt{RS}$

$$Q = 100 \times 60\sqrt{3.33 \times S_e} \Rightarrow 50 = 100 \times 60 \times \sqrt{3.33 \times S_e}$$

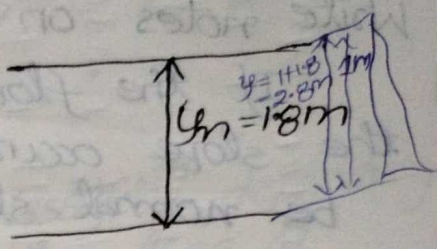
$$S_e = 2.08 \times 10^{-5}$$

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - \frac{Q^2 T}{gA^3}}$$

$$= \frac{1/4000 - 2.08 \times 10^{-5}}{1 - \frac{(50)^2(20)}{9.81\,(100)^3}}$$

$$\frac{dy}{dx} = 2.303 \times 10^{-4}$$

A channel of rectangular section having width 12m carries water at a normal depth of 1.8m & $S_0 = 1/1800$ due to an obstruction in the form of a dam the water level near the obstruction raises by 1m. find the slope of water surface with respect to horizontal $n = 0.05$.



Sol: Given:

Width (b) = 12m

Depth (y_n) = 1.8m

$S_0 = 1/1800$

$y_1 = 2.8$m

$S_w = ?$

$n = 0.05$

$A = b \times y_n = 12 \times 1.8 = 21.6$; $P = b + 2y_n = 12 + 2(1.8) = 15.6$

$\rightarrow R = \frac{21.6}{15.6} = 1.38$m

$\rightarrow Q = A \cdot \frac{1}{n} R^{2/3} \cdot S_0^{1/2}$

$= (21.6) \cdot \frac{1}{0.05} \cdot (1.38)^{(2/3)} (1/1800)^{1/2}$

$Q = 25.24$ m^3/sec.

$\rightarrow q = \frac{Q}{b} = \frac{25.24}{12} = 2.10$ m^2/sec;

$\rightarrow y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(2.10^2)}{9.81}\right)^{1/3} = 0.766$m

$\rightarrow$

$$\frac{dy}{dx} = \frac{S_0\left(1 - \left(\frac{y_n}{y}\right)^{10/3}\right)}{1 - (y_c/y)^3}$$

$$= \frac{1/1800\left(1 - \left(\frac{1.8}{2.8}\right)^{10/3}\right)}{1 - \left(\frac{0.766}{2.8}\right)^3}$$

$$\frac{dy}{dx} = 4.37 \times 10^{-4}$$

$$S_w = S_0 - \frac{dy}{dx}$$

$$= \frac{1}{1800} - 4.37 \times 10^{-4}$$

$$= 1.18 \times 10^{-4}$$

Write notes on channel slopes

If the flow in open channels is uniform then the slope occured in uniform flow is said to be normal slope.

If the flow in channels is critical flow the channel slope is said to be critical slope.

Based on particular, dischare, roughness coefficients, the channel slopes are classified using critical slope criteria. The various channel slopes are

(i) mild slope; $S_0 < S_c$
(ii) steep slope; $S_0 > S_c$
(iii) critical slope; $S_0 = S_c$
(iv) Horizontal slope; $S_0 = 0$
(v) Adverse slope; S_0 is -ve

(i) Mild slope channel:

If the bed slope of channel is less than critical slope.

"$S_0 < S_c$"

(ii) steep slope channel:

If the bed slope of channel is greater than critical slope. $S_0 > S_c$

(iii) critical slope:

If the bed slope of channel is equal to the critical flow.

$S_0 = S_c$

(iv) Horizontal slope:
If the bed slope of channel is equal to zero. "$S_0 = 0$"

(v) Adverse slope:
If the bed slope of channel is negative.
"S_0 = -ve" i.e $S_0 < 0$.

Classification of surface profiles:
Surface profiles are to be classified depending upon normal depth line & critical depth line & the space exists with NDL, CDL.

Normal depth line: (NDL) (y_n):
A line is drawn parallel to normal depth.

Critical depth line: (CDL) (y_c):
A line is drawn parallel to critical depth.

Surface profiles are classified as zone vice;
Zone-(i): space above NDL & CDL
Zone-(ii): space between NDL & CDL
Zone-(iii): space below NDL & CDL

S/NO	Type of channel	Relation between y_n, y_c	Relation between S_0, S_c	Nature of profile	condition	Type of profile	Example
1.	Mild slope	$y_n > y_c$	$S_0 < S_c$	M_1	$y > y_n > y_c$	Back water curve	Flow behind a dam in mild slope channel.
				M_2	$y_n > y > y_c$	Draw down curve	Tile end of a mild submerged in a reservoir
				M_3	$y_n > y_c > y$	Back water curve	Flow under sluis gates in a mild slope channel
2.	steep slope	$y_n < y_c$ (or) $y_c > y_n$	$S_0 > S_c$	S_1	$y > y_c > y_n$	BWC	same above examples but "steep" not mild
				S_2	$y_c > y > y_n$	DDC	
				S_3	$y_c > y_n > y$	BWC	
3.	critical slope	$y_n = y_c$	$S_0 = S_c$	C_1	$y > y_n = y_c$	→ BWC, $\frac{dy}{dx}$ is +ve	It occurs when the bottom slope changes from critical to mild slope
				C_3	$y_n = y_c > y$	→ BWC	It occurs under sluis gates on a critical slope channel
4.	Horizontal slope	$y_n = 0$	$S_0 = 0$	H_2	$y > y_c$	→ DDC	H_2 profile forms at the down stream end of Horizontal channel
				H_3	$y_c > y$	→ BWC	The profile occurs under a sluis gate on a Horizontal channel.
5.	Adverse slope	y_n does not exist	$S_0 < 0$	A_2	–	→ DDC	At the downstream end of a channel with Adverse slope.
				A_3		→ BWC	The profile under a sluis gate is Adverse slope channel.

Classify the slope of a channel whether it has width of 2m & Having depth 1m and the rate of flow in the channel is $2m^3/sec$. Bed slope of channel is 0.0004. Mannings N = ~~0.0004~~ 0.014

Given data:

$b = 2m$

$y = 1m$

$Q = 2m^3/sec$

Bed slope (S) = ~~0.0004~~

N = ~~0.0004~~ 0.014

$\Rightarrow y_c = \left(\frac{q^2}{g}\right)^{1/3}$

$\because q = \frac{Q}{b} = \frac{2}{2} = 1m^2/sec$

$y_c = \left(\frac{1}{9.81}\right)^{1/3}$

$y_c = 0.467m$

$y_c > 0$

$\rightarrow Q = A \cdot \frac{1}{n} R^{2/3} S_0^{1/2}$

$2 = (2 \times y_n) \cdot \frac{1}{0.014} \cdot \left(\frac{2y_n}{2+2y_n}\right)^{2/3} (0.0004)^{1/2}$

$2 = 2y_n \times 1.4285 \frac{(y_n)^{2/3}}{(1+y_n)^{2/3}}$

$(1+y_n)^{2/3} 2 = 2.857 y_n (y_n)^{2/3}$

$(1+y_n)^{2/3} \times 2 = 2.85 y_n^{5/3}$

$\left[(1)^{2/3} + y_n^{2/3}\right] 2 = 2.85 y_n^{5/3}$

$2 + 2y_n^{2/3} = 2.85 y_n^{5/3}$

$2 = 2.85 y_n^{5/3} - 2y_n^{2/3} \Rightarrow 2.85 y_n^{5/3} - 2y_n^{2/3} - 2 = 0$

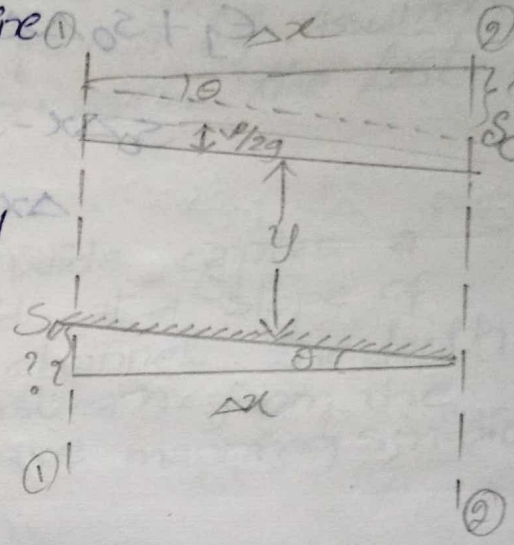
$y_n = 1.29m$

$\therefore y_n > y > y_c$

It belongs to Zone-II
The Normal depth line is greater than critical depth line
∴ It is a Mild Slope channel.

Determination of Length of surface flow in gradually varied flow:

In practise it is often to determine the distance upto which the surface profile of Gradually varied flow extends.



For example; If a weir is constructed across a mild slope river than it is required to determine the Length or distance upto which the M₁ profile extends. In order to solve the problem integration of Dynamic Equation is necessary.

There are different ways by which the integration of Dynamic equation is done

(i) Direct step method
(ii) Numerical Integration method
(iii) Graphical Integration method

(i) Direct step method:

The surface profile or total length is divided into No of reaches & each reach Length is calculated then summing of all the total Length of profile can be obtained.

Equating the total energy at two sections

At section -①: Total energy equation is $y_1 + \frac{V_1^2}{2g} + z_1$

$= y_1 + \frac{V_1^2}{2g} + S_o \Delta x$

$= E_1 + S_o \Delta x$.

At section ②: Total energy equation is $y_2 + \frac{V_2^2}{2g} + Z_2$

$= y_2 + \frac{V_2^2}{2g} + S_E \Delta x$

$= E_2 + S_E \Delta x$

Equating two sections;

$$E_1 + S_0 \Delta x = E_2 + S_E \Delta x$$

$$S_0 \Delta x - S_E \Delta x = E_2 - E_1$$

$$\Delta x (S_0 - S_E) = E_2 - E_1$$

$$\Delta x = \frac{E_2 - E_1}{S_0 - S_E}$$

$$\Delta x = \frac{\Delta E}{S_0 - S_E}$$

$$\therefore \boxed{\Delta x = \frac{\Delta E}{S_0 - S_E}}$$

S_E is Average Energy line slope $S_E = \frac{S_{e_1} + S_{e_2}}{2}$

For Calculation of length of Surface profile.

This are necessary (i) Discharge
(ii) channel shape & size.
(iii) Bottom slope
(iv) Roughness coefficient
(v) Depth of flow at any section

procedure:

1. At know depth of flow 'y_1'. calculate Area of flow, perimeter, Hydraulic mean radius, specific energy
2. Calculate the velocity of flow & the velocity head.
3. Calculate the energy line slope S_{e_1} by using Chezy's or Mannings equation.

4. Choose another section which is nearer to first section.

5. Again calculate the parameters mentioned above.

6. Use the equation $\Delta x = \frac{\Delta e}{S_0 - S_f}$

where; $S_f = \frac{S_{f1} + S_{f2}}{2}$

7. Calculate Δx; & repeat whole process by Assuming different depths of flow until you reach the last section.

1. A rectangular channel of 10m wide carries a discharge of $30 m^3/sec$ it is laid at a slope of 0.0001 if at a section in this channel the depth is 1.6m how far upstream or downstream from the section will the depth '2' 2m take manning's n=0.015

Sol: Given data:

Width of channel, b = 10m

Discharge, Q = $30 m^3/sec$

slope, S_0 = 0.0001

$y_1 = 1.6m$

$y_{n2} = 2m$

n = 0.015

critical depth; $y_c = \left(\frac{q^2}{g}\right)^{1/3}$

$\Rightarrow q = \frac{Q}{b} = \frac{30}{10} = 3 m^3/sec$

$y_c = \left(\frac{3^2}{9.81}\right)^{1/3}$

$y_c = 0.9714$

Discharge; $Q = A \cdot \frac{1}{n} R^{2/3} S^{1/2}$

$30 = 10y_n \cdot \frac{1}{0.015} \left(\frac{10y_n}{10+2y_n}\right)^{2/3} (0.0001)^{1/2}$

$30 = 6.66y_n \frac{(10)^{2/3}(y_n)^{2/3}}{(10+2y_n)^{2/3}}$

$3(10+2y_n) = 6.66y_n \times 10^{2/3} \times y_n^{2/3} \Rightarrow y_n = 2.97$

$y_n > y > y_c$

M_2 profile

Depth of flow (y)	Area (A) $A=by$	perimeter (P) $P=b+2y$	H.M.R $R=\frac{A}{P}$	velocity $V=\frac{Q}{A}$	Kinetic Head $\frac{V^2}{2g}$	specific energy $\epsilon = y+\frac{V^2}{2g}$	slope of energy line $Se=\left(\frac{nQ}{AR^{(2/3)}}\right)^2$
1.6m	$A=1.6\times10$ $=16$	$P=10+2\times1.6$ $P=13.2$	$R=\frac{16}{13.2}$ $R=1.212$	$V=\frac{30}{16}$ $V=1.875$	$\frac{(1.875)^2}{2\times9.81}$ $=0.179$	$\epsilon=1.6+\frac{(1.875)^2}{2\times9.81}$ $G=1.779$	$Se=\left(\frac{0.015\times30}{16\times(1.212)^{(2/3)}}\right)^2$ $Se_1=6.13\times10^{-4}$
1.7m	$A=1.7\times10$ $=17$	$P=10+2\times1.7$ $P=13.4$	$R=\frac{17}{13.4}$ $R=1.268$	$V=\frac{30}{17}$ $V=1.764$	$\frac{(1.764)^2}{2\times9.81}$ $=0.158$	$\epsilon=1.7+0.158$ $\epsilon_2=1.858$	$Se=\left(\frac{0.015\times30}{17\times(1.268)^{(2/3)}}\right)^2$ $Se_2=5.14\times10^{-4}$
1.8m	$A=1.8\times10$ $=18$	$P=10+2\times1.8$ $P=13.6$	$R=\frac{18}{13.6}$ $R=1.323$	$V=\frac{30}{18}$ $V=1.666$	$\frac{(1.666)^2}{2\times9.81}$ $=0.141$	$\epsilon=1.8+0.141$ $\epsilon_3=1.94$	$Se=\left(\frac{0.015\times30}{18\times(1.323)^{(2/3)}}\right)^2$ $Se_3=4.316\times10^{-4}$
1.9m	$A=1.9\times10$ $=19$	$P=10+2\times1.9$ $P=13.8$	$R=\frac{19}{13.8}$ $R=1.376$	$V=\frac{30}{19}$ $V=1.578$	$\frac{(1.578)^2}{2\times9.81}$ $=0.127$	$\epsilon=1.9+0.127$ $\epsilon_4=2.02$	$Se=\left(\frac{0.015\times30}{19\times(1.376)^{(2/3)}}\right)^2$ $Se_4=3.665\times10^{-4}$
2.0m	$A=20\times10$ $=20$	$P=10+2\times2.0$ $P=14$	$R=\frac{20}{14}$ $R=1.428$	$V=\frac{30}{20}$ $V=1.5$	$\frac{(1.5)^2}{2\times9.81}$ $=0.114$	$\epsilon=2.0+0.114$ $\epsilon_5=2.11$	$Se=\left(\frac{0.015\times30}{20\times(1.428)^{(2/3)}}\right)^2$ $Se_5=3.14\times10^{-4}$

$Se' = \frac{Se_1+Se_2}{2}$ $\quad \Delta\epsilon=\epsilon_2-\epsilon_1$ $\quad \Delta x=\frac{\epsilon_2-\epsilon_1}{So-Se}$

(y)	Se'	$\Delta\epsilon$	Δx
1.6			
1.7	$=\frac{6.13\times10^{-4}+5.14\times10^{-4}}{2}$ $=5.63\times10^{-4}$	$\Delta\epsilon=1.858-1.779$ $=0.079$	$\Delta x=\frac{0.079}{0.0001-5.63\times10^{-4}}=-170.62$
1.8	$=\frac{5.14\times10^{-4}+4.316\times10^{-4}}{2}$ $=4.72\times10^{-4}$	$\Delta\epsilon=1.94-1.858$ $=0.082$	$\Delta x=\frac{0.082}{0.0001-4.72\times10^{-4}}=-220.43$
1.9	$=\frac{4.316\times10^{-4}+3.66\times10^{-4}}{2}$ $=3.98\times10^{-4}$	$\Delta\epsilon=2.02-1.94$ $=0.08$	$\Delta x=\frac{0.08}{0.0001-3.98\times10^{-4}}=-268.45$
2.0	$=\frac{3.66\times10^{-4}+3.14\times10^{-4}}{2}$ $=3.4\times10^{-4}$	$\Delta\epsilon=2.11-2.02$ $=0.09$	$\Delta x=\frac{0.09}{0.0001-3.4\times10^{-4}}=-375$

2. A rectangular flume 2m wide carrying discharge at the rate of $2m^3/sec$. the bed of the flume is 0.0004 at a certain section the depth of flume is 1m. calculate the distance of the section downstream when the depth of flow is 0.9m. Assume $n=0.014$. solve by single step method if the channel slope mild or steel and classify the surface profile.

Sol: Given: $b = 2m$

$Q = 2m^3/sec$

$S_0 = 0.0004$

$y_1 = 1m$

$y_2 = 0.9m$

Mannings $n = 0.014$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$q = \frac{Q}{b} = \frac{2}{2} = 1m^2/sec$$

$$y_c = \left(\frac{1^2}{9.81}\right)^{1/3}$$

$$y_c = 0.467m$$

Discharge $Q = A \cdot \frac{1}{n} R^{2/3} \cdot S_0^{1/2}$

$$2 = 2y_n \cdot \frac{1}{0.014} \frac{(2y_n)^{2/3}}{(2+2y_n)^{2/3}} (0.0004)^{1/2}$$

$$2 = 2.857 y_n \cdot \frac{y_n^{2/3}}{(1+y_n)^{2/3}}$$

$$2(1+y_n)^{2/3} = 2.857 y_n^1 \cdot y_n^{2/3}$$

$$2+2y_n^{2/3} = 2.857 y_n^{5/3}$$

$$2.857 y_n^{5/3} - 2y_n^{2/3} - 2 = 0$$

$$y_n = 1.29m$$

$y_n > y > y_c$

$\therefore M_2$ profile

Depth of flow(y)	Area(A)	perimeter	Hydraulic mean radius	velocity	Kinetic Head	specific energy	slope of energy line
	$A=by$	$P=b+2y$	$R=A/P$	$V=\frac{Q}{A}$	$V^2/2g$	$\epsilon=y+\frac{v^2}{2g}$	$Se=\left(\frac{nQ}{AR^{2/3}}\right)^2$
1	$A=2\times1$	$P=2+2\times1$	$R=2/4$	$V=\frac{2}{2}$	$P^2/2\times9.81$		
	$A=2$	$=4$	$R=0.5$	$V=1$	$=0.05$	$\epsilon=1.05$	$Se=\left(\frac{0.014\times2}{2\times(0.5)^{2/3}}\right)^2$
							$Se=4.93\times10^{-4}$
1.1	$A=2\times1.1$	$P=2+2\times1.1$	$R=\frac{2.2}{4.2}$	$V=\frac{2}{2.2}$	$\frac{0.9^2}{2\times9.81}$	$\epsilon=1.1+0.04$	$Se=\left(\frac{0.014\times2}{2.2\times(0.52)^{2/3}}\right)^2$
	$A=2.2$	$=2+2.2$					
		$P=4.2$	$R=0.52$	$V=0.9$	$=0.04$	$\epsilon=1.14$	$Se=3.87\times10^{-4}$
1.2	$A=2\times1.2$	$P=2+2\times1.2$	$R=\frac{2.4}{4.4}$	$V=\frac{2}{2.4}$	$\frac{0.8^2}{2\times9.81}$	$\epsilon=1.2+0.03$	$Se=\left(\frac{0.014\times2}{2.4\times(0.54)^{2/3}}\right)^2$
	$A=2.4$	$P=4.4$	$R=0.54$	$V=0.8$	$=0.03$	$\epsilon=1.23$	$Se=3.09\times10^{-4}$

$$Se=\frac{Se_1+Se_2}{2} \qquad \Delta\epsilon=\epsilon_2-\epsilon_1 \qquad \Delta x=\frac{\epsilon_2-\epsilon_1}{So-Se}$$

1

1.1 $Se=\dfrac{4.93\times10^{-4}+3.87\times10^{-4}}{2}$

$Se=4.4\times10^{-4}$

$\Delta\epsilon=1.14-1.05$

$=0.09$

$\Delta x=\dfrac{0.09}{0.0004-4.4\times10^{-4}}$

$\Delta x=-2250$

1.2 $Se=\dfrac{3.87\times10^{-4}+3.09\times10^{-4}}{2}$

$Se=3.48\times10^{-4}$

$\Delta\epsilon=1.23-1.14$

$=0.09$

$\Delta x=\dfrac{0.09}{0.0004-3.48\times10^{-4}}$

$\Delta x=1730.76$

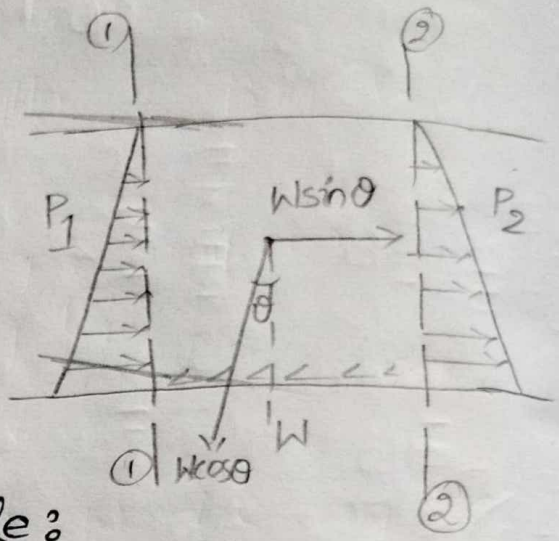
Rapid Varied flow:

The flow in open channel is said to be rapid varied flow if there is a sudden change in depth occurs for a small length of the channel.

Ex: Hydraulic jump

Define Momentum principle:

The force is applied on fluid in any direction is equal to rate of change of momentum in the same direction.



Expressions for Momentum principle:

The force applied on fluid in any direction is equal to rate of change of momentum in the same direction.

From the above figure;

The momentum of the fluid at section ① is equal to mass of the fluid at section ① & velocity at section ① $\rho Q \times V_1 \Rightarrow \frac{W}{g} Q V_1$

The Momentum of the fluid at section ② is equal to mass of the fluid at section ② x velocity at section ② $\rho Q \times V_2 \Rightarrow \frac{W}{g} Q V_2$

Where; ρ = mass density of fluid

The various forces acting on the fluid in between section ①-① & section ②-②

P_1, P_2 are pressure forces at ① & ②

$W\sin\theta, W$ are Weight components are at flow & also friction force (R_f).

Momentum principle equation is

$$P_1 - P_2 + W\sin\theta - R_f = \frac{W}{g} Q V_2 - \frac{W}{g} Q V_1.$$

Define specific force & Derive Expression.

Consider a small length of the channel. The momentum principle applied to this channel is

$$P_1 - P_2 + W\sin\theta - R_f = \frac{W}{g}QV_2 - \frac{W}{g}QV_1 \qquad [\sin\theta = 0;\ R_f = 0$$

$$P_1 - P_2 = \frac{W}{g}QV_2 - \frac{W}{g}QV_1$$

$$\Rightarrow P_1 = WA_1\bar{z}_1 ; \rightarrow P_2 = WA_2\bar{z}_2$$

$$WA_1\bar{z}_1 - WA_2\bar{z}_2 = \frac{W}{g}(QV_2 - QV_1)$$

$$A_1\bar{z}_1 - A_2\bar{z}_2 = \frac{Q}{g}V_2 - \frac{Q}{g}V_1$$

$$A_1\bar{z}_1 + \frac{Q}{g}V_1 = \frac{Q}{g}V_2 + A_2\bar{z}_2$$

$$A_1\bar{z}_1 + \frac{Q}{g}\cdot\frac{Q}{A_1} = \frac{Q}{g}\cdot\frac{Q}{A_2} + A_2\bar{z}_2$$

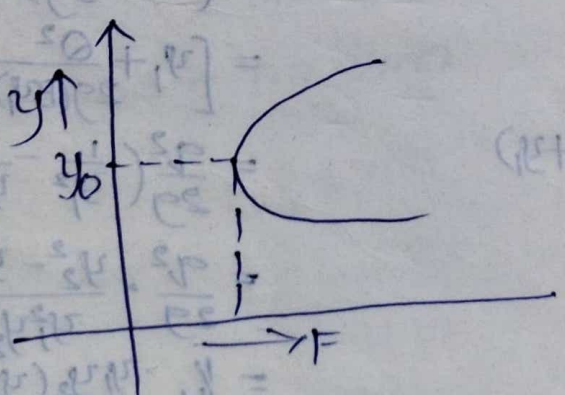
$$A_1\bar{z}_1 + \frac{Q^2}{gA_1} = \frac{Q^2}{gA_2} + A_2\bar{z}_2$$

$$\therefore F_1 = \frac{Q^2}{gA_1} + A_1\bar{z}_1 \quad ; \quad F_2 = \frac{Q^2}{gA_2} + A_2\bar{z}_2$$

The depth at were specific force is minimum. The depth is critical depth. The depths where the specific force is equal to depths (y_1, y_2) are called conjugated depths.

Where; $\frac{Q^2}{Ag}$ = the moment of the flow passing through the channel.

$A\bar{z}$ = force per unit weight of water.



Specific force Diagram

Hydraulic jump:

It is defined as sudden & turbulent passage of flow from super critical to sub critical.

Hydraulic jump take place whenever the flow changes from "super critical to subcritical".

It occurs in a very short length of channel.

Assumptions:

* The flow before & after the jump is uniform
* The pressure distribution is Hydrostatic
* As the length of the channel is small so that frictional force is neglected.
* The flow is Horizontal so that weight component is neglected.
* For the Analysis of Hydraulic jump momentum Equation is applicable.

Expression for Hydraulic jump:

$$P_1 - P_2 + W\sin\theta - R_f = \frac{WQ}{g}(V_2 - V_1)$$

$$P_1 - P_2 = \frac{WQ}{g}(V_2 - V_1)$$

$$wA_2\bar{z}_2 - wA_1\bar{z}_1 = \frac{w}{g}Q\left(\frac{Q}{A_1} - \frac{Q}{A_2}\right)$$

$$\frac{Q^2}{gA_1} + A_1\bar{z}_1 = \frac{Q^2}{gA_2} + A_2\bar{z}_2$$

$$\frac{Q^2}{g(By_1)} + (By_1)\left(\frac{y_1}{2}\right) = \frac{Q^2}{g(By_2)} + By_2\left(\frac{y_2}{2}\right)$$

$$\frac{Q^2}{gB^2}\left(\frac{1}{y_1} - \frac{1}{y_2}\right) = \frac{1}{2}(y_2^2 - y_1^2)$$

$$\frac{q^2}{g}\left(\frac{y_2 - y_1}{y_1 y_2}\right) = \frac{1}{2}(y_2 - y_1)(y_2 + y_1)$$

$$\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)$$

$$y_2 = -\frac{y_1}{2} + \sqrt{\left(\frac{y_1}{2}\right)^2 + \frac{2q^2}{gy_1}}$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{8q^2}{gy_1^3}}\right]$$

$$y_2/y_1 = \frac{1}{2}\left[-1 + \sqrt{1 + 8\left(\frac{y_c}{y_1}\right)^3}\right]$$

$$y_2/y_1 = \frac{1}{2}\left[-1 + \sqrt{1 + 8Fr_1^2}\right]$$

$$\Rightarrow y_2 = \frac{y_1}{2}\left[-1 + \sqrt{1 + 8Fr_1^2}\right]$$

Energy Loss

$$\Delta E = \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right)$$

$$= \left[y_1 + \frac{Q^2}{2g(By_1)^2}\right] - \left[y_2 + \frac{Q^2}{2g(By_2)^2}\right]$$

$$= \frac{q^2}{2g}\left(\frac{1}{y_1^2} - \frac{1}{y_2^2}\right) - (y_2 - y_1)$$

$$= \frac{q^2}{2g} \cdot \frac{y_2^2 - y_1^2}{y_1^2 y_2^2} - (y_2 - y_1)$$

$$= \frac{1}{4} \frac{y_1 y_2 (y_1 + y_2)(y_2^2 - y_1^2)}{(y_1 y_2)^2} - (y_2 - y_1)$$

$$\Delta E = \frac{(y_2 - y_1)^3}{4y_1 y_2}$$

① A sluice gate discharges water into a Horizontal rectangular channel with a velocity of 10m/sec & depth of flow is 1m. Determine the depth of flow after the jump & consequent loss in total Head.

Sol:- Given data:

$V = 10 m/sec$

$y_1 = 1m$

$y_2 = ?$

$\Delta E = ?$

Froude Number:

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}}$$

$$= \frac{10}{\sqrt{9.81 \times 1}}$$

$$Fr_1 = 3.192$$

Depth of flow after the jump:

$$y_2 = \frac{y_1}{2}\left[-1 + \sqrt{1 + 8Fr^2}\right]$$

$$= \frac{1}{2}\left[-1 + \sqrt{1 + 8(3.192)^2}\right]$$

$$y_2 = 4.038m$$

consequent Loss of total Head

$$\Delta E = \frac{(y_2 - y_1)^3}{4 y_1 y_2}$$

$$= \frac{(4.038 - 1)^3}{4 \times 4.038 \times 1}$$

$$\Delta E = 1.735m$$

② A Hydraulic jump forms at the downstream and spill way carrying 17.93 m^3/sec (discharge). If the depth of the flow before the jump is 0.8m. Determine the depth after the jump and also loss.

Sol: Given:

Discharge (Q) = 17.93 m^3/sec

$y_1 = 0.8m$

$y_2 = ?$

$\Delta E = ?$

Assume the width of channel; b = 1

$A = b \times y_1 = 1 \times 0.8 = 0.8$

$Q = AV \Rightarrow V = \frac{Q}{A}$

$= \frac{17.93}{0.8}$

$V = 22.41 m/sec$

$\rightarrow Fr_1 = \frac{V}{\sqrt{g y_1}}$

$= \frac{22.41}{\sqrt{9.81 \times 0.8}}$

$Fr_1 = 7.99$

$Fr_1 \simeq 8$

$\rightarrow y_2 = \frac{y_1}{2}\left[-1 \pm \sqrt{1+8Fr^2}\right]$

$= \frac{0.8}{2}\left[-1 + \sqrt{1+8(8)^2}\right]$

$y_2 = 8.65m$

$\Rightarrow \Delta E = \frac{(y_2 - y_1)^3}{4 y_1 y_2}$

$= \frac{(8.65 - 0.8)^3}{4 \times 8.65 \times 0.8}$

$\Delta E = 17.54m$

(3) The width of a stream is 15m and the discharge is $50 m^3/sec$ if passes through a sluices of 70cm & 15m wide. Determine a Hydraulic jump occur or not. If so find the sequent jump after the depth.

Sol: Given:

Discharge (Q) $= 50 m^3/sec$

$y_1 = 70cm = 0.7m$

$b = 15m$

$$\rightarrow q = \frac{Q}{b} = \frac{50}{15} = 3.33 m^3/sec$$

From the critical depth of rectangular section

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$= \left(\frac{(3.33^2)}{9.81}\right)^{1/3}$$

$$y_c = 1.041m$$

$$y_1 < y_c \Rightarrow 0.7 < 1.041$$

From continuity equation; $Q = AV$

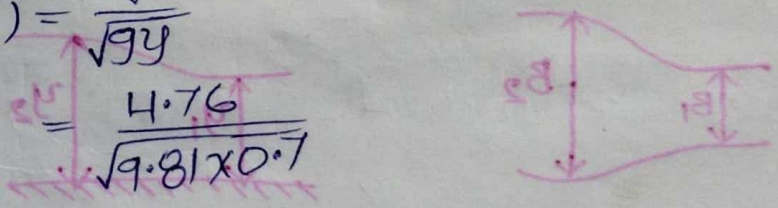
$$V = \frac{Q}{A}$$

$$= \frac{50}{15 \times 0.7}$$

Velocity (V) $= 4.76 m/sec$

Froude Number (F_r) $= \frac{V}{\sqrt{gy}}$

$$= \frac{4.76}{\sqrt{9.81 \times 0.7}}$$



$F_r = 1.816$

$F_r > 1$

Height of Hydraulic jump $= y_2 - y_1 = 1.475 - 0.7$

$= 0.775$

sequent depth:

$$y_2 = \frac{y_1}{2}\left(-1 \pm \sqrt{1 + 8F_r^2}\right)$$

$$= \frac{0.7}{2}\left(-1 \pm \sqrt{1 + 8(1.816)^2}\right)$$

$y_2 = 1.48m$

Define control section & what is its significance.

It is a section where there exists a definite relation between depth of the flow & discharge

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$ (q = Discharge per meter width)

A control section is used for measuring rate of flow.

Define channel transition. What are various types of transition.

It is the portion of channel with varying cross section, which connects one uniform channel to other which may or may not have the same cross sectional flow.

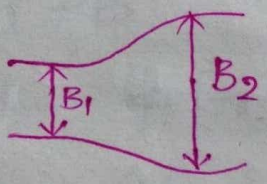
These are of two types:

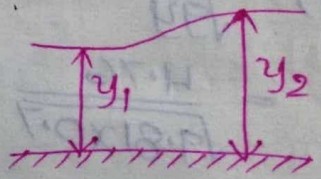
(i) Reduction in width of channel

(ii) channel width rise in water.

Some of the functions of transition:

- Measuring rate of flow
- Desipation of energy
- Reduction or increase in velocity
- Change in Alignment or section with minimum energy desipation.





(4) The depth of flow of water at a certain section of a rectangular channel of 4m wide is 0.5m the discharge through the channel is $16m^3/sec$. If a Hydraulic jump take places on the down stream side. Find the depth of flow after the jump?

Sol: Given data: $b = 4m$

$y_1 = 0.5m$

$Q = 16m^3/sec$

$$\rightarrow q = \frac{Q}{b} \rightarrow \frac{16}{4} \Rightarrow q = 4m^3/sec$$

Depth of flow after the jump:

$$\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)$$

$$\frac{2 \times 4^2}{9.81} = 0.5 y_2 (0.5 + y_2)$$

$$3.261 = 0.25 y_2 + 0.5 y_2^2$$

$$y_2 = 2.316m$$

⑤ The Depth of flow of water at a certain section of a rectangular channel of 2m wide is 0.3m. The Discharge through the channel is 1.5m³/sec. Determine whether a Hydraulic jump will occur. If so find it's height & loss of energy per kg of water.

Note: Hydraulic jump will occur when

→ $fr > 1$

$y_1 < y_c$

Sol: Given data: $b = 2m$

$y_1 = 0.3m$

$Q = 1.5m^3/sec$

$\Delta E = ?$

velocity $(V_1) = \frac{Q}{A} = \frac{1.5}{(0.3 \times 2)} = 2.5m/sec$

Froude Number $fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{2.5}{\sqrt{9.81 \times 0.3}} = 1.45$

$fr_1 > 1$

critical depth $(y_c) = \left(\frac{q^2}{g}\right)^{1/3}$

$= \left(\frac{0.75^2}{9.81}\right)^{1/3}$

$\because q = \frac{Q}{b}$

$= \frac{1.5}{2}$

$q = 0.75 m^2/sec$

$y_c = 0.38$

$y_1 < y_c$

$0.3 < 0.38$

Depth after the Hydraulic jump: $\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)$

$\frac{2 \times (0.75)^2}{9.81} = 0.3 y_2 (0.3 + y_2)$

$0.114 = 0.09 y_2 + 0.3 y_2^2$

$y_2 = 0.48m$

Loss of energy $\Delta E = \frac{(y_2 - y_1)^3}{4 y_1 y_2}$

$= \frac{(0.48 - 0.3)^3}{4 \times 0.48 \times 0.3} = 0.011m$

Height of Hydraulic jump $= y_2 - y_1 = 0.48 - 0.3$

$= 0.18m$

⑥ A sluice gate discharges water into a horizontal rectangular channel with a velocity of 6m/sec and the depth of flow is 0.4m. The width of the channel is 8m. Determine whether a Hydraulic jump will occur and if so find it's Height, loss of energy per kg of water also. Determine the power of loss in Hydraulic jump.

Sol: Given data: $b = 8m$

$y_1 = 0.4m$

$V = 6m/sec$

Discharge $(Q) = A \times V$

$= (8 \times 0.4) \times 6$

$Q = 19.2\ m^3/sec$

$q = \frac{Q}{b} = \frac{19.2}{8} = 2.4 m^2/sec$

Froude Number: $Fr_1 = \frac{V_1}{\sqrt{g y_1}}$

$= \frac{6}{\sqrt{9.81 \times 0.4}}$

$Fr_1 = 3.03$

Depth of flow after jump: $\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)$

$y_2 = +\frac{y_1}{2}(-1 \pm \sqrt{1 + 8Fr^2})$

$\frac{2(2.4)^2}{9.81} = 0.4\, y_2 (0.4 + y_2)$

$1.114 = 0.16 y_2 + 0.4 y_2^2$

$y_2 = 1.524m$

Loss of energy: $\Delta E = \frac{(y_2 - y_1)^3}{4 y_1 y_2}$

$= \frac{(1.524 - 0.4)^3}{4 \times 1.524 \times 0.4}$

$\Delta E = 0.582m$

Height of Hydraulic jump $\Rightarrow y_2 - y_1 = 1.524 - 0.4 = 1.124m$

Length $= 5(y_2 - y_1) = 5(1.524 - 0.4) = 5.6m$

power $= W Q \Delta E$

$= 9.810 \times 19.2 \times 0.582$

$P = 109.62 \times 10^3$ kWatt

(7) The depth of flow of water before & after Hydraulic jump in a 5m wide in rectangular channel are 1.2m, 7.8m. calculate the discharge throught the channel and power loss in the jump.

Sol: Given data:

$b = 5m$

$y_1 = 1.2m$

$y_2 = 7.8m$

$Q = ?$

$P = ?$

Depth of flow after jump: (conjugate depths)

$$\frac{2q_1^2}{g} = y_1 y_2 (y_1 + y_2)$$

$$\frac{2q_1^2}{9.81} = (1.2)(7.8)(1.2+7.8)$$

$$\frac{2q_1^2}{9.81} = 84.24$$

$$q_1 = 20.3 m^2/sec.$$

We know; $q_1 = \frac{Q}{b}$

$Q = q_1 b$

$Q = 20.3 \times 5$

$Q = 101.6 m^3/sec$

Loss of energy $(\Delta E) = \frac{(y_2 - y_1)^3}{4 y_1 y_2}$

$$= \frac{(7.8 - 1.2)^3}{4 \times 7.8 \times 1.2}$$

$\Delta E = 7.67 m$

power loss in the jump $(P) = \rho g Q \Delta E$

$= 9810 \times 101.6 \times 7.67$

$P = 764.46 \times 10^3$ KWatt

⑧ Uniform flow occurs at a depth of 1.5m in a long rectangular channel of 3m wide & laid at a slope of 0.0009. If Manning's $n = 0.015$ (i) calculate the Maximum Height of the Hump on the flow to produce critical depth.
(ii) Width of contraction which will produce critical depth with out increasing upstream depth of flow.

Note: For a Hump; The conditions for solving the problem are (i) $E_1 = E_2 + \Delta Z_{max}$

Where; ΔZ = Height of Hump

(ii) Channel transistion involving reduction in the width of channel. The only equation is $E_1 = E_2$

For minimum Width of channel transistion $E_1 = E_2$

Transistion involving a Hump [rised flow]

$E_1 = E_2 + \Delta Z_{max}$

Sol: Given data:

Depth (y) = 1.5m

Width (b) = 3m

slope (S) = 0.0009

Manning's (n) = 0.015

(i) specific energy $(E_1) = y_1 + \frac{V_1^2}{2g}$

Velocity formula for Manning's $V_1 = \frac{1}{n} R^{2/3} S^{1/2}$

Where; Hydraulic mean radius $(R) = \frac{\text{Wetted Area}}{\text{perimeter}}$

$$R = \frac{by}{b+2y}$$

$$= \frac{1.5 \times 3}{3 + 2(1.5)}$$

$R = 0.75m$

$$\Rightarrow V_1 = \frac{1}{0.015}(0.75)^{2/3}(0.0009)^{1/2}$$

$V_1 = 1.65 m/sec$

$$\Rightarrow E_1 = 1.5 + \frac{(1.65)^2}{2\times 9.81}$$

$$E_1 = 1.638m$$

Minimum specific energy of rectangular channel is $E_c = \frac{3}{2}y_c$

Where; critical depth $(y_c) = \left(\frac{q^2}{g}\right)^{1/3}$

$$q = \frac{Q}{b}$$

$$\Rightarrow Q = AV = 1.5\times 1.65 = 7.425$$

$$q = \frac{7.425}{3} \Rightarrow q = 2.475$$

$$\Rightarrow y_c = \left(\frac{(2.475)^2}{9.81}\right)^{1/3}$$

$$y_c = 0.854m$$

$$\Rightarrow E_c = \frac{3}{2}\times 0.85 = 1.282m$$

From the 1st condition of Hydraulic jump

$$E_1 = E_c + \Delta Z_{max}$$

$$1.638 = 1.282 + \Delta Z_{max}$$

$$\Delta Z_{max} = 1.638 - 1.282$$

$$\Delta Z_{max} = 0.356$$

(iii) $E_1 = E_c$

$$1.638 = E_c$$

$$E_c = \frac{3}{2}y_c$$

$$1.638\times\frac{2}{3} = y_c \Rightarrow y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$y_c = 1.092m \qquad 1.092 = \left(\frac{q^2}{9.81}\right)^{1/3}$$

$$q = 3.574 m^2/sec$$

$$b = \frac{Q}{q} = \frac{7.425}{3.574}$$

$$b = 2.077m$$

∴ Width of contraction (b) = 2.077m

⑨ A rectangular channel is 2.5 wide & carries a flow of $2.75 m^3/sec$ at a depth of 0.9m a contraction of the channel width is proposed at a certain section. Find the smallest allowable contracted width that will not effect the upstream condition

Sol: Given data:

Width (b) = 2.5m

Discharge (Q) = $2.75 m^3/sec$

Depth (y_1) = 0.9m

From the condition of channel transition involving reduction in width of channel is

$$E_1 = E_c$$

$$E_1 = y_1 + \frac{V_1^2}{2g}$$

Discharge (Q) = AV

$$V = \frac{Q}{A}$$

$$= \frac{2.75}{2.5 \times 0.9}$$

$$V = 1.22 m/sec$$

$$E_1 = 0.9 + \frac{1.22^2}{2 \times 9.81}$$

$$E_1 = 0.962m$$

Minimum specific energy of channel

$$E_c = \frac{3}{2} y_c$$

$$\Rightarrow y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$\Rightarrow E_1 = E_c$$

$$0.962 = \frac{3}{2} y_c$$

$$0.962 \times \frac{2}{3} = y_c$$

$$0.641 = y_c$$

$$0.641 = \left(\frac{q^2}{g}\right)^{1/3}$$

$$0.641 \times (9.81)^{1/3} = q^{2/3}$$

$$q^{2/3} = 1.372$$

$$q = 1.607 \, m^2/sec$$

$$\Rightarrow \text{Width } b = \frac{Q}{q}$$

$$= \frac{2.75}{1.607}$$

$$b = 1.71 m$$

∴ The smallest allowable contracted width is

$$b = 1.71 m$$

① A wide rectangular channel carries a flow of 2.76 m³/sec for meter width the depth of flow being 1.524m. Calculate the minimum rise in the flow at a certain section require to produce critical flow condition. What is the corresponding change in the water surface elevation

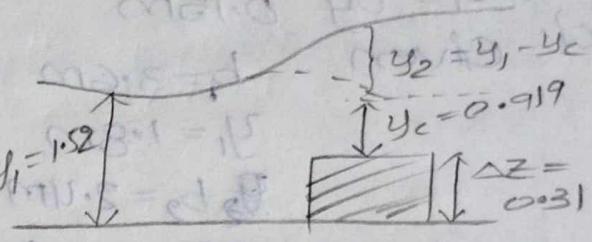
Sol: Given:

Wide (b) = 1m

$q = 2.76 m^2/sec$

$y_1 = 1.524$

$\Delta z = ?$



→ condition: $\epsilon_1 = \epsilon_c + \Delta Z_{max}$

$$\epsilon_1 = y_1 + \frac{V_1^2}{2g}$$

$$V_1 = \frac{Q}{A}$$

$$\rightarrow q = \frac{Q}{b} \rightarrow Q = qb$$

$$Q = 2.76 \times 1$$

$$Q = 2.76 m^3/sec$$

$$V_1 = \frac{2.76}{1 \times 1.524}$$

$$V_1 = 1.81 m^2/sec$$

$$\epsilon_1 = 1.524 + \frac{(1.81)^2}{2 \times 9.81}$$

$$\epsilon_1 = 1.69 m$$

$$\epsilon_c = \frac{3}{2} y_c$$

$$= \frac{3}{2}\left(\frac{q^2}{g}\right)^{1/3}$$

$$= \frac{3}{2}\left(\frac{2.76^2}{9.81}\right)^{1/3}$$

$$\epsilon_c = 1.37 m$$

$$\Longrightarrow \epsilon_1 = \epsilon_c + \Delta z_{max}$$

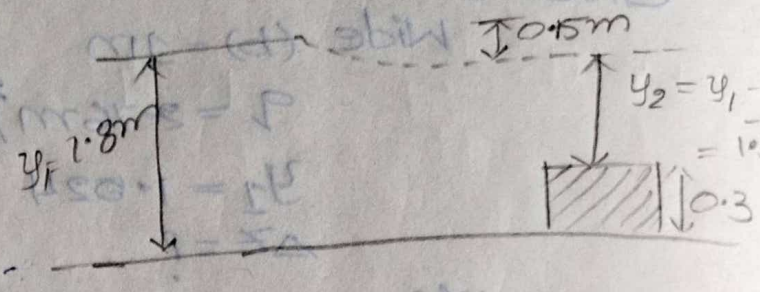
$$\Delta z_{max} = \epsilon_1 - \epsilon_c$$

$$= 1.69 - 1.37$$

$$\Delta z_{max} = 0.32 m$$

corresponding change is $y_2 = y_1 - y_c - \Delta z = 0.295 m$

(11) A 3.6m wide rectangular channel carries water at a depth of 1.8m. In order to measure discharge, the channel width is reduced to 2.4m & a Hump of 0.3m is provided in the bottom. Calculate the discharge if the water surface in contracted section drops by 0.15m



Sol: Given $b_1 = 3.6m$

$y_1 = 1.8m$

$b_2 = 2.4m$

$\Delta Z_{max} = 0.3m$

$Q = ?$

$y_2 = 1.35$

$\rightarrow E_1 = E_2 + \Delta Z_{max}$

$\rightarrow E_1 = y_1 + \frac{V_1^2}{2g}$

$V_1 = \frac{Q}{A} = \frac{Q}{3.6 \times 1.8} = \frac{Q}{6.48}$

$E_1 = 1.8 + \frac{Q^2/(6.48)^2}{2 \times 9.81}$

$= 1.8 + (1.21 \times 10^{-3}) Q^2$

$\rightarrow E_2 = y_2 + \frac{V_2^2}{2g}$

$V_2 = \frac{Q}{A} = \frac{Q}{2.4 \times 1.35} = \frac{Q}{3.24}$

$E_2 = 1.35 + \frac{Q^2/(3.24)^2}{2 \times 9.81}$

$= 1.35 + (4.85 \times 10^{-3}) Q^2$

$\because E_1 = E_2 + \Delta Z_{max}$

$1.8 + (1.21 \times 10^{-3}) Q^2 = 1.35 + (4.85 \times 10^{-3}) Q^2 + 0.3$

$(1.21 \times 10^{-3}) Q^2 - (4.85 \times 10^{-3}) Q^2 = -0.15$

$-3.64 \times 10^{-3} Q^2 = -0.15$

$Q^2 = \frac{0.15}{3.64 \times 10^{-3}}$

$Q = 6.415 m^3/sec$

4. Basics of Turbomachinery

Syllabus:

- Hydrodynamic force of jets on stationary and moving flat.
- Inclined & curved vanes.
- Jet striking centrally and at tip.
- velocity triangles at inlet & outlet.
- Expressions for workdone and efficiency.
- Angular momentum principle.

Impulse momentum principle:

The impulse force exerted on any body is equal to rate of change of its momentum. This principle is obtained from Newton's second law of motion.

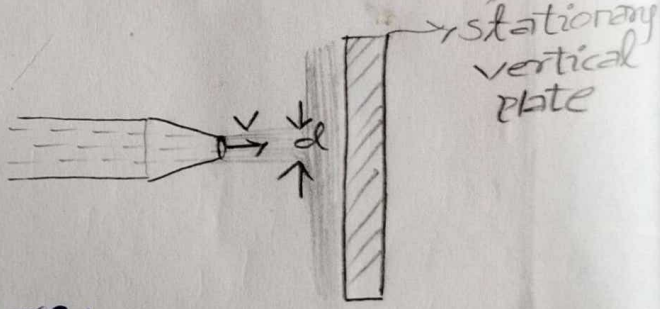
$$F = ma$$
$$= m \cdot \frac{V}{t}$$
$$= \frac{mV}{t}$$
$$F = \rho QV$$

Impact force:

Whenever the liquid jet emerges from nozzle exerts some force on the surrounding objects which causes some change in it's momentum and the emerging force is known as impact force.

Stationary plates:

(i) Force exerted by liquid jet on a stationary vertical plate:-



consider a liquid jet of diameter 'd' & moving with a velocity 'v' which emerges from the nozzle strikes the vertical plate as shown in figure;

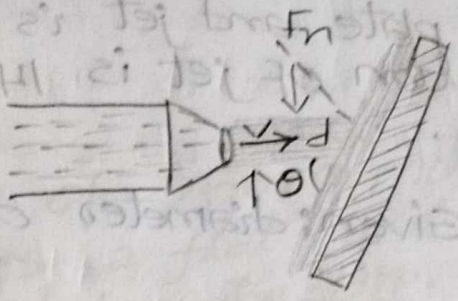
After striking the plate the jet will moves along the plate. The force exerted by the liquid jet in the direction of flow, $F = m$

F = mass of the fluid striking the plate per second x change in velocity of the liquid jet

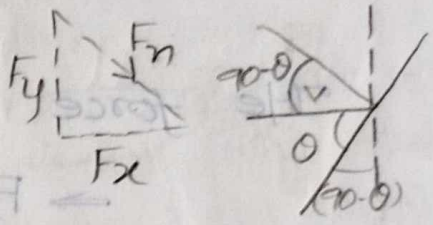
$$= \rho a v [v - 0]$$
$$F = \rho a v^2$$

(ii) Force exerted by liquid jet on a vertical inclined plate:~ (stationarly inclined plate)

Consider a liquid jet of diameter 'd', & moving with a velocity 'v' which emerges from the nozzles strikes the inclined plate as shown in figure.



After striking the plate the jet will moves along the plate inclinely. The force exerted by the liquid jet in the direction of flow



$v\cos(90-\theta)$
$= v\sin\theta$

$$F = \rho a v^2 (v\sin\theta - 0)$$

$$F_n = \rho a v^2 \sin\theta$$

Force exerted by the jet on the plate and direction to the normal to plate

$$F_n = \rho a v^2 \sin\theta$$

Force exerted by the jet on the plate along direction of flow

$$F_x = F_n \sin\theta$$
$$F_x = \rho a v^2 \sin\theta \cdot \sin\theta$$
$$F_x = \rho a v^2 \sin^2\theta$$

Force exerted by the jet on the plate perpendicular to the direction of flow

$$F_y = F_n \cos\theta$$
$$F_y = \rho a v^2 \sin\theta \cdot \cos\theta$$

1. A jet of water of 50mm diameter, strikes a fixed inclined plane in such a way that the angle between the plate and jet is 30° the force exerted in the direction of jet is 14715N. Determine the rate of flow.

Sol: Given: diameter of jet = 50mm = 0.05m

$\theta = 30°$

$F = 14715N$

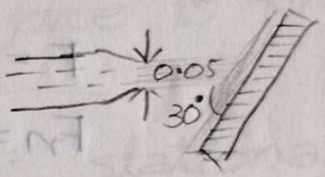
The force exerted in the direction of jet

$\Rightarrow F_x = \rho a v^2 \sin^2\theta$

$a = \frac{\pi}{4}d^2 = \frac{\pi}{4} \times 0.05^2 = 1.96 \times 10^{-3} m^2$

$\Rightarrow 14711.5 = 1000 \times 1.96 \times 10^{-3} \times v^2 \sin^2 30$

$v = 54.8 m/sec$

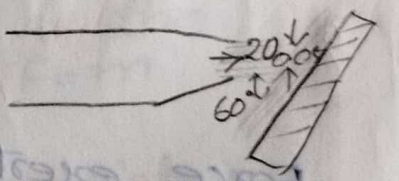


We know; $Q = AV$

$= 1.96 \times 10^{-3} \times 54.8$

$Q = 0.107 m^3/sec$

2. A jet of water of diameter 50mm, moving with a velocity of 20m/sec, strikes a fixed plate in such a way that the Angle between jet & plate is 60°. Find the force exerted by the jet on the plate in the direction normal to the plate (ii) In the direction of jet.



Sol: Given:

Diameter (d) = 50mm = 0.05

Velocity (V) = 20m/sec

$\theta = 60°$

$a = \pi/4 d^2 = 1.96 \times 10^{-3} m^2$

(ii) In the Direction of normal to the plate;

$F_n = \rho a v^2 \sin\theta$

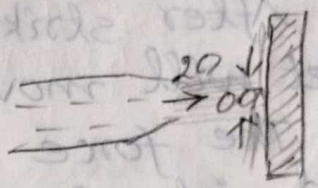
$= 1000 \times 1.96 \times 10^{-3} \times (20)^2 \times \sin 60°$

$F_n = 678.96 KN$

(ii) In the direction of jet:

$$F_x = \rho a v^2 \sin^2\theta$$
$$= 1000 \times 1.96 \times 10^{-3} \times (20^2) \sin^2 60$$
$$F_x = 588 N$$

3. A jet of water having diameter of 75mm moving with a velocity of 20m-sec strikes a flat vertical plate. Determine the force exerted by the plate.



Sol: Given: $d = 0.075m$

$V = 20 m/sec$

$$a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.075)^2 = 0.0044$$

Force exerted by the liquid jet on flat vertical plate

$$F = \rho a v^2$$
$$= 1000 \times 0.0044 \times 20^2$$
$$F = 1767.14 N$$

4. A jet of water flowing through a pipe at the end of which a nozzle is fitted the diameter of the nozzle is 100mm & the head of water at the centre of nozzle is 100m. Find the force exerted by the jet of water on a fixed vertical plate. The coefficient of velocity is given by 0.95.

Sol: Given:

Diameter (d) = 100mm = 0.1m

Head of water (H) = 100m

coefficient of velocity (C_v) = 0.95

Force (F) = ?

$$a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 0.1^2 = 0.0079$$

coefficient of velocity (C_v) = $\frac{\text{Vac}}{\text{Vthe}}$

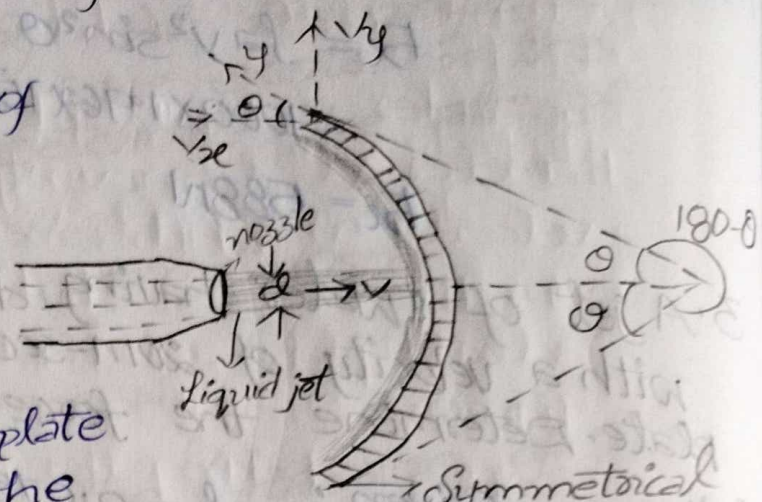
$$\text{Vthe} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 100} = 44.29$$

$$\text{Vac} = 0.95 \times 44.29 = 42.07$$

Force (F) = $\rho a v^2 = 1000 \times 0.0079 \times (42.07)^2 = 13982 N$.

Force exerted by the liquid jet on the curved plate at it's centre:

Consider a liquid jet of diameter 'd' moving with velocity 'V' which emerges from the nozzle and strikes a symmetrical curved plate at its centre.



After striking the jet plate the jet will moves along the plate. The force exerted by the liquid jet along the curved plate is given as

The mass of water striking the curved plate at it's centre per second(M) = $\rho Q = \rho a V$

Force (F) is given by

$$F = \rho a V(V - (-V\cos\theta))$$

$$F = \rho a V(V + V\cos\theta)$$

$$F = \rho a V^2(1+\cos\theta)$$

Whenever $\theta = 90^\circ$; $F = \rho a V^2$

$\theta = 180^\circ$; $F = 0$ (semicircular)

5. A jet of water of diameter 50mm moving with a velocity of 40m/sec strikes a curved fixed plate at centre. Find the force exerted by the jet in the direction of jet. If the jet deflected through an Angle of 120°.

Sol: Given:

Diameter(d) = 50mm = 0.05m ; $a = 1.96 \times 10^{-3}$

Velocity (V) = 40 m/sec

Deflected Angle = 120°

$\Rightarrow \theta = 180 - 120^\circ = 60^\circ$

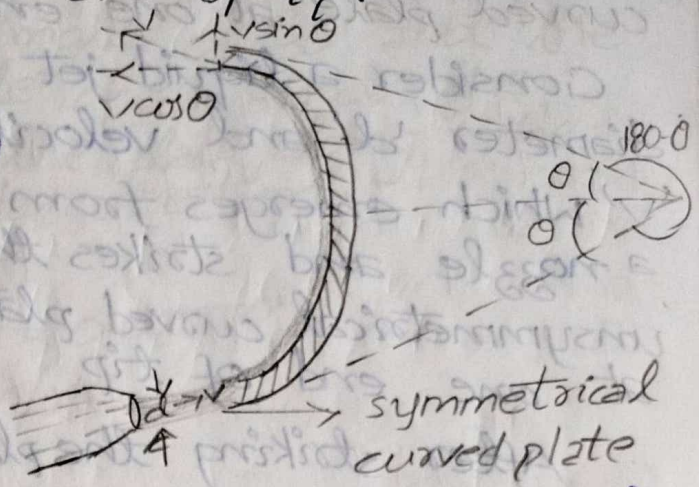
Force exerted by the jet $F = \rho a V^2(1+\cos\theta)$

$= 1000 \times 1.96 \times 10^{-3} \times 40^2(1+\cos 60^\circ)$

$F = 470.4\,N$

(iv) Forced exerted by liquid jet on a stationary symmetrical curved plate at one end of tip:

Consider a liquid jet of diameter 'd' moving with velocity 'v' which emerges from the nozzle and strikes a symmetrical curved plate at one end of tip.



After striking the plate the liquid jet will moves along the plate.

The mass of water striking the curved plate at one end per second $(M) = \rho Q$

$= \rho a V$

Force exerted by the liquid jet in Horizontal direction is $F_x = \rho a V (V\cos\theta - (-V\cos\theta))$

$F_x = 2\rho a V^2 \cos\theta$.

Along vertical direction is $F_y = \rho a V (V\sin\theta - V\sin\theta)$

$F_y = 0$

6. A jet of water of diameter 75mm moving with a velocity of 30m/sec strikes a fixed curved plate at one end tangentially at angle of 30° to the Horizontal and the jet leaves the plate at an angle of 30°. Find the force exerted by the jet on plate.

Sol: Given:

Diameter $(d) = 0.075m$; $a = \frac{\pi}{4} \times (0.\pi)^2 = 0.0044$

Velocity $(V) = 30m/sec$

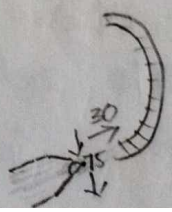
Angle $(\theta) = 30°$

Force exerted by the liquid jet in Horizontal direction is $F_x = 2\rho a V^2 \cos\theta$

$= 2 \times 1000 \times 0.0044 \times (30)^2 \cos 30°$

$F_x = 6888.92N$

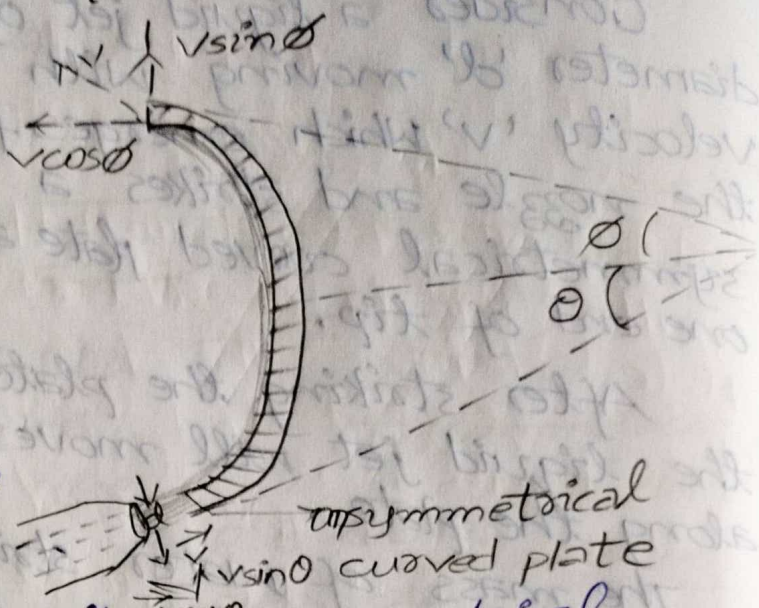
$F_x = 6.85KN$



(V) Force exerted by liquid jet on a unsymmetrical curved plate at one end of tip:

Consider a liquid jet of diameter 'd' and velocity 'V' which emerges from a nozzle and strikes the unsymmetrical curved plate at one end of tip.

After striking the plate the liquid jet moves along the direction of the plate.



The mass of water striking the unsymmetrical curved plate at one end per second $(M) = \rho Q$

$= \rho a V$

The force exerted by liquid jet in Horizontal direction $= \rho a V (V\cos\theta - (-V\cos\phi))$

$$F_x = \rho a V^2 (\cos\theta + \cos\phi)$$

The force exerted by liquid jet in vertical direction $= \rho a V (V\sin\theta - V\sin\phi)$

$$F_y = \rho a V^2 (\sin\theta - \sin\phi)$$

1. A jet of water of diameter 75mm moving with a velocity of 30m/sec strikes a curved fixed plate at one end tangentially at an Angle of 30° and the jet leaves the plate at angle of 20°. Find the force exerted by the jet on the plate along the direction of flow in perperdicular direction.

Sol: Given:

Diameter of jet(d) 75m = 0.075m

velocity(V) = 30m/sec

$\theta = 30°$

$\phi = 20°$

Force exerted by the jet on the plate along the direction of flow

Vertical: $F_y = \rho a V(V\sin\theta - V\sin\phi)$

$= 1000 \times \frac{\pi}{4} \times 0.075^2 \times 30^2(\sin 30° - \sin 20)$

$F_y = 628.14N$

Horizontal: $F_x = \rho a V^2(\cos\theta + \cos\phi)$

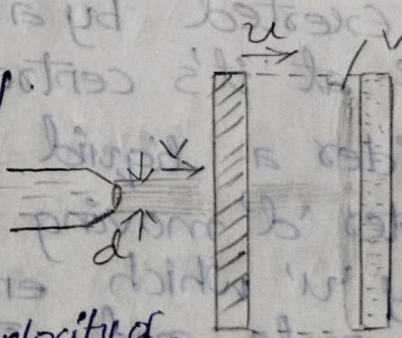
$= 100 \times \frac{\pi}{4} \times 0.075^2 \times 30^2(\cos 30° + \cos 20)$

$F_x = 717.96N$

* Moving plates:

1) Force exerted by a liquid jet on a moving vertical plate:

Consider a liquid jet of diameter 'd' moving with a velocity 'V' which emerges from the nozzle strikes a moving vertical plate as shown figure.



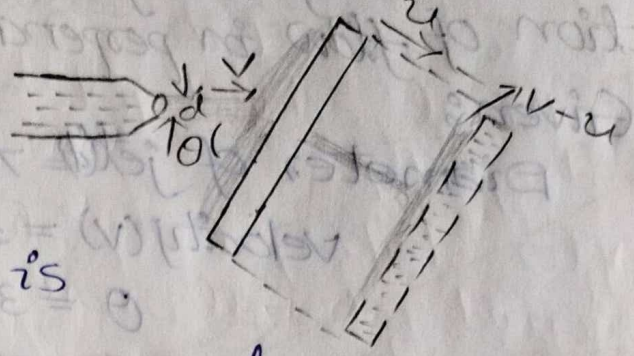
The relative velocity of moving jet is = V-u. (∵ velocity of jet - velocity of moving plate)

Force exerted by the liquid jet on a moving vertical plate = $\rho a(V-u)((V-u)-0)$

$F = \rho a(V-u)^2$

(ii) Force exerted by a liquid jet on an inclinely moving vertical plate:

Consider a liquid jet of diameter 'd' moving with a velocity 'v' which emerges from the nozzle and strikes the moving inclined plate.



The relative velocity of inclinely moving jet is V-u

Force exerted by the liquid jet

$$F_n = \rho a(v-u)((v-u)\sin\theta - 0)$$

$$F_n = \rho a(v-u)^2 \sin\theta$$

Along Horizontal direction;

$$F_x = F_n \sin\theta$$

$$= \rho a(v-u)^2 \sin\theta \cdot \sin\theta$$

$$F_x = \rho a(v-u)^2 \sin^2\theta$$

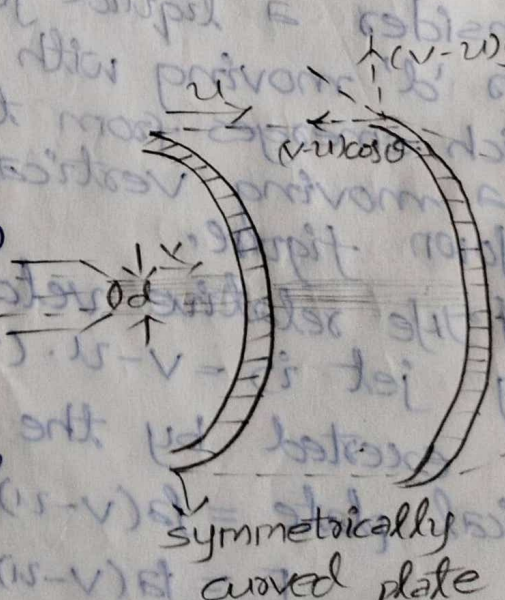
Along vertical direction;

$$F_y = F_n \cos\theta$$

$$F_y = \rho a(v-u)^2 \cdot \cos\theta \cdot \sin\theta$$

(iii) Force exerted by a liquid jet on a moving curved plate at it's centre:

Consider a liquid jet of diameter 'd' moving with a velocity 'v' which emerges from the nozzle and strikes the moving curved plate at it's centre.



The relative velocity of moving jet is 'v-u'.

Force exerted by the liquid jet

$$F = \rho a(v-u)(v-u) \rightarrow (-(v-u)\cos\theta)$$

$$= \rho a(v-u)((v-u) + (v-u\cos\theta)$$

$$F = \rho a(v-u)^2(1+\cos\theta)$$

Whenever $\theta = 90°$; $F = \rho a(v-u)^2$

$\theta = 180°$; $F = 0$

Work done per second by the jet on the moving plate (W) = $F_x \times u$ (Walls)

power of the jet (P) = $\frac{\text{Workdone}}{1000}$

$$= \frac{F_x \times u}{1000} \text{ kW}$$

Efficiency of the jet (η) = $\frac{\text{Workdone}}{\text{Kinetic energy}}$

$$= \frac{F_x \times u}{\frac{1}{2}\rho a v^3} \times 100$$

A jet of water of diameter 10cm strikes a flat vertical plate with a velocity of 15m/sec, the plate is moving with a velocity of 6m/sec in the direction of jet & away from the jet. Find out
(i) force exerted by the jet on the plate
(ii) Workdone per second on the jet
(iii) power of jet
(iv) Efficiency of jet.

Sol: Given: Diameter (d) = 10cm = 0.1m

Initial velocity (v) = 15 m/sec (∵ jet velocity)

Final velocity (u) = 6m/sec (∵ plate velocity)

(i) Force exerted by the jet on plate

$$F = \rho a(v-u)^2$$

$$= 1000 \times \frac{\pi}{4}(0.1)^2(15-6)^2$$

$$F_x = 636.17$$

(ii) Workdone (W) = $F_x \times u$

$= 636.17 \times 6$

$= 3817.03$ W

(iii) power (P) = $\frac{\text{Workdone}}{1000} = 3.817$ KW

(iv) efficiency (η) = $\frac{\text{Workdone}}{\frac{1}{2} \rho a v^3} \times 100$

$= \frac{3817.03}{\frac{1}{2} \times 1000 \times \frac{\pi}{4} (0.1)^2 \times 15^3} \times 100$

$\eta = 79\%$.

2. A 7.5cm diameter jet having a velocity of 30m/sec strikes a inclined plate to the normal of which is inclined at 45° to the Axis of the jet. Find the Normal force on the plate.

(i) When the plate is stationary.

(ii) When the plate is moving with a velocity of 15m/sec and away from jet also determine efficiency of the jet.

Sol: Given:

Diameter of jet (d) = 7.5cm = 0.075

velocity (v) = 30m/s

Angle (θ) = 45°

(i) When the plate is stationary:

Normal force on the plate

$F_n = \rho a v^2 \sin\theta$

$= 1000 \times \frac{\pi}{4} (0.075)^2 \times 30^2 \sin 45°$

$F_n = 2811.5$ N

(ii) When the plate is moving;

$v = 30 m/sec$; $u = 15 m/sec$

Force; $F_n = \rho a (v-u)^2 \sin\theta$

$= 1000 \times \frac{\pi}{4} (0.075)^2 (30-15)^2 \sin 45^\circ$

$F_n = 702.87 kN$

$\Rightarrow F_x = \rho a (v-u)^2 \sin^2\theta$

$= 1000 \times \frac{\pi}{4} (0.075)^2 (30-15)^2 \sin^2 45$ (4.417×10^{-3})

$= 497.009 kN$

Workdone $(W) = F_x \times u = 497.009 \times 15 = 7455.14 W$

power $(P) = \frac{\text{Workdone}}{1000} = 7.45 kW$

Efficiency $(\eta) = \frac{\text{Workdone}}{\frac{1}{2}\rho a v^3} \times 100$

$= \frac{7455.14}{\frac{1}{2} \times 1000 \times \frac{\pi}{4} (0.075)^2 \times 30^3} \times 100$

$= \frac{7455.14}{1988.039} \times 100$

$\eta = 12.5\%$

3. A jet of water of diameter of 7.5cm strikes a curved plate at it's centre with a velocity of 20m/sec. the curved plate is moving with a velocity of 8m/sec. In the direction of jet. The jet is deflected through an angle of 165°. Find force, power, efficiency of jet.

Given: Diameter $(d) = 7.5cm = 0.075$

velocity $(V) = 20 m/sec$

Moving velocity $(u) = 8 m/sec$

Deflected Angle $= 165^\circ$

$\theta = 180 - 165^\circ = 15^\circ$

Force exerted by the curved moving plate is

$$F_x = \rho Q (v - v_1)^2 (1 + \cos\theta) \quad N$$

$$= 1000 \times \frac{\pi}{4} (0.075)^2 (20 - 8)^2 (1 + \cos 15^\circ)$$

$$F_x = 1250.66 N$$

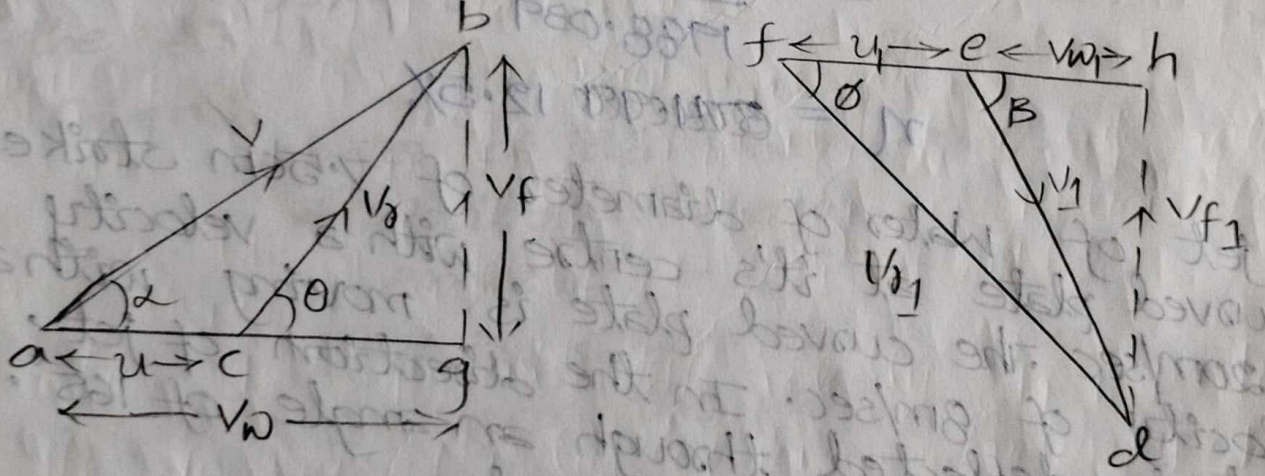
Workdone $(W) = F_x \times v_1 = 10003.38 W$

power $(P) = \frac{W \cdot D}{1000} = 10.003 kW$

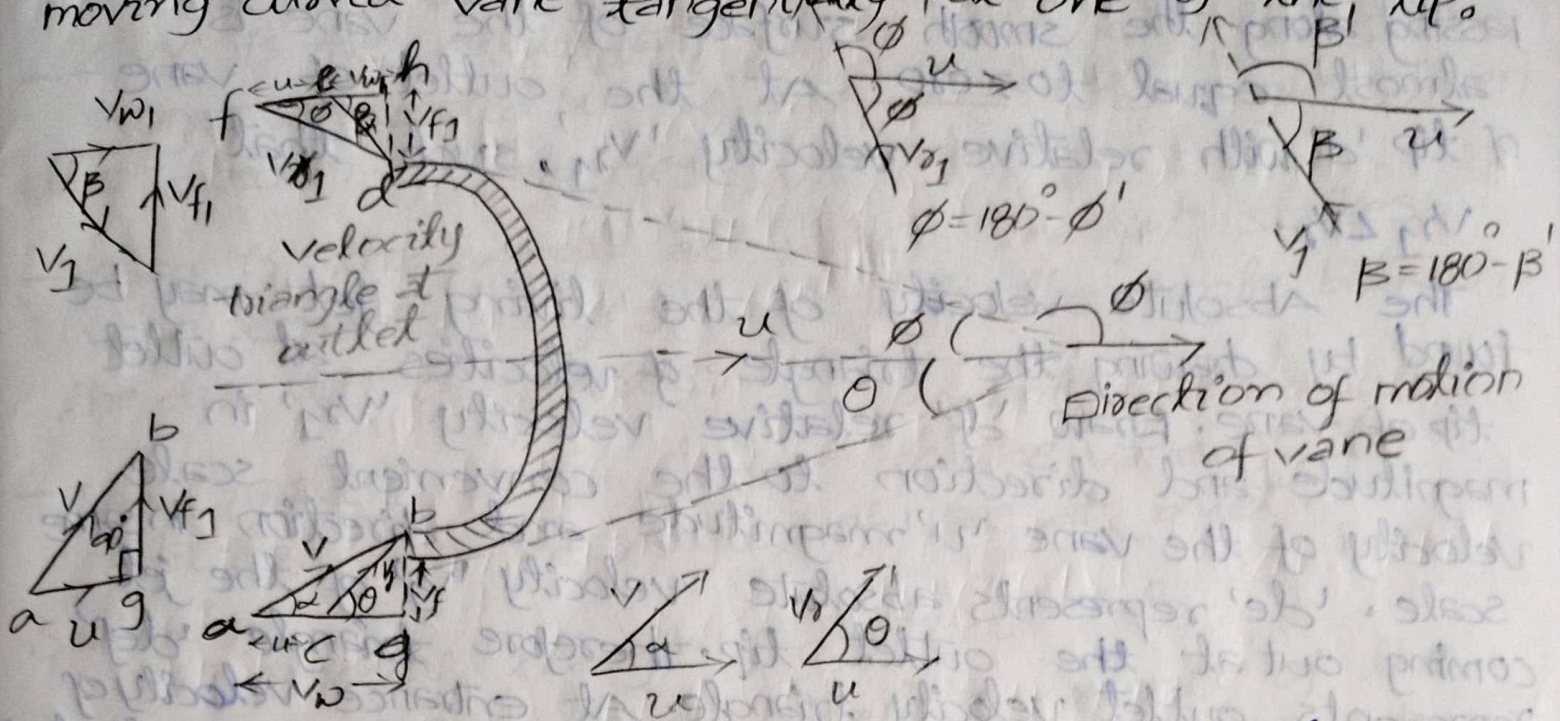
Efficiency $(\eta) = \frac{W \cdot D}{\frac{1}{2} \rho a v^3} \times 100$

$$= \frac{10003.38}{\frac{1}{2} \times 1000 \times 11.417 \times 10^{-3} \times (20)^3} \times 100$$

$$\eta = 56.61\%$$



Force exerted by liquid jet striking on unsymmetrical moving curved vane tangentially, at one of the tip:



Let us consider a curved vane moving with a velocity of 'u' in the 'x' direction. A jet of cross sectional Area 'a' moving with a velocity of 'v' strikes the vane tangentially at one of the tip. In this case the effective velocity of the jet entering the vane will be equal to relative velocity of jet with respect to the vane

V_r is the relative velocity between liquid jet & vane.

Draw a line 'ab' represents the velocity of the jet in magnitude & direction to the convenant scale.

'ac' represents the velocity of vane 'u' in Magnitude & direction. 'cb' represents the relative velocity 'V_r' between the jet & vane, in magnitude & direction to the same scale.

$\triangle ABC$ is the velocity triangle at inlet of tip.

∵ The jet is assumed to impenge (strike) on the vane tangentially, there is no loss of energy because of

impact. Further the friction loss of the flow passing along the smooth surface of the vane is almost equal to zero. At the outlet of vane of tip 'd' with relative velocity 'V_{r_1}', such that

$V_{r_1} < V_r$.

The Absolute velocity of the living jet may be found by drawing the triangle of velocities at outlet tip of vane. Draw 'df' relative velocity 'V_{r_1}' in magnitude and direction to the convenient scale velocity of the vane 'u' magnitude and direction in same scale. 'de' represents absolute velocity 'V_1' of the jet coming out at the outlet tip. therefore triangle 'def' represents outlet velocity triangle. At entrance velocity of the entering jet 'v' may be resolved into two components that are one parallel to other is perpendicular to the direction of motion given

line 'ac' represents component velocity parallel to the direction of motion of vane which is represented by 'V_w' & it is known as velocity of the whierl

'bg' represent component of velocity perpendicular to the direction of motion of vane which is represented as 'V_f' and is velocity of flow at inlet.

Similarly at leaving position

line 'eh' represents component of Absolute velocity V_1 parallel to the direction of motion of vane V_{w_1} is the velocity of vane at outlet

line 'hd' represents the velocity V_{f_1} perpendicular to the direction of motion of vane. i.e velocity of vane at outlet V_{f_1}

In the velocity triangle abc at inlet
- ∠bac is the angle at which the jet strikes the vane represented by 'α' and it is the angle between the direction of velocities V, u.
- ∠bcg is the angle at which the jet strikes the vane represented by 'θ' and it is the angle between the direction of velocities V_j, V_f (velocity of moving plate).
- V_j is parallel to the tangent to the vane at it's inlet tip, 'θ' is the angle of vane at inlet.

Similarly; velocity triangle fhd at out let
- ∠deh is Angle between Absoulute velocity of the jet 'V_1' & direction of motion of vane 'β'.
- ∠dfe is Angle between the direction of velocities V_{r1} & u & direction of motion of vane is 'ϕ'.
- V_{r1} is parallel to the tangent of vane at it's outlet tip. ϕ is Angle between the velocities u_1 & V_1 is β'. But Angle between the velocities is 'β' & it can be represented as $\beta = 180 - \beta'$.
- The Angle between the velocities is u & V_{r1} is ϕ'. But the Angle of vane at outlet tip is considered as ϕ &, it can be written as $\phi = 180 - \phi'$.
- This case can be Analysed by applying the principle of relative motion to the whole system.

For this case the moving vane is brought to stationary state by applying a velocity equal in magnitude to vane but in opposite direction.

i.e -u' on the whole system.

Mass of the fluid striking the vane per second is $\rho Q = \rho a V_j$

Component of velocity 'V_f' at inlet the direction of motion of vane $= V_f \cos\theta = V_w - u$

IIly component of velocity 'V_{f_1}' at outlet in the direction of motion of vane $= -V_{f_1}\cos\phi$

$= -(u + V_{w_1})$, $\beta < 90^\circ$

When $\beta = 90^\circ$; $V_{w_1} = 0$

$\therefore -V_{f_1}\cos\phi = -u$

When $\beta > 90^\circ$; $V_{w_1} = -(u - V_{w_1})$

$-V_{f_1}\cos\phi = -(u - V_{w_1})$

$\therefore$ The force exerted by the liquid jet striking on unsymmetrical moving curved vane tangentially at one of tip

$$F = \rho a V_f ((V_w - u) + (u + V_{w_1}))$$

$$F = \rho a V_f (V_w + V_{w_1})$$

When $\beta = 90^\circ$; $V_{w_1} = 0$

$$F = \rho a V_f \cdot V_w$$

When $\beta > 90^\circ$; $V_{w_1} = -(u - V_{w_1})$

$$F = \rho a V_f ((V_w - u) + (-(u - V_{w_1})))$$

$$F = \rho a V_f (V_w - V_{w_1})$$

$\therefore$ The general expression for 'V_x' becomes

$$V_x = \rho a V_f (V_w \pm V_{w_1})$$

The workdone for second is $V_x \times u$

$$\Rightarrow \rho a V_f (V_w \pm V_{w_1}) u \quad \text{——} (1)$$

The workdone per second is also equal to change in kinetic energy of the jet per second.

Hence Workdone per second is $\frac{1}{2} \rho a V_1 V^2 - \frac{1}{2} \rho a V_2 \cdot V_1^2$

$$= \frac{\rho a V_1}{2} (V^2 - V_1^2) \quad \text{---------- (2)}$$

∴ By equating the above expressions

$$\rho a V_1 (V_w \pm V_{w1}) u = \frac{\rho a V_1}{2} (V^2 - V_1^2)$$

$$V^2 - V_1^2 = 2 (V_w \pm V_{w1}) u.$$

Efficiency of the jet = $\frac{\text{Workdone}}{\frac{1}{2} \rho a V^3} \times 100$

$$\eta = \frac{\rho a V_1 (V_w \pm V_{w1})}{\frac{1}{2} \rho a V^3} \times 100$$

Torque exerted on a Wheel with radial curved vane:

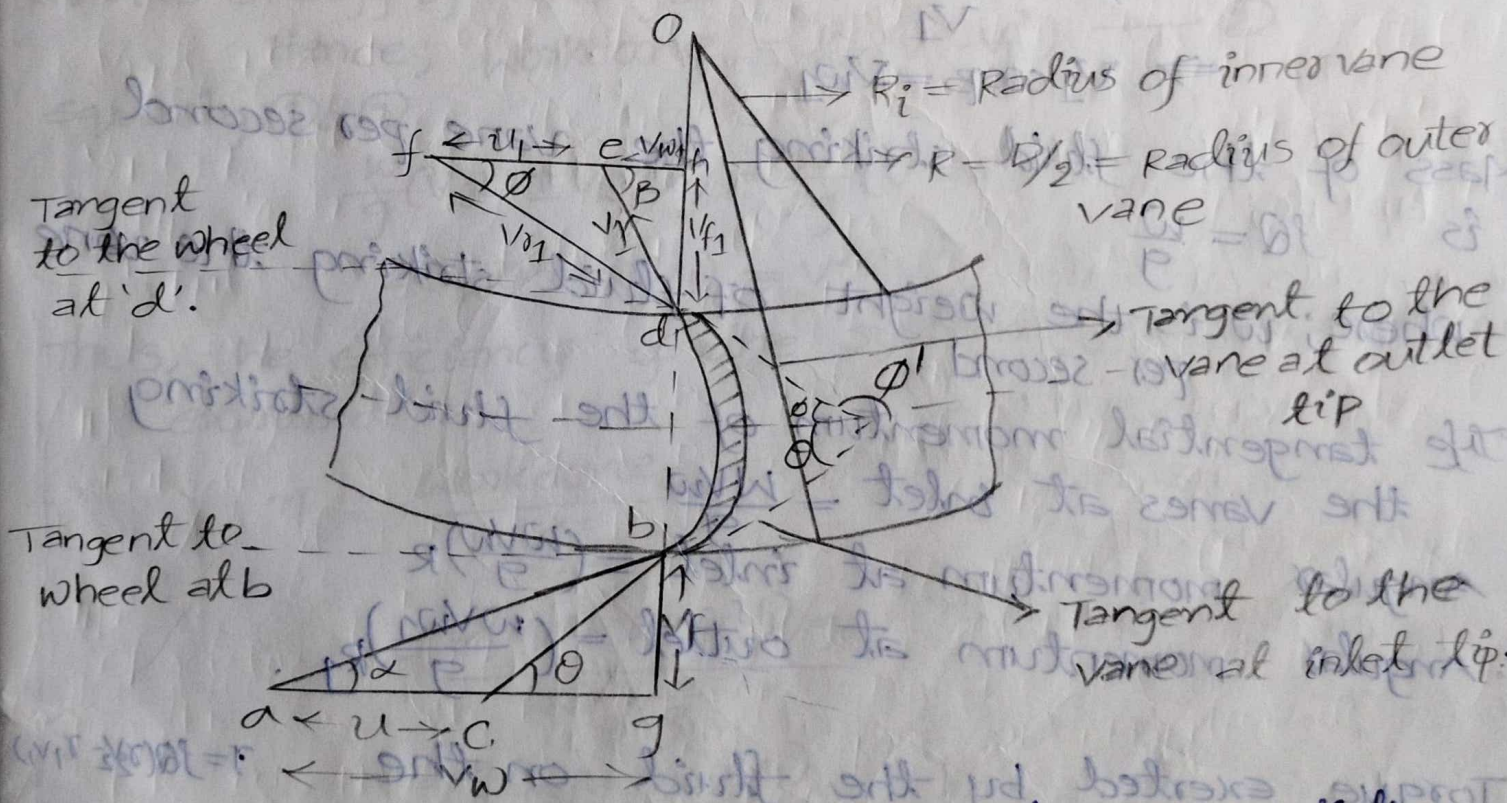


Figure shows part of the wheel of a radial flow Hydraulic turbine. R & R_i are the radius of inner & outlet ends of vane. The entire wheel is rotated

at Angular velocity of 'w', 'u' & u_1 are the tangential velocities at inlet & outlet of tip of vanes.

i.e $u = Rw$

$$u = \frac{2\pi N}{60} R$$

$u_1 = R_i w$; $u_1 = \frac{2\pi N}{60} R_i$

'N' represents the speed of wheel in rpm.

The tangential component or the component of Absolute velocity 'v' of incoming jet in the direction of motion of jet 'u' is

$$\cos\alpha = \frac{V_w}{V}$$

$$\Rightarrow V\cos\alpha = V_w$$

Ilrly $\cos\beta = \frac{V_{w1}}{V_1}$

$$\Rightarrow V_1\cos\beta = V_{w1}$$

Mass of the fluid striking the vane per second is $\rho Q = \frac{w}{g}$

where; w is the weight of fluid striking the vane per second.

The tangential momentum of the fluid striking the vanes at inlet $= \frac{wV_w}{g}$

Angular momentum at inlet $= \left(\frac{wV_w}{g}\right)R$

Angular momentum at outlet $= \left(\frac{wV_{w1}}{g}\right)R_1$.

Torque exerted by the fluid on the wheel to revolve it is

$$T = \left[\frac{wV_w}{g}R - \left(\frac{wV_{w1}}{g}R_1\right)\right]$$

$T = \rho Q(r_2V_2 - r_1V_1)$

$$T = \frac{W}{g}\left[V_w R + V_{w_1} R_1\right]$$

power delivered (or) workdone on the wheel
= Torque × Angular velocity = $T \times \omega$

$$= \frac{W}{g}(V_w R + V_{w_1} R_1)\omega$$

$u = \omega R$; $u_1 = \omega R_1$

$$= \frac{W}{g}\left(V_w \cdot \frac{u}{\omega} + V_{w_1} \cdot \frac{u_1}{\omega}\right)\omega$$

Workdone on the wheel $= \frac{W}{g}(V_w \cdot u + V_{w_1} \cdot u_1)$

$\Rightarrow$ Workdone on the wheel per second $= \frac{W}{g}(V_w u \pm V_{w_1} u_1)$ ——— (1)

If $\beta = 90°$; $V_{w_1} = 0$

Workdone $= \frac{W}{g}(V_w \cdot u)$

If there is no loss of energy, the workdone on the wheel per second is equal to the change in kinetic energy of the jet per second

Hence; Workdone $= \frac{W}{2g}(V^2 - V_1^2)$ ——— (2)

Equating (1) & (2)

$$\frac{W}{g}(V_w u \pm V_{w_1} u_1) = \frac{W}{2g}(V^2 - V_1^2)$$

$$2(V_w u \pm V_{w_1} u_1) = V^2 - V_1^2$$

Thus, the efficiency of the system may also be expressed as

$$\eta = \frac{\text{Workdone}}{WV^2/2g}$$

$$= \frac{W(V_w u \pm V_{w_1} u_1)/g}{WV^2/2g}$$

$$= \frac{2(V_w u \pm V_{w_1} u_1)}{V^2}$$

$$= \frac{V^2 - V_1^2}{V^2}$$

$$\eta = \left[1 - \left(\frac{V_1}{V}\right)^2\right]$$

1. A jet of water having a velocity of 45m/sec strikes without shock a series of vanes moving at 15m/sec, The direction of motion of vanes being inclined at 20° to that of the jet. The relative velocity at outlet is 0.9 times the relative velocity of inlet, and the Absolute velocity of the water at exit is to be normal to motion of the vanes. Find vane Angles at entrance & exit. Find workdone on vanes per unit weight of water supplied by the jet & also find Hydraulic efficiency.

Sol: Given:

velocity of jet is (V) = 45m/sec,

velocity of moving vane (u) = 15m/sec,

Direction of motion ($\alpha = 20°$)

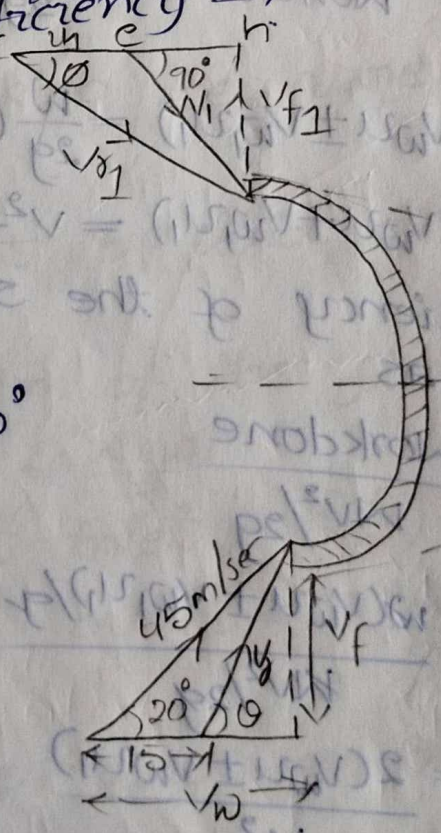
Condition: $V_{f_1} = 0.9 V_{r_1}$.

vane at exist is normal (β) = 90°

θ, ϕ = ?

workdone = ?

Hydraulic efficiency = ?



At inlet velocity triangle;

Cosine rule at inlet velocity triangle;

$$V_r^2 = V^2 + u^2 - 2Vu\cos\alpha$$

$$= 45^2 + 15^2 - 2(45)(15)\cos 20°$$

$$V_r = 31.33 \text{ m/sec}$$

$$V_{r_1} = 0.9 V_r = 28.19 \text{ m/sec}$$

(i) θ = ?:

$$\tan\theta = \frac{V_f}{V_w - u}$$

$$\sin\alpha = \frac{V_f}{V}$$

$$V_f = V\sin\alpha = 45\sin 20° = 15.39$$

$\cos\alpha = \frac{V_w}{V}$

$V_w = V\cos\alpha = 45\cos 20^\circ = 42.28$

$\Rightarrow \tan\theta = \frac{15.39}{42.28 - 15}$

$\boxed{\theta = 29^\circ 25'}$

From outlet velocity triangle;

$\cos\phi = \frac{u_1}{v_{r1}}$

$= \frac{15}{28.19}$

$\boxed{\phi = 57^\circ 51'}$

(ii) Workdone;

Workdone by unit weight of water is

$= \frac{V_w u}{g}$

$= \frac{42.28 \times 15}{9.81}$

$W = 64.65$

(ii) Efficiency:

$\eta_0 = \frac{\text{Workdone}}{\text{kinetic energy of water } (V^2/2g)} \times 100$

$= \frac{64.65}{\left(\frac{45^2}{2\times 9.81}\right)} \times 100$

$\eta = 62.63\%$

2. A jet of water moving at 20m/sec strikes an symmetrical curved vane shaped to deflect the jet through 120°. (i.e vane Angles θ & ϕ are each = 30°) If the vane is moving at 5m/sec, Find the Angle of the jet so that there is no shock at inlet & also determine absolute velocity of exit in magnitude & direction & the workdone.

Sol: Given:

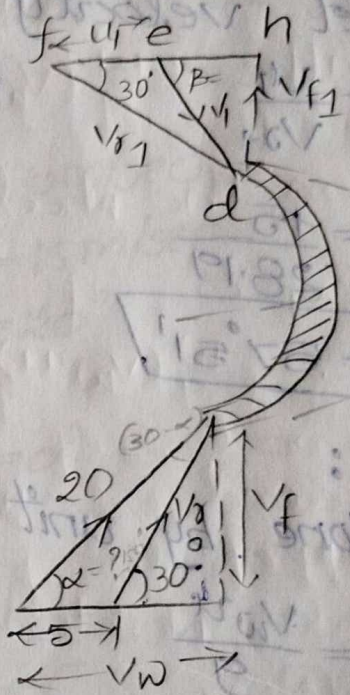
Velocity of jet (V) = 20m/sec

$\phi, \theta = 30°$

Velocity of vane (u) = 5m/sec

Angle at inlet α = ?

velocity at outlet (V_1) = ?



At inlet velocity triangle;

Apply sine rule;

$$\frac{V}{\sin 150^\circ} = \frac{u}{\sin(30-\alpha)} = \frac{V_r}{\sin\alpha}$$

$$\Rightarrow \frac{20}{\sin 150^\circ} = \frac{5}{\sin(30-\alpha)}$$

$$20 \times \sin(30-\alpha) = 2.5$$

$$30 - \alpha = 7.18$$

$$\alpha = 22.49^\circ$$

$$\Rightarrow \frac{20}{\sin 150^\circ} = \frac{V_r}{\sin 22.49^\circ}$$

$$V_{r_1} = V_r = 15.51 \quad (\because \text{velocity is constant})$$

$$\tan\theta = \frac{V_f}{V_w - u}$$

$$\rightarrow \sin\alpha = \frac{V_f}{V}$$

$$V_f = V \sin\alpha = 20$$

$$\Rightarrow \cos 30^\circ = \frac{V_f}{V_r}$$

$$V_r \cos 30^\circ = V_f$$

$$\therefore V_f = 15.51 \cos 30^\circ$$

$$V_f = 13.43$$

$\Rightarrow \sin\alpha = \frac{13.143}{20}$

$\alpha = 42°10'$

$\xrightarrow{2.5} \cos\alpha = \frac{V_w}{V}$

$V_w = V \cdot \cos\alpha$

$= 20 \cdot \cos 42°10'$

$V_w = 14.82$

From outlet velocity triangle applying the cosine rule

$V_1^2 = u_1^2 + V_{r_1}^2 - 2u_1 V_{r_1} \cos 30°$

$20^2 = u_1^2 + (15.51)^2 - 2u_1 \cdot 20 \cos 30°$ $V_1^2 = 5^2 + (15.51)^2 - 2 \times 5 \times 15.51 \cos 30°$

$159.43 = 11^2 - 34.64u_1$ $V_1 = 11.216 m/s$

from outlet velocity triangle;

$\tan\beta = \frac{V_{r_1} \sin 30°}{V_{r_1} \cos 30° - u_1}$

$= \frac{15.51 \sin 30°}{15.51 \cos 30° - 5}$

$\beta = 42°36'$

Workdone per unit weight of fluid

$W = \frac{1}{g}[V_w u \pm V_{w_1} u_1]$

$\frac{V_w}{V} = \cos\alpha \Rightarrow V\cos\alpha = V_w \Rightarrow V_w = 20 \cos 22°49' = 18.44 m/sec$

$\frac{V_{w_1}}{V_1} = \cos\beta \Rightarrow V_1 \cos\beta = V_{w_1} \Rightarrow V_{w_1} = 11.216 \cos 42°36' = 8.43 m/sec$

$\rightarrow W = \frac{1}{g}[V_w + V_{w_1}] u$ ($\because V_{w_1}$ is opposite to the motion of vane.

$= \frac{5}{9.81}[18.44 + 8.43]$

$W = 13.70 Nm$

5. Hydraulic turbine's

Syllabus:

- Layout of typical Hydropower installation
- Heads & efficiencies
- classification of turbines
- pelton wheel
- Francis turbine
- kaplan turbine working,
- working propositions
- velocity diagram
- Workdone & efficiency
- Hydraulic design, draft tube
- theory & efficiency
- Governing of turbines
- surge tanks
- units & specific quantities
- selection of turbines
- performane characteristics
- Geometric similarity
- cavitation.

Turbine's: (Mechanical energy → Electrical energy)

These are the Hydraulic machines which convert Hydraulic energy to Mechanical energy. -this mechanical energy is used in running an electric generator which is directly coupled to the shaft of turbine.

Classification of Turbines:

- According to type of energy at inlet
  - Impulse
  - Reaction
- According to direction of flow of water
  - Tangential flow turbines
  - Axial flow turbines
  - Radial flow turbines
    - Inward
    - outward
  - Mixed flow turbines
- According to water head & quantity of water available
  - High head & small quantity of flow of water
  - Medium head & medium flow rate
  - Low head & large flow rate.
- According to the specific speed of the turbine
  - Low specific speed turbine (<60)
    - Ex: pelton wheel
  - Medium specific speed turbine (60 to 400)
    - ex: Francis wheel
  - High specific speed turbine (>400)

Impulse turbine:

If at the inlet of the turbine, the energy available is only kinetic energy, the turbine is known impulse turbine.

Ex: Pelton wheel turbine

Reaction turbine:

If at the inlet of the turbine, the water posses kinetic energy as well as pressure energy, the turbine is known as Reaction turbine.

Ex: Francis turbine, Kaplan turbine

Radial flow turbine:

If the water flow in the radial direction through the runner, the turbine is called radial flow turbine.

Inward radial flow turbine:

If the water flows from outward to inward radially, the turbine is known as inward radial flow turbine.

Ex: old francis turbine, Thomson turbine

Outward radial flow turbine:

If the water flow radially from inward to outwards, the turbine is known as outward radial flow turbine.

Ex: Fourneyron turbine

Axial flow turbine:

If the water flow through the runner along the direction parallel to the axis of rotation of the runner, the turbine is called Axial ~~wheel~~ flow turbine.

Ex: Francis turbine.

Mixed flow turbine:

If the water flows through the runner in the radial direction but leaves in the direction parallel to Axis of rotation of runner, the turbine is called mixed flow turbine.

Ex: Modern Francis turbine.

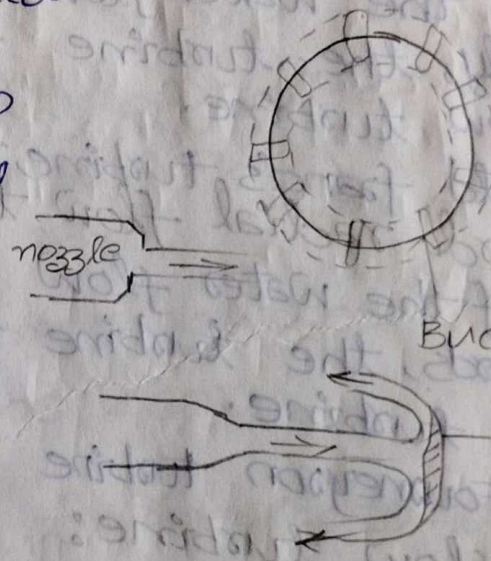
Tangential flow turbine:

If the water flows along the tangent of the runner, the turbine is called Tangential flow turbine.

Ex: pelton wheel turbine.

→ Impulse turbine & reaction turbine:

- Water from penstock to turbine via one or more nozzle.
- pressure energy is converted into kinetic energy.
- Rotation of wheel due to tangential force created by impact of jet.



⇒ Reaction turbine:

- Both kinetic & pressure energy.
- Feeding of water through fixed blades in casing.
- static pressure at inlet is greater than outlet of runner.
- Rotation due to impulse reaction and due to reaction in pressure so it is called Reaction Turbine.

Classification According to speed:

specific speed:

It is the speed of a geometrically similar turbine that would develop 1KW power when working under a head of 1m.

According to Head & quantity of water:

- High head turbines:

this requires very high heads (generally > 250m) require relatively less quantity of water.

Ex: Impulse turbine

- Medium head turbines:

Heads ranges from 60m to 250m, it requires large quantity of water.

Ex: Modern francis turbines.

- Low head turbines.

Head ranges < 60m, it requires large quantity of water.

Ex: kaplan turbine.

Classification according to disposition of shafts:

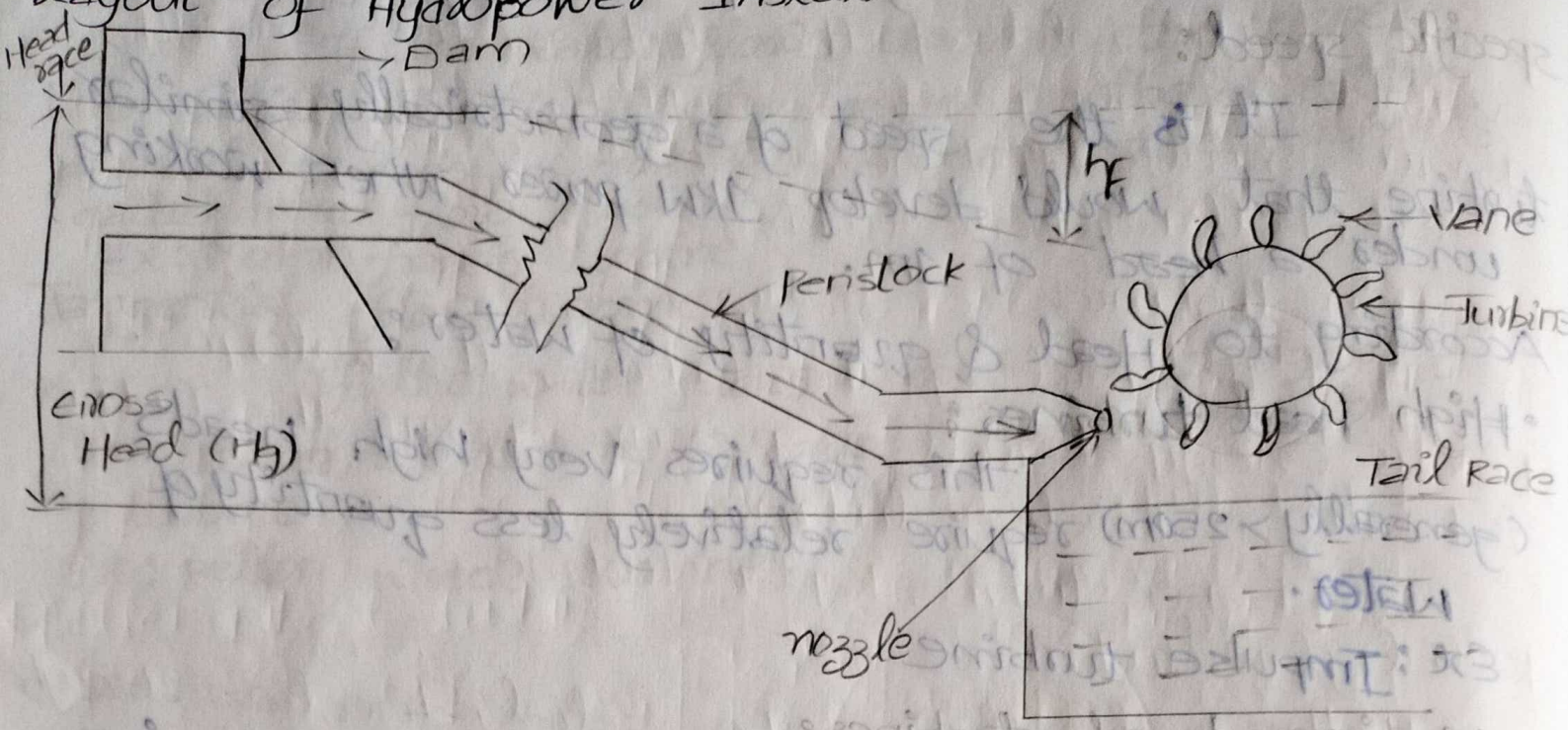
(i) Turbines with Horizontal shafts:

Ex: pelton wheel turbine

(ii) Turbines with vertical shafts:

Ex: kaplan turbine.

Layout of Hydropower Installation:



Penstock:

These are the large diameter pipes carrying water under pressure from a storage reserviour to turbine.

Head race:

It is the level of water surface in the storage reserviour.

Tail race:

It is the channel which carries water away from the power house after it has passed through the turbine.

Forebay:

It is the storage reserviour at the head of pen stocks. It's purpose is to store water temperorly when it is require by the turbine & supply the same when require.

Gross head (Hg):

It is the difference between the Head race level & tile race level. When no water is flowing. It is also termed as static Head or total Head.

Net (or) effective Head: (H)

It is the Head available at the entrance to the turbine.

$$H = H_g + H_f$$

Where; H_f = total Head loss between Head race & the entrance of turbine.

Efficiency's of Turbine:

(i) Hydraulic efficiency:

The ratio of power developed by the runner (i.e. water horse power) to ~~(ratio of power developed~~ the net power supplied by the water at the entrance to the turbine.

$$\eta_h = \frac{W H_P}{W Q H}$$

Where; Q = quantity of water actually striking the runner.

(ii) Mechanical efficiency:

The ratio of power obtained from shaft of turbine (shaft horse power or Brake horse power) to the power developed by runner.

$$\eta_m = \frac{S_{HP}}{W H_P}.$$

(iii) volume efficiency:

The ratio of volume of water actually striking the runner to the volume of water supplied to the turbine.

$$\eta_V = \frac{Q}{Q + \Delta Q}$$

Where; ΔQ = quantity of water (i.e discharge directly to the tail race without striking turbine)

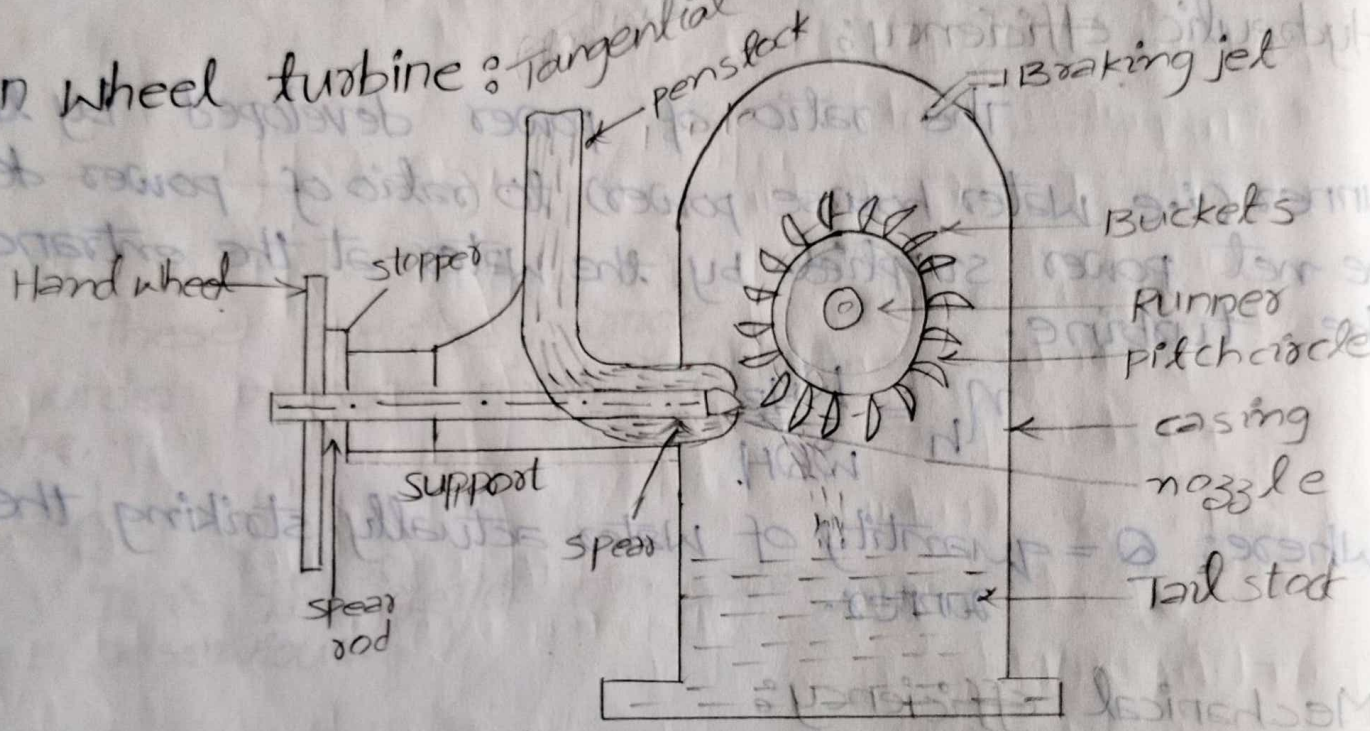
Overall efficiency:

It is the ratio of power available at the turbine shaft to the power supplied by water by the entrance to the turbine:

$$\eta_o = \frac{SHP}{\text{Net power available at entrance}}$$

$$\eta_o = \eta_h \times \eta_v \times \eta_m.$$

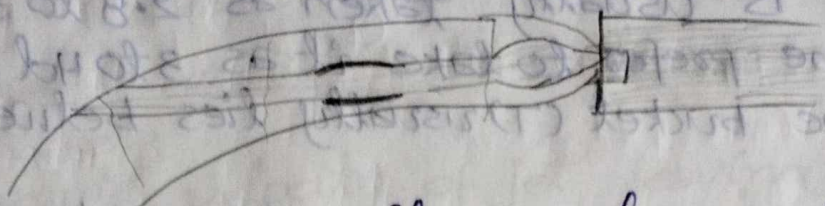
*pelton wheel turbine: Tangential



Components of pelton Turbine:

Nozzle with Guide mechanism:

- The function of nozzle of a pelton wheel is to "convert the available pressure energy into high velocity energy in the form of jet."
- The quantity of water required is propotional to the load on the turbine. Therefore, to control the flow through the nozzle, some sort of a regulating or a governing mechanism is necessary. this is generally done by spear inside the nozzle. The movement of spear inside the nozzle changes the area of flow through it, thus varying the discharge.



- The nozzle is usually made up of cast iron or cast steel.
- sometimes, a small brake nozzle is also used in case of large turbines. When the wheel is to stopped, besides cutting off the supply of water through the main nozzle, the brake nozzle also directs the water on the back of buckets to bring the wheel quickly to rest.

(ii) Runner with buckets:

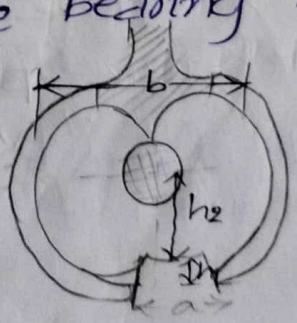
- The runner of a pelton turbine consists of a number of double cupped buckets fixed to the periphery of the wheel.
- Each bucket has a sharp edge at the centre called the splitter.
- The jet strikes each bucket at this splitter and is divided into two sides, thus avoiding any unbalanced thrust on the shaft.
- As the splitter takes the full impact of the jet, so it has to be quite strong.

(iii) Casing:

- It is made of cast iron or fabricated steel plates.
- The casing does not perform any Hydraulic function. However, a casing is necessary to avoid accidents, splashing of water, to lead the water to the tail race and to support the housing for the bearing and the nozzle.

Dimensions of bucket:

- The minimum possible width B of the bucket is Approximately 2d, where 'd' is diameter of jet.



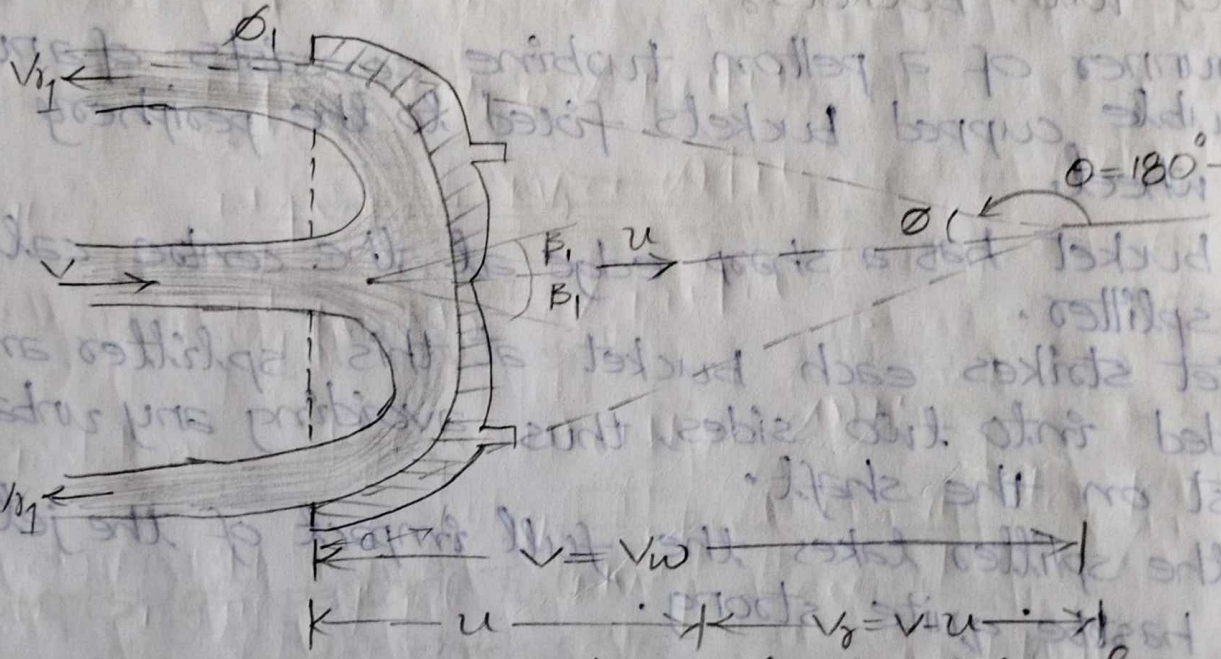
This dimension is usually taken as 2·8 to 3·2d to avoid losses. some prefer to take it as 3 to 4d.

- The depth of the bucket (T) usually lies between 0·8 to 1·2d.
- The Height (H) of the bucket is 2.4d to 2·8d.

(iv) Number of jets:

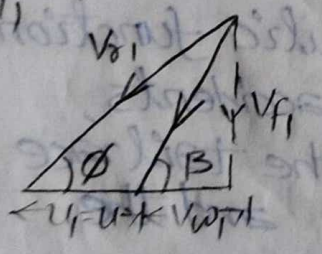
Number of jets may be increased, usually upto 4, but never more than 6 because higher number of jets makes the governing complicated.

Workdone & efficiency of pelton wheel turbine:



Inlet velocity triangle

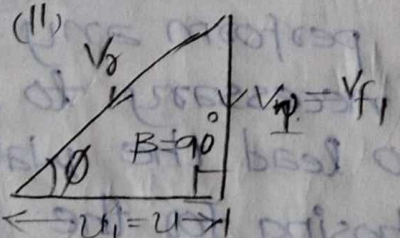
(i)



$B = 90°$,

V_w is -ve

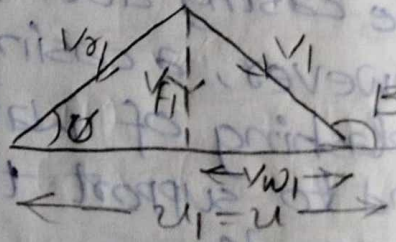
(ii)



$B = 90°$,

$V_{w_1} = 0$

(iii)



$B > 90°$,

V_{w_1} +ve

From the Figure;

V = Absolute velocity of jet before striking the bucket.

V_1 = Absolute velocity of jet leaving the bucket.

u = Absolute velocity of bucket considered along the direction tangential to the pitch circle.

V_r = Relative velocity of the incoming to jet relative to bucket.

V_{r_1} = velocity of the jet leaving the bucket relative to the bucket.

V_w = velocity of wheel at inlet tip of bucket.

V_{w_1} = velocity of wheel at outlet tip of bucket.

θ = Angle through which the jet deflected by the bucket.

$= 180° - \phi$

ϕ = Angle of the bucket at the outlet tip

In case of pelton wheel turbine, V & u are collinear; the velocity triangle at the inlet tip of the bucket is a straight line and

$V_r = V - u$ & $V = V_w$

The Inlet & outlet tips are at the same radial distance, the tangential velocity of bucket at both the tips is same.

i.e $u = u_1$.

By the consideration of loss of energy $V_{r_1} < V_r$.

$\therefore V_{r_1} = KV_r$

$= K(V - u)$

From outlet velocity triangles;

From Fig-(i) $V_{w_1} = V_{r_1}\cos\phi - u_1$ $(\beta < 90°)$

$= KV_r\cos\phi - u$

From Fig-(iii):

$$V_{w_1} = u_1 - V_{r_1}\cos\phi \quad (\beta > 90^\circ)$$
$$= u - KV_1\cos\phi$$

From Fig-(ii):

$$V_{w_1} = 0 \quad (\because \beta = 90^\circ)$$

If 'W' is the weight of water per second which strikes the bucket then Workdone per second on the wheel $\frac{W}{g}[V_w u \pm V_{w_1} u_1]$.

If losses are neglected then $K = 1$

$\beta < 90^\circ$; $= \frac{W}{g}[V - (V_{r_1}\cos\phi - u)]u$

$= \frac{W}{g}[V - KV_{r_1}\cos\phi + u]u$

$\beta > 90^\circ$; $= \frac{W}{g}[V + (u - KV_1\cos\phi)]u$

$= \frac{W}{g}[V - KV_1\cos\phi + u]u$

$$\text{Workdone} = \frac{W}{g}((V-u)(1+K\cos\phi))u$$

The energy supplied to the wheel is in the form of kinetic energy of jet $= \frac{WV^2}{2g}$ per second

$\therefore$ The Hydraulic efficiency $\eta_h = \frac{\text{Workdone}}{\text{kinetic energy}}$

$$\eta_h = \frac{2((V-u)(1+K\cos\phi))u}{V^2}$$

If $u = 0$; $\eta_h = 0$ (wheel is stationary)

The velocity of bucket is equal to velocity of liquid jet ($u = V$).

Hydraulic efficiency ($\eta_h = 0$).

Condition for Hydraulic efficiency is maximum

$$\frac{d(\eta_h)}{du} = 0$$

$u \cdot \frac{dv}{du} + v \cdot \frac{du}{du}$

$v^2 \cdot \frac{d}{du}$

$$\frac{d}{du}\left[\frac{2((v-u)(1+\cos\phi)u)}{v^2}\right] = 0$$

$$\Rightarrow u - 0.5v$$

$$\boxed{u = v/2}$$

$$\eta_h = \frac{2[(v-0.5v)((1+k\cos\phi)(0.5v))]}{v^2}$$

$$= \frac{2[(0.5v)(1+k\cos\phi)(0.5v)]}{v^2}$$

$$= 2\frac{0.25v^2(1+k\cos\phi)}{v^2}$$

$$= 0.5(1+k\cos\phi)$$

$$\eta_h = \frac{(1+k\cos\phi)}{2}$$

If there is no loss of energy, the workdone per second = change in kinetic energy per second

$\therefore$ change in kinetic energy $\frac{w}{2g}(v^2 - v_1^2)$

$\therefore$ efficiency $(\eta_h) = \frac{v^2 - v_1^2}{v^2}$

mechanical efficiency $(\eta_m) = \frac{P}{((v-u)(1+\cos\phi))u}$

The Hydraulic efficiency in pelton wheel ranges from 97 to 99%

The overall efficiency for large pelton wheels is 85 to 90%

Working proportions of pelton wheel turbine:

(i) the Ideal velocity of the jet usually known as spouting velocity ($\sqrt{2gH}$)

Where; H = Net head

However the actual velocity of the jet is slightly less due to the friction in the nozzle.

∴ velocity of jet (V) = $K_V\sqrt{2gH}$

(ii) The Maximum efficiency η_H is attained whenever the velocity of wheel 'u' at pitch circle,

$$\boxed{(\eta_H)_{max}\ u = 0.5V}$$

In actual practice the maximum efficiency occurs when the value of 'u' is about 0.46 times of velocity of jet

i.e $u = 0.46V$

$$\boxed{u = K_u V} \quad (\because K_u = 0.43 \text{ to } 0.47)$$

(iii) Angle through which jet of water gets deflected in buckets is 165° each unless otherwise stated.

(iv) The least diameter of the jet is given by

$$\boxed{d = \left(\frac{4Q}{\pi K_V\sqrt{2gH}}\right)^{1/2}} \quad \left[\because Q = AV = \pi/4\, d^2 K\sqrt{2gH}\right]$$

Where; Q = Discharge through the jet in m^3/sec

taking $K_V = 0.98$ then we obtain

$$d = 0.542 \cdot \left(\frac{Q}{\sqrt{H}}\right)^{1/2} \text{(meter)}$$

(v) Mean diameter (or) pitch diameter 'D' of the pelton wheel may be obtained as follows

If the wheel rotates at 'N' rpm,

$$\boxed{D = \frac{60u}{\pi N}}$$

(vi) the ratio of pitch diameter 'D' of the wheel to the jet diameter 'd' is known as jet ratio & it is represented by 'm'

i.e $\boxed{m = \frac{D}{d}}$

the jet diameter is an important phenomenon in the design of pelton wheel. For maximum efficiency the jet ratio to be 11 to 14 & normally at jet ratio of 12 is adopted.

(vii) some of the main dimensions of the bucket of pelton wheel is expressed interms of jet diameter are noted below.

B = 4 to 5 times of 'd'.

G = (0.81 to 1.05)d

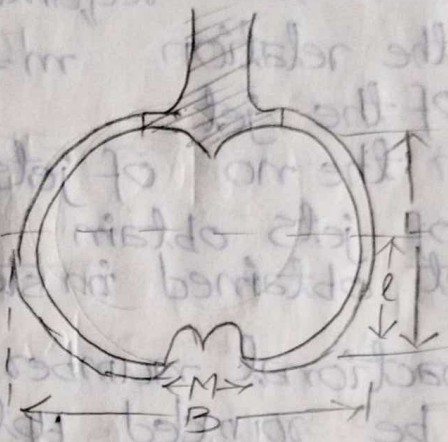
M = (1.1 to 1.25)d

L = (24 to 32)d

l = (1.2 to 1.9)d

ϕ = 10° to 20°

β_1 = 5° to 8°



(viii) The no of buckets is usually more than 15 & that can be calculated by

$$Z = \frac{D}{2d} + 15$$

Design of pelton wheel turbine runner:

A pelton wheel turbine runner require to be design to develop a known power 'P' when running at known speed 'N' rpm under a known head 'H' the probable values of 'η_0', K_v, K_u & 'm' are assumed. The various steps involved in the design are as follows:

(i) Determine the discharge required 'Q' from the relation $P = \eta_0 WH$

$P = \eta_0 (\omega Q H)$ $(\because W = \omega Q)$

(ii) From the equation $V = K_v \cdot \sqrt{2gH}$, Calculate velocity 'V' of the jet.

(iii) Calculate the Total Area of jets required by using the relation $a = Q/V$.

(iv) Calculate the pitch circle diameter 'D' from the relation $D = \frac{60v}{\pi N}$

(v) Calculate the required diameter 'd' of the jet from the relation $m = \frac{D}{d}$ & also calculate corresponding area of the jet.

(vi) Obtain the no of jets required by dividing the total area of jets obtain in step-(iii) by the Area of each jet obtained in step-(v).

(vii) The fractional number of jets obtain in step-(vi) may be rounded upto the appropriate integral number & the corresponding diameter of each jet to be Actually provided may be calculated.

(viii) Calculate the no of buckets to be provided from the expression $Z = \frac{D}{2d} + 15$ (or) $Z = 0.5m + 15$ and the bucket dimensions from the relavent equations.

1. A pelton wheel has to be designed for the following data power to be developed P = 6000 kW net head (H) available is 300 m speed (N) 550 rpm, ratio of jet diameter to wheel diameter of jets is 1/10. overall efficiency (η_h) is 85%. Find no of jets ; diameter of jet 'd', diameter of wheel 'D', & quantity of water required. $K_v = 0.98$; $K_u = 0.46$

Sol: Given data ;

Power (P) = 6000 kW = 6000×10^3 W

Net Head (H) = 300 m

speed (N) = 550 rpm

overall efficiency (η_h) = 85% = 0.85

Ratio of jet diameter to wheel diameter = 1/10

$\frac{d}{D} = \frac{1}{10}$

We know that ; power (P) = $\eta_o W H$

$P = \eta_o \rho g Q \times H$

$6000 \times 10^3 = 0.85 \times 1000 \times 9.81 \times Q \times 300$

$Q = 2.39 \text{ m}^3/\text{sec}$

(Total area) Velocity of jet (V) = $K_v \sqrt{2gH}$

$= 0.98 \sqrt{2 \times 9.81 \times 300}$

$V = 75.18 \text{ m/sec}$

velocity of bucket (u) = $K_u \sqrt{2gH}$

$= 0.46 \sqrt{2 \times 9.81 \times 300}$

$(u) = 35.29 \text{ m/sec}$

Diameter of wheel (D) = $\frac{60u}{\pi N} = \frac{60 \times 35.29}{\pi \times 550} = 1.225 \text{ m}$

Given $\frac{d}{D} = \frac{1}{10}$

Dia of jet $d = \frac{D}{10} = \frac{1.225}{10} = 0.1225 \text{ m}$

Total Area of jets $(a) = \frac{Q}{V} = \frac{2.399}{75.19} = 0.031 m^2$

Area of each jet $(m) = \frac{D}{d} = \frac{1.225}{0.1225} = 10$

No of jets $= \frac{\text{Total Area of jet}}{\text{Area of each jet}}$

$= \frac{0.031}{10}$

Area of jet $= \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 0.1225^2 = 0.011$

No of jets $= \frac{\text{Total Area of jet}}{\text{Area of each jet}} = \frac{0.031}{0.011} = 2.8 \simeq 3$

2. A pelton wheel has a mean bucket speed of 12m/sec and is supplied with water at a rate of 750lit/sec under a head of 35m if the bucket deflects the jet through an Angle of 160°. Find the power developed by the turbine & its Hydraulic efficiency. consider $K_v = 0.98$ neglect friction in bucket also determine overall efficiency of turbine. If mechanical efficiency is 80%.

Sol: Given data:

Bucket Mean speed $(u) = 12 m/sec.$

Discharge $(Q) = 750 lit/sec = 0.75 m^3/sec$

Head $(H) = 35m$

Deflects at Angle $(\theta) = 160°$

$K_v = 0.98$

$K = 1$

mechanical efficiency $(\eta_m) = 80\%$.

power $(P) = ?$

Hydraulic efficiency $(\eta_h) = ?$

overall efficiency(η_o) = ?

$\Rightarrow \phi = 180 - \theta = 180 - 160 = 20^\circ$

We know; power $P = \frac{\text{Workdone}}{1000}$

Workdone $= \frac{W}{g}((V-u)(1+k\cos\phi))u$

$W = \rho g Q = 9810 \times 0.75 = 7357.5$

velocity $(V) = k_v\sqrt{2gH} = 0.98\sqrt{2\times 9.81 \times 35} = 25.68 m/sec$

Workdone $= \frac{7357.5}{9.81}((25.68-12)(1+1.\cos 20^\circ))12$

$= 238814.95$

power $= \frac{W}{100} = 238.81 kW$

Hydraulic efficiency $(\eta_h) = \frac{2((V-u)(1+k\cos\phi))u}{V^2}$

$= \frac{2(25.68-12)(1+1\cos 20^\circ))12}{25.68^2}$

$= 0.96$

$\eta_h = 96.56\%$

$\frac{\frac{W}{g}((V-u)(1+k\cos\phi))u}{\frac{WV^2}{2g}}$

overall efficiency $(\eta_o) = \eta_h \times \eta_m$

$= 0.96 \times 0.80$

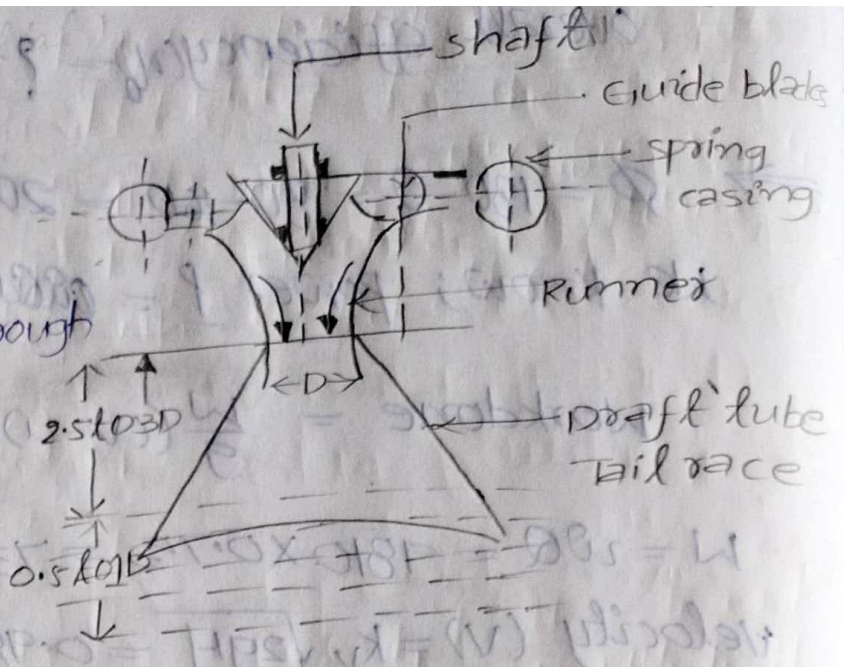
$= 76.8\%$.

* Francis turbine: Radial flow

components of francis turbine:

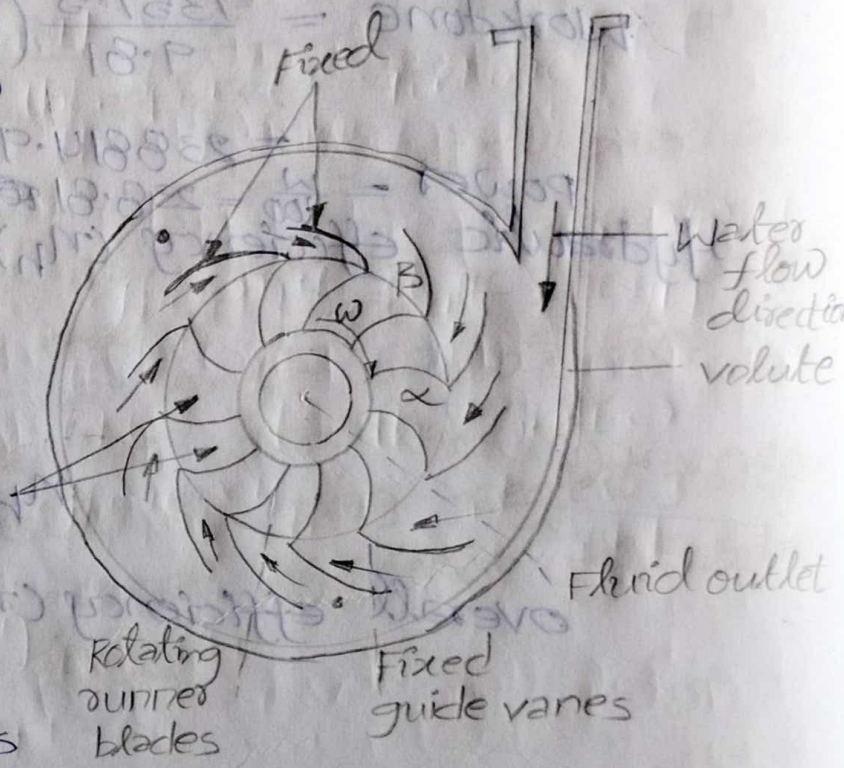
penstock:

Large diameter tube through Water from dam comes to turbine inlet it is made from steel.



spiral casing:

- It is a closed passage whose diameter gradually decreases along flow of direction. Area is maximum at inlet and nearly zero at exit.
- To maintain constant flow rate, numerous openings provided.
- The purpose of casing to distribute water over guide vanes and prevent formation of eddies.
- Made of cast steel, plates steel or concrete depends on Head.



Guide vanes:

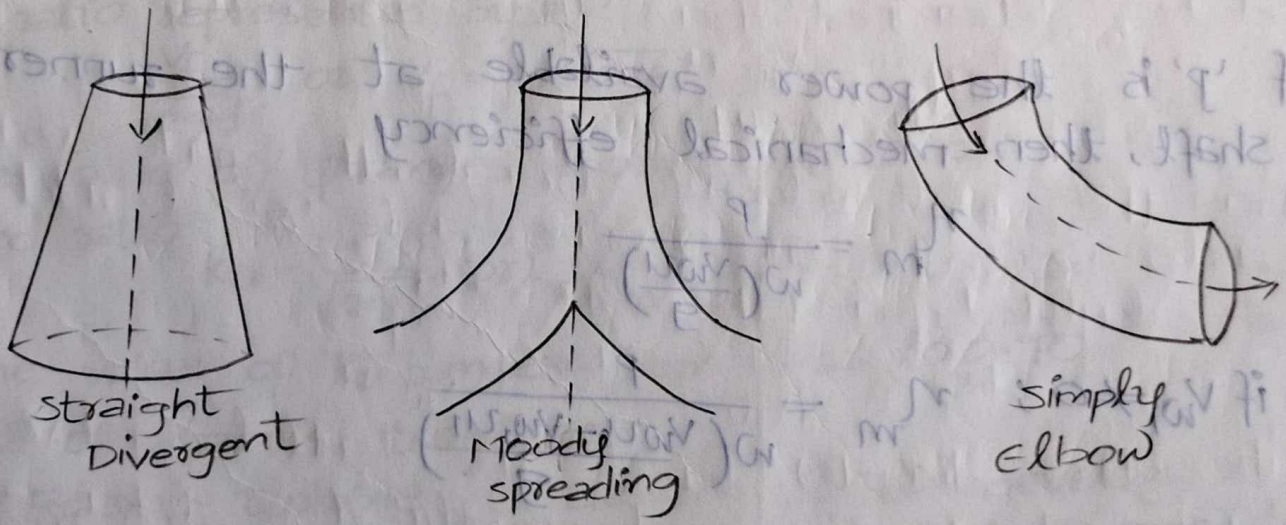
- It is Aierofoil shaped vanes fixed between two rings.
- It convert a part of pressure energy into kinetic energy.
- Each guide vane can rotate about it pivot centre hence it also serves to direct the flow at design Angles to the blade runners.

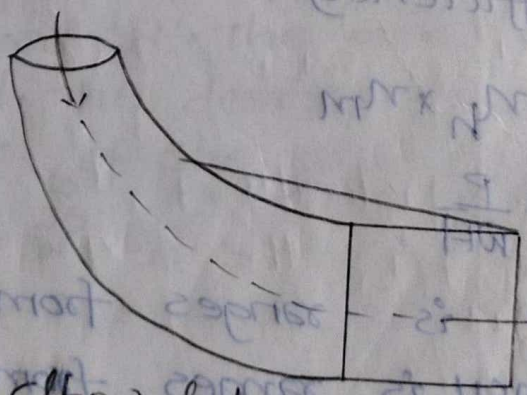
Governing mechanism:
It can change the position of guide vanes to vary the flow on turbine.

Runner:
- profile is designed so that water enter inwards in radial direction & leave it in axial direction.
- Runner rotate due to impulse & reaction effects.
- It is classified according to fast, medium & slow.
- Made of cast iron, stainless steel, bronze.
- It is keyed to shaft of turbine.

Draft tube:
- It is gradually expanding tube which discharge water passing through the runner to tail race.
- Generally with increasing diameter in flow direction.

{Draft tube:
- It is gradually expanding t}





elbow tube having circular correction of inlet & Rectangular at outlet.

Workdone & efficiency of francis turbine:

If 'W' is the weight of water per second which strikes the runner.

∴ The workdone per second on the runner $= \frac{W}{g}(V_{w2}u - V_{w1}u_1)$

Maximum output under specified conditions is obtained by making the velocity of wheel by exit = 0

Then, Workdone $= \frac{W}{g}(V_{w2}u)$ or

$$\frac{\text{Workdone}}{\text{unit weight of water}} = \frac{V_{w2}u}{g}$$

If 'H' is the net head then the input energy per second for the runner $= WH$

∴ Hydraulic efficiency for the francis turbine

$$\eta_h = \frac{V_{w2}u}{gH}$$

If $V_{w1} \neq 0$ Then, Hydraulic efficiency $\eta_h = \frac{V_{w2}u - V_{w1}u}{gH}$

If 'P' is the power available at the runner shaft, then mechanical efficiency

$$\eta_m = \frac{P}{W\left(\frac{V_{w2}u}{g}\right)}$$

if $V_{w1} \neq 0$; $\eta_m = \frac{P}{W\left(\frac{V_{w2}u - V_{w1}u_1}{g}\right)}$

Then overall efficiency

$$\eta_o = \eta_h \times \eta_m$$

$$\eta_o = \frac{P}{WH}$$

Overall efficiency is ranges from 80-90%.

Hydraulic efficiency is ranges from 85-95%.

Degree of reaction: (ρ)

It is defined as the ratio of pressure drop in the runner to the Hydraulic Workdone on the runner. Thus, if P and P_1 are the pressures at inlet & outlet of the runner then

$$\rho = \frac{(P/w - P_1/w)}{\left(\frac{V_{w2}u - V_{w1}u_1}{g}\right)}$$

If V_{w1} is 0 then; $\rho = \frac{P/w - P_1/w}{(V_{w2}u/g)}$

Working proportions of francis turbine:

(i) The ratio of width B of the runner to the diameter D of the runner represented by n

i.e, $n = B/D$

The value of n ranges from 0.10 to 0.45.

(ii) The ratio of velocity of flow at inlet tip V_f of the vane to the spouting velocity ($\sqrt{2gH}$) is known as flow ratio represented by 'ψ'

$$\psi = \frac{V_f}{\sqrt{2gH}} \quad \text{or} \quad V_f = \psi.\sqrt{2gH}$$

The value of ψ ranges from 0.15 to 0.1

(iii) speed ratio

$$K_u = \frac{u}{\sqrt{2gH}} \quad (\text{or}) \quad u = K_u\sqrt{2gH}$$

The value of K_u ranges from 0.60 to 0.90.

Design of francis turbine:

Francis's turbine is design to be develop a known power 'P', when running at a known speed 'N', under a known head 'H'. The probable value of η_h, η_0, η & θ are Assumed. The design of runner which involves the determination of the size & vane angles are carried out as

(i) Determine the required discharge in form the reaction.

(ii) If 'Z' is the no of vanes in the runner, 't' is the thickness of the vane at inlet & 'B' is the width of the wheel at inlet, then the area of flow section at the wheel inlet $= (\pi D - Zt)B = K\pi BD$

Where; 'k' is a factor which allows for thickness of vanes. then,

$$\text{Discharge } (Q) = K\pi BDV_f$$

$$Q = K\pi nD^2 V_f$$

since $B = nD$

then Assuming a suitable value of K, the diameter D, & the width B of the runner can be determined.

(iii) the Tangential velocity of the runner at inlet may be determined from the following equation

i.e, $u = \frac{\pi DN}{60}$

(iv) The velocity of wheel 'V_w' at inlet of the runner can be determined from the expression

$$\eta_h = \frac{V_w \cdot u}{gH}$$

(v) From the inlet velocity triangle, the guide vane angle 'α' & the runner vane angle 'θ' at inlet can be calculated from the expressions

$$\text{Tan}\,\alpha = \frac{V_f}{V_w}; \quad \text{Tan}\,\theta = \frac{V_f}{V_w - u}$$

(vi) The runner diameter 'D_1' at the outlet end varies from $\frac{1}{3}D$ to $\frac{2}{3}D$. and usually it is taken as

$\frac{1}{3}D$ to $\frac{2}{3}D$

$\frac{1}{2}D$

Thus; the tangential velocity of the runner at outlet may be determined from the expression

$$u_1 = \frac{\pi D_1 N}{60}$$

(vii) If t_1 & B_1 are respectively thickness of the vane & the width of runner at outlet then

$$Q = K\pi B_1 D_1 V_{f1}$$

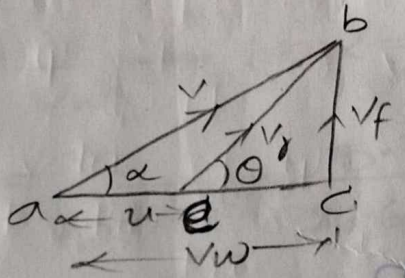
From the discharge expressions $\frac{V_f}{V_{f1}} = \frac{K\pi B_1 D_1}{K\pi BD}$

Normall it is assumed that $V_f = V_{f1}$ & $K = K_1$ then $B_1 = 2B$ since $D_1 = D/2$.

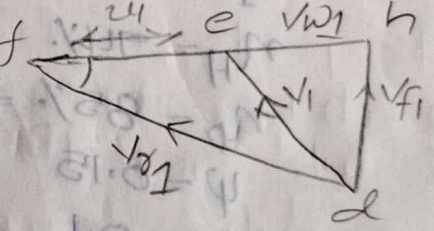
(viii) Generally the runner is designed to have the velocity of wheel V_{w_1} at outlet is equal to zero. i.e, $V_{w_1} = 0$ & $\beta = 90^\circ$ then from outlet velocity triangle the runner vane angle ϕ at outlet may be determined from the expression $\tan\phi = \frac{V_{f1}}{u_1}$

- the no of runner vanes should be either one more or less than the number of guide lines, in order to avoid setting up of periodic impulse.

Inlet velocity triangle



outlet velocity triangle at $\beta = 90^\circ$



Design a francis turbine runner with the following data; Net head (H) = 68m,
Speed (N) = 750 rpm,
output power (op) = 330 kW;
Hydraulic efficiency (η_H) = 94%
overall efficiency (η_0) = 85%.
Flow area ration (ψ) = 0.15
Breadth area ratio (n) = 0.1
Internal diameter of runner is half of the outer diameter also Assume 6% of circumference area of the runner to be occupied by the thickness of the vanes velocity of the flow remains constant throughout & flow is radial at exist.

Sol: Given data:

$H = 68m$

$N = 750 rpm$

$op = 330 kW = 330 \times 10^3 W$

$\eta_H = 94\%$

$\eta_0 = 85\% = 0.85$

$\psi = 0.15$

$n = 0.1$

$D = \frac{1}{2} D_1 \Rightarrow D = 0.5 D_1 \Rightarrow \frac{1}{2} D$

We know that $P = \eta_0 \rho g Q H$

$$Q = \frac{330 \times 10^3}{0.85 \times 9810 \times 68}$$

$$Q = 0.581 \ m^3/sec.$$

$$\Rightarrow V_f = \psi \sqrt{2gH}$$

$$= 0.15 \sqrt{2 \times 9.81 \times 68}$$

$$V_f = 5.47 \ m/sec$$

$\Rightarrow Q = K\pi n D^2 V_f$

$$D^2 = \frac{Q}{K\pi n V_f}$$

$$= \frac{0.581}{0.95 \times \pi \times \frac{0.1}{0.1} \times 5.47}$$

$$D = 0.596 \simeq 0.6m$$

$$\therefore B = nD = 0.1 \times 0.596 = 0.059 \simeq 0.06$$

$$\Rightarrow u = \frac{\pi DN}{60} = \frac{\pi \times 0.596 \times 750}{60} = 23.56 m/sec$$

$$\Rightarrow \eta_h = \frac{V_w u}{gH}$$

$$\frac{\eta_h \times g \times H}{u} = V_w \rightarrow V_w = \frac{0.94 \times 9.81 \times 68}{23.56} = 26.615 m/sec$$

$$\tan\alpha = \frac{V_f}{V_w} \rightarrow \tan\alpha = \frac{5.47}{26.61} \rightarrow \alpha = 11°36'$$

$$\tan\theta = \frac{V_f}{V_w - u} \Rightarrow \tan\theta = \frac{5.47}{26.61 - 23.56} \rightarrow \theta = 60°51'$$

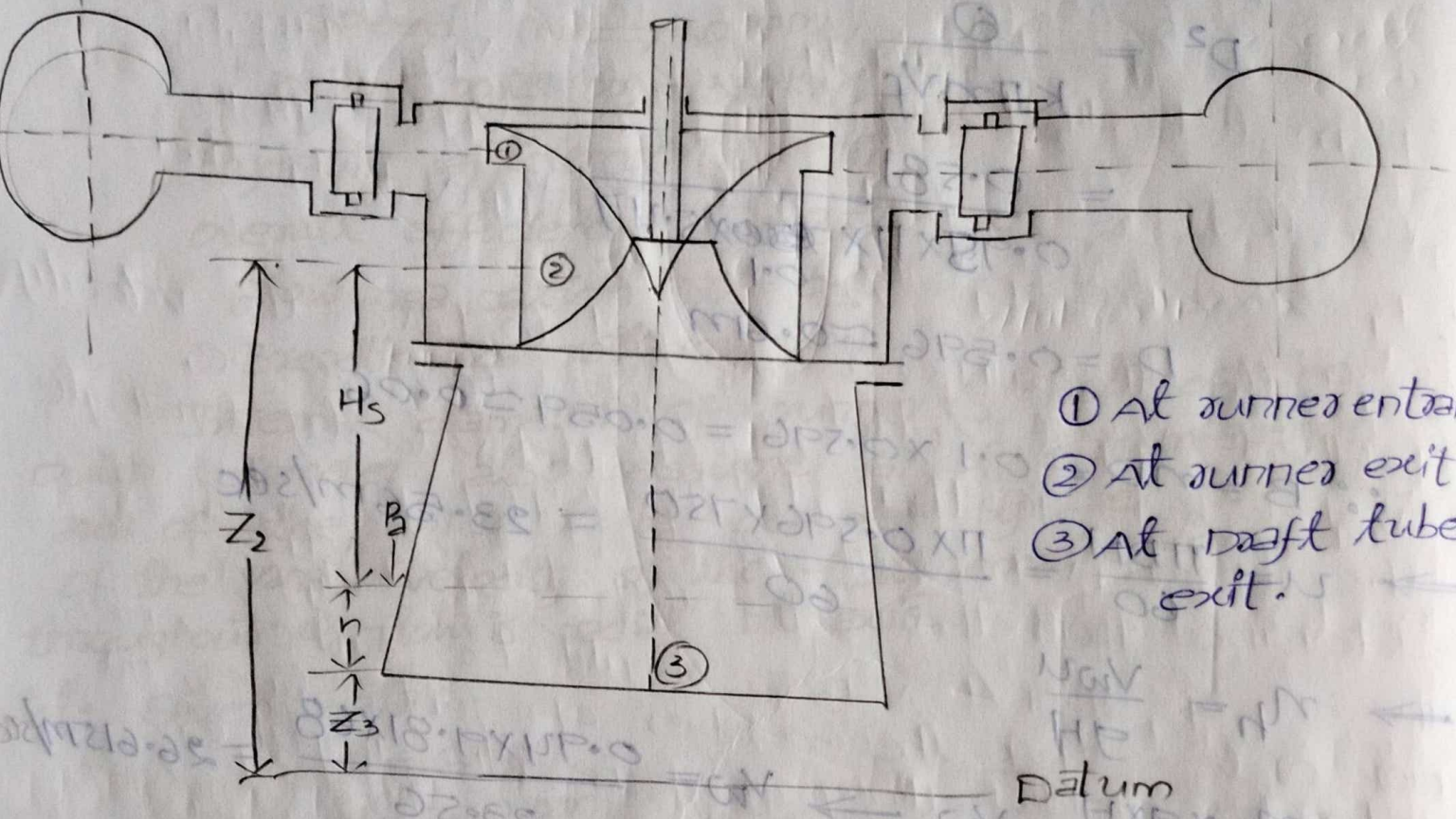
$$\tan\phi = \frac{V_{f1}}{u_1} \Rightarrow \tan\phi = \frac{5.47}{11.78} \rightarrow \phi = 24.9'$$

$$u_1 = \frac{\pi D_1 N}{60} \qquad D_1 = D/2$$

$$= \frac{\pi (\frac{0.6}{2}) 750}{60}$$

$$u_1 = 11.78 m/sec$$

Draft tube theory:



① At runner entrance
② At runner exit
③ At Draft tube exit.

From figure;

The points ①, ②, ③ have been considered at the runner entrance, runner exit & the outlet end of the draft tube respectively.

Apply Benouli's equation for section - ② & ③

$$\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 = \frac{P_3}{\rho g} + \frac{V_3^2}{2g} + Z_3 + h_f \quad \{w = \rho g\}$$

Where; P_2, P_3 & V_2, V_3 & Z_2, Z_3 are pressures & velocities & datum heads at 2 & 3 respectively.

'h_f' is the loss of head in the draft tube

$$\Rightarrow \frac{P_2}{w} = \frac{P_3}{w} - (Z_2 - Z_3) - \left(\frac{V_2^2 - V_3^2}{2g}\right) + h_f \quad \ldots (1)$$

But $\frac{P_3}{w} = \frac{P_a}{w} + h$

$$\frac{P_2}{w} = \frac{P_a}{w} + h - (Z_2 - Z_3) - \left\{\frac{V_2^2 - V_3^2}{2g}\right\} + h_f$$

$$\frac{P_2}{w} = \frac{P_a}{w} - (Z_2 + Z_3 - h) - \left(\frac{V_2^2 - V_3^2}{2g}\right) + h_f$$

$$\frac{P_2}{w} = \frac{P_a}{w} - H_s - \left(\frac{V_2^2 - V_3^2}{2g}\right) + h_f$$

Where; h_f = Height of runner exit about tail race level

$$\rightarrow \frac{P_2}{w} = \frac{P_a}{w} - \left(H_s + \left(\frac{V_2^2 - V_3^2}{2g}\right)\right) + h_f$$

$\frac{V_2^2 - V_3^2}{2g}$ = Dynamic suction Head (negative head - net)

$$h_f = k\left(\frac{V_2^2 - V_3^2}{2g}\right)$$

$$\Rightarrow \frac{P_2}{w} = \frac{P_a}{w} - \left(H_s + \left(\frac{V_2^2 - V_3^2}{2g}\right)\right) + k\left(\frac{V_2^2 - V_3^2}{2g}\right)$$

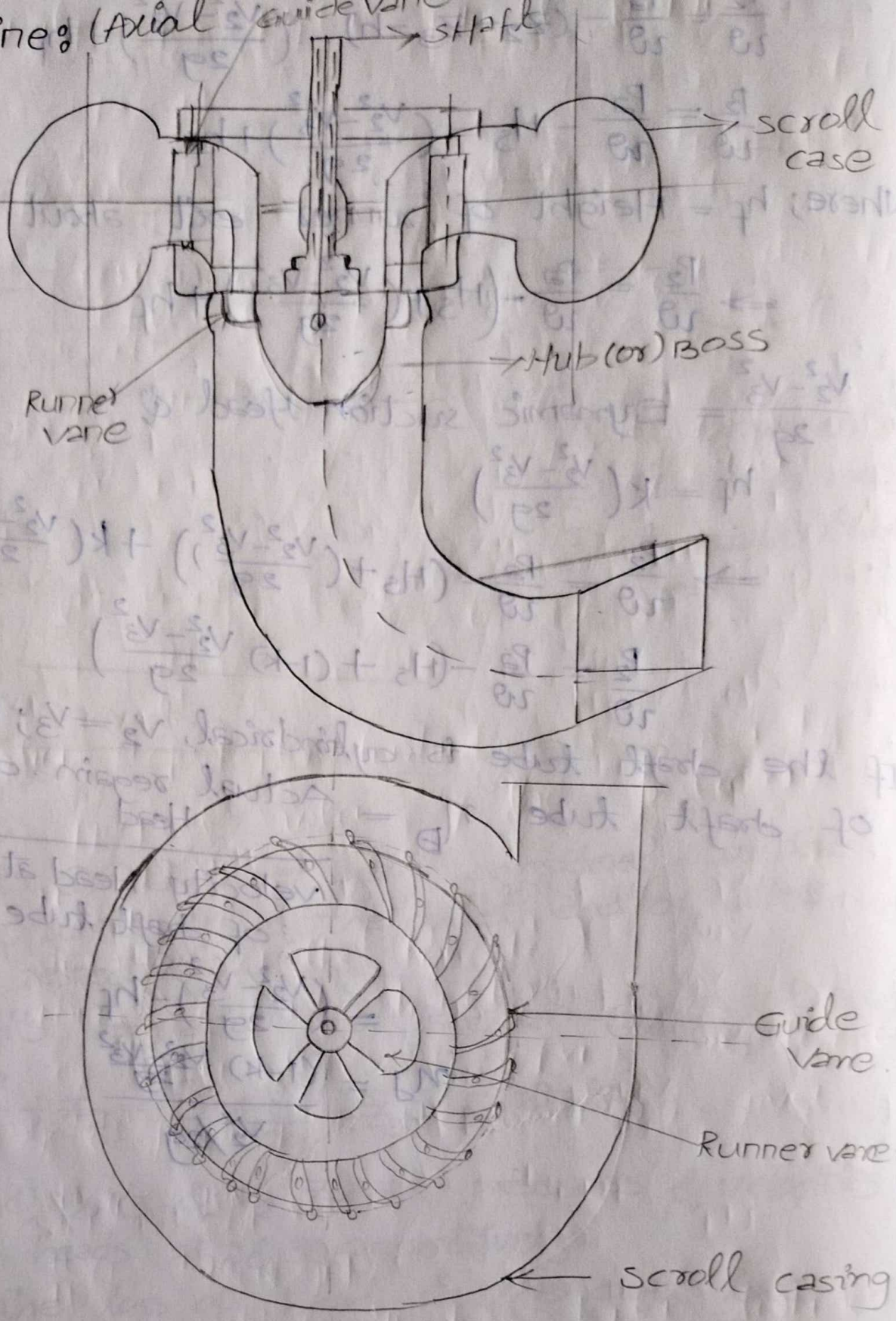
$$\frac{P_2}{w} = \frac{P_a}{w} - \left(H_s + (1-k)\frac{V_2^2 - V_3^2}{2g}\right)$$

If the draft tube is cylindrical, $V_2 = V_3$; Efficiency of draft tube η_D = $\frac{\text{Actual regain of pressure Head}}{\text{velocity Head at the entrance of draft tube.}}$

$$= \left(\frac{V_2^2 - V_3^2}{2g}\right) - h_f$$

$$\eta_d = \frac{(1-k)\frac{V_2^2 - V_3^2}{2g}}{V_2^2/2g}$$

Kaplan turbine: (Axial



- The kaplan turbine is a low head turbine & used for heads of less than 80 meters.
- The kaplan is of the propeller type, similar to an airplane propeller.
- It is a pure axial flow turbine uses basic aerofoil theory.

components of Kaplan Turbine:—

Scroll casing:

It is the casing in which guides the water & control in water passage.

Guide Vanes:

It is the blade in which guides the water and control the water passage.

Draft tube:

After passing through the runner the water is discharged to the tail race through a gradually expanding tube.

Runner:

It is connected to the shaft of the generator.

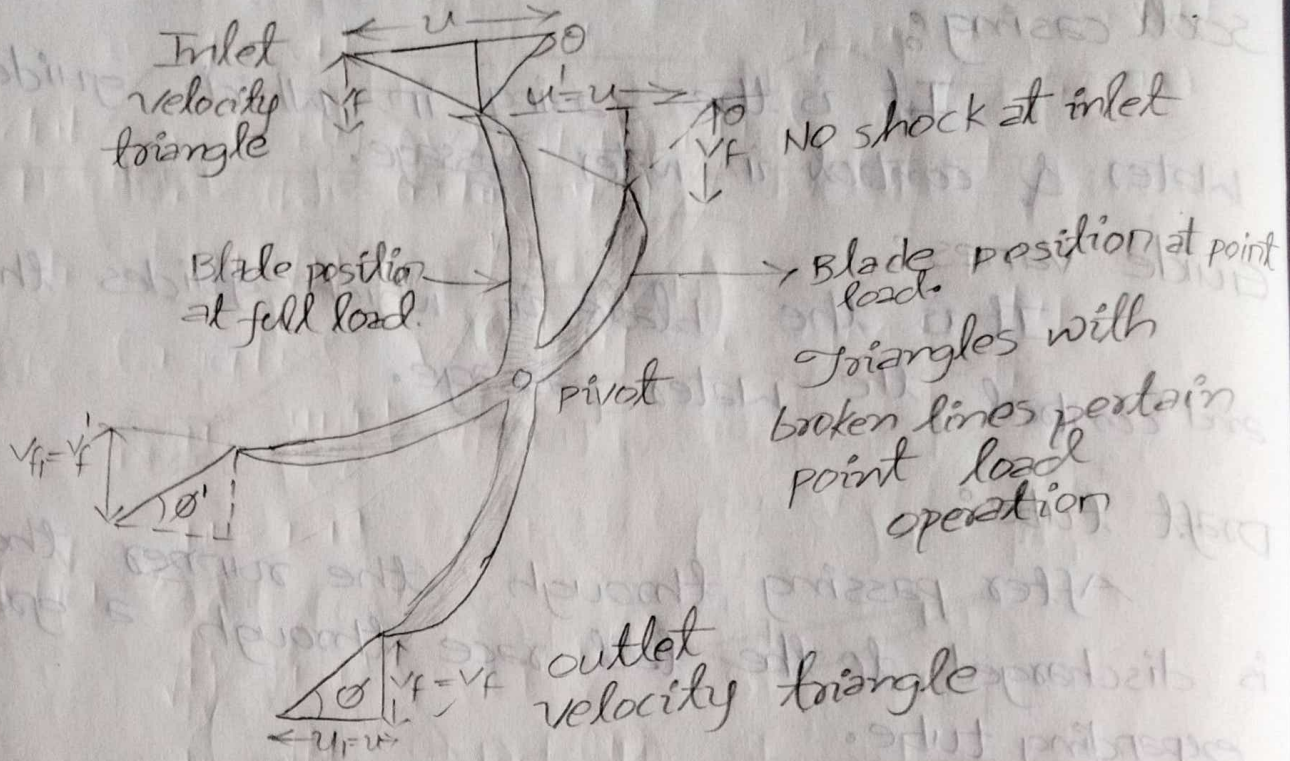
Hub:

It is the part of the runner in which blades are mounted.

Working principle:—

- Water under pressure flush draft tube through the guide vanes & falls on the adjustable blades, which enables adjust the flow.
- After crossing propeller is the kinetic energy of water converted into rotational motion shaft. The shaft powered generator producing electric power.

Velocity triangles for kaplan Wheel turbine:



Working proportions of kaplan turbine:

(i) Choose an appropriate value of the ratio

$$n = \frac{d}{D}$$

Where; d = diameter of the hub or boss

D = Runner outside diameter.

The value of 'n' usually varies from 0.35 to 0.60.

(ii) The Discharge 'Q' flowing to the runner is given by

$$Q = \frac{\pi}{4}(D^2 - d^2) V_f$$

$$= \frac{\pi}{4}(D^2 - d^2) \psi \sqrt{2gH}$$

The value of flow ratio 'ψ' for a kaplan turbine is around 0.70.

(iii) The runner blades of kaplan turbine runner are twisted, the blade Angle being greater at the outer tip then at the Hub.

This is because the peripheral velocity of the blades being directly propotional to radius,

it will vary from section to section along the blade hence in order to have shock free entry & exit of water the blades with Angles varying from section to section have to be designed.

Surge tanks:

A surge tank is a water storage device used as a pressure neutralizer in Hydropower water conveyance system in order to dampen the excess pressure varience.

- In Hydro electric power uses, a surge tank is an additional storage space (or) reservior fitted between the main storage reservior & the power house (as close to the power house as possible).
- Surge tanks are usually provided in High or medium head plans where there is a considerable distance between water source & power unit along with the penstock.
- The main functions of surge tanks are (i) when the load decreases, the water moves backward & gets stored in it.

(ii) When the load increases, additional supply of water will be provided by surge tanks.

Specific speed of turbine:

It is defined as the speed of a turbine which is identical in shape, geometrical dimensions, blade angles, gate openings etc with the actual turbine but of such a size that it will develop unit power when working under unit head.

Expression for specific speed:

The overall efficiency of turbine is given by $\eta_o = P/\rho g Q H$

$\Rightarrow P = \eta_0 w Q H$

$P \propto QH$ ——— ① ($\because \eta_0, w$ are constants)

The Absolute velocity (V), Tangential velocity (u) and head on the turbine 'h' are related as

$u \propto \sqrt{H}$ ——— ②

But the tangential velocity is given by

$$u = \frac{\pi DN}{60}$$

$u \propto DN$ ——— ③

From equations ② & ③;

$\sqrt{H} \propto DN$

$\Rightarrow D \propto \frac{\sqrt{H}}{N}$ ——— ④

The Discharge through the turbine is given by

$Q = AV$

$A \propto B \times D$

$A \propto D^2$ ($\because B = \pi D$, $B \propto D$)

$Q \propto AV$

$Q \propto D^2 \sqrt{H}$ ——— ⑤

From Equations ④ & ⑤;

$Q \propto \left(\frac{\sqrt{H}}{N}\right)^2 \sqrt{H}$

$Q \propto \frac{H\sqrt{H}}{N^2}$

$Q \propto \frac{H^{3/2}}{N^2}$ ——— ⑥

From eq-① & ⑥;

$P \propto \frac{H^{3/2}}{N^2} H$

$$P = K\frac{H^{5/2}}{N^2} \quad \text{——} (7)$$

For N_S ($P=1$, $H=1$)

$$1 = K\cdot\frac{1^{5/2}}{N_S^2}$$

$$N_S^2 = K \quad \text{——} (8)$$

From (7) & (8);

$$P = \frac{N_S^2 \cdot H^{5/2}}{N^2}$$

$$N_S = \frac{N\sqrt{P}}{H^{5/4}}$$

Important for competative exams

— specific speed is used to credit the behaviour of turbine.

A water turbine has a velocity of 6m/sec at the entrance to the draft tube & a velocity of 1.2m at the exit for friction losses of 0.1m & tail water 5m below the entrance to the draft tube. Find the pressure head at entrance.

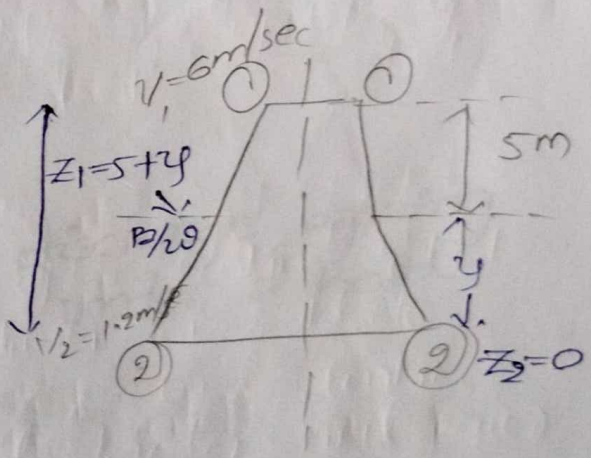
Sol: Given data:

Velocity (V_1) = 6m/sec at entrance

Velocity (V_2) = 1.2m/sec at exit

Friction losses (h_f) = 0.1m

Tail head = 5m



$$\frac{P_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + Z_2 + h_f$$

$$\frac{P_1}{w} + \frac{6^2}{2g} + 5+y = \frac{P_2}{w} + y + \frac{1.2^2}{2\times 9.81} + 0.1$$

$$\frac{P_1}{w} + 6.83 + y = \frac{P_2}{w} + y + 0.173$$

If; $\frac{P_2}{\rho g}$ is taken as zero then we will get $\frac{P_1}{\rho g}$ as vaccum pressure head at inlet of draft tube.

If $\frac{P_2}{\rho g} = 10.3$ m of water, then we will get $\frac{P_1}{\rho g}$ as absolute pressure at the inlet of draft tube.

$$\Rightarrow \frac{P_1}{w} + 6.83 - 10.3 + 0.173$$

$$\frac{P_1}{w} = 3.643$$

Absolute pressure $\frac{P_1}{\rho g} = 3.643$

Vaccum pressure $\frac{P}{\rho g} = -6.65$

Unit quantities:

(i) Unit speed:

It is defined as the speed of turbine working under a unit head (i.e, under a head of 1m) it is denoted by N_u. The expression for unit speed is obtained as below.

The tangential velocity, Absolute velocity and the head of a turbine is related as

$$u \propto V$$

$$u \propto \sqrt{H} \quad \text{——— (1)}$$

We know; Tangential velocity $u = \frac{\pi DN}{60}$

Assuming the diameter 'D' for a turbine is constant

then $u \propto N$ ——— (2)

From (1) & (2) equations;

$$N \propto \sqrt{H}$$

$$\Rightarrow N = K_1\sqrt{H} \quad (K = \text{constant})$$

$$N = K_1 \quad (H = 1)$$

$$N_u = K_1$$

$$\Rightarrow N = N_u\sqrt{H}$$

$$\boxed{N_u = \frac{N}{\sqrt{H}}}$$

(ii) Unit Discharge:

It is defined as the discharge passing through a turbine, which is working under a head of 1m. It is denoted by Q_u.

Expression:

The discharge passing through a given turbine under a head 'H' is given by

$$Q = \text{Area of flow} \times \text{velocity}$$

But for a turbine, the Area of flow is constant

and velocity is propotional to $\sqrt{H}$.

$$Q \propto \sqrt{H}$$

$$Q \propto v$$

$$Q = K_2\sqrt{H} \quad \text{——} (1)$$

From the defination of unit discharge

$$Q_u = K_2\sqrt{1}$$

$$Q_u = K_2 \quad \text{——} (2)$$

$$Q = Q_u\sqrt{H}$$

$$\boxed{Q_u = \frac{Q}{\sqrt{H}}}$$

(iii) Unit power:

It is defined as the power developed by turbine working under a head of 1m. It is denoted by P_u.

$$\eta_0 = \frac{P}{wQH}$$

$$P \propto QH$$

$$Q \propto \sqrt{H}$$

$$P \propto \sqrt{H} \cdot H$$

$$P \propto H^{3/2}$$

$$P = K_3 H^{3/2} \quad \text{——} (1)$$

From the defination of unit power

$$P_u = K_3(1)^{3/2}$$

$$P_u = K_3$$

$$\longrightarrow \boxed{P_u = \frac{P}{H^{3/2}}}$$

If $H_1, H_2 \ldots$ are the Heads under which a turbine works $N_1, N_2 \ldots$ are the corresponding speeds, $Q_1, Q_2 \ldots$ are the corresponding discharges, $P_1, P_2 \ldots$ are the power developed by turbines

then $$N_u = \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

$$Q_u = \frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}}$$

$$P_u = \frac{P_1}{(H_1)^{3/2}} = \frac{P_2}{(H_2)^{3/2}}$$

A turbine develops 9000kW when running at 100 rpm the head on the turbine is 30-m. If the Head on the turbine reduces to 18m. Determine the speed and power developed by the turbine.

Sol:

Power (P_1) = 9000 kW

Speed (N_1) = 100 rpm

Head of turbine (H_1) = 30m

Reduced Head (H_2) = 18m

N_2, P_2 = ?

$$\Rightarrow N_u = \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

$$N_u = \frac{100}{\sqrt{30}} = \frac{N_2}{\sqrt{18}}$$

$$N_u = 18.25 \text{ rpm}$$

$$\Rightarrow N_u = \frac{N_2}{\sqrt{18}}$$

$$N_2 = 77.42 \text{ rpm}$$

$$\rightarrow P_u = \frac{P_1}{\sqrt{H_1}} = \frac{P_2}{\sqrt{H_2}}$$

$$P_u = \frac{9000}{\sqrt{30}} = \frac{P_2}{\sqrt{18}}$$

$$P_u = 1643.16 = 1.641 \text{ kW}$$

$$\Rightarrow P_u = \frac{P_2}{\sqrt{18}}$$

$$P_2 = 6971.37 \text{ kW}$$

$$P_2 = 6.97 \text{ kW}$$

A turbine is to operate under a head of 25m at 200rpm. The discharge is 9m³/sec (given). If the efficiency is 90%. Determine the performance of turbine under a head of 20m.

Sol: Given:

Head (H_1) = 25m

speed (N_1) = 200rpm

Discharge (Q_1) = 9m³/sec

efficiency (η_0) = 90%.

performance Head = 20m

$P_2, Q_2, N_2 = ?$

$$N_u = \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}} \qquad Q_u = \frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}}$$

$$N_u = \frac{200}{\sqrt{25}} = \frac{N_2}{\sqrt{20}} \qquad Q_u = \frac{9}{\sqrt{25}} = \frac{Q_2}{\sqrt{20}}$$

$N_u = 40$ rpm

$N_2 = 178.88$ rpm

$Q_u = 1.8$ m³/sec

$Q_2 = 8.04$ m³/sec

$$\eta_0 = \frac{P}{\omega Q H}$$

$$P_1 = 0.9 \times 9810 \times 9 \times 25$$

$$P_1 = 1.98 \times 10^6 = 1986.5 \text{ kW}$$

$$P_u = \frac{P_1}{\sqrt{H_1}} = \frac{P_2}{\sqrt{H_2}}$$

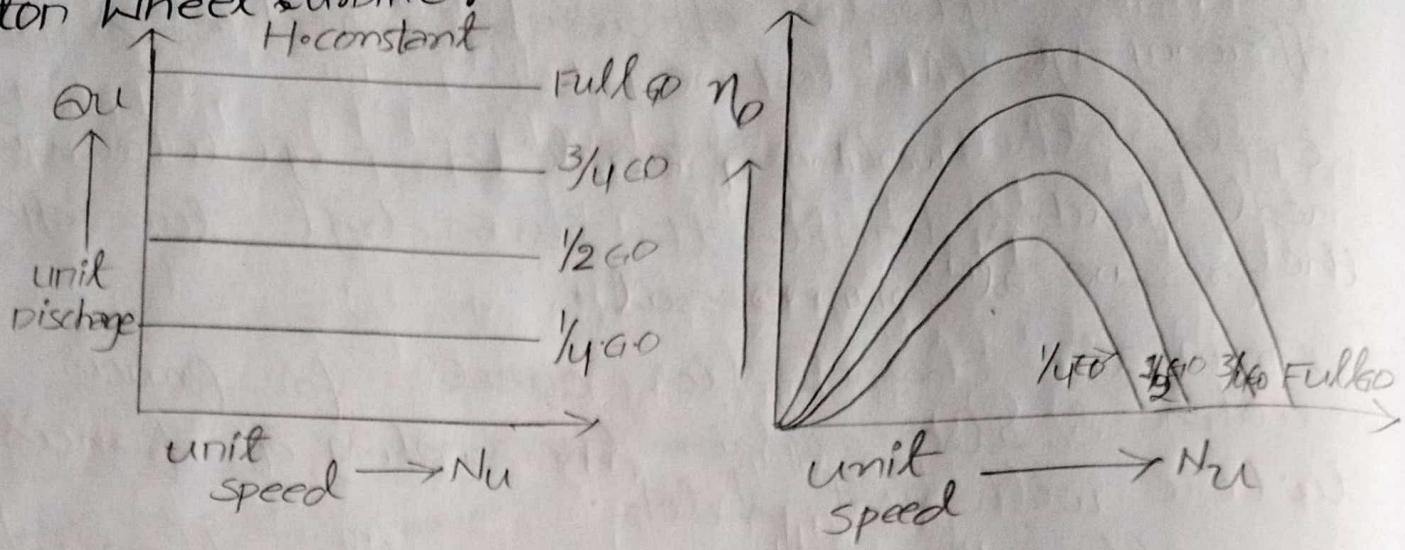
$$P_u = \frac{1.98 \times 10^6}{\sqrt{25}} = \frac{P_2}{\sqrt{20}}$$

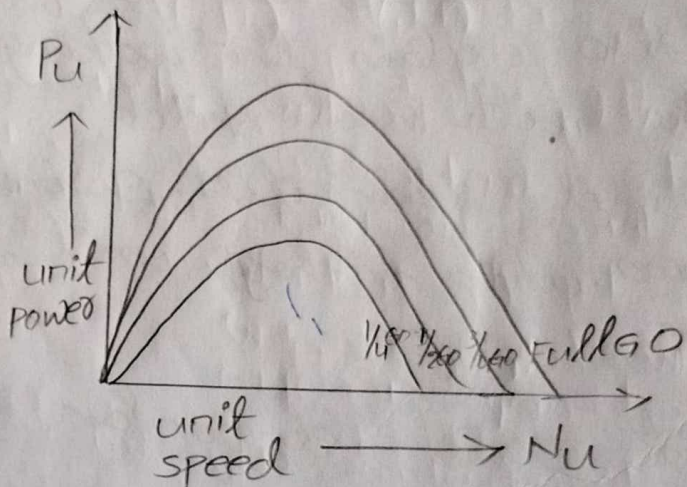
$$P_u = 397.3$$

$$P_2 = 1776.77 \text{ kW}$$

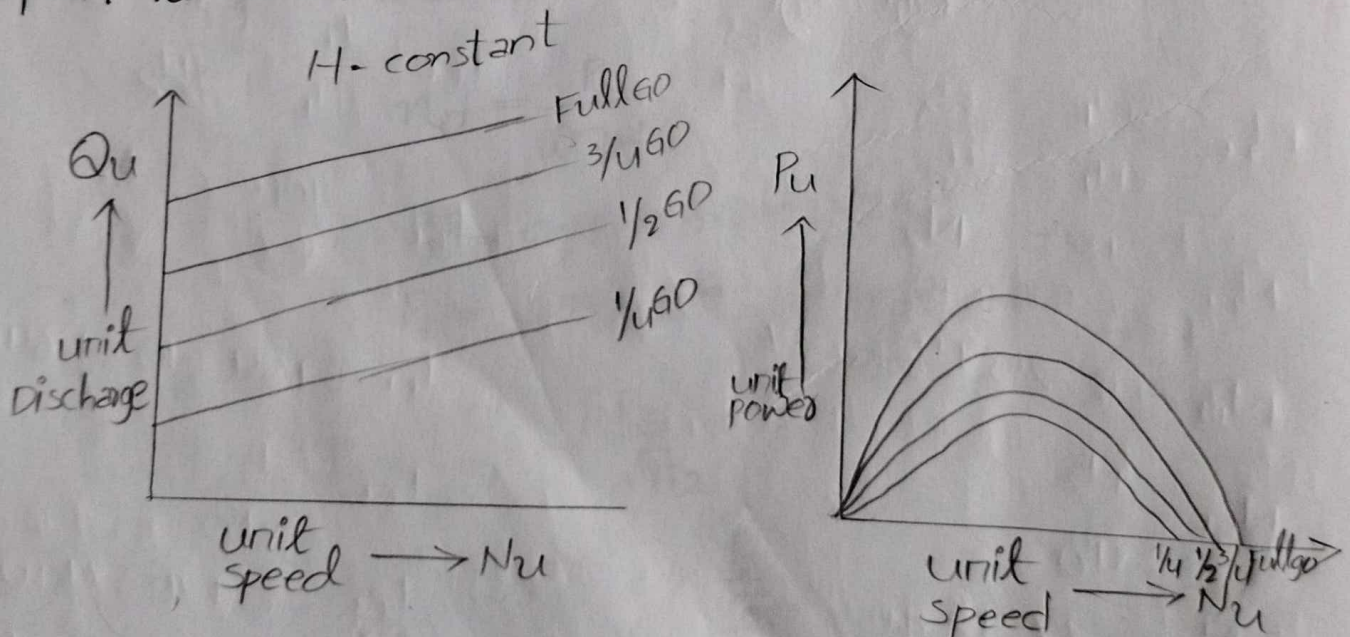
Characteristics curves of turbine:

Pelton Wheel turbine:

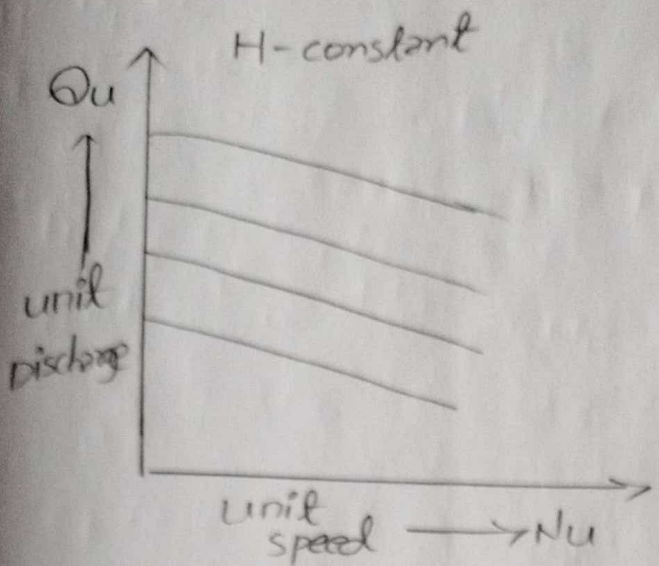


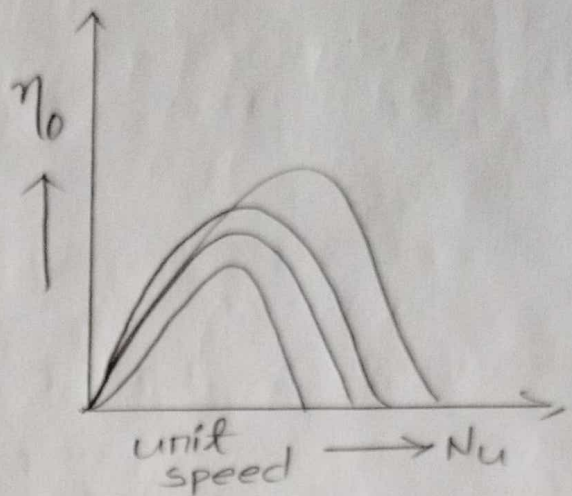


kaplan turbine:

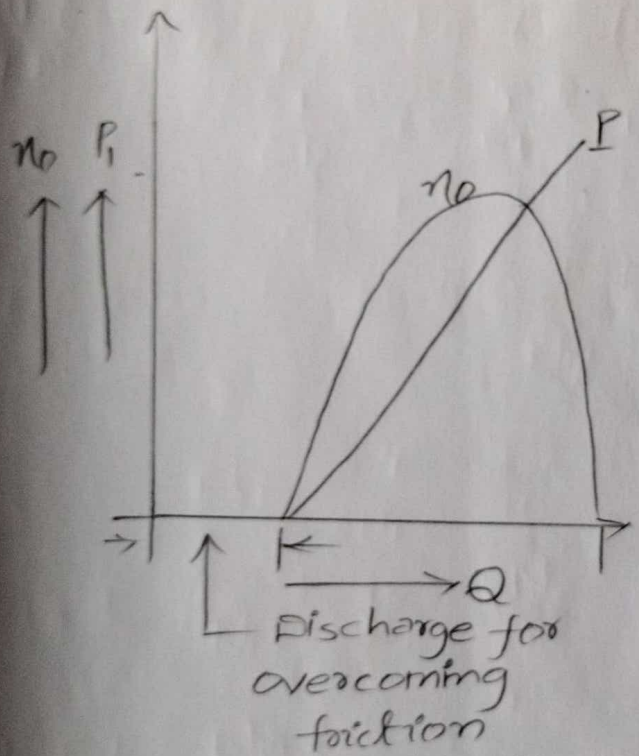


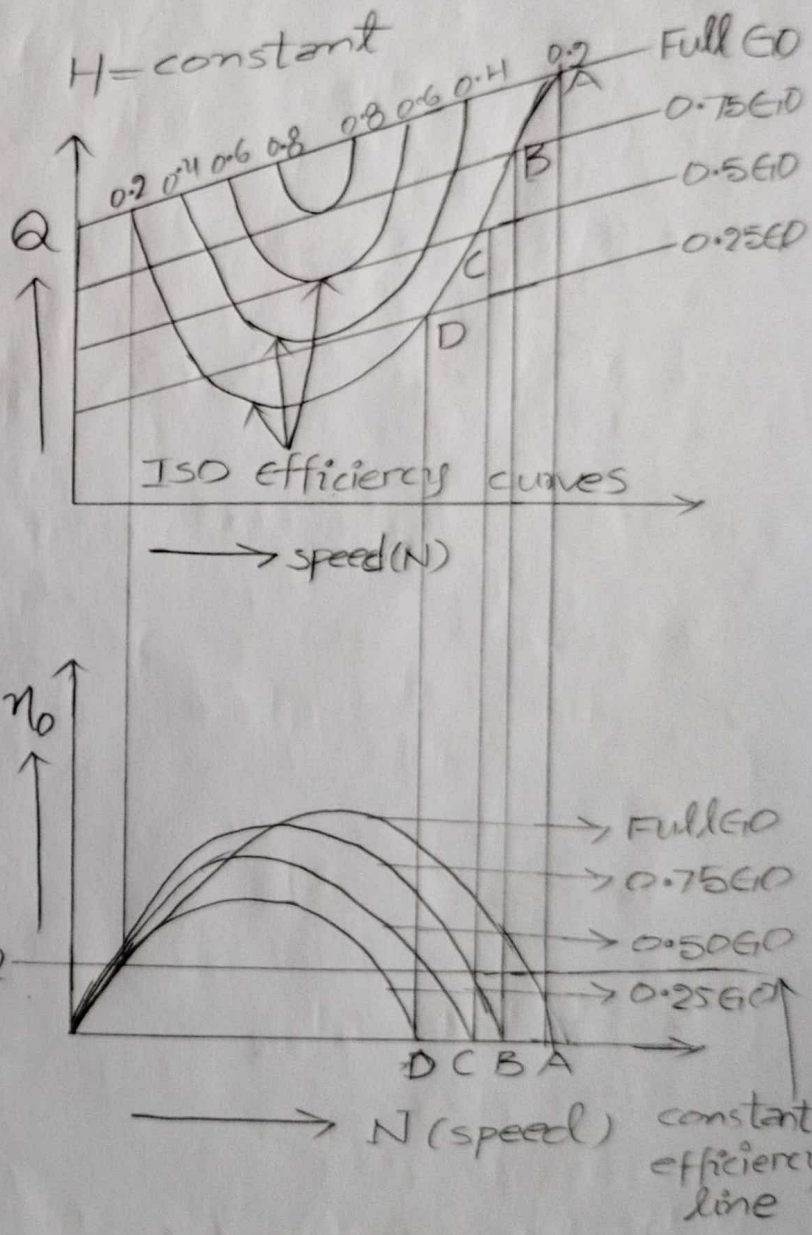
Francis turbine:





H-constant, N-constant





6. Centrifugal pumps

Syllabus:

- Pump installation details
- classification
- Work done
- Manometric heead
- minimum starting speed
- Losses and efficiencies
- specific speed.
- Multistage pumps
- pumps in parallel and series
- performance of pumps
- characteristic curves
- NPSH - cavitation

Reciprocating pumps

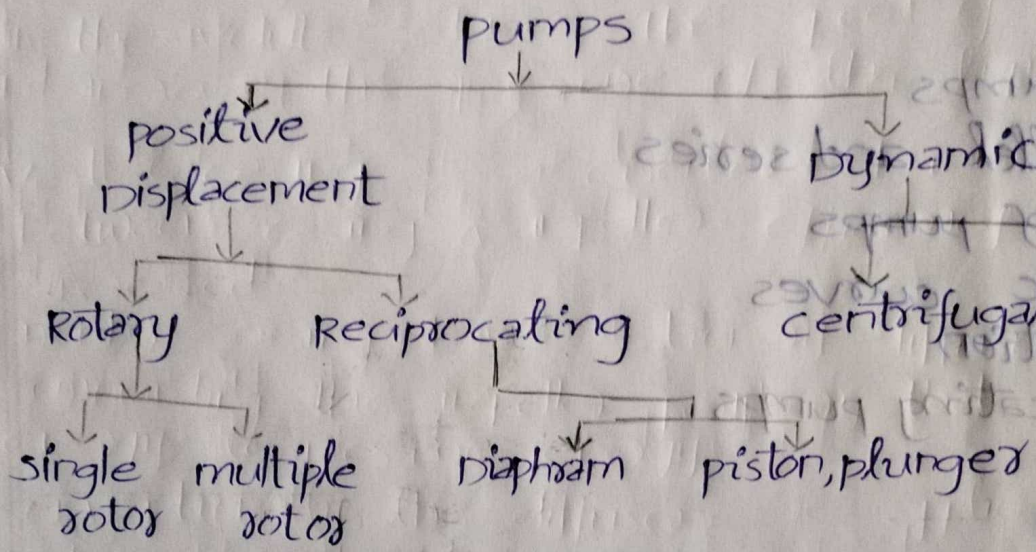
- Introduction
- classification
- components
- Working
- Discharge
- Indicator diagram
- Workdone & slip

Pump:

A pump is a device used to move fluids, such as liquids, gases or slurries. It increases the mechanical energy of the fluid. the additional energy can be used increase

- velocity
- pressure
- elevation

Classification of pumps:



positive displacement pump:

positive displacement pumps apply pressure directly to the liquid by a reciprocating piston, or by rotating members.

1. can handle shear sensitive liquid.
2. Use for high pressure appliction
3. Use for variable viscosity application

Types:

- Reciprocating pump
- Rotary pump

Centrifugal pump:

- It converts mechanical energy into Hydraulic energy by virtue of centrifugal force.
- flow in radial outward direction.

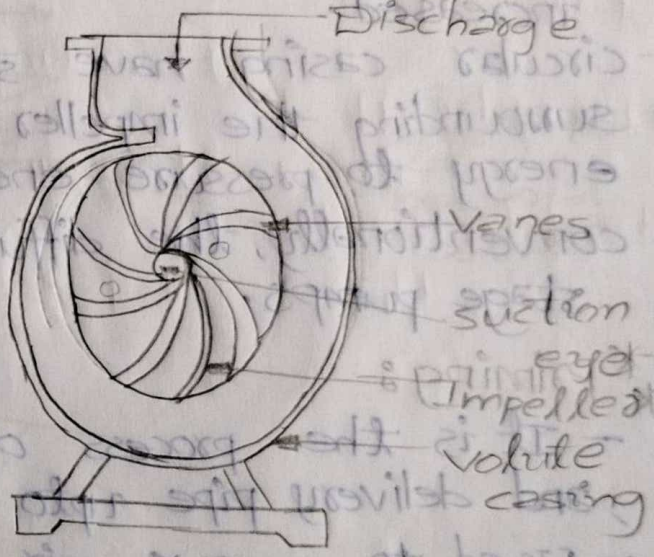
- It works on principle of forced vortex flow.
- Uses include Water, sewage, petroleum & petrochemical pumping.

Working principle:

- It works on the principle of forced vortex flow means when a certain mass of fluid is rotated by external torque rise in pressure head takes place.
- conversion of energy occur by virtue of two main parts of the pump:
  (a) Impeller (b) Casing
- Impeller converts driver energy into the kinetic energy & diffuser converts the kinetic energy into pressure energy.

components:

- Impeller
- casing
- suction pipe
- Foot valve & strainer
- Delivery pipe



- A centrifugal pump has two main components:
  (i) A rotating component: impeller, shaft
  (ii) A stationary component: casing, casing cover & bearings

Impeller: The impeller is the main rotating part that provides the centrifugal acceleration to the fluid

Shaft: Its purpose is to transmit the torques encountered when starting and during operation. supports the impeller & other rotating parts.

Casing: The main purpose of casing is to convert kinetic energy to pressure energy.

casings are generally of 3 types:

(i) volute: used for Higher head, eddy currents formed

(ii) vortex: eddy currents are reduced

(iii) circular: used for lower head.

- volute is a curved funnel is increasing in area to the discharge part. As the area of the cross-section in the volute reduces the speed of the liquid and increase the pressure of the liquid.
- vertex casing is a circular chamber is introduced between casing & impeller. efficiency of pump is increased.
- circular casing have stationary diffusion vanes surrounding the impeller periphery that convert velocity energy to pressure energy.
- conventionally, the diffusers are applied to multi stage pumps.

** priming:

- It is the process of filling suction pipe, casing, and delivery pipe upto delivery valve with water.
- Used to remove air from these parts
- It is of two types:

(a) positive priming:

The one which speeds up processing.

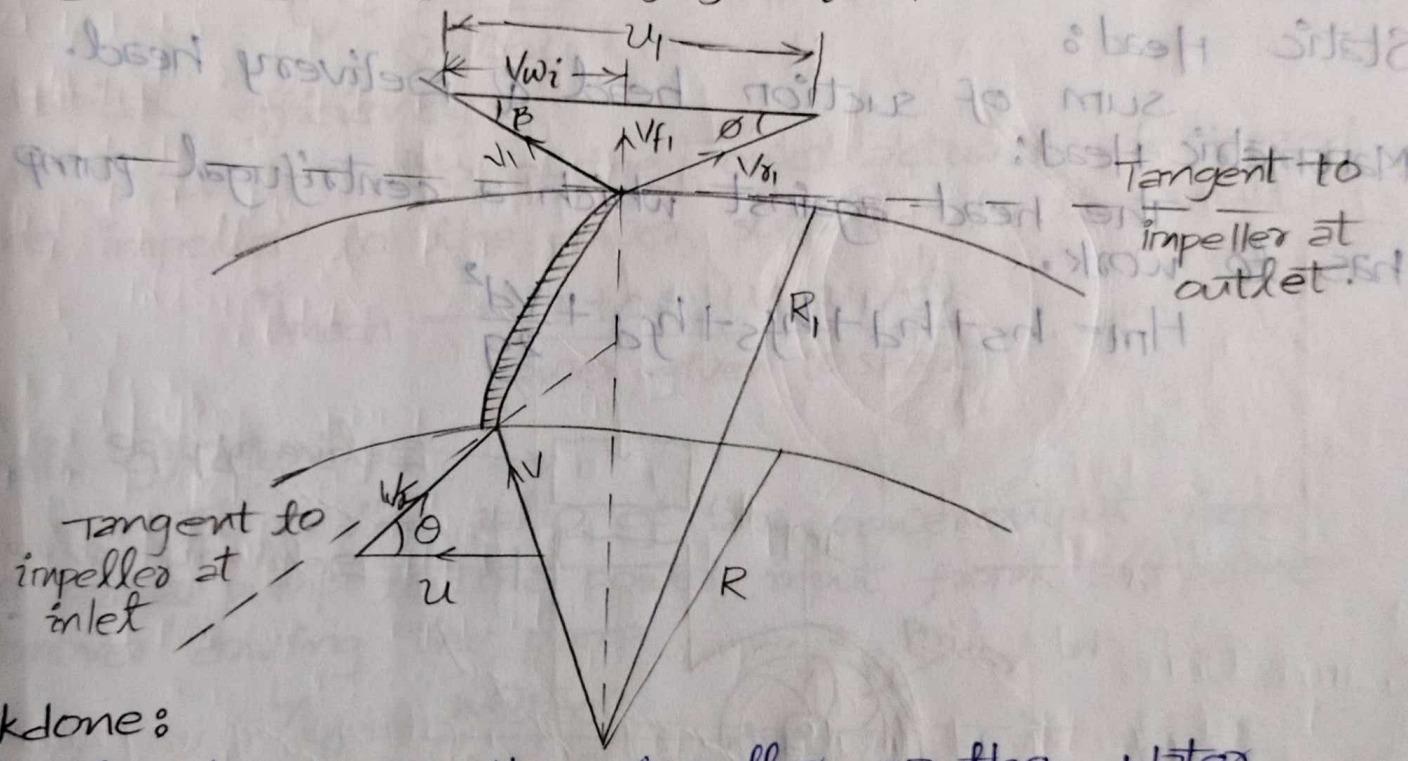
(b) Negative priming:

The one which slows down the processing.

Working of centrifugal pump:

- Liquid forced into impeller.
- Vanes pass kinetic energy to liquid, liquid rotates and leaves impeller.
- Volute casing converts kinetic energy to pressure energy.
- It consists of an Impeller rotating with casing.
- Liquid directed into the centre of the rotating impeller is picked up by the impeller vanes and accelerated to a higher velocity by the rotation of the impeller & discharged by centrifugal force into the casing.

Velocity triangle of centrifugal pump:



Workdone:

Work is done by the impeller on the water

$$W = [V_{w_1}u_1 - V_w u]/g$$

Where; W = Workdone per unit weight of water per sec

V_{w_1} = whirl component of Absolute velocity of jet at outlet.

u_1 = Tangential velocity of impeller at outlet.

V_w = whirl component of Absolute velocity at inlet.

u = Tangential velocity of impeller at inlet.

As water comes radially:

Guide blade angle at inlet $\alpha = 90^\circ$

$V_w = 0$

then $W = \frac{V_{w_2} u_2}{g}$

Heads in centrifugal pump:

Suction Head:

Vertical Height of center line of centrifugal pump above the water surface to the pump from which water to be lifted.

Delivery Head:

Vertical distance between centre line of the pump and the water surfaces in the tank to which water is delivered.

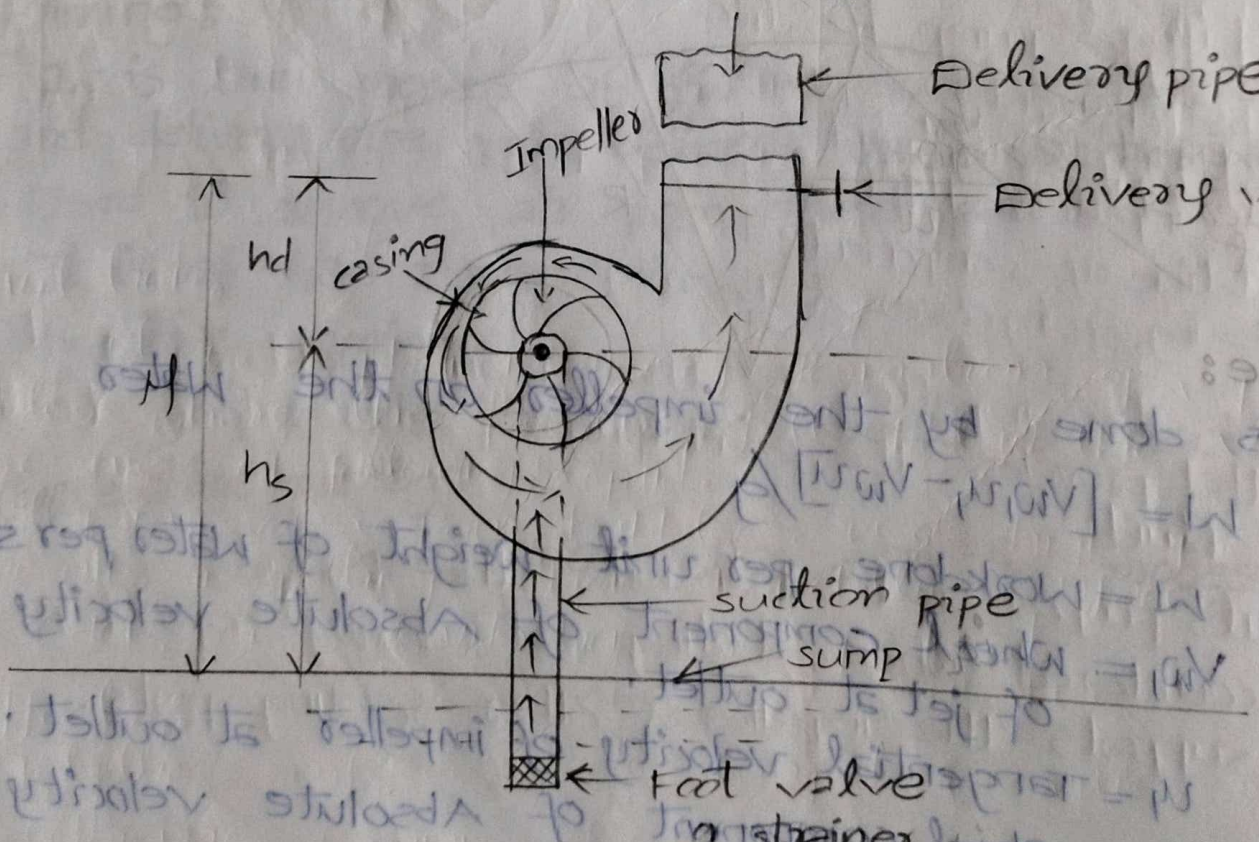
Static Head:

sum of suction head & delivery head.

Manometric Head:

The head against which a centrifugal pump has to work.

$$H_m = h_s + h_d + h_{fs} + h_{fd} + \frac{V_d^2}{2g}$$



Efficiencies:

Manometric efficiency:

The ratio of manometric head to the head imparted by impeller.

$$\eta_{man} = \frac{H_m g}{V_{w_1} u_1}$$

Volumetric efficiency:

It is the ratio of the quantity of liquid discharged per second from the pump to the quantity passing per second through the impeller

$$\eta_v = \frac{Q}{Q + \Delta Q}$$

Mechanical efficiency:

It is ratio of the power actually delivered by the impeller to the power supplied to the shaft.

$$\eta_{mech} = \frac{w(Q + \Delta Q)(V_{w_1} u_1 / g)}{\text{power given to shaft}}.$$

Overall efficiency:

It is the ratio of the poweroutput from the ~~power~~ pump to the power input from the prime mover driving the pump

$$\eta_0 = \frac{wQH_m}{\text{power given to shaft}}$$

Characteristic Curve:

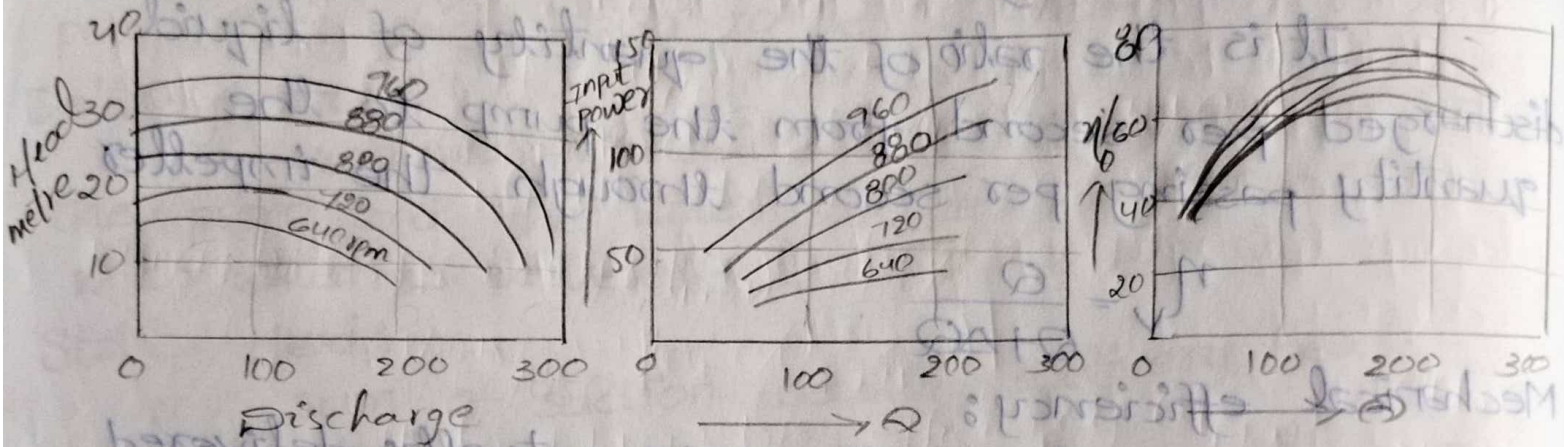
These are required to predict the performance & behaviour of pump working under different head, flow rate & speed.

Following are the important curves:

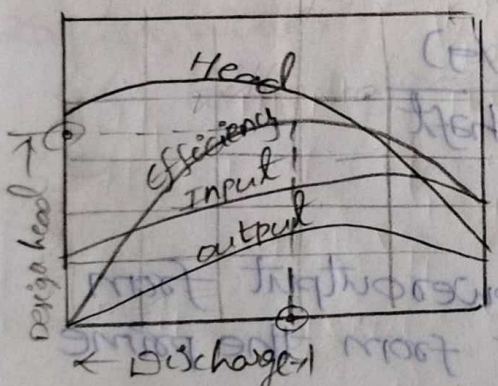
(a) Main characteristic curve

(b) operating characteristic curve

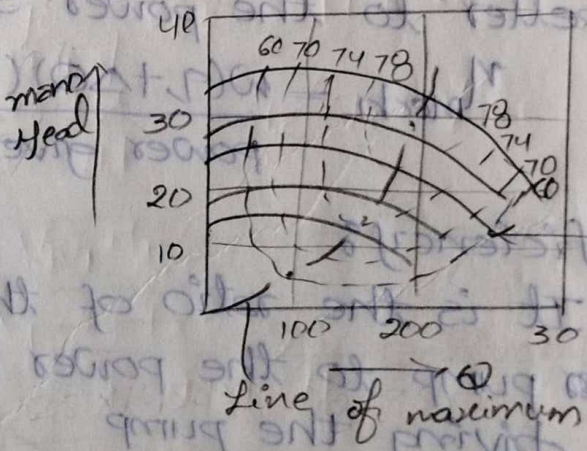
(c) Muschel (or) constant efficiency curve.



operating C.C:



constant EC:



**

Cavitation:

It is a phenomena of formation of vapour bubbles where the pressure falls below the vapour pressure of following liquid.

- collapsing of vapour bubble causes high pressure results in pitting action on metallic surface.
- Erosion, noise & vibration are produced.

Effects:

- Metallic surface are charged & cavities are formed
- efficiency of pump decreases.
- unwanted noise and vibrations are produced.

1. The internal & external diameters of impeller of a centrifugal pump are 200mm & 400mm respectively. The pump is running at a speed of 1200 rpm the vane angles at inlet & outlet are 20° & 30° respectively. The water enters the impeller radially and velocity of flow is constant. Determine the Workdone by the impeller per unit weight of water.

Sol: Given data:

Inner diameter (D) = 200mm = 0.2m

external diameter (D_1) = 400mm = 0.4m

Speed (N) = 1200 rpm

Inlet Angle (θ) = 20°

outlet Angle (ϕ) = 30°

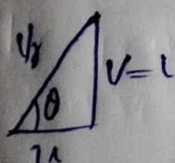
Velocity of flow (V_f) is constant

Workdone (W) = ?

We know; Workdone for centrifugal pump = $\frac{V_{w_1} u_1}{g}$

$\Rightarrow u = \frac{\pi DN}{60} = \frac{\pi \times 0.2 \times 1200}{60} = 12.56 m/sec$

$\Rightarrow u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.4 \times 1200}{60} = 25.13 m/sec$

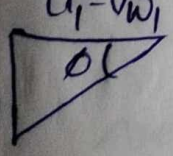


$\tan\theta = \frac{V_f}{u}$

$V_f = u \tan\theta$

$V_f = 12.56 \tan 20°$

$V_f = 4.57 m/sec$



$\tan\phi = \frac{V_f}{u_1 - V_{w_1}}$

$\tan 30°(25.13 - V_{w_1}) = 4.57$

$V_{w_1} = 17.21 m/sec$

∴ Workdone $W = \frac{17.21 \times 12.56}{9.81} = 22.03$ N-m/N

$W_1 = \frac{17.21 \times 25.13}{9.81} = 44.09$ N-m/N

2. A centrifugal pump is to discharge 0.118m³/sec at a speed of 1450rpm against a head of 25m the impeller diameter is 250mm its width at outlet is 50mm and manometric efficiency is 75%. Determine the vane angle at the outer periphery of impeller.

Sol: Given:

Discharge (Q) = $0.118 m^3/sec$

Speed (N) = 1450rpm

Head (H_m) = 25m

Diameter (D) = 250mm = 0.25m

Width (B) = 50mm = 0.05m

Manometric Efficiency (η_{man}) = 75%.

Vane Angle (ϕ) = ?

We know that; Manometric efficiency $(\eta_{man}) = \frac{gH_m}{V_{w_1} u_1}$

Discharge $(Q) = \pi B D V_f$

$$\Rightarrow u_1 = \frac{\pi DN}{60} = \frac{\pi \times 0.25 \times 1450}{60} = 18.98 m/sec$$

$$\Rightarrow Q = \pi B D V_{f_1}$$

$$0.118 = \pi \times 0.05 \times 0.25 V_f$$

$$V_{f_1} = 3 m/sec$$

$$\Rightarrow \eta_{man} = \frac{gH_m}{V_{w_1} u_1}$$

$$\frac{75}{100} = \frac{9.81 \times 25}{V_{w_1} \times 18.98}$$

$$V_{w_1} = 17.2 m/sec$$

$$\Rightarrow \tan\phi = \frac{V_{f_1}}{u_1 - V_{w_1}}$$

$$= \frac{3}{18.98 - 17.2}$$

$\tan\phi = 1.685$

$\phi = 59°13'$

Multistage centrifugal pump:

- It consists of two or more impellers

There are two types:

(a) series - To produce high speed

(b) parallel - To discharge large quantity of liquid.

3. A centrifugal pump has an impeller 0.5m outer diameter and when running at 600 r.p.m discharges water at the rate of 8000 lit/minute against a head of 8.5m. The water enters the impeller without whirl and shock. The inner diameter is 0.25m, and the vanes are set back at outlet at an angle of 45° and the area of flow which is constant from inlet to outlet of the impeller is $0.06m^2$. Determine

(a) the Manometric efficiency of the pump

(b) the vane angle at inlet &

(c) the least speed at which the pump commences to work.

Sol: Given data:

outer diameter (D_1) = 0.5m

speed (N) = 600 rpm

Discharge (Q) = 8000 lit/min

Head (H) = 8.5m

Inner diameter (D_1) = 0.25

Vane Angle = 45°

Area of flow = 0.06

$Q = \frac{A}{V}$

$V = \frac{Q}{A}$

$$\Rightarrow V_f = V_{f1} = \frac{Q}{A} = \frac{8000 \times 10^{-3}}{60 \times 0.06} = 2.22 m/sec$$

$$\Rightarrow u = \frac{\pi DN}{60} = \frac{\pi \times 0.25 \times 600}{60} = 7.85 \text{ m/s}$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.50 \times 600}{60} = 15.71 \text{ m/s}$$

From the inlet velocity triangle

$$\tan\theta = \left(\frac{V_f}{u}\right)$$

$$= \frac{2.22}{7.85}$$

$$\theta = 15^\circ 47'$$

From the outlet velocity triangle

$$V_{w_1} = \left[u_1 - \left(\frac{V_{f_1}}{\tan 45^\circ}\right)\right]$$

$$= \left[15.71 - \frac{2.22}{\tan 45^\circ}\right]$$

$$V_{w_1} = 13.49 \text{ m/s}$$

$$\text{Efficiency } (\eta_{mano}) = \frac{gH}{V_{w_1} u_1}$$

$$= \frac{9.81 \times 8.5}{13.49 \times 15.71}$$

$$\eta_{mano} = 39.35\%.$$

Least speed $\frac{\pi N^2}{64}(D_1^2 - D^2) = 2gH$

$$\frac{\pi N^2}{60}\left((0.50)^2 - (0.25)^2\right) = (2 \times 9.81 \times 8.5)$$

$$N = 570 \text{ rpm}.$$

4. A centrifugal pump has the following characteristics
outer diameter of impeller = 800mm
Width of impeller vanes at outlet = 100mm
Angle of impeller vanes at outlet = 40°.
The impeller runs at 550 rpm and delivers 0.98 cubic meters of water per second under an effective head of 35m. A 500 KW motor is used to drive the pump. Determine the manometric, mechanical and overall efficiencies of the pump. Assume water enters the impeller vanes radially at inlet.

Sol: Given data:

Discharge $Q = 0.98 m^3/sec$

outer diameter $(D_1) = 800mm = 0.8m$

speed $(N) = 550 rpm$

Width of vane $(B_1) = 100mm = 0.1m$

Angle of impeller vane $(\phi) = 40°$

Head $(H) = 35m$

power $(P) = 500kW$

$\Rightarrow$ We know; $u_1 = \frac{\pi D_1 N}{60}$

$= \frac{\pi \times 0.8 \times 550}{60}$

$= 23.04 m/s$

$V_{f1} = \frac{Q}{\pi D_1 B_1} = \frac{0.98}{\pi \times 0.8 \times 0.1} = 3.90 m/s$

$V_{w_1} = (u_1 - V_{f_1} \cot\phi)$ (∵ From velocity triangle)

$= (23.04 - 3.90 \cot 40°)$

$= 18.39 m/s$

Manometric efficiency is given by $(\eta_{mano}) = \frac{gH_m}{V_{w_2}u_2}$

$$\eta_{mano} = \frac{9.81 \times 0.98 \times 35}{500 \times 10^3}$$

$$\eta_{mano} = 67\%$$

Mechanical efficiency $(\eta_{mech}) = \frac{\eta_o}{\eta_{mano}}$

$$= \frac{0.67}{0.81}$$

$$= 83\%$$