14-9-22 Fluid mechanics

* Properties of fluid:-

→ It is the branch of science which deals with the behaviour of fluids whenever fluid is in static position and also in motion

→ Matter exists in three states
1. solids
2. liquids
3. gaseous

→ Fluid:- It is the substance which deforms continuously when ever we applying shear stress on it
Ex:- liquids, gaseous

→ shear stress:- A force per unit area, acting parallel to an infinitesimal surface element.
units for shear stress = $\frac{N}{m^2}$

→ properties of fluids:-
1. Mass density
2. weight density
3. specific volume
4. specific gravity
5. Viscosity
6. surface tension
7. vapour pressure
8. compressibility
9. capillarity
10. cohesion
11. adhesion.

Mass density:-(ρ) It is defined as the mass of fluid per unit volume

$$\rho = \frac{\text{Mass of Fluid}}{\text{unit Volume}}$$

units for mass density is kg/m3 - Dimensional Formula = ML^{-3}

If temperature increases mass density decreases

As pressure increases mass density increases

2. **Weight density (or) specific weight (w) or (γ):-**

It is defined as the weight of fluid per unit volume

weight = weight of × volume

$$w \text{ (or) } \gamma = \frac{\text{Weight of fluid}}{\text{volume}}$$

units for weight density $= \frac{N}{m^3}$ | $1N = \frac{kg\text{-}m}{sec^2}$

Dimensional formula is ML^{-3}

The relation between weight density (or) specific weight is specific weight = mass density × gravity

$$w = \rho \times g$$

When temperature increases specific weight also decreases

When pressure increases specific weight increases

→ the mass density of water (ρ_{water}) = 1000 kg/m3

specific weight of water $= \rho \times g$

$$w_{(water)} = 1000 \times 9.81 \times m/sec^2$$

$$w = 9810 \frac{kg \times m/sec^2}{m^3}$$

$$w = 9810 \frac{N}{m^3}$$

$$\boxed{w = 9.81 \frac{kN}{m^3}}$$

3. Specific Volume:- Specific volume is reciprocal of mass density

specific volume = $\frac{1}{\text{massdensity}} = \frac{1}{\rho}$

units for specific volume is m3/kg, D.F = L^3M^{-1}

specific gravity:- It is the ratio of Massdensity (or) specific weight of unknown fluid to the mass density of standard fluid

Specific gravity = $\frac{\text{massdensity of unknown fluid}}{\text{massdensity of standard fluid}}$

Here standard fluid = Water

for specific gravity there are no units

Problem:-

→ If $5m^3$ of oil weighs 98kN calculate specific weight mass density & specific gravity of that oil.

sol:- given data

{ $1m^3$ = 1000 liters
1 liter = $10^{-3}m^3$ }

volume of oil = $5m^3$

weight of oil = 98 kN

specific weight = $\frac{\text{weight of fluid}}{\text{volume}} = \frac{98kN}{5m^3} = 19.6\frac{kN}{m^3}$

mass density = $\frac{\text{mass of the fluid}}{\text{volume}}$

Relation b/w specific weight & mass density is

specific weight = mass density × gravity

mass density = $\frac{\text{specific weight}}{\text{gravity}}$

mass density = $\frac{19.6\ kN/m^3}{9.81\ m/sec^2} = 1997.96\ \frac{N/m^3}{m/sec^2}$

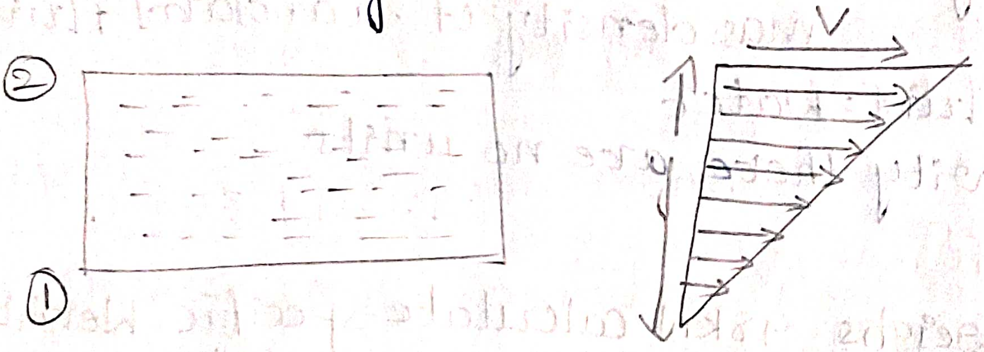
specific gravity = $\frac{\text{mass density of unknown Fluid}}{\text{Mass density of standard Fluid}}$

$= \frac{19.6 \text{ kN/m}^3}{9.81 \text{ kN/m}^3} = 1.99$

Viscosity:-

It is the property by virtue of which offers resistance to motion of layer over the other layer

$\tau = \frac{dv}{dy}$ [change in velocity]



Velocity distribution

→ consider a fluid body is separated between two stationary plates by a distance "y" whenever we applying a shear stress which represented by the letter 'τ' shear is fluid an area of 'A'. As per the Newtons law of viscosity the shear stress (τ) [$\tau = \frac{F}{A}$] is directly proportional to the time rate of deformation that is ($\frac{V}{y}$)

It is mathematically expressed as $\frac{F}{A}$ which is directly propotionally $\frac{F}{A} \propto \frac{V}{y}$

$\tau \propto \frac{V}{y}$

Here μ is dynamic viscosity

The units for dynamic viscosity is

$\mu = \frac{N\text{-sec}}{m^2}$

The dimensional formula is $ML^{-1}T^{-1}$ & in FTL^{-1} in (FLT) system

If we consider small fluid particle b/w the two static plates which is (represented) separated by a distance of dy moving with a velocity dv then the expression is

$$\tau = \mu \frac{dv}{dy}$$

where $\frac{dv}{dy}$ is also called as rate of shear strain and also called velocity gradiant.

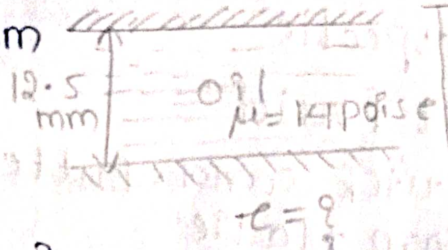
Problem:-

→ two horizantal plates are placed 12.5mm apart the space b/w then is filled with oil of viscosity 14poise calculate shear stress (τ) in the oil. If the upper plate is moving with a velocity of 2.5m/sec.

Sol:- Given data

distance b/w plates = 12.5mm

$= 12.5 \times 10^{-3}$m



1m = 1000mm
1m = 10^3mm

viscosity of oil = 14 poise

1 poise = 0.1 N-sec/m²

14×0.1 N-sec/m²

velocity of moving plate (v) = 2.5m/sec.

$$\text{shear stress } (\tau) = \mu \frac{v}{y} = 14 \times 0.1 \times \frac{2.5\text{m/sec}}{12.5 \times 10^{-3}\text{m}}$$

$\tau = 280$ N/m²

Classification of fluids Based on Newtons law of viscosity

Fluids are classified based on Newton's law of viscosity into two types that are

① Newtonian fluid:- (Time doesn't effect)

The fluids which obey's Newtons law of viscosity then that type of fluids called as Newtonian fluids.

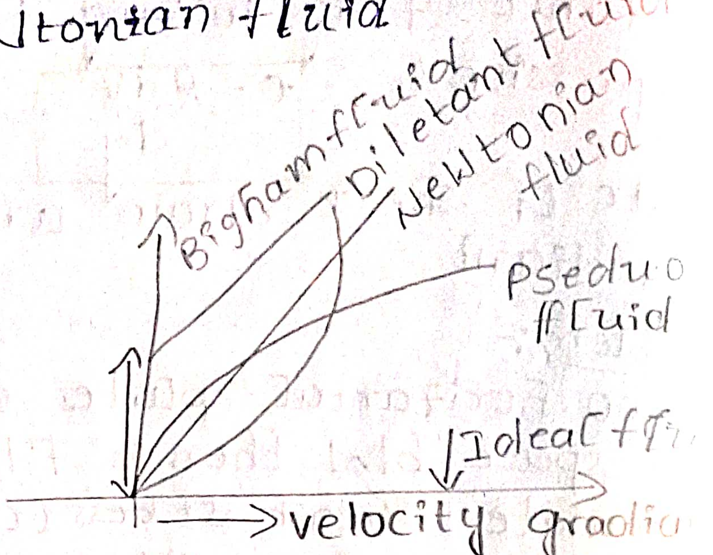
2. Non-Newtonian fluids:-

The fluids which don't obey newton's law of viscosity then that type of fluids are called as non-newtonian fluids

The expression for non-Newtonian fluid

$$\tau = A\left[\frac{dv}{dy}\right]^n + B$$

Non-Newtonian fluid:-



① Time independent

(i) This are Diletant fluid

(ii) Bingham plastic

(iii) pseduo plastic.

② Time dependent

(i) Thixotropic

(ii) Pheopectic

→ Diletant fluid

Ex:- quick sand, Butter, printers ink $n > 1$

→ Bingham plastic

Ex:- sewage sludge, Brilling mud

→ Pseduo plastic:-

Ex:- paper pulp, rubber, susspension paints

① Thixotropic fluid

Ex:- lipstic and certain paints and enamils

② pheopectic

Ex:- gypsum suspensions in water Bentonite solution

→ The velocity distribution of a fluid particle is given $2y - y^2$ and the fluid having a viscosity of 0.1 N-sec/m2 calculate shearstress at boundary and at 15cm away from the boundary

Sol:- given velocity distribution $(u) = 2y - y^2$

Dynamic viscosity $\mu = 0.1$ N-sec/m2

Shearstress (τ), $y = 0$, $y = 15$cm

1m = 100cm

1cm = 10^{-2}m

$y = \frac{15}{100} = 0.15 \, m$

Newton law of viscosity

$\tau = \mu \cdot \frac{du}{dy}_{(y=0)} = \frac{d(2y-y^2)}{dy} = 2-2y = 2-2(0) = 2.$

$\frac{du}{dy}(y=0.15) = 2-2y \Rightarrow 2-2(0.15) = 1.7$

$\tau(y=0) = 0.1 \times 2 = 0.2 \, N/m^2$, $\tau(y=0.15) = 0.1 \times 1.7 = 0.17 \, N/m^2$

→ A piston 796 mm diameter and 200mm long works in a cylinder of 800mm diameter. If the angular space is filled with a lubricating oil of viscosity 5 centipose calculate the speed of desent of piston in verticle position. the weight of piston and the weight of piston and the axial load are 9.8 N

Sol:- given data

Diameter of piston (d) = 796mm

$= 796 \times 10^{-3} = 0.796 \, m$

Length of piston (l) = 200mm = 0.2m

Viscosity of lubricant oil = 5cp

$= 5 \times 10^{-2} \times 0.1 \, \frac{N\text{-}sec}{m^2}$

$\mu = 5 \times 10^{-3} \, N\text{-}sec/m^2$

Shear force (F) = 9.8 N, As per Newton's law of viscosity

$\tau = \mu \frac{du}{dy} \rightarrow \tau = \frac{\text{shear force}}{\text{Area}} = \frac{9.8}{\pi \times 0.796 \times 0.2}$

$= 19.594 \, N/m^2 \Leftrightarrow 19.64 \, N/m^2$

$\frac{F}{A} = \mu \cdot \frac{V}{y} \Rightarrow 19.64 = 5 \times 10^{-3} \times \frac{V}{0.2} \Rightarrow V = \frac{19.64 \times 0.2}{5 \times 10^{-3}}$

V = 785.6 m/sec

→ An oil film of thickness 1.5mm is used for lubricating and inclined on b/w a square plate of size 0.9 m × 0.9 m and inclined plane having an inclination of 20° weight of the square plate is 392.4N it slides down the plate with a uniform velocity of 0.2m/sec. find the dynamic viscosity of the oil.

given data
oil film thickness = 1.5mm = 1.5×10^{-3} m
square plate size (A) = 0.9m × 0.9m
Inclination of 20°
weight of square plate = 392.4N
velocity of square plate = 0.2m/sec
$\mu = ?$

$\tau = \mu \frac{V}{y} \Rightarrow \frac{F}{A} = \mu \cdot \frac{V}{y} \Rightarrow F = W\sin\theta \Rightarrow \frac{392.4\times\sin20^\circ}{0.9\times0.9} = \mu \cdot$

$\mu = \frac{392.4\sin20^\circ\times1.5\times10^{-3}}{0.9\times0.9\times0.2} = 1.2426$ N-sec/m2

→ A plate 0.025mm distance from a fixed plate m.. at 50cm/sec and requires a force of 1.471N/m.. maintain the speed determine the fluid viscosi.. b/w the plates

Sol:- Given data, plate distance (y) = 0.025mm = 0.0..
plate moves (V) = 50cm/sec = 50×10^{-2} m/sec
shear stress (τ) = 1.47N/m²

$\tau = \mu \cdot \frac{V}{y}$

$1.47 = \mu \cdot \frac{50\times10^{-2}}{0.025\times10^{-3}m} \Rightarrow \mu = \frac{1.47}{20,000}$

$\mu = 7.355\times10^{-5}$ N-sec/m2

→ A hydraulic lift consists of a 25cm diameter ram which slides in 25.015cm diameter cylinder the angular space being filled with an oil having a kinematic viscosity of 0.025 stokes and specific gravity of 0.85 If the rate of travell of the ramp is 9.15m/min find the frictional resistance when 3.05 m of the ramp is engaged in the cylinder.

Sol:- Kinematic Viscosity:
It is the ratio of dynamic viscosity of any fluid to the mass density of some fluid. It is Represented

By the letter (ν)
kinematic viscosity (ν) = $\frac{\text{Dynamic viscosity}}{\text{mass density}} = \frac{\mu}{\rho}$

The units per kinematic viscosity and also strokes and also $\boxed{m^2/sec}$ 1 stroke = $10^{-4} m^2/sec$ $\left(\frac{\mu}{\rho} = \frac{N\text{-}sec}{m^2 \times kg/m^3}\right)$

$\boxed{\nu = m^2/sec}$ $\Rightarrow \frac{kg\text{-}m \times sec \times m^3}{sec^2 \times m^2 \times kg}$

Sol:- given data,
diameter of hydraulic lift = 25cm
$= \frac{25}{100} m = 0.25m$
diameter of cylinder = 25.015cm
$= 0.25015m$
oil kinematic viscosity = ~~0.25015~~ 0.025 strokes
$= 0.025 \times 10^{-4} m^2/sec$

specific gravity (G) = 0.85
velocity of lift = 9.15 m/min
$= \frac{9.15 m}{60 sec} = 0.1525 m/sec$

Length of the lift = 3.05m
frictional resistance = ?
Newton's law of viscosity $\tau = \mu \cdot \frac{V}{y} = \frac{\text{frictional resistance force}}{\text{surface area}}$

surface area = $\pi d l$
$= \pi \times 0.25 \times 3.05 = 2.395 m^2$

$\nu = \mu/\rho$ $G = \frac{\rho_{unknown}}{\rho_{water}} = \frac{\rho_{unknown}}{1000} = 0.85 = \frac{\rho_{oil}}{1000}$

$\rho_{oil} = 0.85 \times 1000 \Rightarrow \rho_{oil} = 850 kg/m^3$

$0.025 \times 10^{-4} = \frac{\mu}{850} \Rightarrow \mu = 0.025 \times 10^{-4} \times 850$ $\mu = 2.125 \times 10^{-3} N\text{-}Sec/m^2$

$y = \frac{0.25015 - 0.25}{2} \Rightarrow y = 7.5 \times 10^{-5} m$ ~~$\Rightarrow$~~

$\frac{F}{2.398} = 2.125 \times 10^{-3} \times \frac{0.1525}{7.5 \times 10^{-5}} = \frac{2.125 \times 10^{-3} \times 0.152 \times 2.395}{7.5 \times 10^{-5}}$

$\boxed{F = 1.03 \times 10^{-9} N}$

* The lateral stability of a long shaft 150mm diameter is obtained by means of a 250mm stationary bearing

having an internal diameter of 150·25 mm if the space between the bearing and the shaft is filled with a lubricant having a viscosity of 0·245 N-sec/m². What power will be required to overcome the viscus resistance when the shaft is rotated at a constant rate of 180 rpm (shift + mode = 3) → for Normal "D" calculator

sol:- To get the power

1· calculate τ ⇒ $\tau = \mu \frac{V}{y}$ [Newton's law of viscosity expression]

2· calculate force [frictional resistance] $F = \mu \cdot$ surface area, $F = \mu \cdot (\pi \cdot d \cdot l)$

3· calculate torque · T = force × radial distance (or) Radius of moving component

4· power = T × angular speed.

sol:- shear stress (τ) = $\mu \cdot \frac{V}{y}$ where $V = r\omega$ $\boxed{\omega = \frac{2\pi N}{60}}$

$V = r \times \frac{2\pi N}{60}$

radius = $\frac{\text{diameter}}{2} = \frac{150 \times 10^{-3}}{2} = 75 \times 10^{-3}$ mm

velocity = $75 \times 10^{-3} \times \frac{2\pi \times 180}{60}$ ⇒ V = 1·413 m/sec.

$y = \frac{150.25 - 150}{2} \times 10^{-3} = 1.25 \times 10^{-4}$ m

(i) $\tau = 0.245 \times \frac{1.413}{1.25 \times 10^{-4}} = 2767$ N/m²

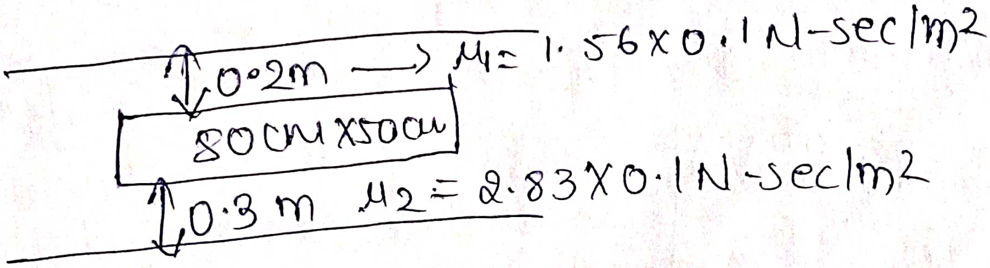
(ii) $F = \tau \times \pi d l = 2767 \times \pi \times 150 \times 10^{-3} \times 250 \times 10^{-3} = 325.97$ N

(iii) torque = F × radius = $325.97 \times \frac{150}{2} \times 10^{-3} = 24.447$ N-m

(iv) P = torque × ω = $24.447 \times \frac{2 \times \pi \times 180}{60} = 460.81$ W

* A think flag plate of size 80cm X 50cm is moved between 2 plane horizantal boundaries at a distance of 0.2m from the top boundary and 0.3m from bottom boundary the viscosity of the fluid above the plate is 1.56×0.1 N-sec/m² and below the plate is 2.83×0.1 N-sec/m² What force is required to grad the plate at a horizantal velocity of 3 m/sec.

Sol:-



Top boundary:-

$$\tau_1 = \mu_1 \times \frac{V}{y}$$

$$\tau_1 = \frac{1.56 \times 0.1 \times 3}{0.2} = 2.34 \ N/m^2$$

$$F_1 = \tau_1 \times A = 2.34 \times \frac{80}{100} \times \frac{50}{100} = 0.936 \ N$$

for Bottom boundary:-

$$\tau_2 = \mu_2 \frac{V}{y} = \frac{2.830 \times 0.1 \times 3}{0.3}$$

$$= 2.83 \ N/m^2$$

$$F_2 = \tau_2 \times A$$

$$F_2 = 2.83 \times \frac{80}{100} \times \frac{50}{100} \Rightarrow F_2 = 1.132 N.$$

Total frictional resistance

$$F = F_1 + F_2 \Rightarrow F = 0.936 + 1.132$$

$$\boxed{F = 2.068 N}$$

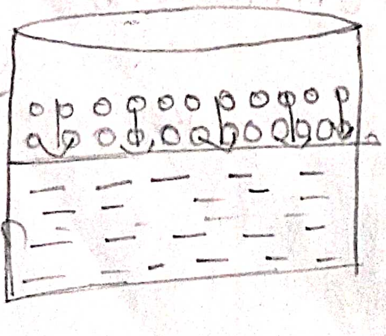
→ In case of Liquids As temperature increases Viscosity is decreases.

→ In case of gases As temperature increases Viscosity is also increases.

→ pressure do not effects on Viscosity But incase of some fluids as the pressure increases viscosity also increasing

Vapour Pressure:-

This state is Nothing but saturation state



→ Vapour Pressure.

→ liquid molecules are constantly exected into the space above the free surface if the space is confined (closed)
→ The exected molecules exercts pressure and it's own surface the pressure is called "Vapour pressure"
→ petrol will be evaporates faster then water
→ A liquid with high vapour pressure evaporation is faster Ex:- petrol i.e 30.4 kN/m2.

The units for vapour pressure is N/m2

→ Mercury has very low vapour pressure i.e 0.106 N/m2.
→ the water vapour pressure 2.5 kN/m2.

compressibility:-

It is the reciprocal of Bulk modulus of elasticity
It may be defined as the property By virtue of which fluids undergo change in its volume under the action of exteral pressure $\therefore C = \frac{1}{k}$

Where k is the Bulk Modulus of elasticity which is equal to change in pressure to the volumatic strain

$$k = \frac{\text{change in pressure}}{\text{volumetric strain}} = \frac{dP}{\left[\varepsilon_v = \frac{\delta v}{v}\right]}$$

The units for compressibility m2/N.
The units for Bulk modulus is N/m2

surface tension:-

surface tension is due to cohesion at the surface of contact b/w a liquid and gas [or two liquids which donot mix with eachother] molecular attraction introduces a force which causes the Intetface acts like a membrane under tension this phenomenon is called surface tension.

It is denoted by "σ" It is the force per unit length

$$\sigma = \frac{force}{length}$$ the units are $\frac{N}{m}$

the effect of surface tension is a small needle is placed on the surface of liquid will not sink but it's flots similarly insects moving on water surface due to surface tension

→ the surface tension in water air contact is 0.0736 N/m

→ At the interface the liquid molecules is attracted downwards so there is a net downword force but due to surface tension the interface will not move down there the tension develops in the interface due to this the interface behaves like a membrane

→ Affects of surface tension:-

Liquid droplet:- When a drop of liquid is separated from the main liquid It takes sphere shape due to surface tension there is an imaginary force which trys the contraction of the droplet the pressure acting on it is resists the surface tensional force therefore surface tension and pressure both balanced

→ If σ is the interal force acting on the perimeter where is P_i is the interal pressure

by equating these two forces, $\sigma(2\pi r) = P_i \times \pi r^2$

$$P_i = \frac{2\sigma}{r}$$

soap bubble:- In case of soap bubbles, its has two surfaces in contact with air in such cases

$\sigma(2\pi r) = P_i \times 2\pi r^2$

$$P_i = \frac{\sigma}{r}$$

Liquid jet:- In case of liquid jet having length (l) and radius (r) If the jet is cut into 2 equal halfs the forces acting on one half will be those due to pressure intensity which is on the projected area "2rl" and the tensile force which is acting on the two sides of liquid jet "2l" [Here the perimeter 2(l+b) the diameter is negleted then "2l"]

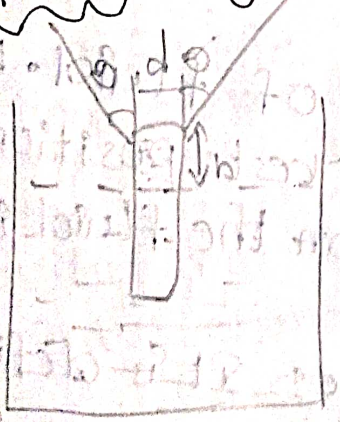
$$\sigma(2l) = P_i(2rl)$$

$$\boxed{P_i = \frac{\sigma}{r}}$$

→ **Cavitation:-** The pressure at any point reduces to vapour-pressure of the flowing liquid, there will be formation of vapour bubbles occured. the collapse of these bubbles when carried to high pressure zone, rushing of the surrounding liquid to the bubble and applies high impact pressure. If this bubble is at the boundary they will be formation of cavities occurs this phenomenon is called cavitation

→ **Capillarity:-** capillary is due to cohesion and adhesion. the phenomenon of rise (or) fall of the liquid when it is touching another body is called capillarity

Expression for capillary rise:-



$$\sigma \cos\theta \times \pi d = W$$

$$w = \frac{W}{V}$$

$$W = w \times V$$

Volume of fluid which is presented in the glass tube $= \frac{\pi}{4} d^2 \times h$

$$W = w \times \frac{\pi}{4} d^2 h$$

$$\sigma \cos\theta \, \pi d = w \times \frac{\pi}{4} d^2 h$$

$$\boxed{h = \frac{4\sigma\cos\theta}{\rho g d}}$$

→ When a glass tube is dipped in water we observe as shown in fig raisin of water in the glass tube

→ the surface tension force tries to pull the water up and gravitational force pull's down therefore the water will rise in the tube to such level, surface tension force is equals to gravitational force

→ component of surface tension in the upward direction = $\sigma \cos\theta$

→ surface tensional force = $\sigma \cos\theta \times \pi d$

gravitational force = weight of water in the portion of height (h) = specific weight × volume

$$h = w \times V$$

$$W = w \times \frac{\pi}{4} d^2 h$$

By equating the above forces;

$$\sigma \cos\theta \times \pi d = W$$

$$\sigma \cos\theta \times \pi d = w \times \frac{\pi}{4} d^2 h$$

$$h = \frac{4\sigma \cos\theta}{wd}$$

where σ = surface tension, d = diameter of glass tube
θ = contact angle, w = specific weight of fluid.

→ If the diameter of tube is more than 5mm then capillarity does not takes place

Fluid statics:-

→ It is the behaviour of study of fluid. When the fluid is stationary are rest position the fluid is the major force which is acting on the fluid is

→ pressure force

→ pressure intensity (or) pressure:- It is defined as pressure force acting per unit area of the fluid.

$$\text{pressure intensity} = \frac{\text{pressure force}}{\text{unit area of fluid}}$$

$P_i = P/A$.

units for pressure intensity = N/m^2.

Dimension formula = FL^{-2}

$= \frac{MT^{-2}L}{L^2}$

$= ML^{-1}T^{-2}$

pressure intensity is always acting Normal to the surface and t

the units may be expressed in terms of pascal's also.

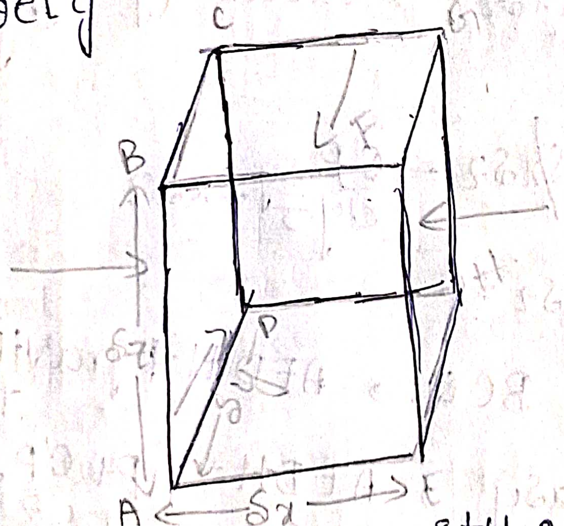
{1 pascal = N/m^2

ex:- 0.1 pa. sec = $\frac{N}{m^2}$-sec}

Pressure Variation in a static fluid:-

→ Consider a rectangular parallelopiped fluid element of having sides $\delta x, \delta y, \delta z$ along x, y, z directions respectively

Black = along y-direction
Blue = along x-direction
pencile = along z-direction



→ Let ρ be the mass density, w be the specific weight of fluid element

→ When fluid is stationary the various forces acting on the fluid elements are ① pressure force and ② gravitational force

gravitational force = weight of fluid element

gravitational force = specific weight × volume of fluid element

= specific weight × volume of fluid element = $\delta x \delta y \delta z$

* The volume of fluid element

Pressure force

consider along x - direction

Let p be the pressure intensity acting at the center of the fluid element

Gravitational force $G = w\delta x \delta y \delta z$

Pressure forces:-

along x-direction :- $p_i = P/A \Rightarrow P = p_i \times A$

ABCD = $(P - \frac{dP}{dx} \cdot \frac{\delta x}{2}) \delta z \delta y$, EFGH = $(P + \frac{dP}{dx} \cdot \frac{\delta x}{2}) \delta z \delta y$

Algebraic sum of these two forces = 0

ABCD − EFGH = 0 (opposite direction)

$$P\delta z\delta y - \frac{dP}{dx} \cdot \frac{\delta x}{2} \cdot \delta z \delta y - P\delta z \delta y - \frac{dP}{dx}\frac{\delta x}{2} \delta z \delta y = 0$$

$$-\frac{dP}{dx} \cdot \frac{\delta x}{2} \cdot \delta z \delta y - \frac{dP}{dx}\frac{\delta x}{2} \delta z \delta y = 0$$

$$-2 \cdot \frac{dP}{dx} \cdot \frac{\delta x}{2} \delta y \delta z = 0$$

$$\frac{dP}{dx} \delta x \delta y \delta z = 0$$

along y-direction:-

ABEF = $(P - \frac{dP}{dy} \frac{\delta y}{2}) \delta x \delta z$, CDGH = $(P + \frac{dP}{dy} \frac{\delta y}{2}) \delta x \delta z$

ABEF − CDGH = 0

$$P\delta x \delta z - \frac{dP}{dy} \frac{\delta y}{2} \delta x \delta z - P\delta x \delta z - \frac{dP}{dy}\frac{\delta y}{2} \delta x \delta z$$

$$\frac{dP}{dy} \delta x \delta y \delta z = 0$$

along z-direction:- BCGF, AEDH + gravitational force

BCGF = $(P - \frac{dP}{dz} \frac{\delta z}{2}) \delta x \delta y$, AEDH = $(P + \frac{dP}{dz} \frac{\delta z}{2}) \delta x \delta y$

Gravitational force = $w\delta x \delta y \delta z$

$$P\delta x\delta y - \frac{dP}{dz}\frac{\delta z}{2} \delta x \delta y - P\delta x \delta y - \frac{dP}{dz}\frac{\delta z}{2} \delta x \delta y - w\delta x \delta y \delta z =$$

$$-\frac{dP}{dz} \delta x \delta y \delta z - w\delta x \delta y \delta z = 0$$

$$-\frac{dP}{dz} - w = 0 \Rightarrow \frac{dP}{dz} = -w \quad \text{I.O.B.S}$$

$$\int \frac{d}{dz}(P) = -\int w\, dz \Rightarrow \boxed{P = -wz + C}$$

Let.

$P = -wz + C \longrightarrow (1)$

for this case

$z = z_0 + H$, $P = P_{atm}$

$P_{atm} = -w(z_0 + H) + C$.

$C = P_{atm} + w(z_0 + H) \rightarrow ②$

substitute eq② in eq① $P = -wz + P_{atm} + w(z_0 + H)$

$P = P_A$, $z = z_0 + H - h$

$P_A = -w(z_0 + H - h) + P_{atm} + wz_0 + wH$

$P_A = -wz_0 - wH + wh + P_{atm} + wz_0 + wH$

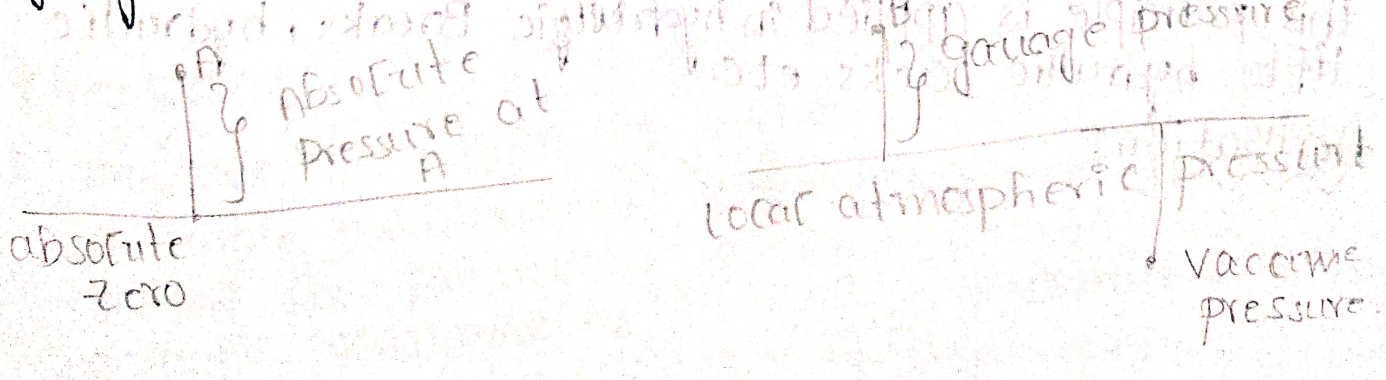
$P_A = wh + P_{atmos}$ { P_{atmos} = constant }

$\boxed{P_A = wh}$ $\boxed{P = wh}$ N/m^2

where h = height from freesurface
w = specific weight
P = pressure intensity at any point.

*Define pressure head :- pressure head (h) = $\frac{P}{w}$
units are meters and cm.

*pressure variation :- pressure at a point can be expressed in two systems
in the first system pressure is measured w.r.t absolute zero (or) complete vaccum in this case clearly it should be specified $4N/m^2$ absoulte
In the second system pressure is measured w.r.t local atmospheric pressure in this case the pressure is above local atmospheric pressure it is called gauge pressure. If the pressure is below local atmospheric pressure it is called vaccum pressure
If nothing is specified always it take it as gauge pressure (w.r.t they don't give than we considered as gauge pressure)



Problems:-

→ An open tank contains water upto a depth of 1.5m and above it oil of specific gravity (G) 0.8 for a depth of 2m. Find the pressure intensity at the interface of liquids at the bottom of the tank

Sol:- (i) pressure intensity at interface $P_A = wh$

Specific gravity $= \frac{W_{oil}}{W_{water}} \Rightarrow W_{oil} = 0.8 \times 9.81$

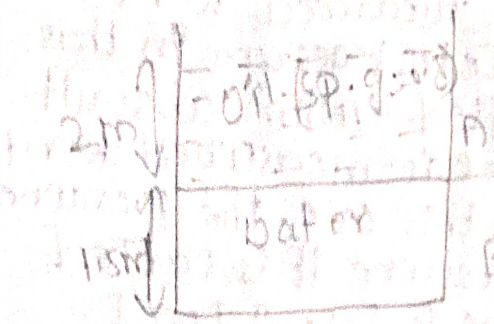
$w_{oil} = 7.848 \text{ kN/m}^3$

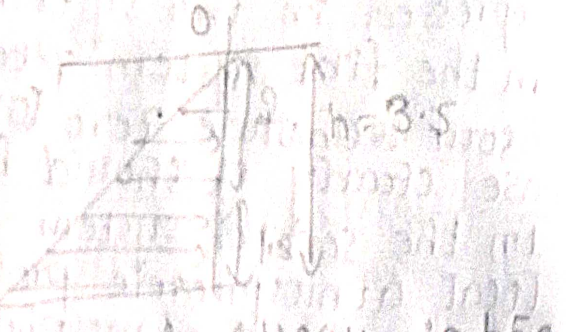
$P_A = wh \Rightarrow P_A = 7.848 \times 2 = 15.69 \text{ kN/m}^2$

(ii) pressure intensity at Bottom of tank

$P_B = wh = 9.81 \times 1.5 = 14.715$

Total pressure intensity $= P_A + P_B = 30.405$



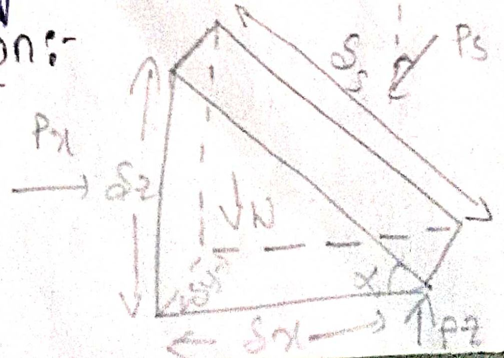


(pressure is always in the form of triangle)

→ **pascal's law:-**

In a static mass of fluid the pressure at any point has same magnitude in all directions. This principle is applied in hydraulic Breaks, hydraulic lift, hydraulic jacks etc.

Derivation:-



→ consider a wedge of sides $\delta x, \delta y, \delta z, \delta s$.
→ And this wedge has pressure acting along x-direction, y-direction, z-direction is P_x, P_y, P_z & P_s
→ weight of the wedge is $W = \frac{w}{2} \delta x \delta y \delta z$

specific weight = $\frac{\text{weight of the fluid}}{\text{volume}}$

weight of the fluid = specific weight × volume

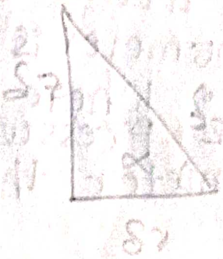
Forces acting along x-direction

$P_x \delta y \delta z - P_s \sin\alpha \, \delta s \, \delta y = 0 \longrightarrow ①$

along z-direction

$P_z \delta x \delta y - P_s \cos\alpha \cdot \delta s \delta y - \frac{w}{2} \delta x \delta y \delta z = 0 \longrightarrow ②$

consider equations ① & ②



$P_x \delta y \delta z - P_s \delta z \delta y = 0$

$P_x = P_s \longrightarrow ③$

$P_z \delta x \delta y - P_s \delta x \delta y - \frac{w}{2} \delta x \delta y \delta z = 0$

from the triangle

$\sin\alpha = \frac{\delta z}{\delta s}$

$\boxed{\delta z = \delta s \sin\alpha}$

$P_z \delta x \delta y - P_s \delta x \delta y = 0$

$P_z = P_s \longrightarrow ④$

$\cos\alpha = \frac{\delta x}{\delta s}$

$\boxed{\delta x = \delta s \cos\alpha}$

from ③ & ④

$\boxed{P_x = P_z = P_s.}$

∴ pressure intensity at all directions are equal.

* Measurement of pressure :- manometers, Mechanical gauges

* they are three types of manometers

→ simple manometer :- simple manometer is used to find the pressure at a single point.

In simple manometer featurely divided into three

(i) piezometer (ii) u-tube manometer

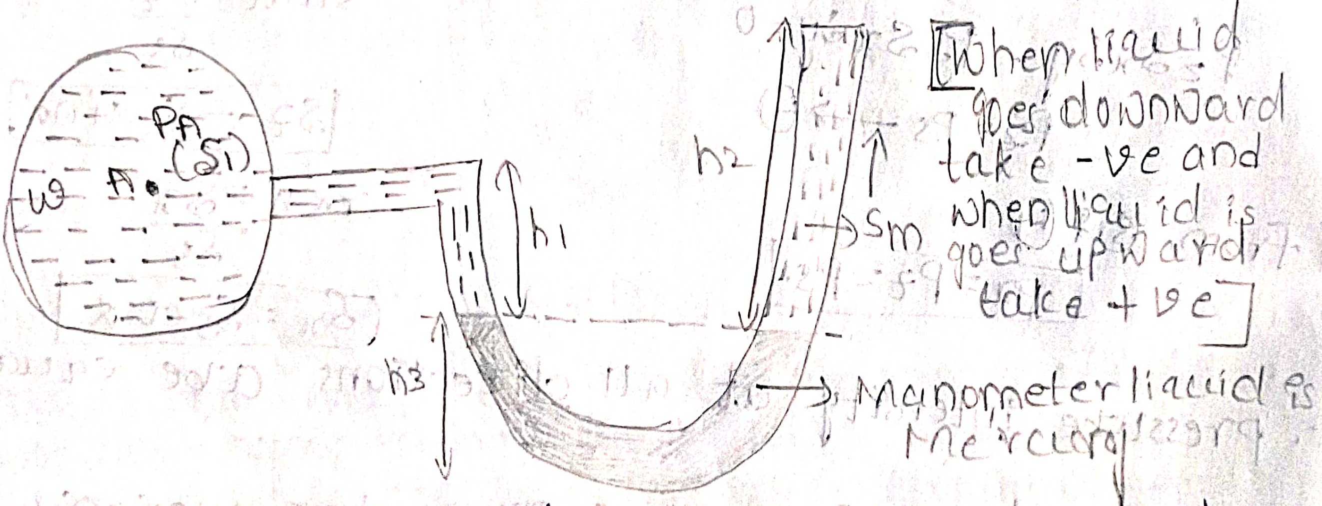
(iii) - single column manometer

Differential manometers :- Differential manometers are used to find the pressure between any two points feurterly divided into three
(i) U-tube differential (ii) inverted U-tube manometer
(iii) micrometers.

Mechanical gauge:- Mechanical gauge is used to find the air in a vechical wheel.
amount of air.

*Manometers:- these are the pressure measuring device works on the principle i.e by balancing the colourn of a liquid against the pressure is to be measured
A manometer is a device which is connected to the point, where the pressure is required and depending on the pressure there will be raise of liquid in the manometer that raise will give the pressure at that point.

*U-tube manometer:-



*single col. We know that pressure intensity p=wh

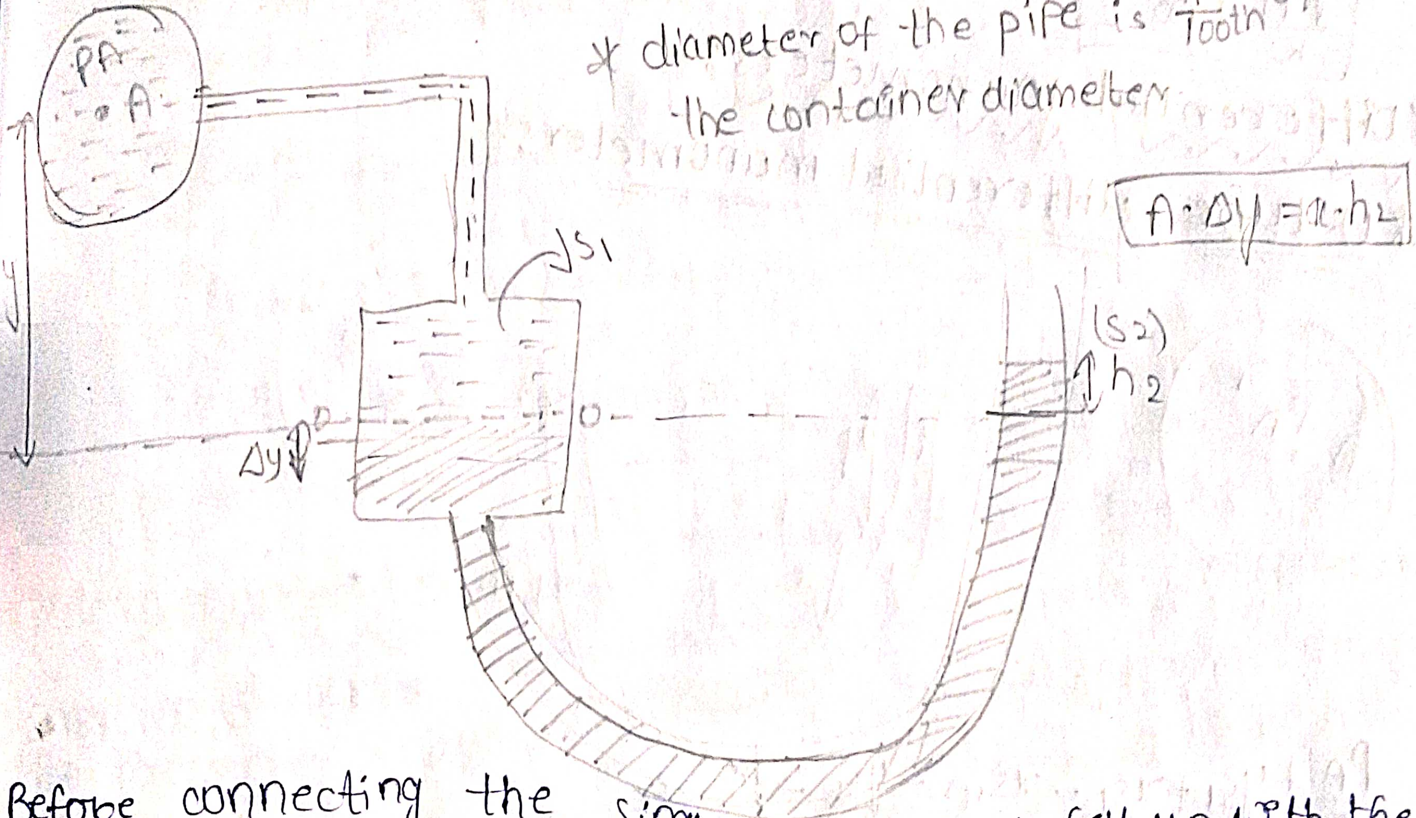
$$\frac{P_A}{w} + S_1h_1 + S_mh_3 - S_mh_3 - h_2S_m = 0$$

$$\frac{P_A}{w} = h_2S_m - h_1S_1$$

$$P_A = w(h_2S_m - h_1S_1)$$

Single column manometer:- It is a modification over U-tube manometer. It consists of a transperent cylinder whose area is 100 times more compare to the pipe. the advantage of this is we don't need to take two Readings like in youtube manometer just one reading is enough

In normal position:-



* Before connecting the single column manometer, in the left link first fill up with the fluid which is flowing in the pipe. Let S_1 is the specific gravity of the liquid flowing in the pipe Because of the reason there is an filing of position to a height y. the reason there is an measurement of level in the Right link that is manometer to the point $yS_1 = h_1S_2$ connect the where pressure is to be measured. there is a decrease in Δy (displacement) in the left link balances the Rising h_2 in the Right link therefore $A \cdot \Delta y = a \cdot h_2$

$$\Rightarrow \boxed{\Delta y = \frac{a \cdot h_2}{A}}$$

$$\frac{P_A}{w} + y s_1 + \Delta y s_1 - \Delta y s_2 - h_1 s_2 - h_2 s_2 = 0 \qquad \boxed{y s_1 = h_1 s_2}$$

$$\frac{P_A}{w} + h_1 s_2 + \Delta y s_1 - \Delta y s_2 - h_1 s_2 - h_2 s_2 = 0$$

$$\frac{P_A}{w} + \Delta y s_1 - \Delta y s_2 - h_2 s_2 = 0 \rightarrow \frac{P_A}{w} + \Delta y (s_1 - s_2) - h_2 s_2$$

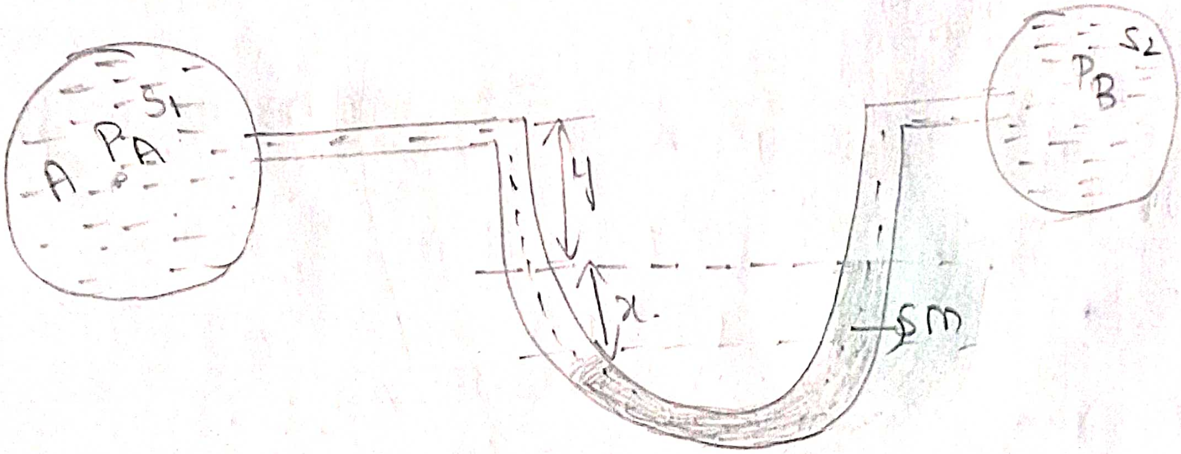
$$\frac{P_A}{w} + \frac{a \cdot h_2 s_1}{A} - \frac{a}{A} h_2 s_2 - h_2 s_2 = 0$$

$$\frac{P_A}{w} - h_2 s_2 \left[\frac{a}{A} + 1\right] + \frac{a}{A} h_2 s_1 = 0 \qquad \boxed{\text{negligible } \frac{a}{A}}$$

$$\boxed{\frac{P_A}{w} = h_2 s_2}$$

*Differential manometers:-

*U-tube differential manometer:-



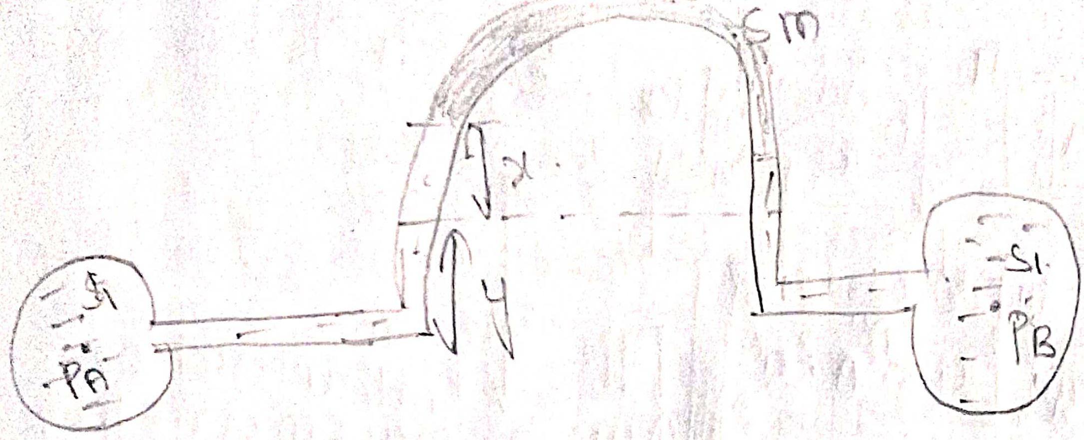
$$\frac{P_A}{w} + y s_1 + x s_1 - x s_m - y s_2 - \frac{P_B}{w} = 0$$

$$\frac{P_A}{w} - \frac{P_B}{w} = y(s_2 - s_1) + x(s_m - s_1)$$

If both end vessels having same fluid $(s_1 = s_2)$

$$\boxed{\frac{P_A - P_B}{w} = x(s_m - s_1)}$$

*Inverted U-tube differential manometer:-



$$\frac{P_A}{w} - yS_1 - xS_1 + xS_m + yS_m + \frac{P_B}{w} = 0$$

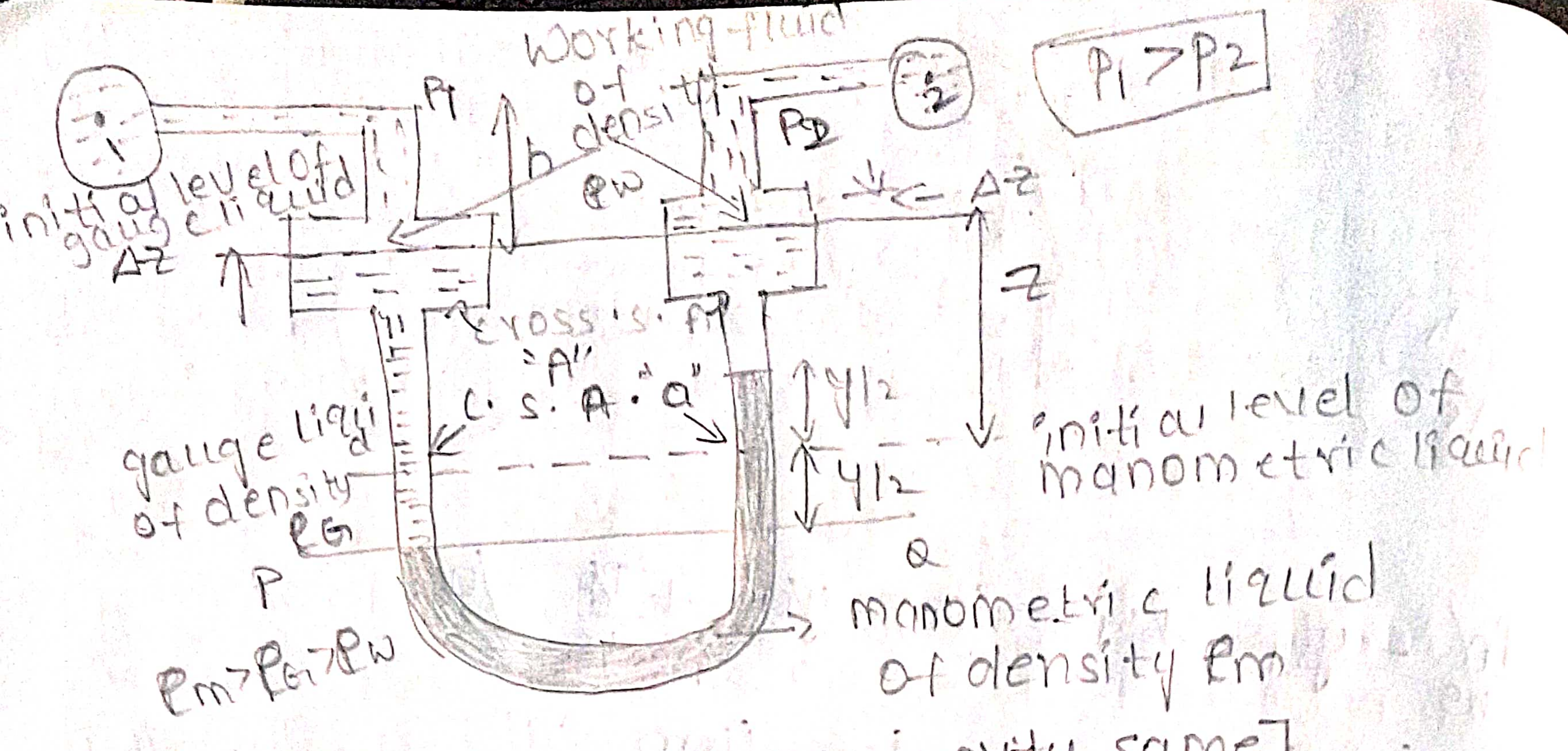
$$\frac{P_B}{w} - \frac{P_A}{w} = yS_1 + xS_1 - xS_m - yS_m$$

$$\frac{P_B}{w} - \frac{P_A}{w} = y(S_1 - S_m) + x(S_1 - S_m)$$

$$\frac{P_A}{w} - yS_1 - xS_1 + xS_m + yS_1 - \frac{P_B}{w} = 0$$

$$\boxed{\frac{P_A}{w} - \frac{P_B}{w} = x(S_m - S_1)}$$

*Micromanometer:- these are used for measuring very small pressure differences this instrument will measure the pressure accuratly. the advantage of micrometers are it gives very small pressure difference with high press. precision. It consistes of two transperent cylinders attached to the left link and Right link respectively. these two cylinders are connected by U-tube in this case. there two manometeric fluids are used of level Δz in the is a decrease in cylinder the increase in the level left portion Balances the right side portion in the to a height of $y/2$ in the Right side portion in the U-tube having an area of 'a' ∴ $A\Delta z = a \cdot y/2$



[specific gravity same]

$$\Delta z = \frac{a}{A}\, y/2 \longrightarrow (1)$$

$$\frac{P_1}{w} + (h+\Delta z)S_2 + (z - \Delta z + \frac{y}{2})S_1 - \frac{P_2}{w} - (h-\Delta z)S_2 - (z - y/2 + \Delta z)S_1 - yS = 0$$

$$\frac{P_1}{w} + hS_2 + \Delta z S_2 + zS_1 - \Delta z S_1 + \frac{y}{2}S_1 - \frac{P_2}{w} - hS_2 + \Delta z S_2 - zS_1 + y/2\, S_1 - \Delta z S_1 - yS = 0$$

$$\frac{P_1}{w} + 2\Delta z S_2 - 2\Delta z S_1 + yS_1 - yS - \frac{P_2}{w} = 0$$

$$\frac{P_1}{w} + 2\left(\frac{a}{A}\cdot\frac{y}{2}\right)S_2 - 2\left(\frac{a}{A}\cdot\frac{y}{2}\right)S_1 + yS_1 - yS - \frac{P_2}{w} = 0$$

$$\frac{P_1}{w} + \frac{a}{A}\cdot y(S_2 - S_1) + y(S_1 - S) - \frac{P_2}{w} = 0$$

$$\boxed{\frac{P_1 - P_2}{w} = y(S - S_1)} \qquad \left(\frac{a}{A}\cdot y = \text{Negligible}\right)$$

→ **Hydrostatic pressure intensity**:- hydrostatic pressure force acting on a body which is summerged in fluid when fluid is stationary from pressure intensity = $\frac{\text{Pressure force}}{\text{Area}}$

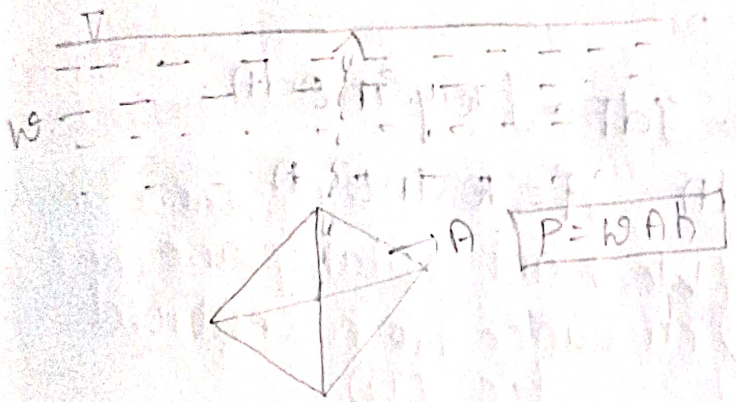
$P_i = \frac{P_f}{A}$ $P_f = P_i A$ where $\boxed{P_i = wh}$

$\boxed{P_F = wh \times A}$

where w = specific weight of liquid
A = Area of object summerged [object = plane surface]
h = centroidal distance of the object from the free surface

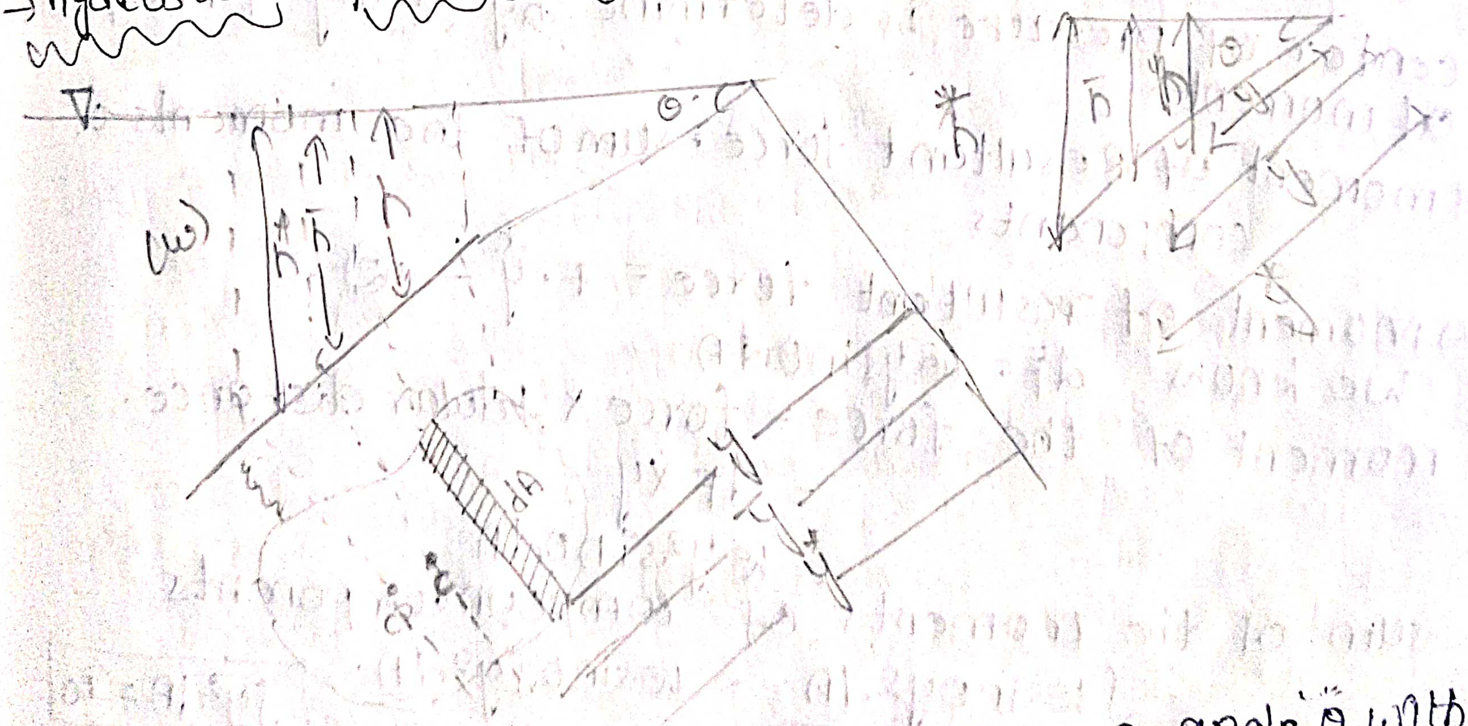
→ we have three types of plane surfaces.
1. Horizantal 2. Vertical 3. inclined.

→ Hydrostatic pressure force acting on a horizantal plane surface



→ The height from the summerged plane to the surface is same at any point. the height of doesn't varies

→ Hydrostatic pressure force acting on inclined plane surface



→ Consider a plane which is making an angle θ with free surface. Let $\bar{h}$ is the distance the of centre of gravity from the free surface, h^* is the distance of the pressure from the free surface. In order to solve the problem consider an element at a distance h from the free surface, extended the inclined surface upto free surface. draw a perpendular line to plane surface. Now w.r.t projected line let $y, \bar{y}, y^*$ be the distance from plane to the centre gravity and centre of pressure.

from figure $\sin\theta = \frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*}$ → ①

Pressure at the element $P = wh$

Pressure force at the element $dF = wh\,dA$

from eq① $h = y\sin\theta$

$dF = wy\sin\theta\,dA$.

Total pressure force $F = \int dF = \int wy\sin\theta\,dA$.

$F = w\sin\theta\int y\,dA$ ∴ $F = w\sin\theta\,\bar{y}A$

$F = wA\bar{y}\sin\theta$.

{ We want total plane then distance from centroid i.e ($\bar{y}$) } distance from

from ① $\bar{h} = \bar{y}\sin\theta$.

$$\boxed{F = wA\bar{h}}$$

center of pressure is determine by using principal of moment.

* moment of Resultant force = sum of the moments of components

→ Moment of resultant force $= F \cdot y^*$ → ②

we know $dF = wy\sin\theta\,dA$

moment of the force = force × ⊥dular distance.

$= dF \times y$

$= wy^2\sin\theta\,dA$.

sum of the moments of compound components

$= \int w\sin\theta\,y^2\,dA = w\sin\theta\int y^2\,dA$. $\boxed{\int y^2 dA = I_0}$

$F \cdot y^* = w\sin\theta\,I_0$

→ from parallel axies therom $I_0 = I_G + A\bar{y}^2$

using the principle of moments $F \cdot y^* = w \sin\theta \cdot I_0$.

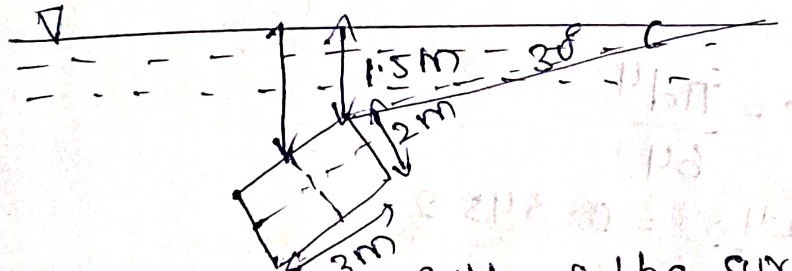
From $F = wA\bar{h}$ then $wA\bar{h}\, y^* = w\sin\theta(I_G + A\cdot\bar{y}^2)$

$$y^* = \frac{I_G\, w\sin\theta}{wA\bar{h}} + \frac{w\sin\theta\, A\bar{y}^2}{wA\bar{h}}.$$

$$y^* = \frac{I_G \sin\theta}{A\bar{h}} + \frac{\bar{y}^2 \sin\theta}{\bar{h}}$$

from the eq (1) $\bar{y} = \frac{\bar{h}}{\sin\theta} \Rightarrow \bar{y}^2 = \frac{\bar{h}^2}{\sin^2\theta}$ = $y^* = \frac{h^*}{\sin\theta}$.

→ A Rectangle plane surface 2m wide 3 m deep lies in water in such a way that It Makes an angle of 30° with the free surface determine total pressure and center of pressure when the upper head is 1.5m Below the water surface



$P_F = wA\bar{h}$

sol:-

Given that width of the surface = 2m

depth of the plate = 3m

the specific weight of water = 9.81 kN/m3

Area of plate = 2×3 = 6m²

$P = wA\bar{h}$ → (1)

$C.P = \bar{h} + \frac{I_G}{A\bar{h}}\sin^2\theta$ → (2)

$= \sin 30° = \frac{x}{1.5}$

$x = \sin 30° \times 1.5 = 0.75m$

$\bar{h} = 1.5 + 0.75 = 2.25m$

$P_A = 9.81 \times 6 \times 2.25 = 132.435$ kN

$I_G = \frac{bd^3}{12} = \frac{2\times 3^3}{12} = 4.5 m^4$

depth of C.P = $2.25 + \frac{4.5}{6 \times 2.25} \times \sin^2 30°$

$\boxed{cp = 2.83}$

* Determine the total pressure on a circular plate of diameter 1.5m which is placed vertically in water in such a way that center of the plate is 3m below the free surface find the center of pressure also.

Sol:- Diameter of circular plate = 1.5 m
specific weight (w) = 9.81 kN/m³
Area of circular plate $A = \pi r^2 = \pi \times (0.75)^2 = 1.77$

$$\boxed{C.P = \bar{h} + \frac{I_G}{A\bar{h}}}$$

$\bar{h} = 3m$ $P = 9.81 \times 1.77 \times 3 = 52.09 \text{ kN}$

$I_G = \frac{\pi d^4}{64} = 0.2485 m^4$

I_G of the circle $= \frac{\pi d^4}{64}$

$C.P = 3 + \frac{0.2485}{1.767 \times 3} = 0.3452$

$\boxed{C.P = 3.0452 m}$

* A Rectangular plane surface is 2m wide 3m deep is lies in a verticle plane in water determine the total pressure and position of centre of pressure on the plane surface when it's upper edge is horizontal and coincide's with water pr-surface and also calculate the same if upper edge is 2.5m below the free surface

Sol:- given width = 2m
deep = 3m
$A = 2 \times 3 = 6m^2$

when the plane surface is coincide with water surface then $\bar{h}$ is the centroidal distance from centroidal surface.

then $P = wA\bar{h}$ $\therefore = (9.81 \times 6 \times \frac{3}{2}) = 88.29$

$I_G = \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5 m^4$

then $c.p = \bar{h} + \frac{I_G}{A\bar{h}} = 1.5 + \frac{4.5}{6 \times 1.5} = 2m$

When the plane surface is below the 2.5 m from surface

$\bar{h} = 2.5 + \frac{3}{2} = 4m$

$P = wA\bar{h} = 9.81 \times 6 \times 4 = 235.44 kN$

$c.p = \bar{h} + \frac{I_G}{A\bar{h}}$ where $I_G = 4.5 m^4$

$c.p = 4 + \frac{4.5}{6 \times 4} = 4.18m$

* A circular plate 3m diameter immersed in water in such a way that it's greater and least depths below the surfaces are 4m and 1.5m respectively determine the total pressure on one face of the plate and position of centre of pressure.

Sol:- given diameter $d = 3m$

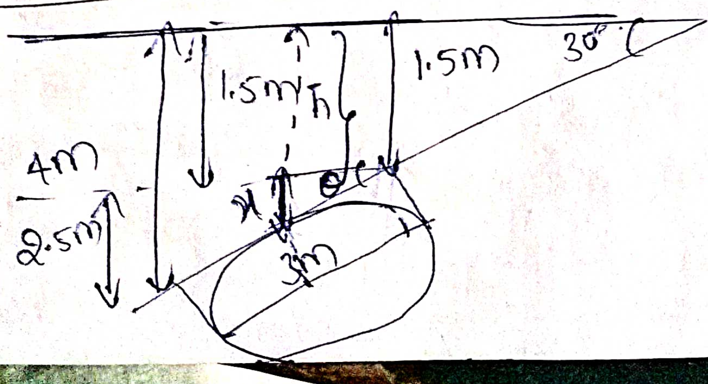
$r = 1.5m$

$A = \pi r^2 = \pi \times (1.5)^2 = 7.06 m^2$

(greter is) = 4m

When the deepth of circular plate

then $\bar{h} = 4m +$



$P = wA\bar{h}$, $w_{water} = 9.81 kN/m^3$, Area of the circle $= \pi r^2$
$= \pi(1.5)^2 = 7.06 m^2$

$\bar{h} =$

$\sin\theta = \frac{2.5}{3}$ $\theta = 56°44'$

$\sin 56°44' = \frac{x}{1.5}$

$x = 1.224 m$

$\bar{h} = 1.5 + 1.224$ $\bar{h} = 2.72 m$

$P = 9.81 \times 7.06 \times 2.72 = 188.36 kN$

$C.P = \bar{h} + \frac{I_G}{A\bar{h}} \sin^2\theta$ $\Rightarrow I_G = \frac{\pi d^4}{64} = \frac{\pi(3)^4}{64} = 3.976 m^4$

then $C.P = 2.72 + \frac{3.97}{(7.06 \times 2.72)} \times \sin^2(56°44')$

$C.P = 2.9 m$

UNIT-II - Fluid kinematics (25-10-2022)

→ It is the branch of science which deals with the behaviour of fluids when fluid is in motion.
When fluid is in motion, we need to analyse the velocity, accleration and discharge (volume/time)

Types of flows (or) classification of fluid flow:-

Steady flow:- The fluid flow is said to be steady if at any point the flow characterstics such as velocity, the fluid characterstics such as pressure, density & temperature which describes the behaviour of fluid in motion doesn't change with time mathematically expressed as $\frac{\partial v}{\partial t}=0$, $\frac{\partial p}{\partial t}=0$, $\frac{\partial \rho}{\partial t}=0$, $\frac{\partial T}{\partial t}=0$

unsteady flow:- The fluid flow is said to be unsteady if at an point the flow characterstics such as velocity, the fluid characterstics such as pressure, density & temperature which describes the behaviour of fluid in motion it changes does change with the time mathematically expressed $\frac{\partial v}{\partial t}\neq 0$, $\frac{\partial p}{\partial t}\neq 0$, $\frac{\partial \rho}{\partial t}\neq 0$, $\frac{\partial T}{\partial t}\neq 0$

uniform flow:- If the flow is said to be uniform flow the velocity doesn't changes from one point to another point along the length at a given time. $\frac{\partial v}{\partial s}=0$.

non uniform flow:- If the flow is said to be non uniform flow the velocity changes from one point to another point along the length at given time

$$\frac{\partial v}{\partial s}\neq 0$$

Rotational flow:- flow is said to be Rotational if the fluid particles while moving in the direction of flow Rotate about their mass centers that is Rotational flow

Irrotational flow:- flow is said to be irrotational if the fluid particles while moving in the direction of flow doesn't rotate about their mass centers that is irrotational flow.

1D, 2D, 3D flow:- Vairous character stics of flowing fluid
velocity, pressure, density etc are functions of
space and time.
space in cartiezein coordinates means (x, y, z)
three dimensional flow:- If velocity is a function
of $V = f(x, y, z, t)$ the flow in three dimensional
flow means velocity components are present in
x-direction, y-direction, z-direction.

$u = \frac{dx}{dt}$, $v = \frac{dy}{dt}$, $w = \frac{dz}{dt}$.

Two dimensional flow:- If the velocity is a functio
of $V = f(x, y, t)$ the flow in two dimensional flow
means velocity components are present in
x-direction, y-direction [along length & along depth]

$u = \frac{dx}{dt}$, $v = \frac{dy}{dt}$

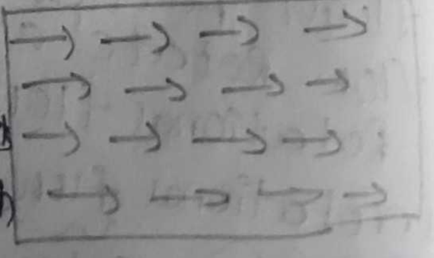
one dimensional flow:- It is not practically possible.
having component in one direction i.e in length
direction

*compressiable flow:- If density the fluid varies then
the fluid flow is said to be compressiable flow

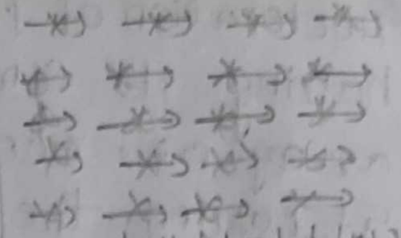
*In compressiable flow:- If the density of fluid is
constant doesn't varies then the fluid flow is said to
be In compressiable flow

* laminar flow:- the fluid flow is said to be laminar
if the fluid particals are moving in straight paths
(or) in parallel layers (without crossing each other)

*turblent flow:- fluid flow is said to be
turblent flow if the fluid particles travel
in zigzag manner (or) mixing with other
(or) crossing with eachother



turbent flow (or) laminar flow are based on the Reynold's number (only in pipe flows)! If the Reynolds number is $Re < 2000$ then it is laminar, $2000 < Re < 4000$ then it is transssion flow, $Re > 4000$ then it is trublent flow



tubblent flow

Reynolds number (Re):- $\left(\frac{\text{Interia force}}{\text{viscus force}}\right)$

$$Re = \frac{\text{Interia force}}{\text{viscous force}}$$

(m/sec²) = (LT⁻²) → D.F

Interia force := mass × acceleration $\left(\rho = \frac{m}{v}\right) \cdot \boxed{m = \rho v}$

(ρ) (M3) = L3 → D.F

= ρ × volume × acceleration

$= \rho \times L^3 \times LT^{-2}$ (Dimensional formula)

$= \rho \times L^4 \times T^{-2} = \rho \times (L^2 T^{-1})^2$ (velocity)

$= \rho \times (LT^{-1})^2 \times L^2$

$= \rho \times v^2 \times L^2$

viscous force = shear stress × Area

$= \mu \cdot \frac{V}{L} \times L^2 = \mu \cdot V \cdot L$

$$Re = \frac{\rho \times v^2 \times L^2}{\mu \cdot V \cdot L}$$

$$\boxed{Re = \frac{\rho}{\mu} \cdot VL}$$

L = any geometrical dimension

V = velocity

*Velocity and accleration of fluid:-

Velocity:- Velocity of fluid denpends on Both position of fluid particle and time. In Rectilinear coordinates velocity is having it's components (u, v, w). along x-direction $u = \frac{dx}{dt}$, along y direction $v = \frac{dy}{dt}$, along z-direction $w = \frac{dz}{dt}$ and the vector quantity $\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$ and magnitude of vector $V = \sqrt{u^2 + v^2 + w^2}$

Accleration of fluid particles:- like velocity accleration having its components along xyz directions, those are a_x, a_y, a_z. acceleration along xyz directions are a_x, a_y, a_z then
It having the function $f(x, y, z, t)$ then

$$a_x = \frac{du}{dt} = \frac{du}{dx}\cdot\frac{dx}{dt} + \frac{du}{dy}\cdot\frac{dy}{dt} + \frac{du}{dz}\cdot\frac{dz}{dt}$$

$$a_x = u\frac{du}{dx} + v\frac{du}{dy} + w\frac{du}{dz} + \frac{du}{dt}$$

$$a_y = u\cdot\frac{dv}{dx} + v\frac{dv}{dy} + w\frac{dv}{dz} + \frac{du}{dt}$$

$$a_z = u\frac{dw}{dx} + v\frac{dw}{dy} + w\cdot\frac{dw}{dz} + \frac{dw}{dt}$$

local acceleration:- It is the acceleration due to change of velocity w.r.t time. in the above equation components. $\frac{du}{dt}, \frac{dv}{dt}, \frac{dw}{dt}$ are the local accleration components.

Convective acceleration:- The accleleration due to change of velocity w.r.t position of the fluid part is convective component.

in $a_x = u\frac{du}{dx} + v\frac{du}{dy} + w\frac{du}{dz}$ is convective component

in $a_y = u\frac{dv}{dx} + v\frac{dv}{dy} + w\frac{dv}{dz}$ is the convective component

in $a_z = u\frac{dw}{dx} + v\frac{dw}{dy} + w\cdot\frac{dw}{dz}$ is the convective component

∴ The total acceleration is the sum of convective and local accleration

Problems:-

→ The fluid flow is described by the velocity filed $V = 5x^3\bar{i} - 15x^2y\bar{j} + t\bar{k}$ find the velocity and acceleration components at a point $(1, 2, 3)$ in the field at the time $t = 1$.

Sol: $V = 5x^3\bar{i} - 15x^2y\bar{j} + t\bar{k}$ (1,2,3) $t=1$

$V = u\bar{i} + v\bar{j} + w\bar{k}$

$u = 5x^3$ (at $x=1$) $\quad v = -15x^2y \quad w = t = 1$

$= 5 \quad v = -15(1)^2(2) \quad w = 1$

$= -30$

$V = \sqrt{u^2+v^2+w^2} = \sqrt{(5)^2+(-30)^2+(1)^2} = 30.43$ m/sec.

$a_x = u.\frac{\partial u}{\partial x} + v.\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$.

$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(5x^3) = 15x^2 = 15$

$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(5x^3) = 0$, $\frac{\partial u}{\partial z} = 0$, $\quad a_x = 5(15) = 75$

$\boxed{a_x = 75}$

$a_y = u\frac{\partial v}{\partial x} + v.\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$.

$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(-15x^2y) = -30xy = -30(1)(2) = -60$

$\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}(-15x^2y) = -15x^2 = -15$, $\frac{\partial v}{\partial z} = \frac{\partial}{\partial z}(-15x^2y) = 0$

$= 5(-60) - 30(-15) = \boxed{a_y = 150}$

$a_z = u.\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w.\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$.

$\frac{\partial w}{\partial z} = 0$, $\frac{\partial w}{\partial y} = 0$, $\frac{\partial w}{\partial z} = 0$, $\frac{\partial w}{\partial t} = 1$

$\boxed{a_z = 1}$

Total acceleration $= \sqrt{(75)^2 + (150)^2 + (1)^2} = 167.70$ m/sec^2

~~The velocity vector (V) =~~

→ The Velocity vector $\bar{V} = (6xt + yz^2)\bar{i} + (3t + xy^2)\bar{j} + (xy - 2xyz - 6tz)\bar{k}$

find velocity and acceleration at point (1,2,3) given time $t=2$.

Sol:- $V = (6xt + yz^2)\bar{i} + (3t + xy^2)\bar{j} + (xy - 2xyz - 6tz)\bar{k}$

given $t = 2$

$V = (12x + yz^2)\bar{i} + (6 + xy^2)\bar{j} + (xy - 2xyz - 12z)\bar{k}$

where $u = 12x + yz^2$, $v = 6 + xy^2$, $w = xy - 2xyz - 12z$

$= 12(1) + (2)(3)^2$, $v = 6 + 1(2)^2$, $w = (1)(2) - 2(1)(2)(3) - 12(3)$

$= 12 + 18$ $\quad v = 6 + 4$

$\boxed{u = 30}$ $\quad \boxed{v = 10}$

$w = 2 - 12 - 36$

$\boxed{w = -46}$

$V = \sqrt{u^2 + v^2 + w^2} = \sqrt{3116} = 55.82$

$a_x = u \cdot \frac{du}{dx} + v\frac{du}{dy} + w\frac{du}{dz} + \frac{du}{dt}$

~~$\frac{du}{dx} = \frac{d}{dx}(12x + yz^2) = 12$~~ | ~~$\frac{du}{dy} = \frac{d}{dy}(12x + yz^2) = 0 + 2yz\ z^2 = 2yz$ $\quad \frac{du}{dy} = z^2$~~

~~$\frac{du}{dz} = \frac{d}{dz}(12x + yz^2)$~~

$\frac{du}{dx} = \frac{d}{dx}(6xt + yz^2)$

$= 6t$

$\boxed{\frac{du}{dx} = 12}$

$\frac{du}{dy} = \frac{d}{dy}(6xt + yz^2)$

$\boxed{\frac{du}{dy} = z^2}$ $= 9$

$\frac{du}{dz} = \frac{d}{dz}(6xt + yz^2)$

$\boxed{\frac{du}{dz} = 2yz}$

$\frac{du}{dt} = (6xt + yz^2)$

$= 6x$

$\frac{du}{dt} = 6$

$a_x = 30(12) + 10(3)^2 + (-46)(2(2)(3)) + 6$

$a_x = 360 + 90 - 552 + 6$

$\boxed{a_x = -96}$

$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$

$\frac{\partial v}{\partial t} = \frac{\partial}{\partial t}(3t + xy^2)$

$\frac{\partial v}{\partial x} = \frac{\partial}{\partial x}(3t + xy^2)$

$\frac{\partial v}{\partial x} = y^2$

$= 4$

$\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}(3t + xy^2)$

$= 2xy$

$= 2(1)(2)$

$= 4$

$\frac{\partial v}{\partial z} = \frac{\partial}{\partial z}(3t + xy^2)$

$\frac{\partial v}{\partial z} = 0$

$\frac{\partial v}{\partial t} = 3$

$= 30(4) + 10(4) + 3 = 163$ $a_y = 163$

$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$

$\frac{\partial w}{\partial x} = \frac{\partial}{\partial x}(xy - 2xyz - 6tz)$

$= y - 2yz$

$= 2 - 2(2)(3)$

$= -10$

$\frac{\partial w}{\partial y} = \frac{\partial}{\partial y}(xy - 2xyz - 6tz)$

$= (x - 2xz)$

$= 1 - 2(1)(3)$

$= 1 - 6 = -5$

$\frac{\partial w}{\partial z} = \frac{\partial}{\partial z}(xy - 2xyz - 6tz)$

$= -2xy - 6t$

$= -2(1)(2) - 6(2)$

$= -4 - 12$

$= -16$

$\frac{\partial w}{\partial t} = \frac{\partial}{\partial t}(xy - 2xyz - 6tz)$

$= -6z$

$= -6(3)$

$= -18$

$a_z = 30(-10) + ~~(-8)~~ 10(-5) + (-46)(-16) - 18$

$= -300 - 50 + 736 - 18$

$a_z = 368$

Total acceleration $= \sqrt{(-96)^2 + (163)^2 + (368)^2}$ $143.77\ m/sec^2$

Total acceleration $=$ ~~412.60~~ m/sec^2

Continuity equation:- It is noting but conservation of mass

* Mass can be neither be created nor destroyed
* According to the continuity equation: the rate of increase of fluid mass in the fixed regin is equal to the difference of rate at which the fluid mass enters the region and the rate at which the fluid mass leaves the region.

rate at which the fluid mass enters the region minus (-) rate at which fluid mass leaves the region equals to rate of increase of fluid mass

(=) rate of increase of fluid mass in the region is equals to zero (=0)

If the fluid flow is steady, then the rate of increase of fluid mass in the region is equals to zero (=0) that is the fluid mass entering per second is equals to fluid mass leaving per second

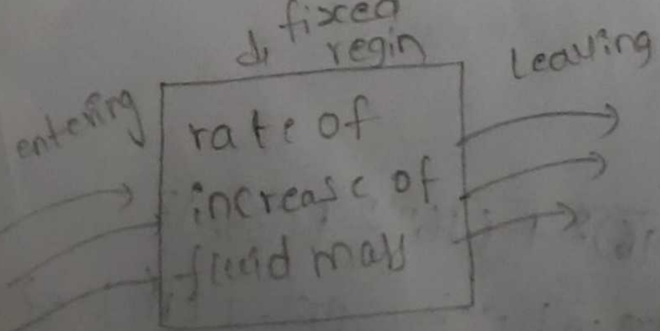
Derivation of continuity equation for three dimensional flow in cartisiean coordinates:

consider a Rectangular parallelopiped fluid element having sides $\delta x, \delta y, \delta z$ along $x-y, z$ directions. Let e be the mass density. Let u, v, w are the velocity components are the velocity along $x-y-z$ directions. mass of the volume

Mass of the fluid element. $= \delta x \delta y \delta z$ { density $(e) = \frac{mass}{volume}$, $e = \frac{m}{v}$, $m = ev$ }

Mass of the fluid $= e \times v$

Mass of the fluid element $= e \times \delta x \delta y \delta z$.

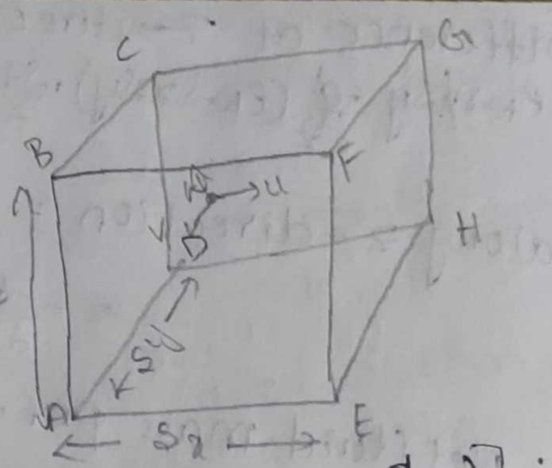


* mass of the fluid (flowing) passing per second at the center of the element along x-direction $= e u \delta z \delta y$
* mass of the fluid entering per second through face ABCD in x-direction

$$ABCD = e u \delta y \delta z - \frac{\partial}{\partial x}(e u \delta y \delta z)\frac{\delta x}{2}$$

mass of the fluid leaving per sec through EFGF

$\rho u \delta y \delta z + \frac{\partial}{\partial x}(\rho u \delta y \delta z)\frac{\delta x}{2}$



Difference of mass of fluid entering per sec and mass of fluid leaving per sec in x-direction

$\rho u \delta y \delta z - \frac{\partial}{\partial x}(\rho u \delta y \delta z)\frac{\delta x}{2} - \left[\rho u \delta y \delta z + \frac{\partial}{\partial x}(\rho u \delta y \delta z)\frac{\delta x}{2}\right]$

$\rho u \delta y \delta z - \frac{\partial}{\partial x}(\rho u \delta y \delta z)\frac{\delta x}{2} - \rho u \delta y \delta z - \frac{\partial}{\partial x}(\rho u \delta y \delta z)\frac{\delta x}{2}$

along x-direction $= -2\frac{\partial}{\partial x}(\rho u \delta y \delta z)\frac{\delta x}{2}$

$= -\frac{\partial}{\partial x}(\rho u \delta y \delta z \delta x) \rightarrow (1)$

Similarly in y-direction difference of mass of fluid entering per second and fluid mass leaving per second

$ABEF = \rho v \delta x \delta z - \frac{\partial}{\partial y}(\rho v \delta x \delta z)\cdot\frac{\delta y}{2}$

$DHCG = \rho v \delta x \delta z + \frac{\partial}{\partial y}(\rho v \delta x \delta z)\cdot\frac{\delta y}{2}$

Difference of y-direction ABEF - DHCG (By C.E)

$\rho v \delta x \delta z - \frac{\partial}{\partial y}(\rho v \delta x \delta z)\cdot\frac{\delta y}{2} - \rho v \delta x \delta z - \frac{\partial}{\partial y}(\rho v \delta x \delta z)\cdot\frac{\delta y}{2}$

$= -2\frac{\partial}{\partial y}(\rho v \delta x \delta z)\cdot\frac{\delta y}{2}$

along y-direction $= -\frac{\partial}{\partial y}(\rho v \delta x \delta y \delta z) \rightarrow (2)$

Similarly in z-direction difference of mass of fluid entering per second and fluid mass leaving per second

$BFCG = \rho w \delta x \delta y - \frac{\partial}{\partial z}(\rho w \delta x \delta y)\cdot\frac{\delta z}{2}$

$ADEF = \rho w \delta x \delta y + \frac{\partial}{\partial z}(\rho w \delta x \delta y)\cdot\frac{\delta z}{2}$

Difference of z-direction = BFCG - ADEF

$\rho w \delta x \delta y - \frac{\partial}{\partial z}(\rho w \delta x \delta y) \cdot \frac{\delta z}{2} - \rho w \delta x \delta y - \frac{\partial}{\partial z}(\rho w \delta x \delta y) \cdot \frac{\delta z}{2}$

along z-direction $= \frac{\partial}{\partial z}(\rho w \delta x \delta y) \cdot \frac{\delta z}{2}$

$= -\frac{\partial}{\partial z}(\rho w \delta x \delta y \delta z) \longrightarrow (3)$

∴ Net fluid mass per second with in the fluid element

eq(1) + eq(2) + eq(3)

$= -\frac{\partial}{\partial x}(\rho u \delta x \delta y \delta z) - \frac{\partial}{\partial y}(\rho v \delta x \delta y \delta z) - \frac{\partial}{\partial z}(\rho v \delta x \delta y \delta z) \longrightarrow (4)$

According to continuity equation the sum of difference of fluid mass entering per sec and leaving per sec in all directions is equal to rate of increase of fluid mass

* Rate of increase of fluid mass equals $= \frac{\delta}{\delta t}(\rho \cdot \delta x \delta y \delta z)$

$\frac{\delta}{\delta x} \rho u \delta x \delta y \delta z - \frac{\delta}{\delta y} \rho v \delta x \delta y \delta z - \frac{\delta}{\delta z} \rho w \delta x \delta y \delta z - \frac{\partial}{\partial t} \rho \delta x \delta y \delta z$

$-\frac{\delta u}{\delta x} - \frac{\delta v}{\delta y} - \frac{\delta w}{\delta z} - \frac{d\rho}{dt} = 0$

$= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} + \frac{\delta \rho}{\delta t} = 0$

This is the general continuity equation for three dimensional unsteady compressiable fluid flow

case(i):- If the flow is steady, compressiable then $\frac{d\rho}{dt} = 0$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

case(ii):- If the flow is incompressiable as density not equals to zero

* This equation is useful for determing whether the flow is continuous (or) not and also this equation is useful to determine wheather the flow exist or not.

* for two dimensional flow $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

* for one dimensional flow & $\frac{\partial u}{\partial x} = 0$. This is nothing but u is constant

there fore discharge $Q = A_1V_1 = A_2V_2$ this is the continuity for one dimensional flow

* the product of Area x velocity is called discharge.

* discharge is also defined as the volume of the fluid flowing per sec ∴ Discharge $(Q) = \frac{\text{volume}}{\text{time}}$.

The units of discharge is m3/sec.

The dimensional formula is L^3T^{-1}

* A fluid flow is given by $\vec{V} = x^2y\bar{i} + y^2z\bar{j} - (2xyz + yz^2)\bar{k}$ is a possible case of fluid flow prove it.

Sol:- given velocity vector $V = x^2y\bar{i} + y^2z\bar{j} - (2xyz + yz^2)\bar{k}$

where $u = x^2y$, $v = y^2z$, $w = -(2xyz + yz^2)$

& the continuity equation of three dimensional flow is $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \longrightarrow (1)$

$\frac{\partial}{\partial x}(x^2y) = 2xy$, $\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}(y^2z) = 2yz$

$\frac{\partial w}{\partial z} = -(2xyz + yz^2) = -2xyz - yz^2$

$= \frac{\partial}{\partial z}(-2xyz - yz^2) = -2xy - 2yz$.

substitute in eq (1)

$2xy + 2yz - 2xy - 2yz = 0$

to zero then The equation is becomes equation

it's satisfies the continuity equation

* the following case Represents two velocity components determine the third component of velocity such that they satisfies contiuity equation

Q) $u = x^2+y^2+z^2$, $v = xy^2 - yz^2 + xy$

Sol:- given that the components along x-direction and y-direction

$u = x^2+y^2+z^2$, $v = xy^2 - yz^2 + xy$ then

the continuity equation is $\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 0 \rightarrow (1)$

$$\frac{du}{dx} = \frac{d}{dx}(x^2+y^2+z^2) = 2x.$$

$$\frac{dv}{dy} = \frac{d}{dy}(xy^2 - yz^2 + xy) = 2xy - z^2 + x.$$

substitute in eq (1)

$$2x + 2xy - z^2 + x + \frac{dw}{dz} = 0$$

$$\frac{dw}{dz} = 3x + 2xy - z^2.$$

$$w = \int -3x - 2xy + z^2 \, dz.$$

$$\boxed{w = \frac{z^3}{3} - 3xz - 2xyz + f(z)}$$

(ii) $v = 2y^2$, $w = 2xyz$.

Sol:- $v = 2y^2$, $w = 2xyz$

$$\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 0 \rightarrow (1)$$

$$\frac{dv}{dy} = \frac{d}{dy}(2y^2) = 2(2y) = 4y, \quad \frac{d}{dz}(2xyz) = 2xy$$

$$\frac{du}{dx} + 4y + 2xy = 0$$

$$\frac{du}{dx} = -4y - 2xy$$

$$u = \int -4y - 2xy \, dx.$$

$$u = -4xy - 2 \cdot \frac{x^2}{2} y$$

$$\boxed{u = -4xy - x^2y}$$

* Define velocity potential function give it's characterstics.

* the velocity potential function is represented by letter (ϕ)
* The negitive derivative of velocity potential function in any direction gives the velocity component in that direction

for example:- With respect to x, y, z directions the velocity components are $-\frac{\partial \phi}{\partial x} = u$, $-\frac{\partial \phi}{\partial y} = v$,

$-\frac{\partial \phi}{\partial z} = w$.

→ characterstics:-

(i) consider continuity equation $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ → ①

where $u = -\partial\phi/\partial x$, $v = -\partial\phi/\partial y$, $w = -\partial\phi/\partial z$.

By ① $\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial \phi}{\partial z}\right) = 0$

$\Rightarrow -\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial y^2} - \frac{\partial^2 \phi}{\partial z^2} = 0$

$\Rightarrow \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$ It satisfies laplace equation

* this means the velocity potential satisfies the laplace equation in case of possible fluid flow

(ii) velocity potential function exists in case of irrotatial flow only that means rotation is equal to zero and the rotational components are zero rotational components, rotational components $= w_x, w_y, w_z$ are $u v w$ and along x, y, z direct let the components

x y z → u v w ; y z x → v w u ; z x y → w u v

$w_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right] = 0$ $w_y = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right] = 0$ $w_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = 0$

are zero

If the rotational components $w_x = w_y = w_z = 0$ then

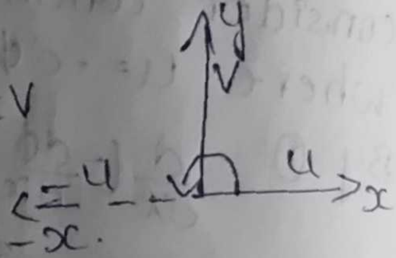
$\omega_x = 0$ then $\frac{1}{2}\left(\frac{dw}{dy} - \frac{dv}{dz}\right) = 0$ $\quad \frac{dw}{dy} = \frac{dv}{dz}$

$\omega_y = 0$ then $\frac{1}{2}\left[\frac{du}{dz} - \frac{dw}{dx}\right) = 0 \cdot$ $\quad \frac{du}{dz} = \frac{dw}{dx}$

$\omega_z = 0$ then $\frac{1}{2}\left[\frac{dv}{dx} - \frac{du}{dy}\right] = 0$, $\quad \frac{dv}{dx} = \frac{du}{dy}$

Equi potential line:- if the velocity potential function is constant on a line then that line is called equi potential line that means ϕ = constant

→ screen function:- (ψ) It is defined as scalar function of space and time such that it's derivative in any direction gives the velocity component at right angles to it measured in anticlock wise direction. then $\frac{d\psi}{dy} = -u$, $\frac{d\psi}{dx} = v$



ψ is always considered in two dimensional flow only

→ properties of screen function:-

* ψ satisfies the continuity equation that means screen function exists possible case of flow

→ The velocity components in a two dimensional flow are $u = \frac{y^3}{3} + 2x - x^2y$ and $v = xy^2 - 2y - \frac{x^3}{3}$ show that these components represents a possible case of irrotational flow then findout the velocity potential function.

sol:- given $u = \frac{y^3}{3} + 2x - x^2y$, $v = xy^2 - 2y - \frac{x^3}{3}$

To find irrotational flow component (or) not:-

$\omega_z = \frac{1}{2}\left[\frac{dv}{dx} - \frac{du}{dy}\right]$

to find $\frac{dv}{dx} = \frac{d}{dx}\left(xy^2 - 2y - \frac{x^3}{3}\right]$

$\frac{\partial v}{\partial x} = y^2 - 3\frac{x^2}{3} = y^2 - x^2$, $\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\frac{y^3}{3} + 2x - x^2 y\right)$

$\frac{\partial u}{\partial y} = \frac{3y^2}{3} + 0 - x^2$ $\frac{\partial u}{\partial y} = y^2 - x^2$

$\omega_z = \frac{1}{2}\left[y^2 - x^2 - y^2 + x^2\right]$ $\omega_z = 0$. then irrotational flow is possible.

to find velocity potential function ϕ:- from the definition $u = -\frac{\partial \phi}{\partial x}$, $v = -\frac{\partial \phi}{\partial y}$ {we have to find out $\phi = ?$}

$\frac{\partial \phi}{\partial x} = -u \rightarrow \frac{\partial \phi}{\partial x} = -\left[\frac{y^3}{3} + 2x - x^2 y\right] = -\frac{y^3}{3} - 2x + x^2 y$

$\phi = \int x^2 y - 2x - \frac{y^3}{3} dx = \frac{x^3}{3}y - \frac{2x^2}{2} - \frac{xy^3}{3} + c$ —①

$\phi = \frac{x^3}{3}y - x^2 - \frac{xy^3}{3} + c$

$-\frac{\partial \phi}{\partial y} = v \rightarrow -\frac{\partial}{\partial y}\left(\frac{x^3}{3}y - x^2 - \frac{xy^3}{3} + c\right] = -\frac{x^3}{3} + \frac{3xy^2}{3} - \frac{\partial c}{\partial y}$

$v = -\frac{x^3}{3} + xy^2 - \frac{\partial c}{\partial y}$

$xy^2 - 2y - \frac{x^3}{3} = -\frac{x^3}{3} + xy^2 - \frac{\partial c}{\partial y}$ $\frac{\partial c}{\partial y} = 2y$ $\boxed{c = y^2}$

$c = \int 2y\, dy$

$c = \frac{2y^2}{2}$

substitute in eq ①

$\boxed{\phi = \frac{x^3}{3}y - x^2 - \frac{xy^3}{3} + y^2}$. This is the required velocity potential function

→ If the expression for the stream function is given by $\psi = x^3 - 3xy^2$ determine wheater the flow is rotational (or) irrotational. If the flow is irrotation then calculate the velocity potential function

Sol:- given $\psi = x^3 - 3xy^2$.

By the defination of stream fuction

$\frac{\partial \psi}{\partial y} = -u$, $\frac{\partial \psi}{\partial x} = v$

$\frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}(x^3 - 3xy^2) = 0 - 6xy = -u = -(-6xy) = 6xy$ fro

$\boxed{u = 6xy}$

$\frac{\partial \psi}{\partial x} = v = \frac{\partial}{\partial x}(x^3 - 3xy^2) = 3x^2 - 3y^2 = 3(x^2 - y^2)$

$v = 3(x^2 - y^2)$

$w_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] \rightarrow (1)$

Now $\frac{\partial v}{\partial x} = \frac{\partial}{\partial x}(3x^2 - 3y^2) = 6x.$

$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(6xy) = 6x.$ By $w_z = \frac{1}{2}(6x - 6x) = \boxed{w_z = 0}$

It Represents Irrotational flow.

to find ϕ:- $u = -\frac{\partial \phi}{\partial x}$, $v = \frac{\partial \phi}{\partial y}$.

$-\frac{\partial \phi}{\partial x} = u \Rightarrow \frac{\partial \phi}{\partial x} = -u \Rightarrow \frac{\partial \phi}{\partial x} = -6xy$ $\phi = \int -6xy \, dx.$

$\phi = -6 \cdot \frac{x^2}{2} y + c$ $\phi = -3x^2y + c \rightarrow (1)$

$-\frac{\partial \phi}{\partial y} = v \Rightarrow \frac{\partial \phi}{\partial y} = 3x^2 - 3y^2$ $= \frac{\partial}{\partial y}(-3x^2y + c) = 0 + \frac{\partial c}{\partial y}$

$= -\frac{\partial}{\partial y}(-3x^2y + c) = +3x^2 - \frac{\partial c}{\partial y}$

$v = -3x^2$ $3x^2 - 3y^2 = 3x^2 - \frac{\partial c}{\partial y}$

$\frac{\partial c}{\partial y} = 3y^2$ $c = \frac{3y^3}{3}$ $\boxed{c = y^3}$

By eq (1) $\boxed{\phi = -3x^2y + y^3.}$

→ the stream function and the velocity potential function for a flow is given by $\psi = 2xy$, $\phi = x^2 - y^2$ show that the conditions of continuity and irrotational flow are satisfy.

Sol:- given $\psi = 2xy$

6xy from the defination of velocity $-\frac{d\phi}{dx} = u.$

$-\frac{d}{dx}(x^2 - y^2) = u.$

$-2x = u.$ $\boxed{u = -2x}$

$-\frac{d\phi}{dy} = v \Rightarrow -\frac{d}{dy}(x^2 - y^2) = -(-2y)$ $v = 2y$

$w_x = \frac{1}{2}\left[\frac{dw}{dy} - \frac{dv}{dz}\right]$ Now we have to calculate.

By the continuity equation

$\frac{du}{dx} + \frac{dv}{dy} = 0.$

$\frac{d(-2x)}{dx} + \frac{d(2y)}{dy} = 0$ $-2 + 2 = 0$

$\boxed{= 0}$

continuty equation is satisfied.

Given that $\psi = 2xy$ By the difinition of screen function $\frac{d\psi}{dy} = -u.$ $\frac{d}{dy}(2xy) = -u.$

$2x = -u$ $\boxed{u = -2x}$, $\frac{d\psi}{dx} = v \Rightarrow \frac{d}{dx}(2xy) = v$

$\boxed{v = 2y}$

By the screen function

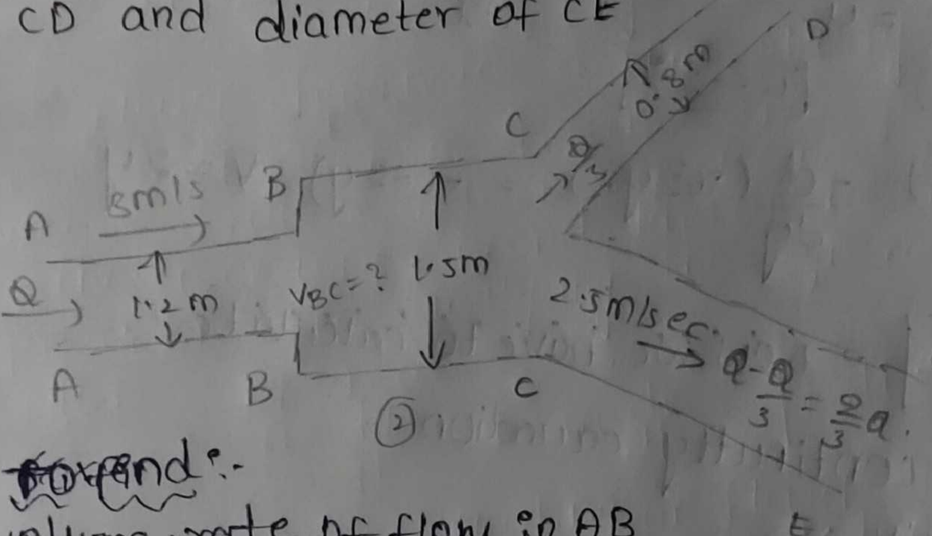
$\omega_z = \frac{1}{2}\left[\frac{dv}{dx} - \frac{du}{dy}\right] = \frac{dv}{dx} = \frac{d}{dx}(2y) = 0$

$\frac{du}{dy} = -2x = 0$

$w_z = \frac{1}{2}[0-0] = 0$ $\boxed{w_z = 0}$

then It represents Irrotation flow

→ water flows through a pipe AB 1.2m diameter at 3m/s and then passes through a pipe BC 1.5m diameter at C the pipe branches Branch CD is 0.8m in diameter and carries $\frac{1}{3}$rd of the flow in AB The flow velocity in Brach CE is 2.5m/sec find the volume rate of flow in AB, the velocity in BC, the velocity in CD and diameter of CE



To find:-

Volume rate of flow in AB

Velocity in BC

Velocity in CD

Diameter of CE

By the definition of Discharge $Q = A_1V_1 = A_2V_2$

$Q = A_1V_1 = \frac{\pi}{4}d^2 \times V_1 = \frac{\pi}{4} \times (1.2)^2 \times 3 = 3.39\ m^3/s.$

↳ volume rate of flow in AB.

$A_1V_1 = A_2V_2 = Q \qquad Q = A_2V_2$

$3.39 = \frac{\pi}{4} \times (1.5)^2 \times V_2$

velocity of flow in BC $(V_2) = \frac{4 \times 3.39}{\pi \times (1.5)^2} = 1.9\ m/sec$

To find V_{CD}:- By discharge $Q = AV \quad V = \frac{Q}{A}$

$V = \frac{Q/3}{A} = \frac{3.39}{3 \times \frac{\pi}{4} \times (0.8)^2} = 2.24\ m/sec$

Diameter of CE:-

$Q = AV = \frac{\pi}{4}d^2 \times V \Rightarrow d^2 = \frac{4Q}{\pi \times V} = \frac{4 \times \frac{2}{3}(3.39)}{\pi \times 2.5}$

$d = \sqrt{\frac{4 \times \frac{2}{3}(3.39)}{\pi \times 2.5}}$ $\boxed{d = 1.07m}$

Stream line:- It is an imaginary line drawn in flow flied. the tangent drawn at any point on line Represents the direction of velocity vector. The differential equantion of stream line is $\boxed{\frac{dx}{u} = \frac{dy}{v}}$

12/10 Path line:- A path line is the locus of fluid particle as it moves along in other words a path line is curved traced by a single fluid particle during it's motion.

* Streak line:- When a die is injected in a liquid (or) smoke in a gas, so as to trace the subsequent motion of fluid particle passing a fixed point the path followed by die (or) smoke is called streak line.

* Stream tube:- If streak line drawn a closed curve they form a boundary surface across which fluid cannot penetrate such a surface bounded by streak lines is a sort of tube and is known as streak tube.

UNIT - III - fluid Dynamics (02-01-2022)

→ fluid dynamics is the study of flowing materials such as liquids and gases and the various forces which effects the fluid flow the Newtons second law of motion which prescribes the analysing the dynamic behaviour of flow

$F = ma.$

$a = \frac{dv}{dt}$ $F = m \cdot \frac{dv}{dt}$ $\boxed{F \cdot dt = m \cdot dv}$

various forces acting on fluid that are gravitational force (F_g) pressure force (F_p) and force due to viscosity (viscous force) (F_V), trublent F_t force, compressable force (F_C), surface tensional force.

along x-direction = $\Sigma F_x = (F_g)_x + (F_p)_x + (F_V)_x + (F_t)_x + (F_c)_x + (F_{st})_x = m \cdot a_x \rightarrow ①$

along y-direction

$\Sigma F_y = (F_g)_y + (F_p)_y + (F_V)_y + (F_t)_y + (F_c)_y + (F_{st})_y = ma_y \rightarrow ②$

along z-direction

$\Sigma F_z = (F_g)_z + (F_p)_z + (F_V)_z + (F_t)_z + (F_c)_z + (F_{st})_z = ma_z \rightarrow ③$

By neglecting suface tensional force, compressible force. the above equations Becomes

$\Sigma F_x = (F_g)_x + (F_p)_x + (F_V)_x + (F_t)_x = ma_x \rightarrow ①$

$\Sigma F_y = (F_g)_y + (F_p)_y + (F_V)_y + (F_t)_y = ma_y \rightarrow ②$

$\Sigma F_z = (F_g)_z + (F_p)_z + (F_V)_z + (F_t)_z = ma_z \rightarrow ③$

} these equations are Reynold's equation

for flow at low Reynold's number the force due to turbulent is of no significants. ∴ the above equations becoms those are Navier Stokes equations

$\Sigma F_x = (F_g)_x + (F_p)_x + (F_V)_x = ma_x \rightarrow ①$

$\Sigma F_y = (F_g)_y + (F_p)_y + (F_v)_y = ma_y$

$\Sigma F_z = (F_g)_z + (F_p)_z + (F_v)_z = ma_z$

→ In case of Ideal fluid flows there is no viscus force

$\Sigma F_x = (F_g)_x + (F_p)_x = ma_x \rightarrow (1)$

$\Sigma F_y = (F_g)_y + (F_p)_y = ma_y \rightarrow (2)$

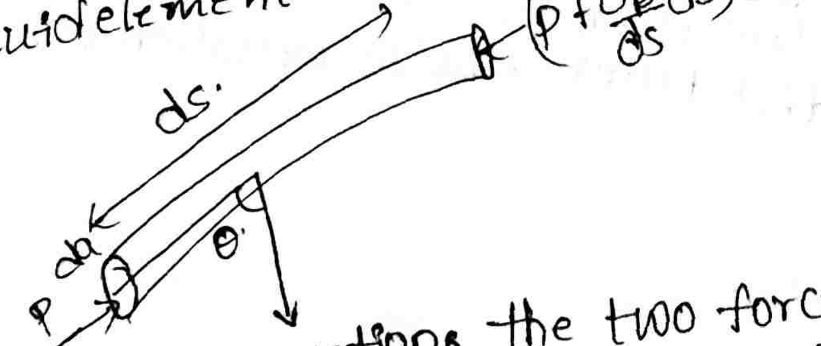
$\Sigma F_z = (F_g)_z + (F_p)_z = ma_z \rightarrow (3)$

} these are Eulers Equations.

Euler's Equations of motion:-

The various forces acting on fluid element present in Euler's Equations of motion are pressure force and gravitational force

consider a fluid element having an area da and length ds Let p be the pressure intensity acting on the one face on the pressure intensity acting on the mass density of that of fluid element. Let ρ be the mass density of that fluid element on a stream line



In Eulers Equations the two forces are acting on a fluid element that is (i) pressure force (ii) gravitational force, gravitational force is nothing but weight of the fluid.

By the pressure intencity $p_i = \frac{P_F}{A}$

Pressure force $P_F = p_i A$.

from the diagram Pressure force $P = p \cdot da$ [one side]

$(p + \frac{dP}{ds} ds) da$.

Gravitational force = weight of the fluid element

$= \gamma \times$ volume

| Weight density (γ) = $\frac{\text{weight of fluid}}{\text{volume}}$

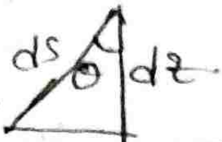
Volume = da ds.

Weight = $\rho g \cos\theta \cdot da \cdot ds$.

| Weight of fluid = $\gamma \times$ volume

From newtons second law of motion $\boxed{F = Ma}$

$$P\,da - \left(P + \frac{dP}{ds}ds\right)da \mp \rho g\,da\,ds\cos\theta = m \cdot a_s.$$



$$P\,da - \left(P + \frac{dP}{ds}ds\right)da \pm \rho g\,da\,ds\cos\theta = \rho\,da\,ds\,a_s.$$

$\boxed{\cos\theta = \frac{dz}{ds}}$

$$P\,da - P\,da - \frac{dP}{ds}ds\,da \pm \rho g\,da\,ds\cos\theta = \rho\,da\,ds\,a_s$$

$$ds\,da\left(-\frac{dP}{ds} \mp \rho g\cos\theta\right) = \rho a_s \cdot da\,ds$$

$$-\frac{dP}{ds} \mp \rho g\frac{dz}{ds} = \rho a_s \quad \text{[divide on Both side with } \rho\text{)}$$

$$-\frac{1}{\rho}\frac{dP}{ds} \pm g \cdot \frac{dz}{ds} = a_s.$$

This is the Eulers equation of a fluid element.

* **Bernoullis equation:-** Bernoullis equation is noting but Integration of Eulers Equation

∴ By Integrating the Euler above equation we get

$$-\frac{1}{\rho}\frac{dP}{ds} - g\frac{dz}{ds} = a_s.$$

$$a_s = \frac{dV_s}{ds} \cdot \frac{ds}{dt} \qquad \left(\frac{ds}{dt} = V_s\right) \qquad \left[\text{dividing on } \tfrac{1}{2}\right]$$

$$= V_s \cdot \frac{dV_s}{ds} \qquad = \frac{1}{2}\frac{d(V_s)^2}{ds}$$

$$\Rightarrow -\frac{1}{\rho}\frac{dP}{ds} = g \cdot \frac{dz}{ds} + \frac{1}{2} \cdot \frac{d(V_s)^2}{ds}$$

$$\int\left(\frac{1}{\rho} \cdot \frac{dP}{ds} + g\frac{dz}{ds} + \frac{1}{2}\frac{dV_s^2}{ds}\right) = c \Rightarrow \frac{P}{\rho} + gz + \frac{V_s^2}{2} = \text{constant}$$

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = C. \qquad \frac{P}{w} + \frac{V^2}{2g} + z = \text{constant}$$

where P = pressure head, V = velocity head, z = Datum head.

when the tube is divided into two halves then it is

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2 + \Delta z$$

Assumptions of Bernoulli's equation:-

* the flow is steady. * It is applicable to Ideal fluid flow * It is applicable to incompressible flow

∴ In steady incompressible Ideal fluid flow the total head [pressure head, velocity head, datum head] constant at a point.

problems:-

A pipe of 30cm diameter carries water at a velocity of 20m/sec. the pressures at point A and B are given as 34.33 N/cm² and 29.43 N/cm² respectively the datum head at A and B are 25m and 28m. Find loss of head Between A and B.

Sol:- Given data is diameter of the pipe = 30cm = 30×10^{-2} = 0.3 m

Velocity of water = 20m/sec.

pressure head of point A $P_A = 34.33\ N/cm^2 = 34.33 \times 10^4\ N/m^2$

pressure head of point B $P_B = 29.43\ N/cm^2 = 29.43 \times 10^4\ N/m^2$

Datum head at A $Z_A = 25m$ loss of pressure head = ?

B $Z_B = 28m$

$$\text{Total head at A} = \frac{P_A}{w} + \frac{V_A^2}{2g} + Z_A = \frac{34.33 \times 10^4}{9.81 \times 10^3} + \frac{(20)^2}{2 \times 9.81} + 25$$

$$= 80.38m \rightarrow ①$$

$$\text{Total head at B} = \frac{P_B}{w} + \frac{V_B^2}{2g} + Z_B = \frac{29.43 \times 10^4}{9.81 \times 10^3} + \frac{(20)^2}{2 \times 9.81} + 28$$

$$= 78.38 \rightarrow ②$$

loss of head b/w A & B = ① - ②

$$= 80.38 - 78.38 = 2m$$ [head units - m]

→ Water flows in a circular pipe of varying cross section at one section the diameter of pipe is 0.4m and pressure is 250 kpascal and velocity is 2.5m/sec and elevation (z) is 5m above ground at the other end elevation (z) is zero and pipe diameter is 0.2m and neglecting loss find the gauge pressure at other End.

Sol:- Given the diameter $d_1 = 0.4m$ and

Pressure $P_1 = 250$ klio-pascal. $= 250 \times 10^3 N/m^2$

Velocity $V_1 = 2.5$ m/sec

given that the diameter $d_2 = 0.2m$, $P_2 = ?$

By the Bernoulli equation

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2 \rightarrow ①$$

continuity equation

$$Q = A_1 \times V_1 = A_2 (V_2) \rightarrow ①$$

$A_1 = \frac{\pi}{4}(0.4)^2 = 0.12.$ $\quad A_2 = \frac{\pi}{4}(0.2)^2 = 0.03$

$V_1 = 2.5$ m/sec $\quad V_2 = ?$

By ① $\frac{0.12 \times 2.5}{0.03} = V_2.$ $\boxed{V_2 = 10}$

By eq ① $\frac{250 \times 10^3}{9.81 \times 10^3} + \frac{(2.5)^2}{2 \times 9.81} + 5m = \frac{P_2}{9.81 \times 10^3} + \frac{(10)^2}{2 \times 9.81} + 0.$

$$\frac{250000}{9810} + \frac{6.25}{19.62} + 5 = \frac{P_2}{9810} + \frac{100}{19.62}.$$

$$25.48 + 0.31 + 5 = \frac{P_2}{9810} + 5.09$$

$$25.7 = \frac{P_2}{9810}$$ $\boxed{P_2 = 252117}$

ns. This is proved as
ll dimensions i.e., dx, dy

The water flowing through a tappered pipe of length 100m having diameter 600mm at the upper end and 300mm at the lower end at the rate of 50 lit/sec. the pipe has a slope of 1 in 30. Find the pressure at the lower end if the pressure at the high end is 19.62 N/cm².

$1\ cm = 10^{-2} m$

$1\ liter = 10^{-3} m^3$

Sol:- given that the value of length of the pipe = 100m

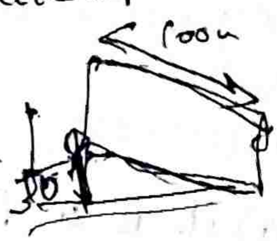
given diameter $d_1 = 600mm$, $d_2 = 300mm$

$d_1 = 600 \times 10^{-3} m$

$d_2 = 300 \times 10^{-3} m$

By the continuity equation

$Q = A_1 V_1$ given "Q" that is rate of flow "Q" $= 50 \times 10^{-3} m^3/sec$.



~~$A_1 = \frac{Q}{V_1}$~~

$V_1 = \frac{Q}{A_1}$

$A_1 = \frac{\pi}{4} d^2 = \frac{\pi}{4} (600 \times 10^{-3})^2$ $A_1 = 0.2827 m^2$.

$A_2 = \frac{\pi}{4} d^2 = \frac{\pi}{4} (300 \times 10^{-3})^2$ $A_2 = 0.0706 m^2$

$V_1 = \frac{50 \times 10^{-3}}{0.2827}$ $V_1 = 0.1768\ m/sec$

Z_1 = slope × length

$Q = A_2 V_2$. $\frac{Q}{A_2} = V_2$. $V_2 = 0.7082\ m/sec$

By the Bernouilli equation

$\frac{P_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + Z_2$

$Z_1 = 100 \times \frac{1}{30}$

$Z_1 = 3.33 m$

$Z_2 = 0$

given pressure $P_1 = 19.62 N/cm^2$

$P_1 = \frac{19.62}{(10^{-2})^2}$ $P_1 = \frac{19.62}{10^{-4}}$

$P_1 = 196200\ N/m^2$.

$\frac{196200}{9810} + \frac{(0.1768)^2}{2 \times 9.81} + 3.33$

$= \frac{P_2}{9810} + \frac{(0.7082)^2}{2 \times 9.81} + 0$.

$$20 + 0.1533 + 3.33 = \frac{P_2}{9810} + \frac{\sout{2.4600} \; \sout{0.25}}{0.025}$$

$$23.48 = \frac{P_2}{9810} + \sout{2.4600}$$

$$\sout{21.02} = \frac{P_2}{9810} \qquad \sout{P_2 = 206208.2 \ N/m^2}$$

$$23.48 - 0.025 = \frac{P_2}{9810} \qquad P_2 = 230093.55 \ N/m^2$$

*** Application of Bernouilli's equation:-**

Venturimeter:- It is a discharge measuring device (or) it is a flow measuring device.

It is a device which is used to measured the discharge through pipe

Working principal:- By reducing crosssectional area of flowpasses pressure difference is created b/w two sections. by measuring the pressure difference the discharge through the pipes can be calculated.

Discription:- Venturimeter is madeup of Brass tube converging cone throught section diverging cone.

- * the converging cone angle lies is b/w 15° to 21°.
- * diverging cone angle should be 5° to 10°.
- * throught length = throught diameter.
- * throught diameter = $\frac{1}{3}$ to $\frac{3}{4}$ of inlet diameter
- * convergent cone. length = $2.7(d_1 - d_2)$

Derivation for discharge:-

Let D_1 be the diameter of inlet and D_2 be the diameter of throught.

* let p_1 be the pressure at inlet, p_2 be the pressure at throught
* let z_1 be the datum head at inlet, z_2 is the datum head at throught

Apply Bernoullis equation b/w section ① & ②

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2.$$

It is Horizantal surface $\boxed{z_1 = z_2}$

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g} \Rightarrow \frac{P_1}{w} - \frac{P_2}{w} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}. \quad \boxed{\frac{P_1}{w} - \frac{P_2}{w} = h}$$

$$h = \frac{1}{2g}(V_2^2 - V_1^2) \rightarrow ①$$

By the continuity equation $Q = A_1 V_1 = A_2 V_2$.

$$V_1 = \frac{Q}{A_1}, \quad V_2 = \frac{Q}{A_2} \rightarrow ② \text{ from ① \& ②}$$

$$h = \frac{1}{2g}\left(\frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2}\right) \Rightarrow h = \frac{Q^2}{2g}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)$$

$$h = \frac{Q^2}{2g}\left(\frac{A_1^2 - A_2^2}{A_2^2 A_1^2}\right) \Rightarrow Q^2 = \frac{2gh(A_2^2 A_1^2)}{A_1^2 - A_2^2}.$$

$$Q_{theritical} = \frac{A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}.$$

Cd = coefficent of discharge.

$$\boxed{Cd = \frac{Q_{actual}}{Q_{theritical}}}$$

*If venturimeter is connected to differential manometer then the difference of pressure head can be estimated By the following formula $h = x\left[\frac{S_m}{S} - 1\right]$

where x is the ~~diffe~~ diffelection in manometer

$x = h_2 - h_1$

S_m is the specific gravity of manometric liquid.

(specific gravity of mecury = 13.6)

S is the specific gravity of fluid.

*A vertical venturimeter has inlet and throught diameter (d_1) (d_2) are 250mm and 125mm Respectively.

A differential mecury manometer connected to inlet and through out point use the Reading of 250mm find Rate of flow If water is flowing through a pipe line.

Sol:- given that the

diameter of inlet $d_1 = 250mm$

$d_1 = 250 \times 10^{-3} m$

$A_1 = \frac{\pi}{4}(250\times10^{-3})^2 = 0.04908\ m^2$

given that $d_2 = 125mm = 125\times10^{-3}\ m$

$A_2 = \frac{\pi}{4}(125\times10^{-3})^2 = 0.0122 m^2$

$h = x\left[\frac{S_m}{S} - 1\right] = 250\left[\frac{13.6}{1} - 1\right] = 250\times10^{-3}(13.6-1)$

$h = 3.15m$

$S = 1$

S = mass density of water / mass density of water

$Q = \frac{A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$

$Q = \frac{0.04908 \times 0.0122\sqrt{2\times9.81\times3.15}}{\sqrt{(0.04908)^2 - (0.0122)^2}}$

$Q = \frac{5.98776\times10^{-4} \times 7.861480}{0.04753}$

$Q = 0.09903\ m^3/sec.$

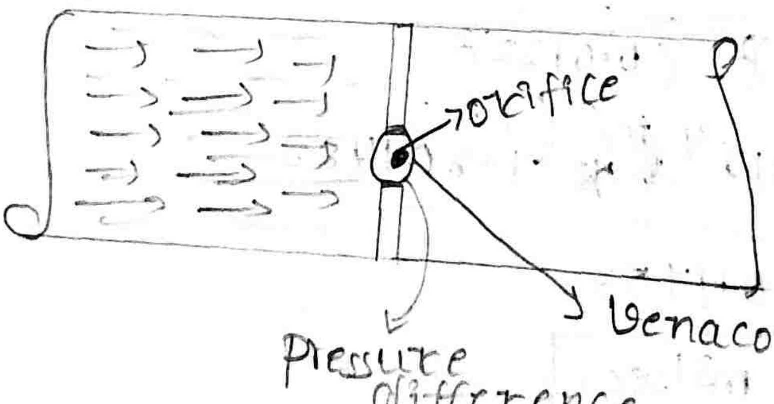
* Orificemeter:- It is a device which is used for measuring discharge and it is a cheap device compared to Venturimeter

Working principle:- The working principle of orifice meter is same as the Venturimeter. By reducing cross sectional area of flow pressure difference is generated. By measuring this pressure difference the discharge can be calculated. Orifice meter consists of a flat or circular plate at the center it is noting but a hole is made to this plate which is fixed in the pipe line. It is generally half of the pipe diameter the pressure difference is measured by using differential manometer which is connected to the pipe.

The inlet section is at 0.98 to 1.0 m diameter of the pipe.

Venacontract:- It is defined as the section at which the cross sectional area of flow is minimum at just outside the orifice

Derive Expression for discharge:-



Apply the Bernoulli's equation Between two sections

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2 \rightarrow (1)$$

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}$$

{It is a horizontal section then $Z_1=Z_2$}

$$\frac{P_1}{\rho g}-\frac{P_2}{\rho g}=\frac{V_2^2-V_1^2}{2g} \Rightarrow h=\frac{V_2^2-V_1^2}{2g} \Rightarrow V_2^2-V_1^2=2gh$$

($\frac{P_1}{\rho g}-\frac{P_2}{\rho g} = h$)

$$V_2=\sqrt{V_1^2+2gh} \rightarrow (1)$$

according to continuity equation discharge

$$Q=a_1V_1=a_2V_2$$

$$V_1 = \left(\frac{a_2}{a_1}\right) V_2 \rightarrow (2)$$

coefficient of contraction (C_c) = $\frac{\text{Area of Vena contract}}{\text{Area of orifice}}$

$$C_c = \frac{a_2}{a_0} \Rightarrow a_2 = C_c a_0 \rightarrow (3)$$

from (2) & (3) $V_1 = \left(\frac{C_c a_0}{a_1}\right) V_2 \rightarrow (4)$. substitute eq (4) in eq (1).

from eq (1)

$$V_2 = \sqrt{\left(C_c \frac{a_0}{a_1} \cdot V_2\right)^2 + 2gh}$$

$$V_2^2 = \left(C_c \frac{a_0}{a_1} V_2\right)^2 + 2gh \Rightarrow V_2^2 - \left(C_c \frac{a_0}{a_1} V_2\right)^2 = 2gh$$

$$V_2^2 = \frac{2gh}{1 - C_c^2 \left(\frac{a_0}{a_1}\right)^2} \qquad V_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \left(C_c \cdot \frac{a_0}{a_1}\right)^2}}$$

C_c = coefficent of contrst

We know discharge $Q = A_2 V_2$ from eq (3)

$$Q = C_c a_0 \sqrt{\frac{2gh}{1 - \left(C_c \frac{a_0}{a_1}\right)^2}}$$

By substuting $C_c = \frac{C_d \sqrt{1 - \left(C_c \frac{a_0}{a_1}\right)^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}$

This is a constant term

substitute.
C_c in discharge expression

$$Q = \frac{C_d \sqrt{1 - \left(C_c \frac{a_0}{a_1}\right)^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}} \cdot \frac{a_0 \cdot \sqrt{2gh}}{\sqrt{1 - \left(C_c \frac{a_0}{a_1}\right)^2}}$$

$$Q = \frac{C_d\, a_0 \sqrt{2gh}}{\sqrt{\frac{a_1^2 - a_0^2}{a_1^2}}} \Rightarrow \boxed{Q = \frac{C_d\, a_0\, a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}}$$

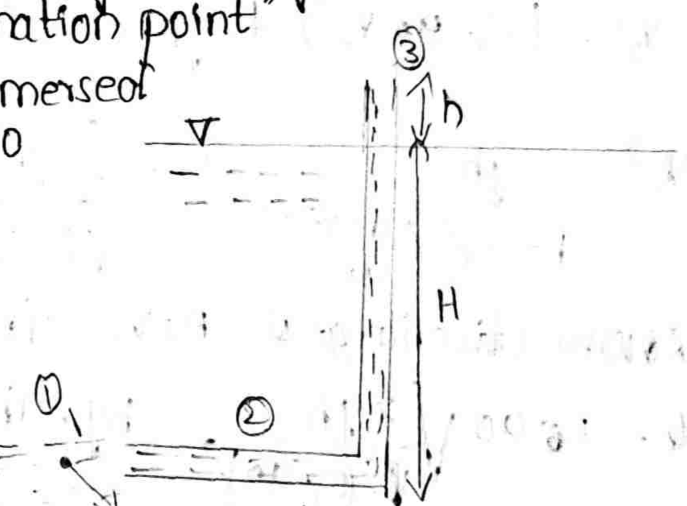
The value of Cd for orifice Meter is about of 0.6 it is Because at the exit of orifice the jet of water suddenly enlarges

*Pitot tube:- It is a device used for measuring of

Velocity of flow in pipes and it is also used for measuring of velocity in open channels (the top surface is opened to sky). A simple pitot tube is a tube which is bent at right angles

Working principle:- If the velocity of flow at a particular point is brought to be zero when the velocity head or kinetic head is converted to pressure head. By measuring the exact pressure head that will give you velocity at that point.

The point where the velocity is brought to be zero that point is called as "stagination point"



→ consider a pitot tube immersed in a fluid consider two sections ① & ②

Apply the Bernoulli's equation B/W ① & ② i.e

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2$$

where $z_1 = z_2$ Because section ① & setion ② is at same height

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g} \Rightarrow \frac{P_1}{w} = \text{pressure head at section ①} = H$$

$\frac{V_1^2}{2g}$ = Velocity head (or) kinetic head at section ①

∴ velocity head is zero.

$\frac{P_2}{w}$ = pressure head at section ② = H+h , $\frac{V_2^2}{2g}$ = Velocity head at section ② $\boxed{\frac{V_2^2}{2g} = \frac{V^2}{2g}}$

$$H + 0 = H + h + \frac{V^2}{2g} \Rightarrow \cancel{H} = \cancel{H} + h + \frac{V^2}{2g} \quad h + \frac{V^2}{2g} = 0$$

$\boxed{V = -\sqrt{2gh}}$ [∴ V varies so this in negative]

If C_V Represents coefficent of velocity then actual velocity $= C_V\sqrt{2gh}$ (0.9) → coefficient of Velocity

Impulse momentum equation:- The net force applied on the fluid in any direction is equals to rate of change of momentum in same direction = change of momentum per second (mass × velocity) in the same direction

= Final momentum per sec - initial momentum per sec

= mass per sec × (final velocity - intial velocity) $\frac{Mass}{sec}$

$= \frac{m}{dt}(V_2 - V_1)$

According to Newtons second law $F = ma = m \times \frac{dv}{dt}$

The impulse momentum equation is Based on conversvation of momentum

* It is used to calculate the dynamic force

Ex:- whenever there is a change in magnitude (or) change in magnitude of velocity (or) direction of velocity or Both

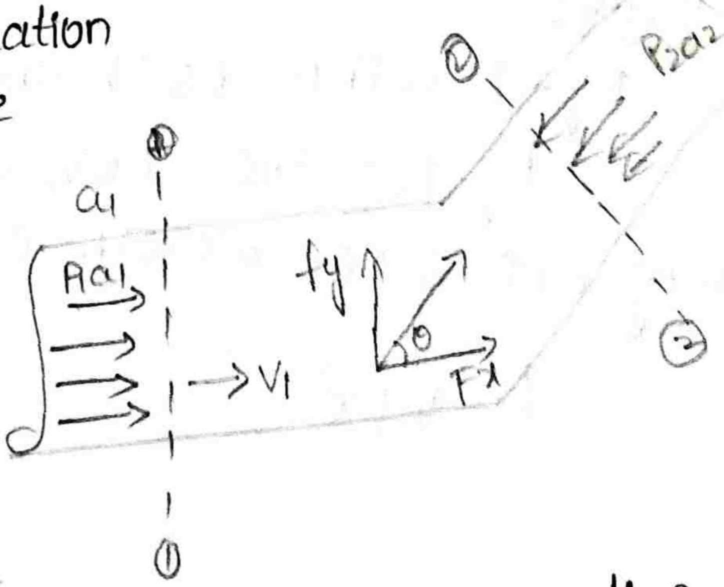
Ex: 1. Force exerted By the fluid on a pipe bend

2. force applied by fluid on a nossle

3. In all hydraulic machinery such as turbines and pumps the impluse Momentum equation is applied

Force applied (Exerted) by the fluid on a pipe bend:-

→ Impluse momentum equation is used to calculate the force exerted by the fluid on a pipe bend. consider a pipe bend as shown in figure



Let "θ" be the angle of the bend.

P_1, P_2 are the pressure intensitys at section ①-① &e section ②-②.

A_1, A_2 area of the pipe line at section ①-① &e section ①-① & ②-②. ②-②, V_1 &e V_2 are the velocitys in section ①-① & ②-②.

* Let ρ be mass density of the fluid.

Let F_x, F_y are the forces applied by the fluid along x-direction and y-direction

According to the impluse momentum equation the net force acting on the fluid between (1)-(1) & (2)-(2) equal to = Rate of change of momentum

Applying it in the x-direction the force applied By the bend on the fluid on the bend in x-y direction are $+F_x$, $+F_y$ respectively

Net force acting on the fluid in x-direction

$P_1a_1 + F_x - P_2a_2\cos\theta$

Net force action on the fluid in y-direction

$0 + F_y - P_2a_2\sin\theta$

$= \frac{mass}{sec}$ (Final velocity - Initial velocity)

$= \rho Q(V_2\cos\theta - V_1)$ $\left(\frac{mass}{sec} = \rho Q\right)$

X-dir eq:-

$P_1a_1 + F_x - P_2a_2\cos\theta = \rho Q(V_2\cos\theta - V_1) \rightarrow (1)$

Y-dir eq:-

$F_y - P_2a_2\sin\theta = \rho Q(V_2\sin\theta - 0)$

$F_y - P_2a_2\sin\theta = \rho Q V_2\sin\theta \rightarrow (2)$.

using eq(1) & eq(2) we can calculate the values of F_x, F_y

$F = \sqrt{F_x^2 + F_y^2}$

The direction of resultant $= \tan^{-1}\left(\frac{F_y}{F_x}\right)$.
with respect to horizontal.

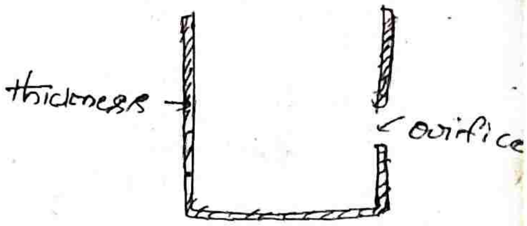
5-10-2009

ॐ UNIT - III

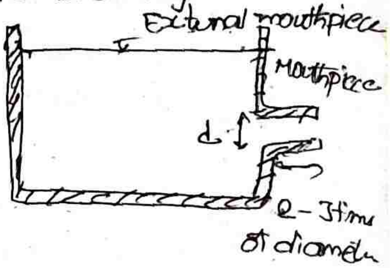
ORIFICES & MOUTHPIECES

To find discharge in case of tanks.

ORIFICE :- An orifice is a small opening made in the side of a tank (or) in the bottom of tank to measure the ~~distance~~ discharge from a tank.

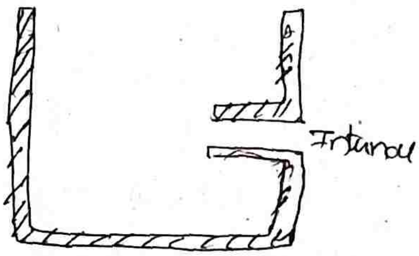


Mouth piece :- A small pipe (length = 2 to 3 times of diameter of opening) attached to the opening to measure discharge from a tank.



→ If the pipe is attached outside the opening of tank the pipe is called external mouth piece.

→ If pipe is attached inside of tank called internal mouth piece



Classification

Orifices :-

1) Based on size → Small, → large

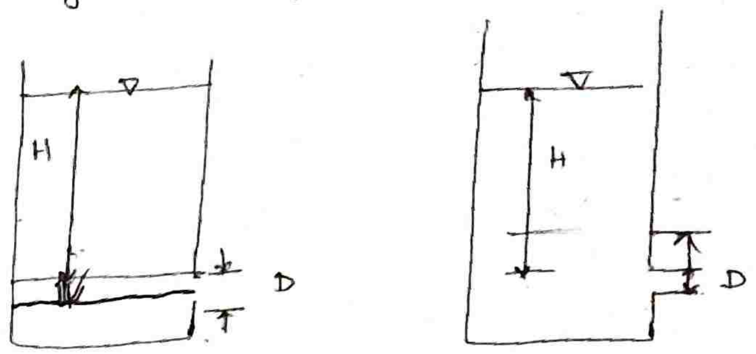
2) Based on shape → Circular, → Square, → Rectangular, → Trapizoidal

3) Based on edge → Sharp edge, → broad edge, → Bell mouth

4) Based on discharge → Freely discharged, → Submerged, → Partially Submerged

→ Based on size.

Distinguish between small & large orifice:-



H = Head above the centre of orifice
D = Depth of orifice

→ If H >> D, we call it as small orifice
→ If H is not significantly high compared to D then called as large orifice

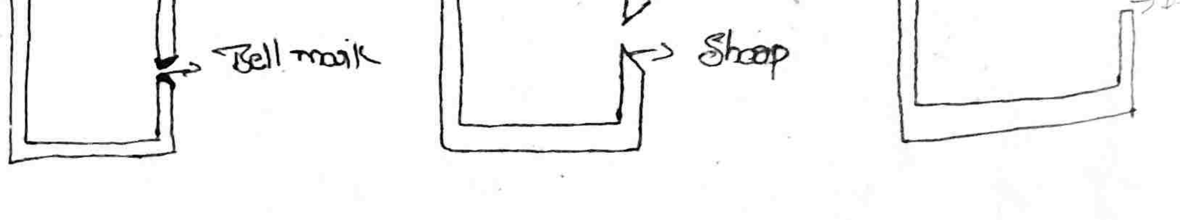
→ In case of small orifice velocity distribution is uniform
→ " " large " " " non "

Small orifice means head above the centre of the orifice is much much higher compared to depth of orifice (H >>> D). If an orifice is considered as small orifice then velocity distribution is uniform.

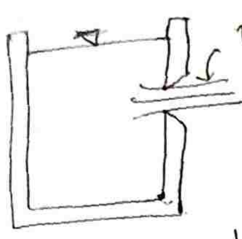
Large orifice means 'h' is not very high compared to depth of orifice. The velocity distribution is non-uniform.

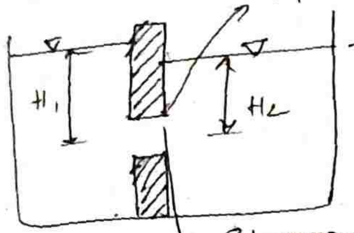
→ The same orifice can behave as a small orifice for some time & behave large orifice for rest of time.

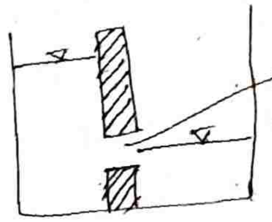
Based on edge



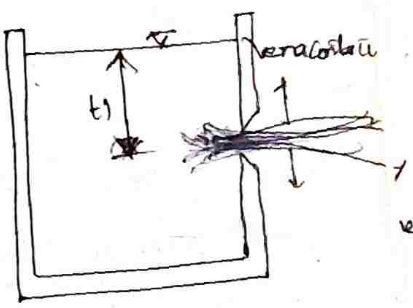
Based on discharge







Derive the equation for discharge through a sharp edge orifice:



Consider a sharp edge orifice fixed in the side of the tank.

Let 'H' be the head above the centre of orifice. As 'H' is very large compared to

Depth of orifice is small orifice.

The liquid leaving the orifice will not be able to suddenly adjust to the path 'z' leaving the orifice converges just outside the orifice.

The area of cross-section of 'z' is minimum at a section called Vena contracta.

At the Vena Contracta section.

i) Cross sectional area of 'z' is minimum.

(ii) Stream lines are parallel.

(iii) Velocity is maximum.

Apply the Bernoulli's equation between the free surface in the tank and vena contracta.

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + Z_2$$

$$0 + 0 + H = 0 + \frac{V_2^2}{2g} + 0$$ (Considering Centre of orifice as datum line)

$$\frac{V_2^2}{2g} = H$$

$V_2 = \sqrt{2gH}$ → Torricelli's Equation.

Theoritical velocity to orifice = $\sqrt{2gH}$

Discharge = Area × velocity

Theoritical discharge Q_{th} = Area of the orifice × velocity of orifice

$Q = a\sqrt{2gH}$

Actual discharge = $C_d \times a\sqrt{2gH}$

Define Hydraulic Coefficients & Derive the relation among them

There are 3 hydraulic coefficients.

1) Coefficient of velocity (C_v)
2) " " Contraction (C_c).
3) " " discharge (C_d)

C_v → It is defined as the ratio of actual velocity of Jet at the Vena Contracta to theoritical velocity of 'Z'.

$$C_v = \frac{\text{Actual velocity of Jet at vena Contracta}}{\text{Theoritical velocity of Jet}}$$

$$C_v = \frac{V_a}{\sqrt{2gH}}$$

Its value varies between 0.95 to 0.99 depending on the size, shape & head. The C_v is due to frictional loss.

ii) C_c : It is defined as the ratio of the area of jet at Vena Contracta to the area of orifice.

$$C_c = \frac{\text{Area of Jet at Vena Contracta}}{\text{Area of orifice}}$$

Its value varies between 0.61 - 0.69

$C_c = \frac{a_c}{a}$

(iii) C_d → It is defined as the ratio of actual discharge to theoridical discharge.

$C_d = \frac{\text{actual discharge}}{\text{Theoridical discharge}} = \frac{Q_a}{Q_t}$

It varies between 0.61 to 0.65

$C_c = C_d \sqrt{2gH}$

$C_v = \frac{V_a}{\sqrt{2gH}}$

★ Relation between C_c, C_d, C_v:

Coefficient of discharge.

$C_d = \frac{Q_a}{Q_{th}} = \frac{\text{actual discharge at vena contracta}}{\text{theoridical discharge}}$

Actual discharge Q_a = Actual area of jet × Actual velocity of jet

= Area of jet at vena contracta × Actual velocity of jet

$Q_a = a_c \times V_a \rightarrow (1)$

we know that $C_c = \frac{a_c}{a}$ & $C_v = \frac{V_a}{\sqrt{2gH}}$

Actual discharge $Q_a = a_c \cdot V_a$

$= (C_c \cdot a)(C_v \sqrt{2gH})$

theoridical discharge = Q_{th} = Area of orifice × Theoritical velocity

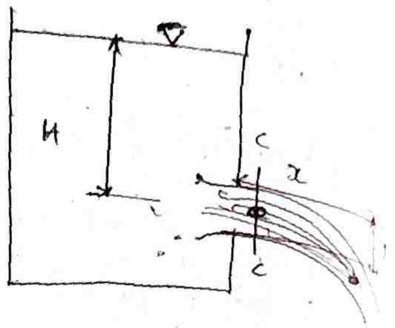
$= a \times \sqrt{2gH}$

$C_d = \frac{Q_a}{Q_{th}} = \frac{(C_c \cdot a)(C_v \cdot \sqrt{2gH})}{a \times \sqrt{2gH}}$

$$\boxed{C_d = C_c \cdot C_v}$$

Explain the method of determining Cd, Cc, Cv:

Coefficient of velocity: (Cv)



The liquid from the tank is leaving to the orifice. let 'H' be the head above the centre of orifice.

Theoritical velocity $= \sqrt{2gH}$

We know that $C_V = \dfrac{\text{actual velocity}}{\text{Theoritical velocity}}$

$$C_V = \frac{V_a}{\sqrt{2gH}}$$

In order to determine actual velocity consider liquid particle at vena contracta. In a small time 't' sec the liquid particle has moved over a horizontal distance 'x' & a vertical distance 'y'. If 'v' is the actual velocity,

$V_a = \dfrac{x}{t} \Rightarrow x = V_a t$ (in horizontal)

$\Rightarrow t = \dfrac{x}{V_a}$

$y = \frac{1}{2} g t^2$

$$y = \frac{1}{2} g \cdot \frac{x^2}{V_a^2}$$

$$y = \frac{g x^2}{2 V_a^2}$$

$$V_a^2 = \frac{g x^2}{2y} \Rightarrow V_a = \sqrt{\frac{g}{2y}}\, x$$

We know that $C_V = \dfrac{V_a}{\sqrt{2gH}}$

$$C_V = \sqrt{\frac{g x^2}{2y}} \Big/ \sqrt{2gH}$$

$$C_V = \sqrt{\frac{x^2}{4yH}}$$

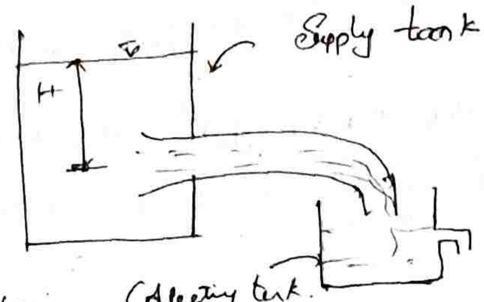
Coefficient of discharge

$$C_d = \frac{Q_{actual}}{Q_{theoretical}}$$

We know that

Q_{theo} = Area of orifice × Theoritical velocity

$= a \times \sqrt{2gH}$



maintain constant head (H) in supply tank.

Let 'a' is the area of orifice then

theoritical discharg $= a\sqrt{2gH}$

actual discharge is to be measured by a collecting tank.

Let 'A' is the cross sectional area of collecting tank.

Let 'R' is the rise in collecting tank. Let 'T' be the time taken for a rise of 'R' which is measured by a stop watch.

∴ volume collected $= A \times R$.

This volume is collected in 'T' sec

∴ Actual discharge $= \frac{A \times R}{T}$

$$C_d = \frac{\text{Actual discharge}}{\text{theoritical discharge}}$$

$Q_a = \frac{AR}{T}$

Table

S.No	Constant head in supply tank (H)	Rise in Collecting tank (R)	Time taken for rise (T)	theoritical discharge $Q_{th} = a\sqrt{2gH}$	Actual discharge $Q_a = \frac{AR}{T}$

$C_d = \frac{Q_a}{Q_{th}}$

Before starting the experiment area of orifice 'a' & area of collecting tank 'A' to be collected

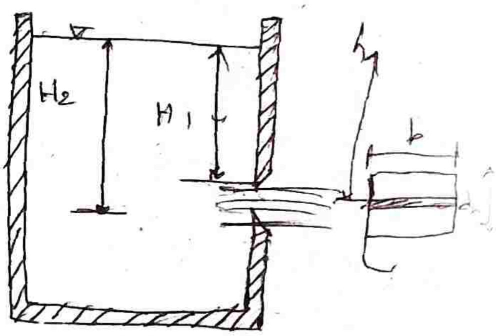
Coefficient of Contraction

We know that $C_c = \frac{C_d}{C_v}$

Large Rectangular orifice

The equation for discharge previously derived

$Q_a = C_d a \sqrt{2gH}$ is applicable for small orifice because for small orifice the velocity distribution is uniform. In case of large orifice " " " non " & hence the above equation is not applicable the separate equation is to be derived for large orifice. Consider a large Rectangular orifice as shown in the figure.



Consider an elementary horizontal strip of depth 'dh' at a height 'h' from free surface as shown in figure.

Area of strip $= b\,dh$

velocity through the strip $= \sqrt{2gH}$

Discharge through " " $dQ = b\,dh \times \sqrt{2gH}$

" " entire orifice $= \int dQ$

$$= \int_{H_1}^{H_2} b\,dh \sqrt{2gh}$$

$$= b\sqrt{2g} \int_{H_1}^{H_2} h^{1/2}\,dh$$

$$= b\sqrt{2g} \left(\frac{h^{3/2}}{3/2} \right)_{H_1}^{H_2}$$

$$= \frac{2b\sqrt{2g}}{3} \left\{ H_2^{3/2} - H_1^{3/2} \right\}$$

Actual discharge $Q_a = C_d \times \frac{2}{3} b \sqrt{2g} \left(H_2^{3/2} - H_1^{3/2}\right)$

For small orifice,

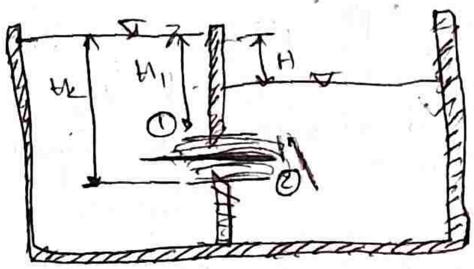
$Q_a = C_d \; a\sqrt{2gH}$

For large Rectangular orifice,

$Q_a = C_d \times \frac{2}{3} b \sqrt{2g} \left(H_2^{3/2} - H_1^{3/2}\right)$

Derive the eqation for discharge through a fully submerged orifice

Fully submerged orifice means on both sides of the orifice the liquid level is above the top of orifice.



Let H be the difference of Water levels in two tanks

Head above the centre of the orifice in the first tank (or) in upstream tank

$$= H_1 + \frac{(H_2 - H_1)}{2}$$

$$= \frac{2H_1 + H_2 - H_1}{2}$$

$$= \frac{H_1 + H_2}{2}$$

Head above the centre of the orifice in the downstream orifice $= \frac{H_1 + H_2}{2} - H$

applying the bernoulli's eqation between ① & ②

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + Z_2$$

As the datum is centre of orifice and is horizontal $Z_1 = Z_2$

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g}$$

$$\frac{P_1}{w} = \frac{H_1+H_2}{2} \quad , V_1 = 0$$

$$\frac{P_2}{w} = \frac{H_1+H_2}{2} - H$$

$V_2 = ?$

$$\frac{H_1+H_2}{2} + 0 = \frac{H_1+H_2}{2} - H + \frac{V_2^2}{2g}$$

$$V_2 = \sqrt{2gH}$$

$b(H_2 - H_1)$

Area of orifice $= b(H_2 - H_1)$

∴ discharge (Q) = area × velocity

$= b(H_2 - H_1) \times \sqrt{2gH}$

Actual discharge $(Q_a) = C_d \; b(H_2 - H_1)\sqrt{2gH}$

In case of submerged orifice the most important parameter is the difference of water levels (H)

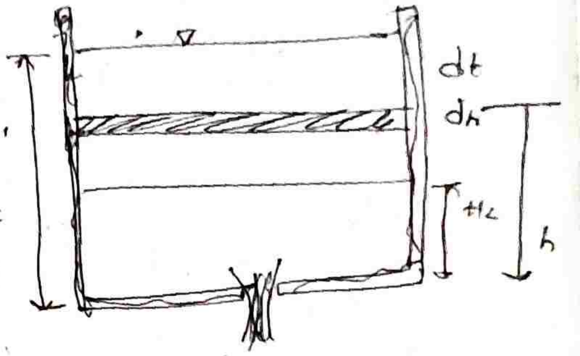
If area of orifice is given then discharge = C_d × area of orifice × $\sqrt{2gH}$

I.MP Q

1) Discharge small orifice
2) " Large rectangular orifice
3) Derivation between C_v, C_d, C_c.
4) Discharge in submerged (sharp orifice)

Time of emptying a tank with an orifice

Consider a cylindrical tank of constant area of cross section. An orifice is fitted at the bottom of the tank.



Let 'A' is the cross section area of the tank. 'a' is the area of orifice. It is required to determine the time taken for liquid to fall from a height H_1 to a height H_2.

In order to determine the time consider an elemental strip of thickness 'dh' at a height 'h' above the centre of the orifice.

At anytime, the water level in the tank is 'h'. In a small time 'dt' the liquid level has fall down to 'y'dk

The volume of liquid leaving the tank in time 'dt' sec = A(dh)

The liquid has to pass through the orifice.

If 'Q' is the discharge through the orifice. Volume of liquid flowing through the orifice in time 'dt' sec. = Q × dt

Equating both above equations.

$-Adh = Qdt$

'−'ve sign indicates as time increases, 'h' decreases.

Cylindrical, Rectangular, Square tanks, area of cross section constant

$-Adh = Q\,dt$

$Q_a = C_d \times a\sqrt{2gh}$

$dt = -\frac{Adh}{Q}$

$dt = -\frac{Adh}{C_d \times a\sqrt{2gH}}$

$t = \int \frac{-Adh}{C_d \times a\sqrt{2gH}}$

$$t = -\int \frac{A\,dh}{C_d\, a\sqrt{2gH}}$$

$$t = -\int_{H_1}^{H_2} \frac{A\,dh}{C_d\, a\sqrt{2gH}}$$

$$t = -\frac{A}{C_d\, a\sqrt{2g}} \int_{H_1}^{H_2} h^{-1/2}\, dh$$

$$t = \frac{-A}{C_d\, a\sqrt{2g}} \left[\frac{h^{-1/2+1}}{-1/2+1}\right]_{H_1}^{H_2}$$

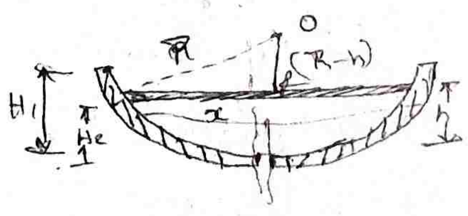
$$= \frac{-2A}{C_d\, a\sqrt{2g}} \left(H_2^{1/2} - H_1^{1/2}\right)$$

$$t = \frac{2A}{C_d\, a\sqrt{2g}} \left[\sqrt{H_1} - \sqrt{H_2}\right]$$

In order to completely empty the tank $H_2 = 0$ $\quad t = \frac{2A}{C_d a\sqrt{2g}}\sqrt{H_1}$

Case (ii) :- Hemi spherical tank (c/s of cricket ball)

In case of hemi spherical tank the c/s area of tank varies with depth. therefore the capacity of problem of problem increases.



Let H_1 is the initial level

H_2 is final level

At any time let 'h' be the height of liquid above the centre of orifice. In time 'dt' a liquid level has fall down by an amount 'dh'. The volume of liquid leaving the tank in dt seconds = Area x dh

If 'x' is the radius the area of c/s at height h $h\ \pi x^2$

volume leaving in dt sec $= \pi x^2 \times dh$

Let R is the radius of the tank.

$x^2 = R^2 - (R-h)^2$

$x^2 = R^2 - R^2 - h^2 + 2Rh$

$x^2 = 2Rh - h^2$

Substitute volume leaving in dt sec $= \pi(2Rh - h^2)dh \rightarrow (1)$

If Qa is the discharge through the orifice, then volume flowing through the orifice in dt seconds = $Qa \times dt$

$$Qa = C_d a \sqrt{2gh}$$

$$\therefore V = C_d a \sqrt{2gh} \times dt$$

We know $-A\,dh = Q\,dt$

$$-\pi(2RH - h^2)dh = C_d a \sqrt{2gh} \times dt$$

$$dt = \frac{-\pi(2RH - h^2)dh}{C_d a \sqrt{2gh}}$$

$$dt = \frac{\pi}{C_d a \sqrt{2g}} \frac{(h^2 - 2RH)}{\sqrt{h}} dh$$

$$= \frac{\pi}{C_d a \sqrt{2g}} [h^{3/2} - 2R.H^{1/2}]dh$$

$$= \frac{\pi}{C_d a \sqrt{2g}} (h^{3/2}dh - 2R.H^{1/2}dh]$$

$\therefore$ Integrating $\therefore$ $t = \frac{\pi}{C_d a \sqrt{2g}} \int_{H_1}^{H_2} h^{3/2} dh - 2R \int_{H_1}^{H_2} H^{1/2}.dh$

$$= \frac{\pi}{C_d a \sqrt{2g}} \left[\frac{2}{5}\left[H_2^{5/2} - H_1^{5/2}\right] - \frac{4R}{3}\left(H_2^{3/2} - H_1^{3/2}\right)\right]$$

$$= \frac{\pi}{C_d a \sqrt{2g}} \left(\frac{2}{5}\left[H_2^{5/2} - H_1^{5/2}\right] - \left[\frac{4R}{3} H_2^{3/2} + \frac{4R}{3} H_1^{3/2}\right]\right)$$

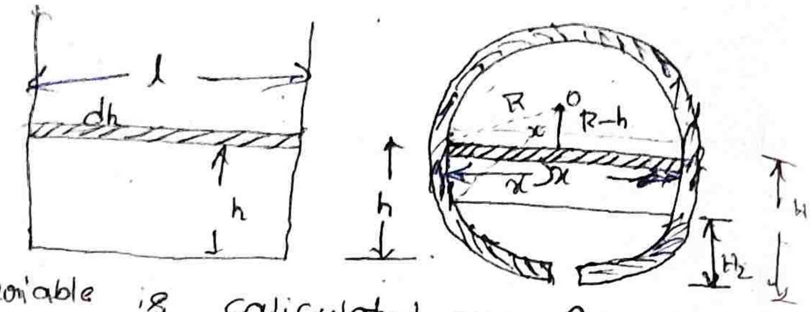
$$= \frac{2\pi}{C_d a \sqrt{2g}} \left[\frac{4R}{3} H_1^{3/2} - \frac{4R}{3} H_2^{3/2} - \frac{2}{5}\left[H_1^{5/2} - H_2^{5/2}\right]\right]$$

to completely the empty the tank H_2 should be zero $\therefore$ $t = \frac{\pi}{C_d a \sqrt{2g}}$

Case - III Cylindrical kept in horizontal direction (Railway oil tank (or) horizontal circular tank]

We know that

$-A\,dh = Q\,dt$



Area A - is variable is calculated as follows:

$\therefore$ Area $= 2x \times L$

$x^2 = R^2 - (R-h)^2$

$x^2 = 2Rh - h^2$, $\quad x = \sqrt{2Rh - h^2}$

$\therefore$ Area of circular of the tank $= 2 \times \sqrt{2Rh - h^2} \times L$

$Q_a = C_d\, a \sqrt{2gh}$

Substitute

$$-2\sqrt{2Rh - h^2}\; L = C_d\, a \sqrt{2gh}\; dt$$

$$dt = \frac{-2}{C_d\, a \sqrt{2gh}} \sqrt{2Rh - h^2} \times L$$

$$dt = \frac{-2L}{C_d a \sqrt{2g}} \left[\frac{\sqrt{2Rh - h^2}}{\sqrt{h}}\, dh \right]$$

$$= \frac{-2L}{C_d a \sqrt{2g}} \left[\sqrt{2Rh - h^2} \times h^{-1/2} dh \right]$$

$$= \frac{-2L}{C_d\, a \sqrt{2g}} \left[\sqrt{2Rh} \times h^{-1/2} - \sqrt{h^2} \times h^{-1/2} \right] dh$$

$$= \frac{-2L}{C_d\, a \sqrt{2g}} \left[\sqrt{2R} - h^{1/2} \right] dh$$

$$dt = \frac{-2L}{C_d\, a \sqrt{2g}} \left[2R - h \right]^{1/2} dh$$

$$dt = \frac{-2L}{C_d a \sqrt{2g}} \left[2R-h\right]^{1/2} dh$$

$$\therefore t = \frac{-2L}{C_d a \sqrt{2g}} \left[\sqrt{2R}\, h - \frac{2}{3} h^{3/2}\right]_{H_1}^{H_2}$$

$$t = \frac{4/3\, L}{C_d a \sqrt{2g}} \left[(2R - H_2)^{3/2} - (2R - H_1)^{3/2} \right]$$

In order to complete the tank, $H_2 = 0$

$$\therefore t = \frac{4L}{3\, C_d a \sqrt{2g}} \left(2R - [2R - H_1]^{3/2} \right)$$

Venturimeter

Orificemeter

pitot tube

Impulse momentum equ force on a pipe load

Determination of C_d, C_c, C_v.

Small orifice discharge, Large orifice discharge, Submerged orifice discharge time of emptying a tank, Const area at C/S, hemispherical horizontal circular pipe

Problems:

A venturimeter has its axis vertical, the inlet & through dia being 150mm & 75mm respectively. The throat is 225mm above inlets k=0.96. petrol Specific gravity 0.78 flows up through the meter at a rate of 0.029 m³/sec. Find the pressure difference between inlet & throat?

Sol:- The discharge through a venturimeter is given by

$$Q = k\sqrt{h}$$

$$= \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{h 2g}$$

$$a_1 = \frac{\pi}{4} \times 0.15^2 = 0.0177\, m^2 \;\&\; a_2 = \frac{\pi}{4} \times 0.75^2 = 0.0044\, m^2$$

$$0.029 = \frac{0.0177 \times 0.0044 \times \sqrt{2 \times 9.81 \times h}}{\sqrt{(0.0177)^2 - (0.0044)^2}}$$

$h = 2.254$ m of oil

$$h = \left(\frac{P_1}{W} + z_1\right) - \left(\frac{P_2}{W} + z_2\right)$$

$$2.254 = \left(\frac{P_1}{W} - \frac{P_2}{W}\right) - (z_2 - z_1)$$

$$= \frac{P_1}{W} - \frac{P_2}{W} - \left[\frac{225}{1000} - 0\right]$$

$P_1 - P_2 = 2.479$ (or) $P_1 - P_2 = 2.479 \times 0.79 \times 9810$

$= 18.969$ kpa

2) 25 lit of gasoline (sp. gr = 0.82) flow/second upwards in an inclined venturimeter fitted to a 300mm & throat is 1.2m. from the entrance along its length pressure gauges inserted at entrance along at throat shows a pressure of 0.141 N/mm² & 0.077 N/mm² respectively. Find the discharge & coeff of ventia

Sol: Instead of pressure guages the entrance & the throat of venturimeter are connected to the 2 limbs of a V-tube mercury manometer. Find its reading in mm of differential mercury column.

The discharge through venturimeter is given by

$$Q = \frac{k\, a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$a_1 = \frac{\pi}{4} \times 0.3^2 = 0.0707\ m^2$

$a_2 = \frac{\pi}{4} \times 0.015^2 = 0.0177\ m^2$

$$h = \left(\frac{P_1}{W} + z_1\right) - \left(\frac{P_2}{W} + z_2\right)$$

$$\frac{P_1}{W} = \frac{0.141 \times 10^6}{9810 \times 0.82} = 17.582 \text{ m. of gasoline}$$

$\frac{P_2}{w} = \frac{0.077 \times 10^6}{9810 \times 0.82} = 9.572$ m. of gasoline

$z_1 = 1.2 \sin 30° = 0.6$ m

$h = (17.528 + 0) - (9.572 + 0.60) = 7.356$ m

$$k = \frac{250 \times 10^{-3} \times \sqrt{0.0707^2 - 0.0177^2}}{0.0707 \times 0.0177 \times \sqrt{2 \times 9.81 \times 7.356}}$$

$k = 0.979$

If U-tube is connected then

$h = x\left[\frac{S_m}{S} - 1\right]$

$7.356 = x\left[\frac{13.6}{0.82} - 1\right] = 0.472$ or 472 mm

Water flows through orifice at 0.147 m³/sec. through a 150 mm dia orifice inserted in 300 mm. diam pipe. If the pressure guage fitted upstream & down stream side. the orifice plate have shown readings of 176.58 kN/m² & 88.29 kN/m² respectively. Find the coefficient of discharge of c of the orifice meter.

Sol:- $$Q = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

$h = \frac{P_1}{w} - \frac{P_2}{w}$

$= \frac{(176.58 - 88.29) \times 10^3}{9810} = 9$ m. of water

$a_1 = \frac{\pi}{4} \times 0.3^2 = 0.0707$ m², $a_0 = \frac{\pi}{4} \times 0.015^2 = 0.0177$ m²

$C_d = 0.605$

2) A 150×75 m.m. Venturimeter width of C_d - 0.98 is to be replaced by an orificemeter having a coeff. discharge 0.6. If both the meter are to give the same diff. mercury manometer reading for a discharge of 100 lit/sec. (8) the inlet dia is to 150 m.m. Find orifice diameter.

Sol:- Given

Venturimeter

$d_1 = 150$ m.m & $d_2 = 75$ mm

$a_1 = \frac{\pi}{4} \times 0.15^2 = 0.0177 m^2$

$C_d = 0.98$m

$a_2 = \frac{\pi}{4} \times 0.075^2 = 0.0044 m^2$

$Q = 100 \times 10^{-3}$ m³/sec

$$100 \times 10^{-3} = \frac{0.98 \times 0.0177 \times 0.0044 \times \sqrt{2 \times 9.81} \times \sqrt{h}}{\sqrt{0.0177^2 - 0.0044^2}}$$

$\sqrt{h} = 5.07$m

$h = 25.71$m

$h = x\left[\frac{S_m}{S} - 1\right]$

$Q = \frac{C_d a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$

$25.71 = x\left(\frac{13.6}{1} - 1\right)$

$x = 2.04$m.

$Q = \frac{C_d a_1 \times a_0 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$

Orifice case

$$100 \times 10^{-3} = \frac{\pi/4 \times 0.15^2 \times a_0 \times \sqrt{2 \times 9.81} \times \sqrt{2.04}}{\sqrt{\frac{\pi}{4} \times 0.15^2 - a_0^2}} \times 0.6$$

$$\frac{0.067 a_0}{\sqrt{0.0177 - a_0^2}} = 100 \times 10^{-3}$$

$$= (0.067 a_0)^2 = (100 \times 10^{-3})^2 \times (0.0177 - a_0)$$

$4.489 \times 10^{-3} a_0^2 + 0.01 a_0 - 1.77 \times 10^{-4} = 0$

$a_0 = 0.0122$

$\frac{\pi}{4} \times d^2 = 0.0122$

$d = 0.124$m

3) 2 pitot tubes are installed in pipe one on the ventre line other 75 mm from the centre line. If the velocities at the 2 point are 3m/s (8) 2m/sec. Calc the reading on manometer connected to 2 ...

$$\frac{P_A}{w} = \frac{P_1}{w} + \frac{V_1^2}{2g} \quad (or) \quad \frac{P_B}{w} = \frac{P_2}{w} + \frac{V_2^2}{2g}$$

$P_1 P_2$ static pressure intensities.

V_1, V_2 - velocities.

$$\frac{P_1}{w} + 0.075 = \frac{P_2}{w}$$

$$\frac{P_B}{w} - \frac{P_A}{w} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + 0.075 \rightarrow (3)$$

Further 'x' is the diff. of levels of mercury column in the 2 limbs of the manometer (or) y - depth of higher mercury column below the centre line of the pipe. then the following equation

$$\frac{P_A}{w} + y + x = \frac{P_B}{w} + y - 0.075 + 13.6x$$

$$\frac{P_B}{w} - \frac{P_A}{w} = -12.6x + 0.075$$

above in equation (3)

$\therefore x = 0.0202 m$ $0.023m$

4. A pitot tube placed in the centre of the 200mm pipe line, has one orifice pointing upper stream (or) the other ⊥ to it. If the pressure diff. b/w 2 orifices is 40mm of water. When the discharge through the pipe is 1365 l/m. Calc. Coeff of the Pitot. Take mean velocity of pipe is 0.83 to central velocity.

Sol: $Q = 1365 \times 10^{-3}/60 = 0.02275 \, m^3/sec.$

The mean velocity of flow $v = \frac{Q}{A} = \frac{0.002275}{\frac{\pi}{4} \times (0.2)^2} = 0.724 m/sec$

Central velocity $= \frac{V}{0.83} = \frac{0.724}{0.83} = 0.872 \, m/sec$

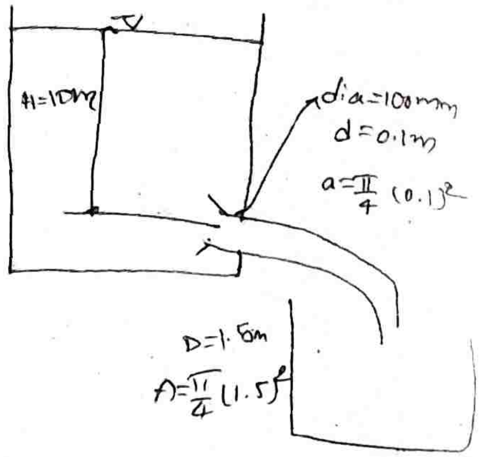
$V = \sqrt{2gh}$

$0.872 = C \sqrt{2 \times 9.81 \times 40 \times 10^{-3}}$

$C =$ ~~0.974~~ 0.984

The head of water over an orifice of diameter 100mm is 10m. The water coming out of the orifice is collected in a circular tank of diameter 1.5m. The rise of water level in this tank is 1m in 25sec. The coordinates of a point on the jet, measured from the contraction are 4.3m from the horizontal and 0.5m vertical. Find the coefficients C_d, C_c, C_v.

Sol:



Theoritical discharge

$= a\sqrt{2gh}$

$= \frac{\pi}{4}\times(0.1)^2\sqrt{2\times 9.81\times 10}$

$= 0.1099$

Rise = 1m.

$A = 1.7671$

Time = 25sec

2) Actual discharge $= \frac{A.R.}{T} = \frac{\frac{\pi}{4}\times 1.5^2\times 1}{25} = 0.0706$

3) Coefficient of discharge $(C_d) = \frac{0.0706}{0.1099} = 0.64$

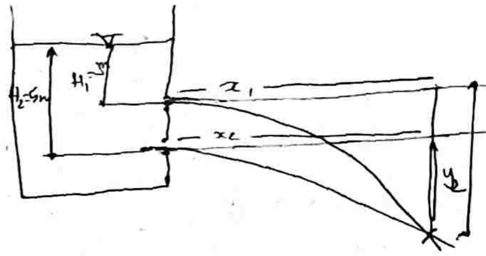
$C_v = \sqrt{\frac{x^2}{4yh}} = \sqrt{\frac{4.3^2}{4\times 0.5\times 10}} = 1.2413$

$C_c = \frac{C_d}{C_v} = \frac{0.64}{1.24} = 0.515$

$C_d = 0.64,\ C_c = 0.515,\ C_v = 1.241$

A tank has two identical orifices in one of its vertical sides. The upper orifice is 3m below water surface & lower one is 5m below water surface. If the value of C_v for each orifice is 0.96. Find the point of intersection of the two jets.

$C_{v_1} = C_{v_2} = 0.96$



Sol: $C_{v_1} = C_{v_2} = 0.96$

$y_1 = y_2 + 2$

$x_1 = x_2$

$$C_{v_1} = \sqrt{\frac{x_1^2}{4y_1h_1}}, \quad C_{v_2} = \sqrt{\frac{x_2^2}{4y_2h_2}}$$

~~$y = y_1 + 2$~~

$y_1 = 2 + y_2$

$C_{v_1} = C_{v_2}$

$$\sqrt{\frac{x_1^2}{4y_1h_1}} = \sqrt{\frac{x_2^2}{4y_2h_2}}$$

$$\Rightarrow \frac{x_1^2}{4y_1h_1} = \frac{x_1^2}{4(y_1-2)h_2}$$

$$4y_1(3) = 4(y_1-2)5$$

$$3y_1 = 5y_1 - 10$$

$$2y_1 = 10 \Rightarrow y_1 = 5, \quad y_2 = 3$$

$4y_1h_1 = 4(y_1-2)h_2$

$4y_1(3) = (y_1-2)(5)$

$3y_1 = 5y_1 - 10$

$2y_1 = 10$

$y_1 = 5 \to (1)$

$y_2 = 3$

$0.96 = \sqrt{\frac{x_1^2}{4y_1}}$

$$C_{v_1} = \sqrt{\frac{x_1^2}{4y_1h_1}}$$

$$0.96 = \sqrt{\frac{x_1^2}{4 \times 5 \times 3}} \Rightarrow 0.9216 = \frac{x_1^2}{60}$$

$$x_1^2 = 55.296$$

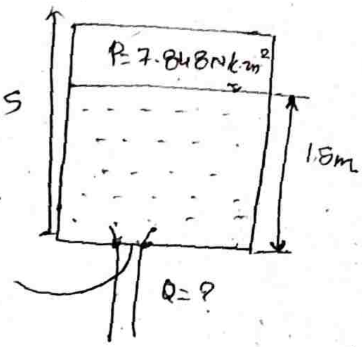
$$x_1 = 7.4361$$

(7.43, 5), (7.43, 3)

3) A closed vessel contains water upto a height of 1.5m and over the water surface there is air having 7.848 N/c.m² above atmospheric pressure. At the bottom of the vessel there is a orifice of diameter 100mm. Find the rate of flow through the orifice take $C_d = 0.6$

Sol:

$C_d = 0.6$



$Q = C_d a \sqrt{2gH}$

$C_d = 0.6$, $a = \frac{\pi}{4}(0.1)^2 = 7.85 \times 10^{-3}$

$H = ?$

= height of water + pressure head in terms of water corresponding to $7.848 N/c.m^2$

pressure head corresponding to $7.848 N/c.m^2$ in terms of water

$= \frac{P}{\text{Specific weight of water}} = \frac{7.848 \times 10^4 N/m^2}{9810 N/m^3} = 8$

total height $H = 1.5 + 8 = 9.5$

$Q = 0.6 \times 7.853 \times 10^{-3} \sqrt{2 \times 9.8 \times 9.5}$

$= 0.06429$

$Q = 0.0643 \; m^3/sec$

h=1.5m
P=7.848
d=0.1m
Cd=0.6
Q=Cd a√2gH
=0.65

d=0.1m

$\frac{P_1}{w} = \frac{7.848}{9800}$ 10.5 lakh

10.5×10^5 cubic per sec

1 sec -> 10.5×10^5

Mouth piece:-

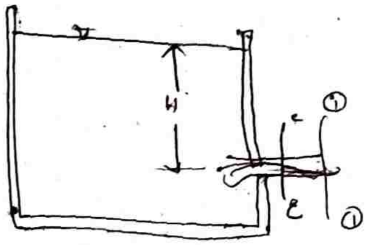
A small cylindrical pipe of length 3-5 times of dia. of an orifice and is attached to an orifice (or) the opening made the side of a tank is called "mouth piece"

Mouth piece:-

∴ External mouth piece:- If the cylindrical pipe is attached outside the tank is called "external mouth piece, otherwise it is attached into the tank (inside) is called "Internal mouth piece"

Derive the equation for the discharge through an external mouth piece:-

Consider an external mouth piece of cross sectional area 'a'. Let H be the head above the centre of mouth piece.



The jet coming out of the tank contraction and then expands to the full cross section of the mouth piece. i.e. the vena contracta is formed well within the mouth piece. using continuity equation :-

$$a_c v_c = a_1 v_1$$

$$v_1 = \frac{A_c v_c}{A_1} \quad (or) \quad v_c = \frac{a_1 v_1}{a_c}$$

$$= \frac{v_1}{(a_c/a_1)}$$

$$\therefore v_c = \frac{v_1}{c_c} \qquad c_c \rightarrow \text{for orifice } 0.62$$

$$v_c = \frac{v_1}{0.62} \rightarrow ①$$

Within the mouth piece, the jet is suddenly expanding from the vena contracta to the original area of the mouth piece. Because of this sudden expansion of the jet there will be loss of energy & this loss of energy due to sudden expansion of jet is given by

head loss due to sudden expansion $h_{L.SE} = \frac{(v_c - v_1)^2}{2g}$

We know that $v_c = \frac{v_1}{0.62}$

$\therefore h_{2}SE = \left(\frac{v_1}{0.62} - v_1\right)^2 / 2g = 0.375 \frac{v_1^2}{2g}$

applying Bernoulli's equation between ① - ① the free surface outlet of the mouth piece

$$\frac{P}{w} + \frac{V_1^2}{2g} + Z = \frac{P_1}{w} + \frac{V_1^2}{2g} + Z_1 + h_2 SE$$

$$0 \quad 0 \quad 1.1 = 0 + \frac{V_1^2}{2g} + 0 + 0.375 \frac{V_1^2}{2g}$$

$$H = \frac{V_1^2}{2g} + 0.375 \frac{V_1^2}{2g}$$

$$V_1 = 1.375 \frac{V_1^2}{2g}$$

But theoritical velocity $V_{th} = \sqrt{2gH}$

$$C_V = V_{act} / V_{th} = \frac{0.853\sqrt{2gH}}{\sqrt{2gH}} = 0.855$$

$$C_V = 0.855$$

in case of external mouth piece the actual area of jet leaving the mouth piece is equal to area of mouth piece. There is no reduction in area of jet while leaving the mouth piece

$\therefore$ C_c for external mouth piece = 1.

$\therefore C_d = C_c \times C_v \Rightarrow \therefore C_d = 0.855$.

$\therefore$ Discharge through external mouth piece $= 0.855\, a\sqrt{2gH}$.

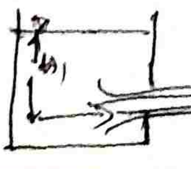
why the C_d of external mouth piece is more than the C_d of an orifice. For an orifice $\rightarrow C_c = 0.62$ where as for mouth piece C_c

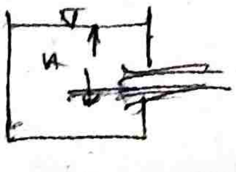
C_c = area of jet / area of mouth piece is equal.

Discharge through an internal mouth piece :- (Borda mouth piece)

There are 2 types 1. Internal mouth piece running full.

2. ~~External~~ Internal mouth piece running free.

① 

② 

Internal mouth piece running

→ Running full $C_c = 1$

$C_v = 0.707$

$C_d = C_c \times C_v = 0.707 \times 1$

$Q = C_d \times a\sqrt{2gH}$

$= 0.707\, a\sqrt{2gH}$

(2) Running free $C_c = 0.5,\ C_v = 1,\ C_d = C_c \times C_v$

$Q = C_d\, a\sqrt{2gH}$

$= 0.5 \times a\sqrt{2gH}$

orifice :-

1) Small orifice → $Q = C_d a\sqrt{2gH}$

2) Large orifice → $Q = \frac{2}{3} C_d b\sqrt{2g}\left[H_1^{3/2} - H_2^{3/2}\right]$

$C_d \to 0.64$; $C_c = 0.62$; $C_v = 0.95 - 0.98$

mouth piece:-

1) External → $Q = C_d a\sqrt{2gH}$ — $C_d = 0.855$; $C_c = 1$; $C_v = 0.855$

2) Internal → $Q = C_d a\sqrt{2gH}$

Running full → $0.707\, a\sqrt{2gH}$ · $C_d = 0.707$; $C_c = 1$; $C_v = 0.707$.

Running free → $0.5\, a\sqrt{2gH}$ $C_d = 0.5$ $C_c = 0.5$, $C_v = 1$

Time of Emptying a tank with an orifice (or) an mouth piece.

1) Constant cross sectional area

2) Hemispherical 3) Horizontal Circular.

1) A sharp edged orifice of 5cm discharges water under a head of 5m. Find the values of C_v, C_c, C_d. If $Q = 0.012\ m^3/sec$. The d of jet at vena contractor 4 cm. ?

$d = 0.05\,m$, $h = 5\,m$

Sol: given

dia. of orifice $= 0.05\,m$

Area " " $= \frac{\pi}{4} \times 0.05^2$

$= 1.96 \times 10^{-3}\,m^2$

dia. of vena contracta $= 0.04\,m$

Area " " $= \frac{\pi}{4} \times 0.04^2$

$= 1.25 \times 10^{-3}\,m^2$

$C_c = \frac{1.25 \times 10^{-3}}{1.96 \times 10^{-3}} = 0.637$

$Q = C_d\, a\sqrt{2gH}$

$C_d = \frac{0.012}{1.96 \times 10^{-3} \times \sqrt{2 \times 9.81 \times 5}} = 0.61$

$C_v = 0.973$

2) A jet of water issues from a sharp edged orifice under constant head of 0.51m. At a certain point issuing jet the horizontal & vertical coordinates measured from vena contracta are 0.406 & 0.085 m respectively. determine C_v & if $C_d = 0.62$

Sol:-

Given

$x = 0.406,\ y = 0.085$

$H = 0.51$

$C_v = ?$

$x = 0.406,\ y = 0.085$

$C_v = \sqrt{\frac{x^2}{4Hy}}$

$C_v = \sqrt{\frac{x^2}{4Hy}} = \sqrt{\frac{0.406^2}{4\times0.5\times0.085}}$

$C_v = 0.975$

$= 0.945$

$C_d = 0.62$

$C_d = 0.62 \Rightarrow C_c = \frac{0.62}{0.975} = 0.64$

$C_c = \frac{0.62}{0.975}$

5)* Water discharges at a rate of 98 lit/sec through 0.12 m as a vertical sharp edged orifice placed under a constant head of 18m. A point on the jet measured from vena contracta of the jet has coordinates 4.5m horizontal & 0.54m vertical. Find C_c, C_v, C_d & C_R for the orifice & power lost of orifice.

$Q = 98\times10^{-3}$

$d_1 = 0.12m$

Sol:- $C_R = \frac{1}{C_v^2} - 1$ & Power lost $= \gamma Q H (1 - C_v^2)$

$(x, y) = (4.5, 0.54)$

$C_v = \sqrt{\frac{x^2}{4Hy}} = \sqrt{\frac{4.5^2}{4\times18\times0.54}} = 0.722$

$C_v =$

$C_c = \frac{\pi}{4}\times(0.12)$

$Q_c = C_d a \sqrt{2gH} \Rightarrow C_d = 9.8\times10^{3} / \frac{\pi}{4}\times0.1^2\times\sqrt{2\times9.81\times18}$

$= 0.46$

$C_c = 0.64$

$Q = C_d a \sqrt{2gH}$

98×10^{-3}

C_d

$C_R = \left(\frac{1}{C_v^2} - 1\right) = 0.92$ & power lost $= 9810\times98\times10^{-3}\times18\times(1-0.7$

$= 8.28\,kN$

4) A reservoir discharges through a sluice 0.915 wide & 1.22 m deep. The top of opening is 0.61 m. below the water level in the bottom of the opening. Calculate discharge through the opening $C_d = 0.6$ (b) % error if the opening is treated as small orifice!

$$Q = \frac{2}{3} C_d b \left[H_2^{3/2} - H_1^{3/2}\right] \sqrt{2g}$$

$H_2 = 1.83,\ H_1 = 0.61$

$h = 1.22$
$H_2 = h + H_1$

$$= \frac{2}{3} \times 0.6 \times 0.915 \left[1.83^{3/2} - 0.61^{3/2}\right] \times \sqrt{2 \times 9.81}$$

$$= 3.2\ m^3/sec$$

$$Q = C_d\, a \sqrt{2gH} = 0.6 \times \frac{\pi}{4} (1.22)^2 \times \sqrt{2g \times 1.22}$$

NOTCH & WEIR $= 3.43\ m^3/sec$.

Notch:- A notch is an opening provided in the side of a tank, such that the water level is always below the top edge. Notch is a metallic plate in which an opening is cut and water must flow below the top edge. A notch is a laboratory device. It is used for measurement of discharge in an open channel.

Notches are classified

According to shape: Rectangular, triangular, Trapezoidal, Stepped notch

Parabolic notch

Notches are also classified based on effect of sheet of water emerging from the notch.

Notch without end contraction: it is possible if the width of the canal is equal to the length of the notch. Then there will be no end contractions. Such a notch is called "Suppressed notch".

Notch with end Contraction :- if the length of notch is less than the width of the canal, then water coming out of the notch will contract at both the edges because of this the effective length of the notch decreases. Discharge also decreases.

Weir :- A weir is a obstruction across a stream river (or) canal constructed with masonry (or) concrete, to raise the water level on up stream. So that water will flow by gravity into the irrigation ~~fetui~~ fields.

A weir is a mainly a diversion works and not a storage works, However its stores temporarily some quantity of water.

Weirs are also classified based on

Shape :- Rectangular, trapezoidal.

Weir are also classified based on the crest → Sharp crest, broad crest

Weirs are also classified based on flow :- Submerged, freely flowing.

Equation

Calculation of V_a :-

$$V_a = \frac{Q}{B(H+z)}$$

$B(H+z)$ is the area of flow in the channel

B channel width

z Height of crest

H head above the crest

$$V_a = \frac{Q}{B(H+z)} \quad V_a = \frac{Q}{B(H+z)} \quad V_a = \frac{Q}{B(H+z)}$$

In any problem, if velocity of approach is to be considered, first cal. the Discharge by neglecting velocity of approach.

$$Q_a = \frac{2}{3} C_d l \sqrt{2g}\, H^{3/2}$$

using this discharge, calculate velocity of approach.

$$V_a = \frac{Q}{B(H+z)}$$ & then cal. H_a $$H_a = \frac{V_a^2}{2g}$$

then calculate Discharge considering velocity of approach

$$Q_a = \frac{2}{3} C_d l \sqrt{2g} \left[(H+h_a)^{3/2} - h_a^{3/2} \right]$$

use this discharge and again calculate V_a, h_a & discharge

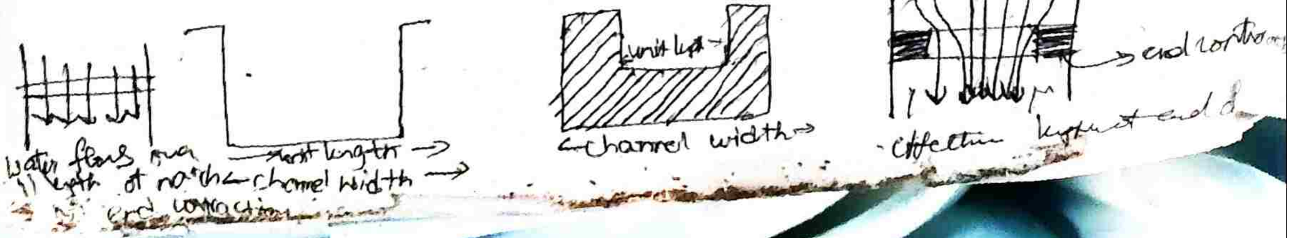
repeat this procedure till the discharge in the consecutive cycles is almost same.

Case (ii): Considering end contractions.

the above two equations are applicable to suppressed notch.

that means the no end contraction, crest length of the notch = width of the channel

If, the crest length is less, compared to channel width then end contractions have to be consider



Because of the end contractions the effective length is reduced

Francis from his experiments found that the reduction in crest length is reduced in the contraction is 0.1 . H

∴ The effective of end contraction is reduction in crest length.

$(l - 0.2H)$

$L.1 - 0.1nH$

$L_e = l - 0.1nH$

'n' no. of end contraction.

∴ The discharge also reduces due to end contraction.

$Q_a = \frac{2}{3} C_d (L - 0.1nH) \sqrt{2g} \, H^{3/2}$

Considering velocity of approach. $Q_a = \frac{2}{3} C_d (l - 0.1nH) \sqrt{2g} \left[(H + h_a)^{3/2} - h_a^{3/2} \right]$

$(H + h_a)$

* 1) Suppressed notch $Q = \frac{2}{3} C_d l \sqrt{2g} \, h^{3/2}$

2) Considering velocity of approach $Q = \frac{2}{3} C_d l \sqrt{2g} \left[(H + h_a)^{3/2} - h_a^{3/2} \right]$

3) Considering end contractions. $\frac{2}{3} C_d \sqrt{2g} \left((H + h_a)^{3/2} - h_a^{3/2} \right)$

(i) Suppressed notch. $Q = \frac{2}{3} C_d (l - 0.1nH) h^{3/2} \sqrt{2g}$

(ii) Velocity of approach $= \frac{2}{3} C_d \left(l - 0.1n (H + h_a) \right) \sqrt{2g}$

$\left[(H+h)^{3/2} - h^{3/2} \right]$

Generally C_d for rectangular notch will be 0.6 – 0.64

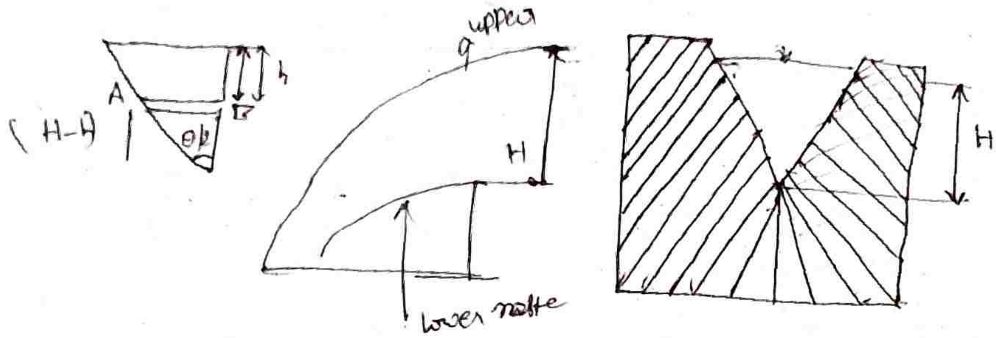
There are empirical formulae also to estimate the discharge through a rectangular notch.

* (i) Francis formula $Q = 1.84 L H^{3/2}$ $\frac{2}{3} C_d l \sqrt{2g} \, H^{3/2}$ → 0.623

(ii) Bazin's formula $Q_a = m L \sqrt{2g} \, H^{3/2}$

m = Bazin's coefficient $= \frac{2}{3} C_d = 0.405 + \frac{0.003}{H}$

Derive the equation for the discharge over a triangular notch (or) V-notch.

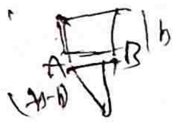


Consider a V-notch of angle θ. Let $\frac{\theta}{2}$ = semi angle.

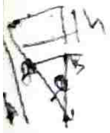
Let 'H' be the head over the vertex.

In order to determine the discharge, consider a horizontal strip of thickness dh at a height 'h' from the free surface.

$\tan\frac{\theta}{2} = \frac{AB}{H-h}$ area of figure $AC \cdot dh$ $\tan\theta/2 = \frac{AB}{(H-h)}$



$AB = (H-h)\tan\theta/2$ $AB = (H-h)$ $AC = 2AB$



$AC = 2AB$ $\tan\theta/2 = \frac{AB}{H-h}$

$= 2(H-h)\tan\theta/2$ $AB = \tan\theta/2(H-h)$

Area of strip $= 2(H-h)\tan\theta/2 \cdot dh$ $AC = 2AB$
$2(\tan\theta/2)(H-h)$

Velocity of flow through the strip $= \sqrt{2gh}$

∴ Discharge through the strip = A.V.

$= 2(H-h)\tan\theta/2 \cdot \sqrt{2gh}\, dh$

∴ total discharge over the V-notch $= \int_0^h 2(H-h)\tan\theta/2 \cdot \sqrt{2gh}$

$= 2\sqrt{2g}\tan\theta/2 \int_0^h H\sqrt{h} - h^{3/2} \cdot dh$

$= 2\sqrt{2g}\tan\theta/2 \left(H\left[\frac{h^{3/2}}{3/2}\right]_0^h - \left[\frac{h^{5/2}}{5/2}\right]_0^h \right)$

$= 2\tan\theta/2\sqrt{2g}\left[2H\left(\frac{h^{3/2}}{3}\right) - \frac{2h^{5/2}}{5}\right]$

$$= 2\tan\theta/2 \sqrt{2g} \left[\frac{2}{3} H^{5/2} - \frac{2}{5} H^{5/2}\right]$$

$$= 2\tan\theta/2 \cdot \sqrt{2g} \left[\frac{4}{15} H^{5/2}\right]$$

$$= \frac{8}{15} C_d \sqrt{2g} \tan\theta/2 \, H^{5/2}$$

$\tan\theta/2 = \frac{AB}{(H-h)}$

$AB = \tan\theta/2 (H-h)$

$\tan\theta/2 (H-h) dh$

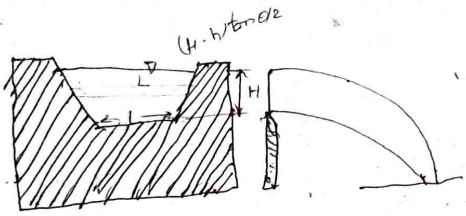
$\sqrt{2gh}$

$Q = A \cdot V$

In case of a right angled V notch $\theta = 90^\circ$

$\therefore$ discharge $= \frac{8}{15} C_d \sqrt{2g} H^{5/2}$

Trapezoidal notch:



Discharge through a Trapezoidal notch

= Discharge through a rectangular portion + Discharge through Triangular portion.

$$Q = \frac{2}{3} C_{d1} L \sqrt{2g} H^{3/2} + \frac{8}{15} C_d \sqrt{2g} \tan\theta/2 \cdot H^{5/2}$$

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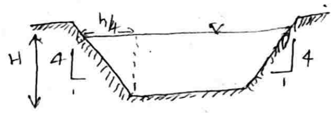
Cipolltti natch (or) weir: It is a Trapizoidal weir with side slopes one horizontal to 4 vertical



$\tan \theta/2 = H/4 / H$

$= 1/4$

$\theta/2 = 14^\circ 2'$

Show that the discharge over a cipolltti weir is equal to the dischage over a suppressed rectangular natch.

The Dischage through a cipolltti weir = Dischage through a rectangular portion + Dischage through Triangular

Dischage through rectangular portion $Q_1 = \frac{2}{3} C_d (L - 0.2H) \sqrt{2g} H^{3/2}$

(Considering end contraction)

" " Δ^{lar} portion $Q_2 = \frac{8}{15} C_d \sqrt{2g} \tan\theta/2 \, H^{5/2}$

For a cipolltti weir $\tan \theta/2 = 1/4$

$\therefore$ Dischage over a cipolltti weir $= \frac{2}{3} C_d (l - 0.2H) \sqrt{2g} H^{3/2}$

$+ \frac{2}{3} C_d \sqrt{2g} \, H^{5/2} \, 1/4$

$= \frac{2}{3} C_d l \sqrt{2g} H^{3/2} + \frac{2}{3} C_d (0.2) H \sqrt{2g} H^{3/2}$

$+ \frac{2}{3} C_d \sqrt{2g} \cdot H^{5/2} \cdot \frac{1}{4}$

$= \frac{2}{3} C_d l \sqrt{2g} H^{3/2}$

= Dischage through a suppressed rectangular natch

In case of a Cipolletti weir the loss in discharge due to end contraction is compensated by providing side slopes 1 Horizontal to 4 Vertical.

∴ Discharge over a Cipolletti weir = Discharge over a suppressed rectangular notch

$$= \frac{2}{3} C_d \, (L ~~- 0.1nH~~) \sqrt{2g} \, H^{3/2}$$

Find the effect on discharge due to error in measurement of head

Rectangular notch Discharge $= \frac{2}{3} C_d L \sqrt{2g} \, H^{3/2}$

$$Q = K H^{3/2} \qquad \left[\because K = \frac{2}{3} C_d L \sqrt{2g}\right]$$

$$dQ = K \cdot \frac{3}{2} H^{1/2} \cdot dH$$

$$\frac{dQ}{Q} = K \cdot \frac{3}{2} H^{1/2} \, dH \Big/ K H^{3/2}$$

$$\frac{dQ}{Q} = \frac{3}{2} \cdot \frac{dH}{H}$$

$$\boxed{\frac{dQ}{Q} \times 100 = \frac{3}{2}\left(\frac{dH}{H} \times 100\right)}$$

$\frac{2}{3} C_d L \sqrt{2g} H^{3/2}$

$Q = K H^{3/2}$

$dQ = K \cdot \frac{3}{2} \cdot H^{1/2} dH$

$\frac{dQ}{Q} = \frac{3}{2}$

$\frac{2}{3} C_d L \sqrt{2g} H^{3/2}$

$(L - 0.2h) \sqrt{2g} \, H^{3/2}$

If 1% error is committed in measurement of head the corresponding discharge $= \frac{3}{2}$

Triangular notch Discharge $= \frac{8}{15} C_d \sqrt{2g} \tan\theta/2 \cdot H^{5/2}$

$$= K \cdot H^{5/2}$$

$$dQ = \frac{5}{2} \cdot K \cdot H^{3/2} \cdot dH$$

$$\frac{dQ}{Q} = \frac{\frac{5}{2} \times K H^{3/2} \times dH}{K H^{5/2}}$$

$$\frac{dQ}{Q} = \frac{5}{2} \frac{dH}{H}$$

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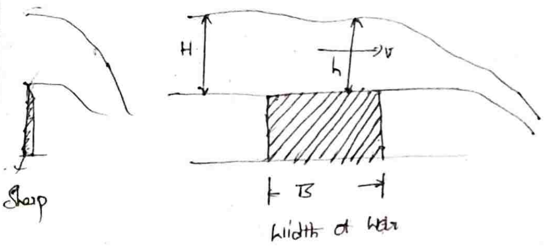
$$\frac{dQ}{Q} \times 100 = \frac{5}{2} \frac{dH}{H} \times 100$$

If 1% error ~~capacity~~ is ~~counted~~ in measurement of head corresponding discharge = 5/2

* Case (ii): Broad crested weir



A weir having a wide crest is called Broad crested weir. Water is in contact with the surface of weir.

Broad crested weir — $B > \frac{H}{2}$

Narrow (or) Sharp weir — $B < H/2$

$\frac{P_2}{w} + \frac{V_2^2}{2g}$ = H-h, $V_2 = \sqrt{2g(H-h)}$

Derivation for discharge over a broad crested weir.

Consider a broad crested weir of width 'B'. Let 'H' be the head on the upstream of the weir. Let 'h' be the head at the centre of the weir. Let 'v' be the velocity on the weir. Applying Bernoulli's equation between upstream and centre of weir.

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2$$

(Velocity of approach neglected)

$$0 + 0 + H = \frac{P_2}{w} 0 + \frac{v^2}{2g} + h$$

$$\frac{v^2}{2g} = H - h \quad (or) \quad v = \sqrt{2g(H-h)}$$

Discharge = $C_d \times$ Area of flow $\times$ velocity of flow

Area of flow = $(L \cdot h)$ [$\because L$ = length of the weir] Width of the channel

Velocity of flow = $\sqrt{2g(H-h)}$.

$\therefore$ Discharge = $C_d (Lh) \sqrt{2g(H-h)}$

In case of a broad crested weir the discharge always adjust it self to max. discharge.

$\therefore$ In order to get the max. discharge $= C_d (Lh) \sqrt{2g(H-h)}$

$$= C_d L \sqrt{2g(Hh^2 - h^3)}$$

as C_d, L & g are constants 'Q' will be Q_{max}. if $(Hh^2 - h^3)$ is maximum. The condition for maximum $(Hh^2 - h^3)$ is $\frac{d}{dh}(Hh^2 - h^3) = 0$

$$2Hh - 3h^2 = 0$$

$$2H\cancel{h} = 3h^2$$

$$\boxed{h = \frac{2}{3}H}$$ Condition for Q maximum

$$Q_{max} = C_d L \sqrt{2g\left(H \frac{4}{9}H^2 - \frac{8}{27}H^3\right)}$$

$$= C_d L \sqrt{2g\left(H^3\left[\frac{12-8}{27}\right]\right)}$$

$$= C_d L H \sqrt{H\, 2g\left(\frac{4}{27}\right)}$$

$$= C_d L H\sqrt{H}\,(1.704) = 1.704 \times C_d L H^{3/2}$$

$$Q_{max} = 1.704 \times C_d L H^{3/2}$$

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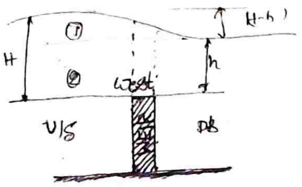
Derive the equation for discharge over a submerged weir or ground weir

Submerged weir: A weir is defined as submerged if the water level on both sides of the weir is above the crest of the weir



The discharge can be determined as follows:

The portion above the crest of the weir is divided into two parts

Q_1 the discharge due to a head of (H-h) which is flowing free.

Q_2 the discharge through the submerged portion

$Q_1 = \frac{2}{3} C_d L \sqrt{2g} (H-h)^{3/2}$

$Q_2 = C_d L h \sqrt{2g (H-h)}$

$Q_2 = C_d L h \sqrt{2g(H-h)}$

$Q = Q_1 + Q_2$

$= \frac{2}{3} C_d L \sqrt{2g} (H-h)^{3/2} + C_d L h \sqrt{2g(H-h)}$

$= \frac{2}{3} C_d L \sqrt{2g} [H-h]^{3/2} + C_d L \sqrt{2g}\, h \sqrt{(H-h)}$

***

1) What are the advantages of a V-notch over a Rectangular notch

Sol: i) V-notch is used for measuring small discharges accurately

ii) In the case of V-notch the emerging nappe is shape is constant for all discharges.

∴ C_d is fairly constant for all heads, but in a Rectangular notch C_d varies with head.

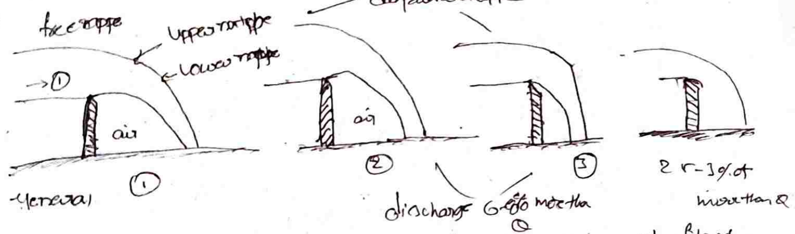
(iii) In the V-notch generally right angled V-notches are used the discharge formula is $Q = \frac{8}{15} C_d \sqrt{2g} \, H^{5/2}$ [$\tan\theta/2 = 1$]

(iv) In most of the cases in V-notch even if V_a is neglected the discharge will be fairly same.

(v) Ventillation of V-notch is not required.

2) What is Ventilation? What is the effect of ventillation.

A:



In the case of suppressed rectangular notch water flows over the crest the bottom is sealed, sides are sealed the top is also sealed. the air present below the lower nappe is slowly absorbed by water. The pressure below the lower nappe decreases. The lower nappe shift towards the notch the discharge increases. The formula that is derived does not hold good. therefore In order to see that the lower nappe doesn't shift. The space below the lower nappe is always filled with air for that ventillation of is necessary in case of suppressed rectangular notch. Ventillation means providing holes to the side walls of the channel for the portion below the lower nappe.

problem

1) Water flows over a rectangular weir of length 1 m. at a depth of 150 mm. and afterwards passes through a right angle triangular weir. Cd for rectangular & triangular weir are 0.62, 0.59 respectively. Find the depth over the triangular weir.

Sol:

$L = 1m$. $h = 150mm = 150\times10^{-3}m$

Cd for rectangular weir $= 0.62$

" " Tri " " $= 0.59$

$\theta = 90°$,

$L = 1m$

$d = 150$

Q for rectangular notch $= \frac{2}{3} Cd \cdot L\sqrt{2g}H^{3/2}$

$= \frac{2}{3}\times 0.62\times 1\sqrt{2\times9.8}\ (0.15)^{3/2}$

$= 1.8308\times(0.15)^{3/2}$

$=$ ~~0.9727~~ 0.10635

$Q_{triangular} = \frac{8}{15} Cd\sqrt{2g}\ \tan\frac{\theta}{2}\ h^{5/2}$

$0.9727 = \frac{8}{15}\times0.59\times\sqrt{2\times9.81}\times h^{5/2}$

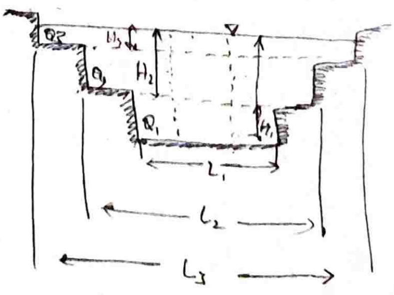
$h = 0.3572$

Stepped notch

$Q_1 = \frac{2}{3} Cd\ L_1\sqrt{2g}\left[H_1^{3/2} - H_2^{3/2}\right]$

$Q_2 = \frac{2}{3} Cd\ L_2\sqrt{2g}\left[H_2^{3/2} - H_3^{3/2}\right]$

$Q_3 = \frac{2}{3} Cd\ L_3\sqrt{2g}\left[H_3^{3/2}\right]$



$Q = Q_1 + Q_2 + Q_3$

2) Water flows through a right angled triangular weir first and then over a rectangular weir of 1m. width. Cd for triangular & rectangular are 0.6 & 0.7. If the depth over the triangular weir is 360mm. Find the depth of water over the rectangular weir.

Sol:

For triangular weir $C_d = 0.6$ $h = 360$ m.m.

$$Q = \frac{8}{15} \cdot C_d \cdot \sqrt{2g} \tan\theta/2 \cdot h^{5/2}$$

$$Q = \frac{8}{15} \times 0.6 \times \sqrt{2 \times 9.81} \times (3.6)^{5/2}$$

$= 34.845$

for rectangular weir $Q = \frac{2}{3} C_d L \sqrt{2g} \, (H)^{5/2}$

$$34.844 = \frac{2}{3} \times 0.7 \times 0.6 \times 1 \times \sqrt{2 \times 9.81} \times (H)^{5/2}$$

$H = 3.8$ m $(H)^{5/2} = 28.10$ $\Rightarrow (H)^{5} =$ ~~[illegible]~~ 789.71 $H =$ ~~[illegible]~~ 78

3) A rectangular notch 40c.m. long is used for measuring a discharge of 30l/sec. An error of 1.5m.m. was made, while measuring the head over the notch. Calculate the percentage error in the discharge. $C_d = 0.6$ $L = 0.4$

Sol:

For Rectangular notch $Q = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}$

$$\frac{dQ}{Q} = \frac{3}{2} \cdot \frac{dH}{H}$$ $$\frac{dQ}{Q} = \frac{3}{2} \cdot \frac{dH}{H}$$ $Q = 30 \times 10^{-3}$ $L = 0.4$m ~~$dQ = 1.5 \times 10^{-3}$ m~~ dQ $dH = 1.5$m

dQ = error in discharge

$\frac{dQ}{Q} \times 100$ = % error in discharge.

dH = error in head

$\frac{dH}{H} \times 100$ = % error in head.

$C_d = 0.6$ $L = 4$ m. $g = 9.81$, $dH = 1.5 \times 10^{-3}$ m.

$Q = 30$ lit/sec $= \frac{30}{1000} = 0.03 m^3/sec$.

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$$Q = C_d L \sqrt{2g}\, H^{3/2}$$

$$0.09 = 0.6 \times 4 \times \sqrt{2 \times 9.81}\, H^{3/2}$$

$$H = 0.1216$$

$$\frac{dQ}{Q} = \frac{3}{2} \times \frac{1.5 \times 10^{-3}}{0.1216}$$

$$\frac{dQ}{Q} = 0.01850$$

$$\%\ \text{error} = \frac{dQ}{Q} \times 100 = 1.8\%.$$

4) A right angled V-notch is used for measuring discharge of 30 lit/sec. An error of 1.5 m.m. was made while measuring the head over the notch. Cal: % error. $C_d = 0.62$

Sol:- For V-notch

$$\frac{dQ}{Q} \times 100 = \frac{5}{2} \times \frac{dH}{H} \times 100$$

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2}\, H^{5/2}$$

$Q = 0.03$, $C_d = 0.62$,

$$dH = 1.5\text{m.m.} = 1.5 \times 10^{-3}$$

$$0.03 = \frac{8}{15} \times 0.62 \times \sqrt{2 \times 9.81} \times H^{5/2}$$

$$H^{5/2} = \frac{0.03 \times 15}{(8 \times 0.62 \times \sqrt{2 \times 9.81})}$$

$$= \frac{0.03 \times 15}{(21.97)}$$

$$H^{5/2} = 0.02, \quad H = 0.211\ m$$

$$\frac{dQ}{Q} = \frac{5}{2} \times \frac{1.8\times10^{-3}}{0.211}$$

$$\frac{dQ}{Q} = 0.0177$$

% error = 0.0177 × 100 = 1.77 %

5) A rectangular weir of crest length 50 c.m. is used to measure the rate of flow of water in a rectangular channel of 80 c.m. wide and 70 c.m. deep. Determine the discharge in the channel if the water level is 80 m.m. above the crest of the weir. Take velocity of approach into consideration & $C_d = 0.62$

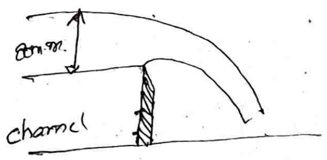
$L = 50$ c.m $= 50\times10^{-2}$ m

$l = 0.5$ m.

$l = 0.8$ m.

$Q = C_d$

$\left(\frac{Q}{M \times L}\right)$



Rectangular notch

Crest length = 50 c.m.

Channel dimension = 80 c.m. width and 70 c.m depth of water

Determine the discharge neglecting velocity of approach

$Q = \frac{2}{3} C_d L \sqrt{2g}\, H^{3/2}$ ← Without v.a.

$Q = \frac{2}{3} C_d L \sqrt{2g}\, \left((H+h_a)^{3/2} - h_a^{3/2}\right)$ ← With v.a.

$$Q = \frac{2}{3} C_d L \sqrt{2g}\, H^{3/2}$$

$$= \frac{2}{3} \times 0.62 \times 0.5 \times \sqrt{2\times9.81}\,(0.080)^{3/2} = 0.0207\ m^3/sec$$

Now considering the velocity of approach $V_a = \frac{Q}{\text{Area of flow in channel}}$

$$V_a = \frac{0.0207}{.8\times0.7} = 0.03696$$

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$h_a = \frac{V_a^2}{2g} = \frac{0.03696}{2 \times 9.81} = 6.97 \times 10^{-5}\, m.$

Discharge $Q = \frac{2}{3} \times C_d L \sqrt{2g} \left((H + h_a)^{3/2} - h_a^{3/2} \right)$

$= \frac{2}{3} \times 0.62 \times 0.5 \times \sqrt{2 \times 9.81} \left((0.080 + 6.97 \times 10^{-5})^{3/2} - (6.97 \times 10^{-5})^{3/2} \right)$

$= \left(5.133 \times 10^{-4} - 3.38 \times 10^{-13} \right)$

$= 0.62074$

15.10.09

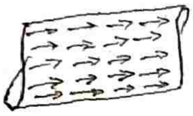
Flow through pipes

** Laminar flow through pipes :- this topic is also called as viscous flow. [velocity law]

possible Reynolds number < 2000

$\frac{\rho V d}{\mu} < 2000$

Laminar flow is possible if μ is high, V is very low }



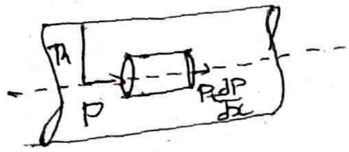
It is defined as the flow in the form of a lamine (plates (or) layers)

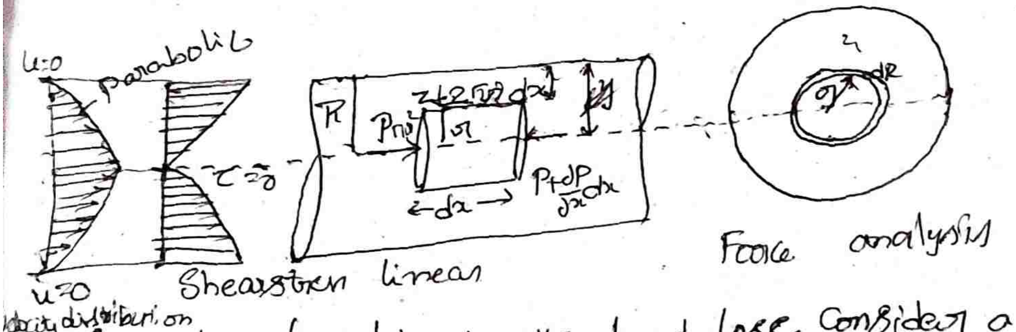
the flow takes place in st. way; fluid partical travels in straight line

Derivation of Equation for head loss in case of

Laminar flow through a pipe: Derive Hagen poisenile equation

Consider a horizontal pipe of radius R. The viscous fluid is flowing from left to right. In order to determine the head loss.





velocity distribution parabolic. In order to determine the head loss, consider a cylindrical element of radius r length dx concentric with the axis of the pipe. The forces acting on the element are

(i) Pressure force.

(ii) Shear force

Shear stress distribution:-

If p is the pressure intensity on the left side of the element

pressure force on left side = $P(\pi r^2)$

the pressure intensity on the right hand side of x = $\left(P + \frac{\partial P}{\partial x} dx\right)$

Pressure force on the right hand side = $\left(P + \frac{\partial P}{\partial x} dx\right)(\pi r^2)$

If 'τ' is the shear stress, the shear force = $\tau(2\pi r)\,dx$

The element is in equilibrium and hence the summation of forces in the direction is zero ∵ flow is steady & uniform. a = 0]

[N - II F = ma = 0]

$$P(\pi r^2) - \left(P + \frac{\partial P}{\partial x}dx\right)\pi r^2 - \tau\, 2\pi r\, dx = 0$$

$$\tau\, 2\pi r\, dx = -\left[P + \frac{dP}{dx}dx\right]\pi r^2 + P\pi r^2$$

$$\tau\, 2\pi r\, dx = \pi r^2\left[-\frac{\partial P}{\partial x}dx\right]$$

$$2\tau\, dx = -r\,\frac{\partial P}{\partial x}dx$$

$$\tau = -\frac{1}{2}\,\frac{r\,\partial P}{dx}$$

$$\tau = -\frac{1}{2}\,\frac{r\,\partial P}{\partial x}$$

$$\boxed{\tau = \left(-\frac{\partial P}{\partial x}\right)\frac{r}{2}}$$

'τ' varies linearly with 'r'. when $r = 0$; $\tau = 0$

$r = R$; $\tau_{max} = \left(\frac{\partial P}{\partial x}\right)\frac{R}{2}$

Shear stress distribution is linear.

Velocity distribution: According to Newton's law of viscosity

$$\tau = \mu\left[\frac{du}{dy}\right]$$

Here 'y' = distance from fixed plate to moving element

$y = (R - r)$

$dy = -dr$

$$\therefore \quad \tau = -\mu \frac{\partial u}{\partial r}$$

Already we know $\tau = -\frac{\partial p}{\partial x} \cdot \frac{r}{2}$

Equating

$$-\mu \cdot \frac{du}{dr} = -\frac{\partial p}{\partial x} \cdot \frac{r}{2}$$

$$\frac{du}{dr} = \frac{1}{\mu} \cdot \frac{\partial p}{\partial x} \cdot \frac{r}{2}$$

Integrating $\quad u = \frac{1}{\mu} \cdot \frac{\partial p}{\partial x} \frac{r^2}{4} + c$

at the boundary $r = R$; $u = 0$ $\qquad u = \frac{1}{4\mu}\left[\frac{-\partial p}{\partial x}\right](R^2 - r^2)$

$$0 = \frac{1}{\mu} \cdot \frac{\partial p}{\partial x} \cdot \frac{R^2}{4} + c$$

$$c = -\frac{1}{4} \cdot \frac{\partial p}{\partial x} \cdot \frac{R^2}{\mu}$$

$$u = \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) r^2 - \frac{1}{4\mu} \cdot \frac{\partial p}{\partial x} R^2$$

$$\therefore \quad u = \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right)\left[R^2 - r^2\right]$$

$u \propto r^2$; the velocity distribution, in the case of laminar flow is <u>Parabolic</u>.

at $r = 0$; $u = u_{maximum} = \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) R^2$

<u>Average velocity</u>: Average velocity $= \frac{\text{discharge}}{\text{Area of pipe}} = \frac{Q}{\pi R^2}$

Determination of discharge :- for determining discharge consider a circular element of thickness dr at a distance r from the centre. Let u is the velocity at this radial distance.

The discharge through the ring dQ = velocity (u) × Area of ring $(2\pi r dr)$

~~dQ = Au.~~

$$dQ = u(2\pi r dr)$$

$$dQ = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)[R^2 - r^2][2\pi r dr]$$

Integrating within the limits $0 - R$, Q can be obtained

$$Q = \int_0^R \frac{1}{4\mu}\left[-\frac{\partial p}{\partial x}\right][R^2 - r^2][2\pi r dr]$$

$$= \frac{2\pi}{4\mu} - \left[\frac{\partial p}{\partial x}\right] \int_0^R (R^2 - r^2)\, r dr$$

$$= -\frac{\pi}{2\mu}\left[\frac{\partial p}{\partial x}\right] \int_0^R (R^2 r - r^3)\, dr$$

$$= \frac{-\pi}{2\mu}\left[\frac{\partial p}{\partial x}\right]\left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]_0^R$$

$$= -\frac{\pi}{2\mu}\left(\frac{\partial p}{\partial x}\right) \cdot \frac{R^4}{4} = -\frac{\pi}{2\mu}\left[\frac{\partial p}{\partial x}\right]\frac{R^4}{4}$$

∴ Average velocity $= \frac{Q}{\pi R^2}$

$$= \frac{\frac{\pi}{2\mu}\left[\frac{\partial p}{\partial x}\right]\frac{R^4}{4}}{\pi R^2} = -\frac{1}{8\mu}\left(\frac{\partial p}{\partial x}\right)R^2$$

$$\therefore \bar{u} = -\frac{1}{8\mu}\left[\frac{\partial p}{\partial x}\right]R^2$$

Relation between Average velocity & max. velocity in cased laminar flow:

$$u_{max} = \frac{1}{4\mu}\left[\frac{-\partial p}{\partial x}\right] \cdot R^2, \quad \bar{u} = \frac{1}{8\mu}\left[\frac{-\partial p}{\partial x}\right] \cdot R^2$$

$$\bar{U} = \frac{1}{2} u_{max}$$

Average velocity = $\frac{1}{2}$ of max. velocity

Head loss:

$$\text{Average velocity} = \frac{1}{8\mu}\left[-\frac{\partial p}{\partial x}\right]R^2$$

$$-\frac{\partial p}{\partial x} = \frac{8\mu \cdot \bar{u}}{R^2}$$

If P_1 & P_2 are the pressures at section ① & section ② and the distance between them is L then $-\frac{\partial p}{\partial x} = \frac{P_1 - P_2}{L}$

$$\left[\frac{P_1 - P_2}{L}\right] = \frac{8\mu \cdot \bar{u}}{R^2} \Rightarrow P_1 - P_2 = 8\mu\bar{u} \cdot \frac{L}{R^2}$$

Pressure of head difference (or) loss of pressure head

Divide both sides by 'w':

$$\frac{P_1 - P_2}{W} = 8\mu \cdot \bar{u} \cdot \frac{L}{WR^2} \text{ (or) head loss} = 8\mu \cdot \bar{u} \cdot \frac{L}{WR^2}$$

as $D = 2R$ Head loss $= 32\bar{u}\mu \cdot \frac{L}{WD^2}$

$$\boxed{\text{Head loss} = 32\bar{u}\mu \cdot \frac{L}{\rho g D^2}}$$

This equation is called Hagen poiseuille equation

This Equation gives the head loss in the case of Laminar flow through a Circular pipe.

Problem:

1) A Crude oil of viscocity 0.97 poise and relative density 0.9 is flowing through a horizontal circular pipe of dia 100m.m and of length 10m. Calculate the difference of pressure at the two ends. if 100k.g. of oil is collected in 30 sec.

Sol: μ: 0.97 poise $= 0.97\times 0.1$ N.sec/m²

$= 0.097$ N.sec/m²

$\rho = 0.9$

$\therefore \rho = 0.9\times 1000 = 900$ kg./m³

$\mu = 0.97\times 10^{-4}$ N.sec/m²

$d = 0.1m$.

$L = 10m$

Pipe diameter $D = 100$ m.m. $= 0.1m$

length $= 10m$.

$P_1 - P_2 = \frac{1}{8\mu}$ ~~$\left(\frac{dp}{dx}\right)^2$~~

$= \frac{8\mu\bar{u}\,L}{R^2}$

$P_1 - P_2 = ?$

If 100 k.g. of oil is collected in 30 sec

Mass = 100 k.g.

$\frac{Mass}{Second} = \frac{100}{30} = 3.33$ kg/sec.

$\frac{Mass}{Second} = \rho \cdot$ discharge

weight/sec $= w.Q$

discharge $= \frac{3.33}{0.9} = 3.7 = 0.0037$ m³/sec

$P_1 - P_2 = 8\mu.\bar{u}\ \frac{L}{R^2}$

$\bar{u} = \frac{discharge}{Area} = \frac{0.0037}{\pi(0.1)^2}\times 4 = 0.47109$

$P_1 - P_2 = 8(0.097)(0.47109)\times\frac{10}{(0.05)^2}$

$P_1 - P_2 = 1461.98 \, N/m^2$

A fluid of $\mu = 0.7 \, N\cdot sec/m^2$ and $\rho = 1.3$ is flowing through a circular pipe of dia 100 mm. the max. shear stress at the pipe wall is given as 196.2 N/m². Find the (i) pressure gradiant
(ii) Average velocity
(iii) Reynolds number.

$\mu = 0.7 \, N\,sec/m^2$ $\quad \rho = 1.3 \times 1000 = 1300 \, kg/m^3, \; w = 1.3 \times 9810$

$\tau_{max} = 196.2 \, N/m^2$ $\qquad$ (i) $\tau = \left[\frac{-\partial p}{\partial x}\right]\frac{R}{2}$

$u_{max} = \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right)R^2$ $\qquad$ $\frac{-\partial p}{\partial x} = \frac{2\tau}{R}$

$\frac{\partial p}{\partial x} =$ $\qquad = \frac{2 \times 196.2}{0.05}$

$\frac{\partial p}{\partial x} = -7848 \, N/m^2 \text{ per m}$

(ii) Average velocity $\quad \bar{u} = -\frac{1}{8\mu}\left[\frac{\partial p}{\partial x}\right]R^2$

$= -\frac{1}{8 \times 0.7}\left[7848\right) 0.05^2$

$\bar{u} = 3.503 \, m/sec.$

(iii) Reynolds number $= \frac{\rho v d}{\mu}$ $\quad \frac{\rho \bar{u} d}{\mu}$

$= \frac{1300 \times 3.503 \times 0.1}{0.7}$

$= 650.65$

3) What power is required per k.m. of a pipe line to over come the viscous resistance to flow of Glycerin through a horizontal pipe of diameter 100 m.m. at rate of 10 lit/sec

take $\mu = 8$ poise and k. viscosity = 6 stokes

stokes = 10^{-4} m^2/s

Sol:

power lost = $\omega Q h_L$

h_L = head loss

$h_L = \frac{32 \mu \bar{u} \cdot L}{\rho g d^2}$ watts

$V = \sigma w$
$= 0.05$

$d = 0.1 m$

$\mu = 8 \times 0.1 = 0.8 N sec/m^2$

$\nu = \frac{\mu}{\rho}$

$\rho = \frac{\mu}{\nu} = \frac{0.8}{6 \times 10^{-4}} = 1333.3$ kg/m^3 = ρ

$w = \rho g$ = 1333.3 × 9.81

$\rho g = 13079.673$ N/m^3

L = k.m. = 1000 m

Q = 10 lit/sec = $10 \times 10^{-3} m^3/s$ = $0.01 m^3/s$

At what distance from the centre of the pipe the Average velocity occurs.

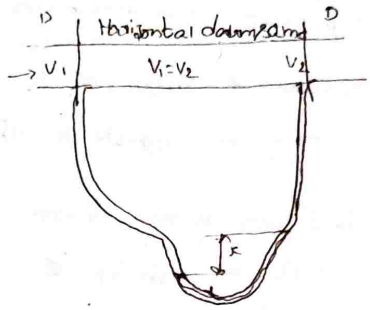
$\bar{u} = \frac{Q}{Area} = \frac{0.01}{\frac{\pi}{4} \times (0.1)^2} = 1.273 m/s$

power required $= wQh_f = \rho g Q h_f = \rho g Q \cdot \frac{32 \mu \bar{u} L}{\rho g D^2}$

$= 13079.673 \times 0.01 \times \frac{32(0.8) \times 1.273 \times 1000}{9810 \times (0.1)^2}$

= 32588.8 watts

Flow through pipes in general:



According to Bernoulli's equation

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$\frac{P_1}{w} = \frac{P_2}{w}; \quad V_1 = V_2; \quad z_1 = z_2$$

Deflection should be zero.

But the deflection in manometer is not zero.

There is loss of energy.

Bernoulli's equation is applicable to real fluids.

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2 + h_{loss}$$

Headloss (energy loss) is taken place when a real fluid flow through pipe.

There are two types of losses.

1) Major loss → due to friction.

2) Minor loss →
1) Loss due to sudden expansion of pipe
2) " " at the entrance of pipe.
3) " " sudden contraction " "
4) " " at the exit " "
5) " " gradual expansion " "
6) Loss " pipe fitting
7) " " obstruction in pipe.

In a long pipe, friction loss is very high and minor losses are negligible.

That's why majority of cases we neglect minor losses because the magnitude of friction loss is very high compared the minor loss.

If the pipe length small, minor losses becomes significant and in that case they cannot be neglected.

Estimation of major loss i.e., friction loss

(i) This loss depends on the type of flow. i.e, Laminar flow(or) Turbulent flow

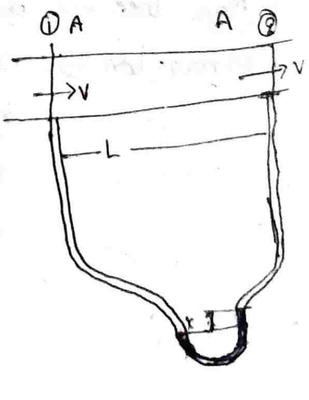
that depends on Reynolds number.

(ii) If the Reynolds no. < 2000 that flow is Laminar and use Haigen poisseille's equation for determining major loss i.e., loss due to friction. and the equation head loss $H = \frac{32 \mu V L}{\rho g d^2}$

The Reynolds no. > 4000. The flow becomes turbulent flow and for turbulent flow the head loss due to friction is estimated by "Darcy - Weisbach equation" which is stated as Head loss friction $h_f = \frac{f L V^2}{2 g d}$

Derivation :

Consider a horizontal pipe line of cross sectional area 'A' and velocity 'v'.



Consider 2 sections ① & ② seperated by a distance 'L'. applying the Bernoulli equation between the two sections

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2 + h_f$$

$V_1 = V_2$, $z_1 = z_2$ (horizontal)

$$\frac{P_1}{w} = \frac{P_2}{w} + h_f$$

$$h_f = \left[\frac{P_1 - P_2}{w}\right] \rightarrow ①$$

Let P_1 be the pressure intensity at ① and
P_2 " " " " " ②

$$P_1 A = P_2 A + \text{frictional resistance} \rightarrow ②$$

ℓbw(r)
ℓbw

Let f' be the friction produce per unit area at unit velocity. P- Perimeter L-length $f'(PL)v^n$

$\therefore$ frictional resistance $= f'(PL)\, v^n$

$$P_1 A = P_2 A + \text{frictional resistance}$$

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2$$

$V_1 = V_2 ; z_1 = z_2$

$h_f = \frac{P_1 - P_2}{w}$

use

the

$$(P_1 - P_2) A = f' P L v^n$$

$$P_1 - P_2 = \frac{f' P L v^n}{A}$$

$P_1 A = P_2 A +$ frictional

$$\frac{P_1 - P_2}{w} = \frac{f' P L v^n}{w A}$$

$w = \rho g$

$A = \pi D^2/4 / (\pi D)$

$$\frac{P_1 - P_2}{w} = \frac{f' L v^n}{w (A/P)}$$

$R = D/4$

$\frac{4f'}{w} = \frac{f}{2g}$

A/P is called as hydraulic mean depth (or) hydraulic radius $= \dfrac{\text{Area of cross section (or) wetted area}}{\text{Wetted perimeter}}$

$\frac{f(A)}{(\pi D)} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4}$

$$\frac{\pi D^2/4}{(\pi D)} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4}$$

$h_f = \frac{4 f' L v^2}{2 g d}$ $R = A/P$

$\frac{4f'}{w} = \frac{f}{2g}$

$$\frac{P_1 - P_2}{w} = \frac{f' L v^n}{w (P/L)} = \frac{4 f'}{w} \frac{L v^n}{D}$$

$\frac{4 f' L v^n}{w D}$

In case of turbulent flow, the n-values varies between 1.72 - 2 and the max. value 2. while estimating loss we use always max. value, $\therefore$ substitute n as 2

$\frac{4f'}{w} = \frac{f}{2g}$

Sub. $\frac{4f'}{w} = \frac{f}{2g}$

$\frac{4f'}{\rho g} = \frac{f}{2g}$ $f' = \frac{f \rho}{8}$

$$\therefore \frac{P_1 - P_2}{w} = \frac{f}{2g} \frac{L v^2}{D} = \frac{f L v^2}{2 g D}$$

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				Extremely informative
				Excellent
				Excellent

and we know that $\frac{P_1 - P_2}{w} = h_f$ from equation ①

$$\therefore h_f = \frac{fLV^2}{2gD}$$

$h_L = \frac{(V_1 - V_2)^2}{2g}$

In this equation 'f' is called as friction factor. The value of 'f' varies from ~~0.001 - 0.004~~ 0.01 - 0.04

In some text books, ~~the~~ friction loss equation is given as

$$h_f = \frac{4fLV^2}{2gD}$$

that f means coefficient of friction and its value will be 0.001, 0.004, 0.005, 0.007

The head loss due to friction in the turbulent flow is given by darcy - weisback equation $h_f = \frac{fLV^2}{2gD}$

We know, for laminar flow head loss $= \frac{32 \mu VL}{\rho g D^2}$ ← Hagen poiseuille equation

equating the above two equation

$$\frac{fLV^2}{2gD} = \frac{32 \mu VL}{\rho g D^2}$$

$f = \frac{h_f}{...}$

$$f = \frac{64 \mu}{\rho D V}$$

$$f = \frac{64 \mu}{\rho D V}$$

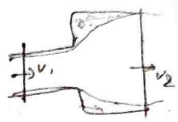
** For laminar flow $f = \frac{64}{Re}$

∴ In case of laminar flow friction factor only depends on Reynolds number.

Minor losses:

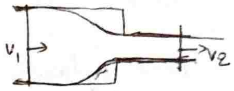
1 Loss due to sudden expansion of the pipe

$$h_L = \frac{(v_1 - v_2)^2}{2g}$$



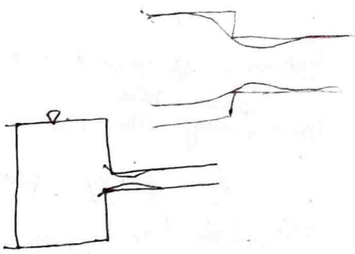
2. Loss due to ~~and~~ sudden contraction of pipe

$$h_L = \frac{0.5 v_2^2}{2g}$$



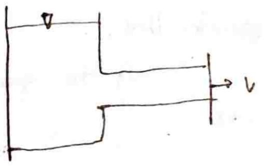
3. Loss due to at the entrance of pipe

$$h_L = 0.5 \frac{v^2}{2g}$$



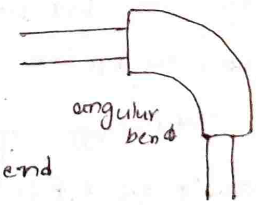
4) Loss due to at the exit of the pipe

$$h_L = \frac{v^2}{2g}$$



5) Loss due to bends

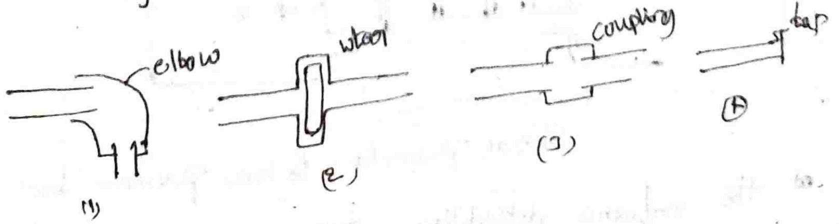
$$h_L = \frac{k \cdot v^2}{2g}$$



k depends on radius of curvature of bend

6) Loss due to various pipe fitting

$$h_L = \frac{k \cdot v^2}{2g}$$



7) loss due to obstruction

$$h_L = \frac{V^2}{2g}\left[\frac{A}{c_c(A-a)} - 1\right]$$

all the minor losses due to change in magnitude (or) direction of the velocity.

1) Define Hydraulic gradient line and energy gradient line.
HGL EGL

Ans. Hydraulic gradient line:- The line joining the various pressure head at various points along the pipe is called H.G.L.

Line joining pressure heads.

Piezometric line:- Line joining the piezometric head {pressure head + Datum head}

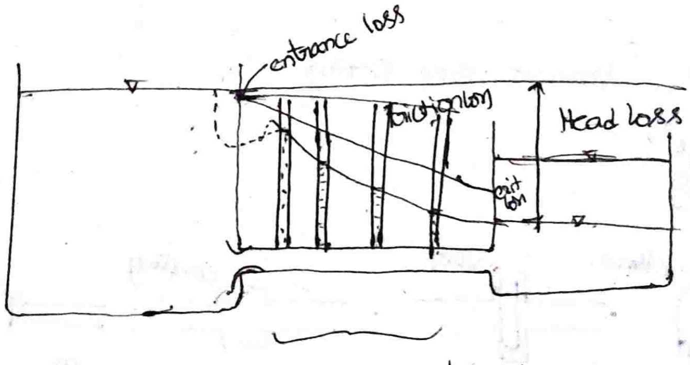
ie, $\left(\frac{P}{w}+z\right)$ at various points along the pipe line is called Piezometric line.

If the pipeline is horizontal The HGL & piezometric line coincides.

EGL:- The Line joining the total energy at various points along the pipe line is called E.G.L.

Total energy = pressure head + velocity head + Datum head

Draw H.G.L. & E.G.L.

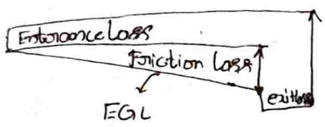


Insert piezometer to know pressure head

at the entrance dotted line reason is vena contracta forms and due to that pressure suddenly drops. The sudden dropped pressure is shown by

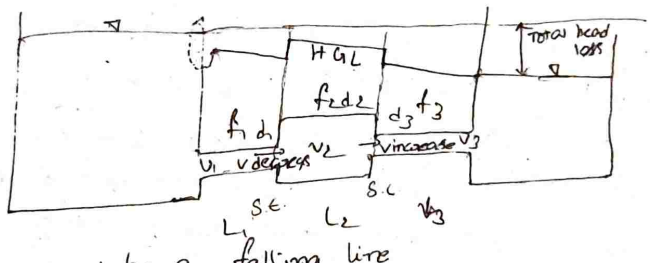
The diff. in water level in two tanks indicates the total head loss. = entrance loss + friction loss + exit loss

$$= 0.5\frac{V^2}{2g} + \frac{fLV^2}{2gD} + \frac{V^2}{2g}$$



$\frac{(V_1 - V_2)^2}{2g}$

$0.5\frac{V_2^2}{2g}$

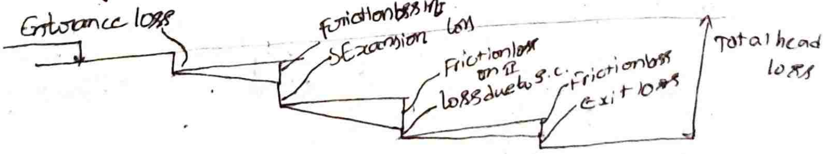


$0.5\frac{V^2}{2g}$

$\frac{V^2}{2g}$

① HGL need not be a falling line

① H.G.L. may rise (or) fall depending on pressure at a point.



$$\text{Total head loss } (H_L) = \frac{0.5V_1^2}{2g} + \frac{f_1 L_1 V_1^2}{2g d_1} + \frac{(V_1 - V_2)^2}{2g} + \frac{f_2 L_2 V_2^2}{2g d_2} + \frac{0.5V_3^2}{2g} + \frac{f_3 L_3 V_3^2}{2g d_3} + \frac{V_3^2}{2g}$$

Neglecting minor losses

$$\text{Total head loss } (H_L) = \frac{f_1 L_1 V_1^2}{2g d_1} + \frac{f_2 V_2^2 L_2}{2g d_2} + \frac{f_3 L_3 V_3^2}{2g d_3}$$

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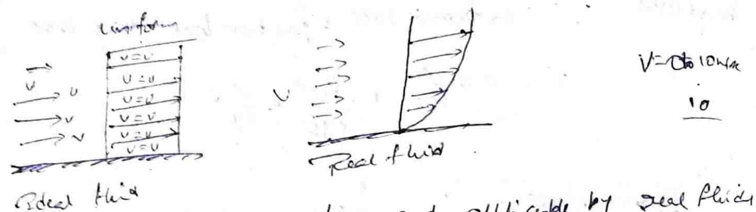
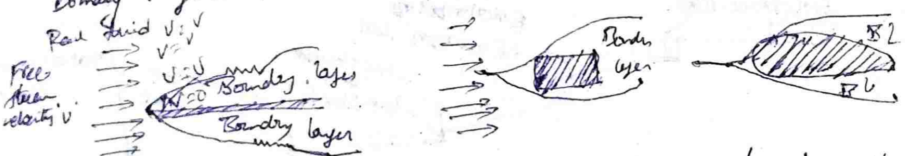
Boundry layer theory

The velocity distribution for ideal and real fluids



The boundry layer theory holds good applicable by real fluids

When a real fluid flows past a solid boundry a layer of fluid which comes incontact with the boundry adheres to the boundry this is called No slip condition that means at the boundry. The fluid layer is retarded. This retarded layer of fluid produces retardation in the adjacent fluid layer. A small region is developed in the immediate vicinity of the boundry where the velocity changes from 0 to V (free stream velocity) & This region is called boundry layer.



When a real fluid flows past a solid boundry the flow above the boundry is divided into two portions

1) flow with in the boundry layer.
2) " outside " " ".

Flow with in the boundry layer:- The velocity changes from 0 at the boundry to free stream velocity V. This change occurs in a relatively very short distance measured perpendicular to the boundry. Therefore velocity gradient is very high. ∴ Even though viscosity is very small. The shear stress $\tau = \mu \cdot \frac{du}{dy}$ is significant. Resistance to flow is also significant.

outside the boundary layer. The velocity is same

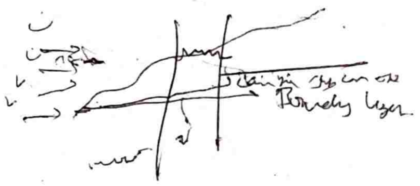
velocity gradient 0. ∴ $\tau = 0$

∴ Resistance to flow is 0 the fluid behaves as ideal fluid

∴ Real fluid effect (δ) limited to a small region adjacent to the boundary which is called as Boundary layer.

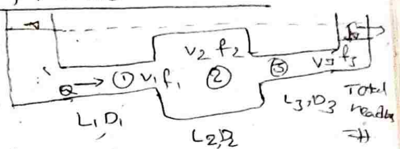
Q. Explain the concept of boundary layer on a flat plate?

Consider a flat plate which kept at 0° angle of incidence to the flow. Let 'v' be the velocity of flow when the flow takes place on the start of a boundary layer formed



(Pipes in Series)
(or)

Compound pipes:- If pipes of different lengths and diff. diameters are connected end to end then the pipes are said to be compound pipes. In this case



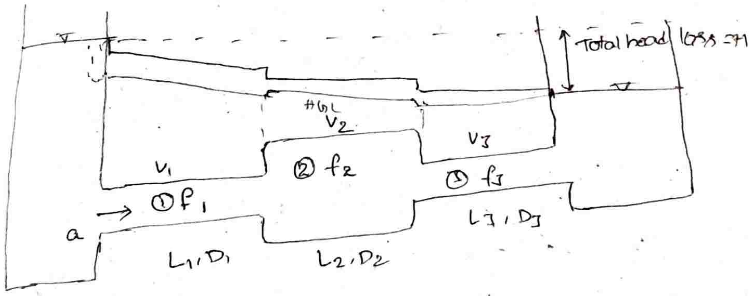
(i) the discharge is constant.

(ii) The total head loss = Sum of the head losses

Total head loss = difference in water level

= Entrance loss + Friction loss in ① pipe + loss due to sudden expansion + Friction loss in ② pipe

+ loss due to sudden contraction + Friction loss in ③ pipe + Exit loss



$$H = \frac{0.5V_1^2}{2g} + \frac{f_1 L_1 V_1^2}{2gD_1} + \frac{(V_1 - V_2)^2}{2g} + \frac{f_2 L_2 V_2^2}{2g D_2} + \frac{0.5V_3^2}{2g} + \frac{f_3 L_3 V_3^2}{2gD_3} + \frac{V_3^2}{2g}$$

$Q = A_1 V_1 =$

Neglecting minor losses

$$H = \frac{f_1 L_1 V_1^2}{2gD_1} + \frac{f_2 V_2 L_2^2}{2gD_2} + \frac{f_3 L_3 V_3^2}{2gD_3}$$

The main question in the pipes in series is determination of discharge.

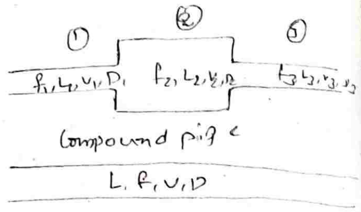
Use the principle.

Total head loss = Sum of the head losses unless otherwise mentioned neglect minor losses use the Continuity equation $Q = A_1V_1 = A_2V_2$

Write V_1 (or) V_2 in terms of Q and solve for Q

→ Define equivalent pipe ?

Ans. A compound pipe (of diff. diameters & diff. lengths are connected to end to end - pipes in series) is replaced by a pipe of uniform diameter. This uniform diameter is called as equivalent pipe. If the head loss in the compound and the head loss in the uniform diameter pipe are same (or) equal.



Head loss in compound pipe =

entrance loss + friction loss in ① pipe + loss due to sudden expansion + friction loss in ② pipe + loss due to sudden contraction + friction loss in ③ pipe + exit loss

$$= \frac{0.5V_1^2}{2g} + \frac{f_1L_1V_1^2}{2gD_1} + \frac{(V_1-V_2)^2}{2g} + \frac{f_2L_2V_2^2}{2gD_2} + \frac{0.5V_2^2}{2g} + \frac{f_3L_3V_3^2}{2gD_3} + \frac{V_3^2}{2g}$$

Head loss in equivalent pipe $= \frac{0.5V^2}{2g} + \frac{fLV^2}{2gD} + \frac{V^2}{2g}$

Neglecting minor losses for equivalent pipe

$$\frac{fLV^2}{2gD} = \frac{f_1L_1V_1^2}{2gD_1} + \frac{f_2L_2V_2^2}{2gD_2} + \frac{f_3L_3V_3^2}{2gD_3}$$

$$h_L = \frac{fLV^2}{2gD}$$

This can be simplified as follows :

i) f

① $f = f_1 + f_2 + f_3$

②
$$\frac{LV^2}{D} = \frac{L_1V_1^2}{D_1} + \frac{L_2V_2^2}{D_2} + \frac{L_3V_3^2}{D_3}.$$

According to continuity Equation $Q = A_1V_1 = A_2V_2 = A_3V_3$

$V = \frac{Q d^2}{\pi 4}$, $V_1 = \frac{Q D_1^2}{4\pi}$, $V_2 = \frac{Q D_2^2}{4\pi}$, $V_3 = \frac{Q V_3^2}{4\pi}$

$$\frac{L\,Q d^2}{4\pi D} + \frac{L_1\,Q d_1^2}{4\pi D_1} + \frac{L_2\,Q d_2^2}{4\pi D_2} + \frac{L_3\,Q d_3^2}{4\pi D_3}$$

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\frac{\pi}{4} D_1^2} = \frac{4Q}{\pi D_1^2}$$

Ily

$$\frac{L}{D}\left(\frac{4Q}{\pi D^2}\right)^2 = \frac{L_1}{D_1}\left[\frac{4Q}{\pi D_1^2}\right]^2 + \frac{L_2}{D_2}\left(\frac{4Q}{\pi D_2^2}\right)^2 + \frac{L_3}{D_3}\left(\frac{4Q}{\pi D_3^2}\right)^2$$

$$\boxed{\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}}$$

Dupiut equation of equivalent pipe.

$L = L_1 + L_2 + L_3$

knowing $L_1, D_1, L_2, D_2, L_3, D_3$

→ D can be calculated.

Problem ①

Determination of equivalent pipe of diameter

Pipes in series (i) Compound pipe

(ii) Q: Constant

(iii) Head loss = sum of Headlosses

(iv) Equivalent pipe =

(v) $\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}$

$L = L_1 + L_2 + L_3$

Pipes in parallel: pipes are Joined in ||el to increase the discharge.

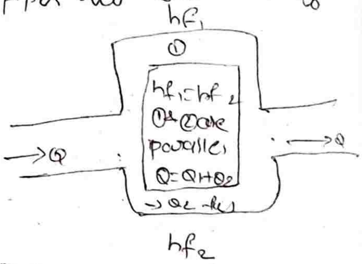
sum of the discharges

In case of parallel pipes Q = discharge (sum of the discharges in ||el pipes)

the in pipes

Head loss is same in ||el pipes.

Pipes are said to be connected in parallel if the pipe divides into two parallel pipes two branches to increase the discharge

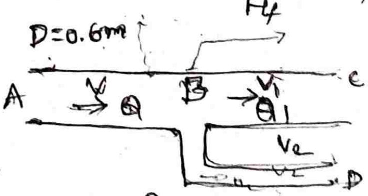


A pipe line of 0.6m dia is 1.5 k.m. long. To increase the discharge another pipe of the same diameter are introduced parallel to the first in the second half of its length. neglecting minor losses. Find the increase in discharge if $f = 0.04$ The total head loss at inlet = 300 m. $h_f = 0.3$

Given data, $D = 0.6m$, $L = 1.5\ km = 1500\ m$

$L_1 = 750m$ $f = 0.04$, $h = 0.3\ m$, $h_f = 0.3m$

1) Without parallel pipe what is discharge



2) with parallel pipe what is the discharge than increase (or) decrease

D

$h_f = \frac{fLV^2}{2gD}$

1)

1) Total head loss $H_f = \frac{fLV^2}{2gD}$ => $V = \sqrt{\frac{H\,2gD}{Lf}}$

$0.3 = \frac{0.04(1500)\,V^2}{2 \cdot 9.8 \times 0.6}$

$V = 0.767$

3(2)(9.81)(0.50)

$\Rightarrow V^2 = \frac{0.3 \times 2 \times 9.8 \times 0.6}{0.4 \times 1500}$ => $V = 0.2426\ m/s$

$Q = \left(\frac{\pi}{4} D^2\right) v = \frac{\pi}{4} (0.6)^2 (0.2426)$

$= 0.0685 \; m^3/sec.$

ii) after connecting the parallel pipe. ($Q = Q_1 + Q_2$) $Q = Q_1 + Q_2$

$h_{f_1} = h_{f_2}$

$Q_1 = \frac{\pi}{4} D_1^2 V_1$

$Q_2 = \frac{\pi}{4} D_2^2 V_2$

for parallel pipes Head loss in BC = Head loss in BD

$\frac{f L_1 V_1^2}{2 g D_1} = \frac{f L_2 V_2^2}{2 g D_2}$

$V_1 = V_2$

When the diameter of the parallel pipe is same as the original pipe and length of two parallel portions and discharge will be same.

$\therefore Q_1 = Q_2$

As $V_1 = V_2$, $D_1 = D_2$, $Q_1 = Q_2$

$Q = Q_1 + Q_2$

$Q = 2Q_1 = 2Q_2$ (or) $Q_1 = Q_2 = \frac{Q}{2}$

Now total head loss ABC = Head loss in AB + Head loss in BC (or) BD

$0.3 = \frac{0.04 \times 750 \times V^2}{2 \times 9.81 \times 0.6} + \frac{0.04 \times 750 \times V_1^2}{2 \times 9.81 \times 0.6}$

$V = Q/A \Rightarrow \frac{Q}{\frac{\pi}{4}(0.6)^2} = 3.536 (Q)$

$V_1 = \frac{Q_1}{A_1} = \frac{Q_1}{\frac{\pi}{4} \times D_1^2} = \frac{Q_1}{\frac{\pi}{4} \times (0.6)^2} = 1.768 Q$

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But we know $Q = Q/2$

$Q = Q/2$

$A_1V_1 = A_2V_2$

$V_1 = V_2$

$V_1 = 3.536\,(Q/2) = 1.768\,Q$

$$0.7 = \frac{0.04 \times 750 \times (3.536Q)^2}{2 \times 9.81 \times 0.6} + \frac{0.04 \times 750 \times (1.763Q)^2}{2 \times 9.81 \times 0.6}$$

$\frac{Q}{A_1} : \frac{Q}{A_2}$

$0.7 = (31.87)Q^2 + (7.92)Q^2$

$Q = 0.0867\ m^3/sec$

increase in discharge = $0.0867 - 0.0685 = 0.0182\ m^3/s$.

→ A pipe of dia 20cm & length 2000m connects two reservoirs having diff. of water levels as 20m. Determine the discharge in the pipe. If an additional pipe of diameter 20cm and length 1200m is attached to the last 1200m of the existing pipe. Find the increase in discharge. take f = 0.06 neglect minor losses.

→

$$H = \frac{fLV^2}{2gd}$$

$f = 0.06$, $L = 2000m$, $d = 0.2m$, $H = 20m$.

$$V^2 = \frac{H \times 2gd}{fL} = \frac{20 \times 2 \times 9.81 \times 0.2}{0.06 \times 2000}$$

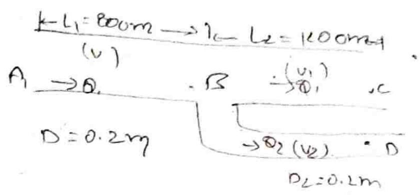
$V^2 = (0.808)^2$

$V = 0.808$

$Q = AV = \frac{\pi}{4} \times (0.2)^2 \times 0.808 =$

$0.025406\ m^3/s$

$Q = Q_1 + Q_2$



All lengths & Dia. of both

parallel pipes are same $Q_1 = Q_2$

$Q = Q_1 + Q_2 = 2Q_1 = 2Q_2$

$Q_1 = Q/2 = 0.0127$, $Q_2 = Q/2$

Total head loss = Head loss in AB + Head loss in BC (or) Head loss in BD

$$20 = \left[\frac{0.06 \times 800 \times v^2}{2 \times 9.81 \times 0.2}\right] + \left(\frac{0.06 \times 1200 \times v^2}{2 \times 9.81 \times 0.2}\right) \to (3)$$

$$V = Q/A = \frac{Q}{\frac{\pi}{4} \times (0.2)^2} = (31.83)Q$$

$$V_1 = \frac{Q_1}{A_1} = \frac{Q_1}{\frac{\pi}{4} \times (0.2)^2} = 31.83 Q_1$$

$Q_1 = Q/2$

$v_1 = 31.83 \times Q/2 = 15.92Q$

$$20 = \left[\frac{0.06 \times 800 \times (31.83\, Q)^2}{2 \times 9.8 \times 0.2}\right] + \left(\frac{0.06 \times 1200 \times (15.92\, Q)^2}{2 \times 9.8 \times 0.2}\right)$$

$Q = 0.0342\ m^3/sec$

increase in discharge $= 0.0342 - 0.0254$

$= 0.0088\ m^3/sec$

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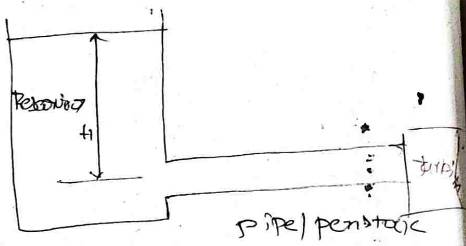
Power transmission through pipes.

In any hydroelectric power plant, pipes are used to transfer water on to the turbines these pipes are ca. penstocks. In this topic the amount of power transmitted through a pipe is to be estimated (or) calculated.

Let 'H' is the gross head in the Valuable Reservoir



power transmitted

= wt. of water flowing per second × Net head available at the exit of the pipe

$P =$ wt of water flowing per second × Net head

• Net head

If 'l' is the length of the pipe, 'd' is the diameter of pipe and 'v' is the velocity of flow, then the loss in the pipe due, neglecting minor loss in friction loss $= \frac{fLv^2}{2gd}$

$\therefore$ Net head at the exit of the pipe $= H - h_f$

$= H - \frac{fLv^2}{2gd}$

wt. of water flowing per second $= (w)Q$

$\therefore$ power transmission through pipe $= (w)Q \times (H - h_f)$

$= wQ \times \left[H - \frac{flv^2}{2gd}\right]$

$= w \times \frac{\pi}{4} D^2 . v \left[H - \frac{fLv^2}{2gD}\right]$

* Condition for max. power transmission

$$\frac{d}{dv}(P) = 0$$

$$\frac{d}{dv}\left[w \times \frac{\pi}{4} D^2 v \left[H - \frac{fLv^2}{2gD} \right] \right] = 0$$

$$w \times \frac{\pi}{4} D^2 \frac{d}{dv}\left[Hv - \frac{fLv^3}{2gD} \right] = 0$$

$$H - \frac{3fLv^2}{2gD} = 0 \Rightarrow H - 3h_f = 0$$

$$H = \frac{3fLv^2}{2gD}$$

$$\boxed{h_f = H/3}$$

Condition for max. power transmission $h_f = H/3$

* Efficiency of power transmission $= \frac{O/P}{I/P} \times 100$

$$= \frac{\text{Net head}}{\text{Gross head}} \times 100$$

$$= \frac{H - h_f}{H} \times 100$$

* Max. efficiency of power transmission $= \frac{H - h_f}{H} \times 100$

for max. power transmission $H_f = H/3$

$$\therefore \text{max.} = \frac{H - H/3}{H} \times 100$$

$$= \frac{2}{3} \times 100$$

$$= 66.6\%$$

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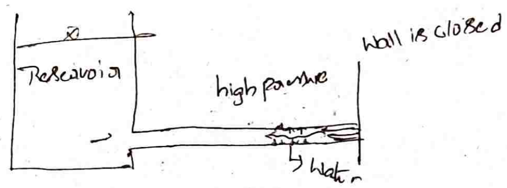
Water hammer

When water flowing in a pipe line, is suddenly stopped by closing the valve, the momentum of water is destroy due to this a high pressure wave travells backward applying applying pressure on the pipe boundries This pressure applied by the water on the pipe due to sudden closer of the wall is called Water hammer pressure.



Water hammer effects generally observed in the penstoke in Hydropower station

Knocking:- The sound that is called due to the closer of the wall.

Determination of Water hammer:

The magnitude of water hammer pressure depends on Velocity of flow in the pipe, length of pipe, Time taken to close the wall, elastic properties (Y) of both pipe material & water (K).

Depending upon the time for closer of the valve two cases are considerd.

(i) Gradual closer of the valve → Time $t > 2l/c$
(ii) Sudden closer of the valve → Time $t < 2l/c$

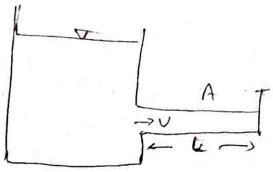
Here 't' means time taken to close the valve

'l' - length of the pipe,

'c' - Velocity of pressure wave $= \sqrt{\frac{K}{\rho}}$

Water hammer pressure in case of gradual closer.

Let A is the crosssection area & L in the length of pipe. 'v' is velocity of flow in the pipe. p is the pressure intensity due to water hammer.



Mass of water flowing in the pipe line = ρ × volume

ρ. volume = ρ A L $\frac{Mass}{volume} = \rho$

Due to gradual closer of the valve, the velocity is brought to rest in time 't' seconds $\frac{v-0}{t} = \frac{v}{t}$

Retarding force = $\rho AL \times \frac{v}{t}$ → ①   F = ma

If 'p' is the pressure intensity due to water hammer, pressure force due to closer of the wall = p.A → ②

Equating ① & ②

$$P.A = \rho AL \times \frac{v}{t}$$

$$p = \frac{\rho L v}{t} \Rightarrow \frac{P}{w} = \frac{\rho L v}{wt} = \text{pressure force}$$

Pressure head due to water hammer = $\frac{\rho L v}{wt} = \frac{\rho L v}{\rho g t}$

$$\frac{P}{w} = \frac{Lv}{gt}$$

Due to gradual close of valve:

Pressure raise = $\frac{Lv}{gt}$ in terms of head

Sudden closer:- for sudden closer substituting t = 0 in the above equation.

the pressure raise = ∞. However, the pressure raise is finite.

In the above Equation the elastic properties of pipe material and water considering them

i) Sudden closer of valve in rigid pipe [pipe expansion neglected] compressibility of water is considered then

$p = \rho v c$ $\left[\because c = \sqrt{\frac{K}{\rho}}\right]$

k = bulk modulus. (Compressibility effect considered)

(ii) Sudden closer of the wall when the pipe is elastic

both elastic (expansion of pipe & compressibility of water considered)

$$p = \sqrt{\frac{\rho v^2}{\frac{1}{k} + \frac{D}{tE}}}$$ → Elastic property

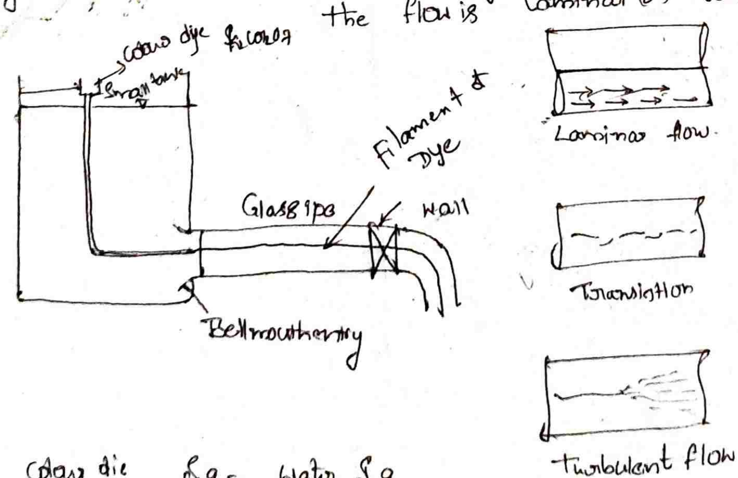
k is compressibility bulk modulus

t ~~thick of th~~ is thickness of pipe.

E : Youngs modulus of pipe material

Conclusion: to resolve the water hammer pressure ~~tak~~ we surge tanks are used in Hydropower stations.

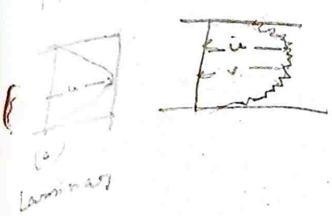
Reynolds number:- [is basically to classify whether the flow is laminar or turbulent]



colour die s.g. = water s.g.

- Laminar flow: Fluid moves in smooth streamlined
- Turbulent flow: by voilent mixing, fluid velocity at a point varies randomly with time.
- Transition to turbulent in a 2in. pipe is at V = 2 ft/s. So

pipe flows are turbulent.



$\frac{I}{V}$ dim

I = Inertia = mass × ac

$= \rho L^3 \cdot L T^{-2}$

$= \rho L^2 (LT^{-1})^2$

$= \rho L^2 v^2$

$= L^2 \times \mu \frac{V}{L}$

In / viscous

mass × acc

$M \times LT^{-2}$

ρL^{-3}

$d = \frac{M}{V}$

m × ac

$\rho L^3 \times LT^{-2}$

$\rho L^2 \cdot L^2 T^{-2}$

$\rho L^2 (LT^{-1})^2$

$= \rho L^2 V^2$

$\mu \frac{V}{L} \cdot L^2$

Reynolds number $R_e = \frac{\text{Inertia force}}{\text{viscous force}}$

Inertia force = mass × acceleration

$= M \times LT^{-2}$

$= \rho L^{+3} \cdot LT^{-2}$

$= \rho L^2 \cdot L^2 T^{-2}$

$= \rho L^2 \cdot (LT^{-1})^2$

$= \rho L^2 \cdot v^2$

Viscous force = shear stress × area

$= \tau \times \text{Area}$

$= \mu \cdot \frac{V}{L} \cdot L^2$

$= \mu \cdot V \cdot L$

$R_e = \frac{\rho L^2 V^2}{\mu V L} = \frac{\rho L}{V}$

2	Kishore	kishoreparagati14@gmail.com	19471A0141	very satisfied	Extremely informative	Excellent	Excellent
3	PASUPULETI VENKATA NAGA NARASIMHA RAO	nagapasupuletinaga@gmail.com	19471A0145	very satisfied	Extremely informative	Excellent	Excellent
4	K.siva	kotapatisiva6@gmail.com	19471A0131	satisfied	moderately informative		
5	Shaik jabeer	jabeersk418@gmail.com					

ρ = mass density

V = average velocity

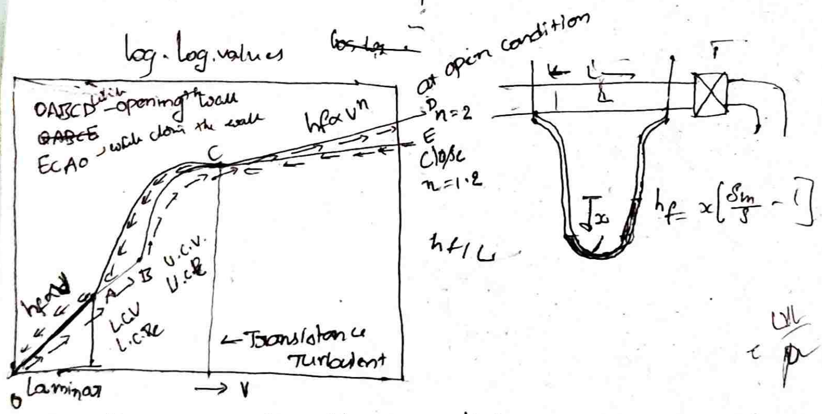
L = characteristic length — for pipe flow = Diameter of pipe

μ = Dynamic velocity

$$R_e = \frac{\rho V L}{\mu} = \frac{\rho V D}{\mu}$$

For pipe flow $R_e = \frac{\rho V D}{\mu}$

log. log. values



· Variation of velocity with head loss.

In this graph, up to A flow is laminar, Beyond the flow is turbulent.

* **Critical velocity:-** The velocity at which the flow changes from laminar flow to turbulent flow.

* **Critical Reynolds number:-** The Reynolds number at which the flow changes from laminar flow to turbulent.

The lower critical velocity is important in Engineering.
The upper " " - not significant.

→ The pipe flow is laminar if Reynolds number < 2000
" " " Turbulent " " > 4000.
2000 - 4000 The pipe flow is Transist.

28-10-09

Turbulent flow in pipes

The flow is called as Turbulent flow if the fluid particles are moving randomly mixing with each other in a half hazard manner.

The only way to know whether that flow is Laminar (or) turbulent is through Reynolds number. as Reynolds no increases the velocity distribution become more & more flatter.

Head loss in Laminar flow:- It is due to viscous shear

$$h_f = \frac{32\mu V L}{\rho g D^2}$$ → Hagen poiseuille equation.

2) Head loss in turbulent flow:-

$$h_f = \frac{f L V^2}{2gD}$$ → Darcy weisback equation

3) Shear stress in turbulent flow:

According to Newton's law of viscosity

$$\tau = \mu \cdot \frac{du}{dy} \rightarrow ①$$

by Boussinesq has given an equation for turbulent shear stress as $\tau_t = \eta \frac{du}{dy}$

'η' is called as Eddy velocity

He could not explain the method to determine η.

He is formulated very limited use.

Reynolds equation for Turbulent Shear Stress:- Reynolds found that shear stress in Turbulent flow is due to fluctuating components of velocity which are responsible for Turbulence and he gave the expression Turbulent shear $= \tau_t = \rho u'v'$

but $u'v'$ are not determined.

4	K.siva	kotapatisiva6@gmail.com	19471A0131	satisfied
5	Shaik jabeer	jabeer...		

*** Prandtl mixing length theory :- Prandtl taken the shearstress equation as follows.

Total shearstress = viscous shear stress + Turbulent shear stress

$= \tau_{viscous} + \tau_{turbulent}$

$$\tau_{total} = \mu \cdot \frac{du}{dy} + \rho \cdot \overline{u'v'}$$

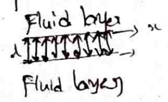
In case of turbulent flow, viscous shear is significant only at the boundary neglecting that the total shear stress

$$\tau = \rho \overline{u'v'}$$

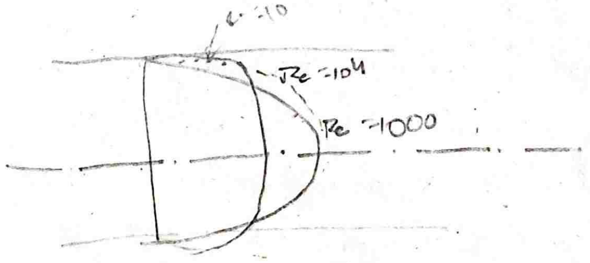
Prandtl define uv as follows:

$$u' = l \cdot \frac{du}{dy} \quad , \quad v' = l \cdot \frac{du}{dy}$$

l = prandtl's mixing length. "It is defined as the distance between two fluid layers in the transverse direction, such that the fluid particles from one layer will move into the other layer and viceversa mix and travel in such a way that momentum of the fluid particles in x-direction is same.



$$\boxed{\tau = \rho \cdot l^2 \left(\frac{du}{dy}\right)^2}$$



velocity distribution

$\tau = \rho \, l^2 \left(\frac{du}{dy}\right)^2$ $l = ky$

$\tau = k^2 y^2 \rho \left(\frac{du}{dy}\right)^2$

$\tau = \rho k^2 y^2 \left(\frac{du}{dy}\right)^2$

Velocity distribution in turbulent flow based on Prandtl Transform mixing length theory:

$$\tau = \rho l^2 \left(\frac{du}{dy}\right)^2$$

Prandtl assumed that mixing length 'l' is a linear function of 'y'.

$l \propto y$ (or) $l = ky$

k → kappa → Karman's universal constant $= 0.4$

$$\tau = \rho k^2 y^2 \left(\frac{du}{dy}\right)^2$$

$$\left(\frac{du}{dy}\right)^2 = \frac{\tau}{\rho k^2 y^2} \quad ; \quad \frac{du}{dy} = \frac{1}{ky}\sqrt{\frac{\tau}{\rho}}$$

Prandtl assumed that the shear stress in turbulent flow is more (or) less constant is equals to boundary shear stress $= \tau_0$

$$\frac{du}{dy} = \frac{1}{ky}\sqrt{\frac{\tau_0}{\rho}}$$

$\sqrt{\frac{\tau_0}{\rho}}$ is called as shear velocity $= u_*$

$u = \frac{u_*}{k} \log_e y + C$

$$\frac{N/m^2}{kg/m^3} = \frac{N}{m^2} \times \frac{m^3}{kg} = Nm/kg$$

$$\sqrt{L^2 T^{-2}} = \sqrt{(LT^{-1})^2} = \text{velocity}$$

u_*

The dimension of $\sqrt{\frac{\tau_0}{\rho}}$ is same as velocity.

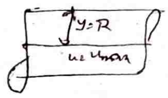
$$\frac{du}{dy} = \frac{1}{ky} u_*$$

$$\boxed{u = \frac{u_*}{k} \log_e y + C}$$

The velocity distribution is Turbulent flow is logarithmic
" " " Laminar " " Parabolic

using the boundary condition at $y = R$

$u = u_{max}$



$\therefore \; u_{max} = \frac{u_*}{k} \log_e R + C$

$(\text{or})\; C = u_{max} - \frac{u_*}{k} \log_e R$

$u = \frac{u_*}{k} \log_e y + u_{max} - \frac{u_*}{k} \log_e R$

$u = u_{max} + \frac{u_*}{k} \left[\log_e (y/R) \right]$

$u = u_{max} + \frac{u_*}{k} \log_e y/R$

Substituting $k = 0.4$

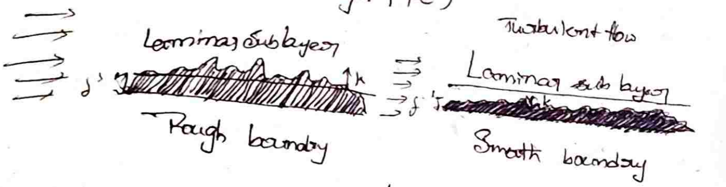
$u = u_{max} + \frac{u_*}{0.4} \log_e (y/R)$

$$\boxed{u = u_{max} + 2.5\, u_* \log_e (y/R)}$$

$$\boxed{u = u_{max} + 5.75\, u_* \log_{10} y/R}$$

This is the general equation for velocity distribution in turbulent flow. The velocity distribution in Turbulent flow is logarithmic, " " " Laminar " parabolic

Distinguish bt Hydrodynamically smooth & Rough boundary axis (Smooth & Rough pipe)



k = average roughness projection
(or) average height of roughness
δ' = Laminar sublayer thickness

In Smooth boundary (Smooth pipe) all the roughness projection are below the Laminar Sublayer (or) with in the laminar sublayer.
The roughness projection are not exposed to the turbulent flow current (Eddy) directly.
The effect of roughness projection is very very small.

In case of rough boundary. The roughness projections penetrate through laminar sublayer and are directly in contact with the turbulent flow because of this the energy loss will be very high.

As the Reynolds number of flow increases, δ' decreases and because of this the roughness projection penetrate through Sub-layer

Mathematically
From experiment
by Nikkuradse

$$\frac{k}{\delta'} < 0.25 \leftarrow \text{Smooth pipe [Smooth boundary]}$$

$$\frac{k}{\delta'} > 6 \leftarrow \text{Rough pipe [Rough boundary]}$$

k = Average height of roughness.
δ' = thickness of Laminar Sublayer

$\frac{k}{\delta'}$, ⇒ 0.25–6 → Transition

$T = \frac{2}{3} C_d \sqrt{2g}\, (h)^{3/2}$

$Q = \frac{8}{15} C_d \sqrt{2g}\, (h)^{5/2}$

The Laminar Sublayer thickness is given by

$$\delta' = \frac{11.6\nu}{u_*}$$

ν = kinematic viscosity = $\frac{\mu}{\rho}$

u_* = Shear velocity = $\sqrt{\frac{\tau_0}{\rho}}$

In case of smooth boundary, the roughness projection will not play any role on the energy loss. whereas in case of rough pipe, the roughness projections play a significant in energy loss.

In terms of roughness Reynolds number $\frac{u_* k}{\nu}$ a pipe boundary is smooth if $\frac{u_* k}{\nu} < 4$

$\frac{k}{\delta'} < 0.25$

$u_* \frac{k}{\nu} > 100$ Rough

$\delta' = 0.6\nu$

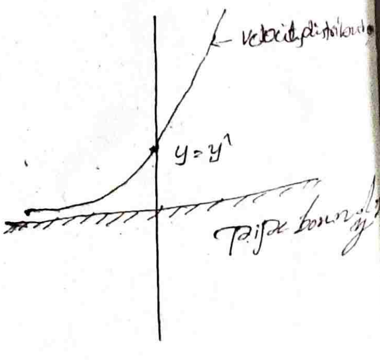
Derive the equation for velocity distribution in smooth pipe.

Considering the general equation for velocity distribution in turbulent flow in pipes $u = \frac{u_*}{k} \log_e y + c$ k 0.04

At the pipe boundary, i.e., $y=0$

At $y=0$
$u = -\infty$

∴ The velocity increases from $-\infty$ to 0 and then positive.



at $y = y'$, $u=0$

$$\therefore 0 = \frac{u_*}{k} \log_e y' + c$$

$$c = -\frac{u_*}{k} \log_e y'$$

$$u = \frac{u_*}{k} \log_e y - \frac{u_*}{k} \log_e y'$$

$$= \frac{u_*}{k} \log_e \left(\frac{y}{y'} \right)$$

$$u = \frac{u_*}{0.4} \log_e (y/y')$$

$$= 0.25\, u_* \log_e (y/y')$$

$$u = 2.5 \times 2.3\, u_* \log_{10} (y/y')$$

$$u = 5.75\, u_* \log_{10} (y/y') \qquad u_* = \frac{u}{5.75} \log (y/y')$$

But, from the experiments it is found that

$$y' = \frac{\delta}{107}$$

$$\delta = \frac{11.6\nu}{u_*} \qquad \frac{u}{u_*} = 5.75 \log_{10} \left[\frac{y}{11.6\nu} \right.$$

$$y' = \frac{11.6\nu}{u_* \times 107} \qquad \frac{y}{11.6\nu} \times \frac{u^*}{107}$$

Subs. y' in the equ. & then simplifying

$$\frac{u}{u_*} = 5.75 \log_{10} \frac{u_* y}{\nu} + 5.55$$

S.NO.	Name	Email	Roll no	... you with this lecture	... find this lecture	session evaluation	... rate your overall experience
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4	K.siva	kotapatisiva6@gmail.com	19471A0131	satisfied	moderately inf...	Excellent	Excellent
5	Shaik jabeer	jabeersk418@gmail.com	[illegible]				

$$\frac{u}{u^*} = 5.75 \log_{10}\left(\frac{y}{y'}\right)$$

$$y' = \frac{11.6\nu}{u^*} \times 10^7$$

$$\frac{u}{u^*} = 5.75 \log_{10}\left(\frac{y \times u^*}{11.6\nu/10^7}\right)$$

$$y' = \frac{0.108\nu}{u_*}$$

$$u = 5.75 \log_{10}\left(\frac{y u_n}{\nu}\right) - 5.5\,u$$

$$\boxed{\frac{u}{u_*} = 5.75 \log_{10}\frac{u_* y}{\nu} + 5.5}$$

velocity distribution in rough pipes

General eqn. for turbulent flow $= (u) = \frac{u_*}{k} \log_e y + c$

at $y = y'$

; $u = 0$

$$0 = \frac{u_*}{k} \log_e y' + c \Rightarrow c = -\frac{u_*}{k} \log_e y'$$

$$u = \frac{u_*}{k} \log_e y - \frac{u_*}{k} \log_e y'$$

$$= \frac{u_*}{k}\left(\log_e (y/y')\right)$$

$$\frac{u}{u_*} = \frac{1}{k} \log_e (y/y')$$

$$\frac{u}{u_*} = 5.75 \log_{10} (y/y')$$

from the experiments, it is found that $y' = \frac{k}{30}$ where

k = roughness height

$$\frac{u}{u^*} = 5.75\left[\log_{10} y - \log_{10}\left(\frac{k}{30}\right)\right]$$

$$= 5.75\left[\log_{10} y - \log_{10} k\right] + 8.5$$

$$= 5.75\left(\log_{10}(y/k)\right) + 8.5$$

Velocity distribution equation for turbulent flow

$$u = \frac{u_*}{k}\log_e y + C$$

General equation for velocity distribution interms of man velocity u_{man} at $[y = R,\ u = u_{man}]$

$$u = u_{man} + 5.75 u_n \log_{10}(y/R)$$

Velocity distribution in smooth pipe : $[u = 0,\ y = y']$

$$2)\ \left(y' = \frac{\delta'}{107},\quad \delta' = \frac{11.6\,u}{u^*}\right)$$

$$\frac{u}{u^*} = 5.75 \log_{10} \frac{u_* y}{v} + 5.55$$

In rough pipe :- $[u = 0$ at $y = y']$

$$y' = \frac{k}{30}$$

$$\frac{u}{u^*} = 5.75\left(\log_{10}(y/k)\right) + 8.5$$

Variation of friction factor with Re, Roughness projection

The resistance to flow results in energy loss (or) Head loss, this head loss can be estimated by using Darcy-weisbach equation.

$$h_f = \frac{fLv^2}{2gd}$$

f = friction factor

it is found that, the friction factor is a function of Re and relative roughness.

f is a function of $f = \phi\left[Re, \frac{k}{D}\right]$

Re = Reynolds number

$k/D = \frac{\text{Height of roughness}}{\text{Dia of pipe}}$ = Relative roughness

D/k = Relative smoothness.

Case (i):- Laminar flow. $f = \frac{64}{Re}$

$$\frac{fLv^2}{2gd} = \frac{32\mu vL}{\rho g d^2} \qquad f = \frac{64\mu}{v\rho d}$$

$$f = \frac{64}{Re}$$

Friction factor in turbulent flow $4000 < Re < 10^5$

(i) In smooth pipe, if Re > 4000 & $Re < \frac{2000}{2000} \times 10^5$

$$f = \frac{0.316}{(Re)^{1/4}}$$

Blassius equation

if $Re > 10^5$; $\frac{1}{\sqrt{f}} = 2\log_{10}(Re\sqrt{f}) - 0.8$ -

Nikuradse

Case (iii) :- Turbulent flow rough pipe

$$\frac{1}{\sqrt{f}} = 2\log_{10}\left[\frac{R}{K}\right] + 1.74$$

R: radius of pipe

K = Height of ~~region~~ roughness

i) In Laminar flow, friction factor depends only on Reynolds number.

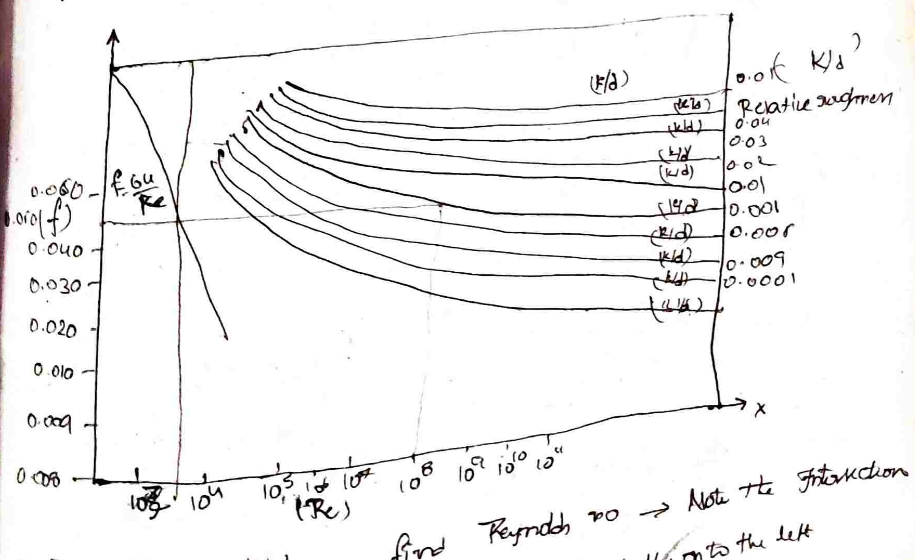
ii) in turbulent flow, in smooth pipe friction factor depends only on Reynolds number..

(iii) In Turbulent flow rough pipe friction factor depends only on Relative roughness.

* Moody's diagram:

Nomogram

It is used to determine the friction factor in the case of Commercial pipes. It is a graphical plot showing Reynolds number, friction factor and Relative roughness. on x-axis, Reynolds number on y-axis friction factor



i) First find (K/d) → find Reynolds no → Note the Intersection point of Reynolds no & (K/d) → project horizontally onto the left that gives Friction factor.

Moody's diagram is a graphical representation of variation of friction factor & Reynolds number for different (k/d) curves.

Its main use is to determine friction factor in Commercial pipe.

→ A pipe line carrying water of average height of irregularities projecting from the surface of the boundary of pipe as 0.15mm. What type of boundary is it? Shear stress developed = $4.9 N/m^2$, $\nu = 0.01$ stokes.

$k = 0.15mm = 0.15 \times 10^{-3} m$

$\tau_0 = 4.9 N/m^2$, $\nu = 0.01 \times 10^{-4} m^2/sec$. $\frac{1}{100}$

$$\delta' = \frac{11.6\nu}{u_*}$$

$$u_* = \sqrt{\frac{\tau_0}{\nu}} = \sqrt{\frac{4.9}{0.01 \times 10^{-4}}} = ~~2213.5~~ \; 0.7 m/sec$$

$$\delta' = \frac{11.6 \times 0.01 \times 10^{-4}}{~~2213.5~~ \; 0.7} = 1.65 \times 10^{-5}$$

$$k/\delta' = \frac{0.15}{1.65 \times 10^{-5}} =$$

$k/\delta' < 0.25$ smooth $k/\delta' > 6$ rough

A liquid of specific gravity 0.88 and viscosity 6.533×10^{-4} N-s/m² flows through a pipe of diameter 0.15m at the rate of 60 liters per second. If the loss of head is 100m length of pipe of is 4.56mm determine whether the pipe is smooth (or) rough.

Des...

as 'k' is not given, In order to determine 'k' the friction factor equation involving 'k' is used. In order to determine friction factor Darcy equation is used.

$Q = AV \Rightarrow V = Q/A = \frac{60 \times 10^{-3}}{\frac{\pi}{4}(0.15)^2} = 3.40$ m/sec.

$h_f = \frac{fLV^2}{2gD} \Rightarrow 4.56 = \frac{f \times 100 \times 3.4^2}{2 \times 9.8 \times 0.15}$

$f = 0.0116$

$Re = \frac{\rho V D}{\mu}$

$= 6.87 \times 10^5$

Assume pipe is rough

$$\frac{1}{\sqrt{f}} = 2\log_{10}\left(\frac{R}{k}\right) + 1.74$$

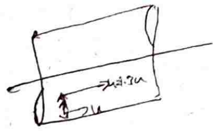
$$\frac{1}{\sqrt{0.0116}} = 2\log_{10}\left[\frac{R}{k}\right] + 1.74$$

$\frac{R}{k} = 5.981$

$$\frac{Re\sqrt{f}}{(R/K)} = \frac{6.87 \times 10^5 \sqrt{0.116}}{(5.981)} = 12.5 \approx 13$$

As $\frac{Re\sqrt{f}}{(R/K)} < 17$ pipe is smooth

A rough pipe is of dia 5cm. The velocity of the point 3cm from wall is 30% more than velocity at a point 1cm from pipe wall. Determine the average height of roughness.



Let velocity at 1cm from pipe wall = u

velocity at 3cm = 1.3u

Velocity distribution in rough pipe : $\frac{u}{u_*} = 5.75 \log_{10}\left[\frac{y}{k}\right] + 8.5$

For a point, 1cm from pipe wall

$\frac{u}{u_*} = 5.75 \log_{10}\left(\frac{1}{k}\right) + 8.5 \rightarrow (1)$ $\frac{1.3u}{u_*} = 5.75 \log\left[\frac{3}{k}\right] + 8.5 \rightarrow (2)$

For a point, 3cm from pipe wall

② → k = 0.7726 cm.
①

→ Water is flowing through a rough pipe of dia 500 mm and L = 4000 m at the rate of 0.5 m³/sec. Find the power required to maintain the flow. Take average height of roughness projection k = 0.40 mm

Sol:

$P = wQh_f$ water

$w = 9810\ N/m^3$

$Q = 0.5\ m^3/sec$

$h_f = \frac{fLV^2}{2gD}$ $L = 4000 m$

$d = 500 \times 10^{-3} m = 0.5 m$

$V = Q/A = \frac{0.5}{\frac{\pi}{4}(0.5)^2} = 2.546\ m/sec.$

f is found out by giving

$\frac{1}{\sqrt{f}} = 2\log\left(\frac{R}{k}\right) + 1.75$

$= 2\log\left[\frac{0.25}{0.4 \times 10^{-3}}\right] + 1.75$ $\sqrt{f} = 0.1364;$ $f = 0.0186$

$h_f = \frac{0.0186 \times 4000 \times (2.546)^2}{2 \times 9.81 \times 0.5} = 49.16 m$

Power required $= wQh_f = 9810 \times 0.5 \times 49.16 = 241.13\ k.N.$

-- Water is flowing through a rough pipe of dia 600 mm at the Q = 600 lit/sec. The wall roughness is k = 3 mm. Find the 'P' for 1 km. length of pipe

Sol:

$P = eQh_f$

$e = 9.810$

$Q = AV$ $V = Q/A = \frac{0.6}{\frac{\pi}{4} \times (0.6)^2} = 2.122\ m/sec.$

$\frac{1}{\sqrt{f}} = 2\log_{10}\left[\frac{R}{k}\right] + 1.74$

$f = 0.03035$

$h_f = \frac{fLV^2}{2gd} = \frac{0.03035 \times 1000 \times 2.123^2}{2 \times 9.81 \times 0.6} = 11.6$

$P = 9810 \times 0.6 \times 11.6 = 68.27\ kN$

Des...

→ A smooth brass pipe line 75mm in d & 900m L. carries water at the rate of 7 lit/sec. $\nu = 0.0175$ Stokes. Calculate loss of head, wall shear stress, centre line, velocity & thickness of laminar sublayer.

Sol:

$Q = 7 \times 10^{-3}$ $\quad \nu = 0.0175 \times 10^{-4} = 1.75 \times 10^{-6}$ N/m²

$h_f = \frac{fLV^2}{2gd}$ $\quad Q = AV \Rightarrow V = \frac{0.7 \times 10^{-2}}{\frac{\pi}{4} \times (0.075)^2} =$ ~~0.63 m/sec~~ 1.584 m/sec

$\frac{1}{\sqrt{f}} =$ ~~2 log~~

Reynolds number $Re = \frac{VD}{\nu} = \frac{1.584 \times 0.075}{0.0195 \times 10^{-4}} = 6.09 \times 10^4$

For smooth pipe, $f = 0.316/(Re)^{1/4}$

$f = \frac{0.316}{(6.09 \times 10^4)^{1/4}} =$ ~~2.85 × 10⁻³~~ 0.0201

$h_f = \frac{0.0201 \times 900 \times 1.584^2}{2 \times 9.81 \times 0.075} = 31m$

$\tau_0 = f \cdot \frac{\rho V^2}{8} = \frac{0.0201 \times 1000 \times 1.584^2}{8} = 6.304 N/m^2$

$u_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{6.304}{1000}} = 0.0794$ m/sec

At $y = R$, $u = v_{max}$

$\frac{u}{u_*} = 5.75 \log_{10}\left[\frac{u_* y}{\nu}\right] + 5.5$ $\quad U_{max} = 1.89$ m/sec

$\delta' = \frac{11.6\nu}{u_*} = 2.85 \times 10^{-4}$ m

→ A smooth pipe of $d = 400$mm & $l = 800$m carries water at the rate of 0.04 m³/sec. Determine the head loss due to friction, wall shear stress, centre line velocity & thickness of laminar sublayer. $\nu =$ ~~1...~~ 0.018 stoke

Sol:

$V = Q/A = \frac{0.04}{\frac{\pi}{4} \times 0.4^2} = 0.3183$ m/sec

$Re = \frac{VD}{\nu} = \frac{0.3183 \times 0.4}{0.018 \times 10^{-4}} = 7.073 \times 10^4$

flow is turbulent

$f = \frac{0.316}{(Re)^{1/4}} = \frac{0.316}{(7.073 \times 10^4)^{1/4}} = 0.0194$

$h_f = \frac{flv^2}{2gD} = \frac{0.0194 \times 800 \times 0.3183^2}{2 \times 9.81 \times 0.4} = 0.20m$

$\tau_0 = \frac{f\rho v^2}{8} = \frac{0.0194 \times 1000 \times 0.3183^2}{8} = 0.245$ N/m²

$\frac{u_{max}}{u_*} = 5.75 \log_{10}\left[\frac{u_* R}{\nu}\right] + 5.5$

$u_* = \sqrt{\frac{\tau_0}{\rho}} = 0.0156$

$\frac{u_{max}}{0.0156} = 5.75 \log_{10}\left[\frac{0.0156 \times 0.2}{0.018 \times 10^{-4}}\right]$ $\quad u_{max} = 0.377$ m/s

$\delta' = \frac{11.6\nu}{u_*} = \frac{11.6 \times 0.018 \times 10^{-4}}{0.0156} = 1.\ldots$