

***NARASARAOPETA ENGINEERING COLLEGE  
(AUTONOMOUS)***



***ENGINEERING PHYSICS MATERIAL***

***NAME:*** -----

***ROLL NO:*** -----

***BRANCH:*** ----- ***SECTION:*** -----

***YEAR OF STUDY:*** -----

**Engineering Physics**  
**(Common to CE&ME)**

<i>I B.TECH</i>	L	T	P	INTERNAL MARKS	EXTERNAL MARKS	TOTAL MARKS	CREDITS
I/II SEMESTER	4	0	0	30	70	100	3
<b>Code:</b>	<b>Engineering Physics</b> <b>(Common to CE&amp;ME)</b>						

**COURSE OBJECTIVES:**

- To impart knowledge in basic concepts of wave optics, fiber optics, properties of solid crystal materials and magnetic materials, acoustics, superconductors.
- To familiarize the applications of materials relevant to engineering field.

**COURSE OUTCOMES:**

After successful completion of this course, the students will be able to:

**CO 1:** Recognize the experimental evidence of wave nature of light and interference in thin films, Diffraction grating and Polarisation in various fields. **(Remember) K1**

**CO 2:** Analyse and understand various types of lasers & optical fibers. **(Analyse) K4**

**CO 3:** Identifies the crystal structures and XRD techniques. **(Remember) K1.**

**CO 4:** Knowing the applications of magnetic and superconducting materials in engineering field. **(Remember) K1**

**CO 5:** Identifies the use of Acoustics and Ultrasonics in engineering field. **(Analyse) K4**

**UNIT– I**

**Interference & Diffraction:** Introduction -Interference in thin films by reflection – Newton’s rings, introduction to diffraction – difference between Fresnel’s and Fraunhofer diffraction - Fraunhofer diffraction at single slit (qualitative) - Diffraction grating.

**Polarization:** Introduction – Types of Polarization – Double refraction – Nicol’s prism-Quarter wave plate and Half Wave plate - Applications

**UNIT–II**

**Lasers:** Introduction – Characteristics of lasers – Spontaneous and Stimulated emission of radiation – Population inversion – Ruby laser – Helium Neon laser-Applications.

**Fiber Optics:** Introduction- Basic Structure and Principle of optical fiber - Acceptance angle – Acceptance cone - Numerical Aperture-Applications.



### UNIT-III

**Crystallography** : Introduction – Space lattice – Basis – Unit Cell – Lattice parameters – Bravais lattices – Crystal systems – Structures and packing fractions of SC, BCC and FCC.

**X-Ray Diffraction**: Directions and planes in crystals – Miller indices – Separation between successive (h k l) planes – Bragg's law.

### UNIT-IV

**Magnetic materials**: Magnetic dipole moment- Magnetization- Magnetic Susceptibility- Magnetic permeability – Classification of Magnetic materials – Dia, Para, and Ferro – Hysteresis Loop- Soft and Hard magnetic materials – Applications of Magnetic materials

**Superconductivity**: Introduction- Properties, Meissner effect - Type-I and Type-II superconductors- BCS theory (Qualitative) – AC and DC Josephson effects - Applications of Superconductors

### UNIT-V

**Acoustics**: Introduction – requirements of acoustically good hall– Reverberation – Reverberation time – Sabine's formula - Absorption coefficient and its determination – Factors affecting acoustics of buildings and their remedial measures.

**Ultrasonics**: Introduction - Properties - Production by Magnetostriction and Piezoelectric methods – Non Destructive Testing (Qualitative) – Applications.

### TEXT BOOKS:

6. A.J. Dekker, "Solid state Physics", ISBN 10: 0333918339 / ISBN 13: 9780333918333, Mc Millan India Ltd, First edition, 2000.
7. M.N. Avadhanulu & P.G. Kshirasagar, "A text book of Engineering Physics", ISBN 81-219-0817-5, S. Chand publications, First Edition, 2011.
8. P. K. Palanisamy, "Engineering Physics", ISBN: 9788183714464, Scitech Publishers, 4th Edition, 2014
9. M.R. Srinivasan, "Engineering Physics", ISBN 978-81-224-3636-5, New Age international publishers, 2<sup>nd</sup> Edition, 2014



### REFERENCE BOOKS:

1. Charles Kittel, "Introduction to solid state physics" ISBN: 9788126578436, Wiley India Pvt.Ltd, 5<sup>TH</sup> edition, 2012.
2. M.Arumugam, "Applied Physics", ISBN: 81-89638-01-7, Anuradha Agencies, 4<sup>th</sup> edition, 2013.
3. D.K.Bhattacharya, "Engineering Physics", ISBN: 0198065426, 9780198065425, Oxford University press, 2<sup>nd</sup> edition, 2010.
4. Sanjay D Jain and Girish G Sahasrabudhe "Engineering Physics", University Press ISBN: 8173716781, 1<sup>st</sup> edition, 2010.
10. B.K.Pandey & S. Chaturvedi "Engineering Physics" ISBN: 8131517616, Cengage Learning, 1<sup>st</sup> edition, 2012.

### Web References:

1. <http://link.springer.com/physics>
2. <http://www.thphys.physics.ox.ac.uk>
3. <http://www.sciencedirect.com/science>
4. <http://www.e-booksdirectory.com>

### E-Books:

1. <http://www.peaceone.net/basic/Feynman>
2. <http://physicsdatabase.com/free-physics-books>
3. <http://www.damtp.cam.ac.uk/user/tong/statphys/sp.pdf>
4. <http://www.freebookcentre.net/Physics/Solid-State-Physics-Books.html>



Subject Code: R20CC1203

I B.Tech. - II Semester Regular Examinations, September-2021

Engineering Physics

(ME, CE)

Time: 3 hours

Max. Marks: 70

Note: Answer All FIVE Questions.  
All Questions Carry Equal Marks (5 X 14 =70M)

Q. No.	Questions	Marks
<b>Unit-I</b>		
1	A	I) What is interference of light? Prove that the diameter of the $n^{\text{th}}$ dark ring in a Newton's ring set-up is directly proportional to the square root of the ring number. [10M]
		II) In Newton's rings experiment, the diameter of $4^{\text{th}}$ and $12^{\text{th}}$ dark rings is 0.4cm and 0.7cm respectively. Find the diameter of $20^{\text{th}}$ dark ring. [4M]
	<b>OR</b>	
	B	I) Explain the phenomenon of double refraction in a calcite crystal. Give the construction and theory of (a) Half wave plate and (b) Quarter wave plate. [10M]
	ii) Find the thickness of a Half wave and Quarter wave plate when the wavelength of light is 5890 Å. ( $\mu_o = 1.55$ and $\mu_E = 1.54$ ) [4M]	
<b>Unit-II</b>		
2	A	I) Differentiate spontaneous and stimulated emissions. [4M]
		II) With the help of suitable diagram, explain the principle, construction and working of Ruby laser. [10M]
	<b>OR</b>	
	B	I) Derive an expression for acceptance angle and discuss the concept of acceptance cone for an optical fiber. [10M]
	II) If an optical fiber has refractive indices of core and cladding are 1.54 and 1.43 respectively. Calculate the critical angle and acceptance angle of the given optical fiber. [4M]	
<b>Unit-III</b>		
3	A	I) Define packing fraction and coordination number. Obtain the expression for packing fraction of SC, BCC and FCC. [10M]
		II) Copper has FCC structure and its atomic radius is 0.1278 nm, calculate its lattice constant and density. [4M]
	<b>OR</b>	
	B	I) Derive an expression for the interplanar spacing between two adjacent planes of Miller indices (h k l) in a cubic lattice of edge a. [10M]
	II) Draw the planes of the cubic unit cell: (011) and (101) [4M]	
<b>Unit-IV</b>		
4	A	I) What is hysteresis? Draw and explain the hysteresis curve (B-H curve) for a ferromagnetic material. [10M]
		II) Distinguish between Hard and Soft magnetic materials on the basis of B-H curve. [4M]



		<b>OR</b>	
	B	I) Discuss the electron-phonon interaction and the formation of Cooper pairs in superconductors on the basis of the BCS theory.	[10M]
		II) Discuss any four applications of superconductors.	[4M]
		<b>Unit-V</b>	
		I) What is absorption coefficient and how can it be determined?	[10M]
	A	II) A lecture hall has volume of $120000 \text{ m}^3$ . It has reverberation time of 1.5 s. What is the average absorbing coefficient of the surface, if total sound absorbing surface is $25000 \text{ m}^2$ .	[4M]
		<b>OR</b>	
5		I) Explain the production of ultrasonic waves by Piezoelectric method.	[10M]
	B	II) A piezo-electric crystal with vibrating length $3 \times 10^{-3} \text{ m}$ has density $3.5 \times 10^3 \text{ Kg/m}^3$ . If it is made of a material of Young's modulus $8 \times 10^{10} \text{ N/m}^2$ , what is its fundamental frequency?	[4M]

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Subject Code: R20CC1203

I B.Tech. - II Semester Supple Examinations, March-2022

Engineering Physics  
(ME,CE)

Time: 3 hours

Max. Marks: 70

Note: Answer All FIVE Questions.  
All Questions Carry Equal Marks (5 X 14 = 70M)

1. A) I) State and explain the principle of Superposition of waves. (5 M)  
II) Write the necessary theory to determine the wavelength of a given monochromatic light by using Newton's Rings method. (9 M)
- OR**
- B) I) Describe the construction and action of Nicol prism. Explain how Nicol Prism acts as an analyser (10 M)  
II) What is a half wave plate? Write an expression for its thickness. (4 M)
2. A) I) Describe the (i) Spontaneous emission and (ii) Stimulated Emission. (4M)  
II) With the help of suitable diagrams explain the principle, construction and working of He-Ne gas laser. (10 M)
- OR**
- B) I) What is meant by acceptance angle for an optical fiber? Obtain mathematical expression for acceptance angle and numerical aperture. (9 M)  
II) The numerical aperture of optical fiber is 0.2 when surrounded by air. Given the refractive index of the cladding is 1.59. Find the acceptance angle when the fiber is in water. Assume the refractive index of water as 1.33. (5 M)
3. A) I) Show that FCC is the most closely packed of the three cubic structures by working out the packing fractions. (12 M)  
II) What is primitive cell? How does it is different from unit cell. (2M)
- OR**
- B) I) State and explain Bragg's law of X-ray diffraction (7 M)  
II) Derive an expression for inter planar distance between successive (h k l) planes. (7 M)
4. A) I) Define the terms magnetic susceptibility, permeability and obtain the relation between them. (5M)  
II) Explain in detail the classification of magnetic materials into dia, para and ferro. (9 M)
- OR**
- B) I) Describe the BCS theory of superconductivity. (10 M)  
II) What are the applications of superconductors? (4 M)
5. A) I) What is meant by reverberation time? Derive the Sabine's formula for the reverberation time. (10 M)  
II) An auditorium has a volume of  $8400 \text{ m}^3$  required to have reverberation time 0.2 seconds. What is the total absorption in the hall? (4 M)
- OR**
- B) I) What are ultrasonic waves? Write the properties of ultrasonic waves. (6 M)  
II) Describe the production of ultrasonic wave by Magnetostriction method. (8 M)

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# UNIT – I

Chapter – I: Interference &  
Diffraction.

Chapter – II: Polarization

Course Outcome:

CO 1: Recognize the experimental evidence of wave nature of light and interference in thin films, Diffraction grating and Polarization in various fields. **(Remember) K1**



# ENGINEERING PHYSICS

## UNIT-I

### INTERFERENCE

#### **Introduction:-**

Interference of light was first demonstrated by Thomas Young in 1802.

- Many observations in our day to day life are due to interference of light.

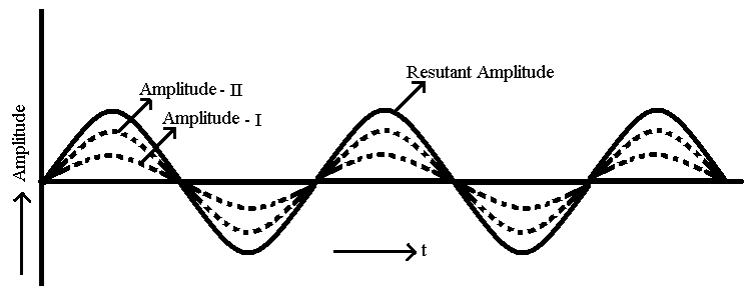
Ex: Multiple colors on soap bubbles

Thin layers of floating oil when viewed in sunlight.

- Interference is based on the principle of superposition of waves.
- Interference is the optical phenomenon in which brightness and darkness images are produced by two exactly similar light waves meeting.

#### **Definition:-**

When light waves of same frequency and having constant phase difference, the resulting the superposition of two light waves is called interference.



- The pattern of bright and dark fringes (images) produced is called interference patterns.
- These interference is of two types, they are
  1. Constructive Interference
  2. Destructive Interference

#### **Constructive Interference:-**

When the resultant amplitude is the sum of the amplitudes due to two light waves is known as Constructive Interference.

## Destructive Interference:-

When the resultant amplitude is equal to the difference of two amplitudes is known as Destructive Interference.

## Conditions for interference fringes:-

1. The two light sources emitting light waves should be coherent.
2. The two sources must emit continuous light waves of same wavelength and frequency.
3. The sources must be monochromatic.
4. The sources should be narrow, i.e. they must be small.
5. To view interference fringes, the background should be dark.
6. The amplitude of light waves should be equal or nearly equal.

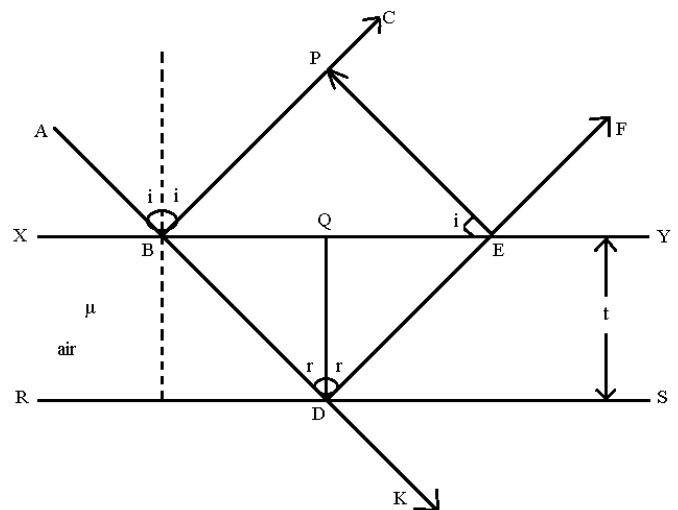
## Interference due to thin films:-

When white light is reflected by thin films like soap bubbles, oil layers on water and oxide layers on metal surface a variety of color could be seen. This is due to interference between light waves reflected by the front and back surface of these films.

A phase change of  $\pi$  (or) a path difference of " $\lambda/2$ " is introduced when light travel from rarer to denser medium. But no changes can occur when light travel from denser to rarer medium.

### Explanation:-

Consider a thin film of thickness " $t$ " and refractive index " $\mu$ " bounded by two planes surface XY and RS. A ray of light AB incident on the surface XY at an angle " $i$ " is partly reflected along BC and partly refracted into the medium along BD making



an angle of refraction “r” at D, it is again partly reflected along DE-inside, the medium and partly refracted out of the medium along DK. Similarly reflections and refractions occur as shown in figure.

### Interference in reflected system:-

Draw EP perpendicular to BC. Now BC and EF are the coherent waves which are going to interference, path difference between BC and EF are

$$\text{Path difference} = \mu (BD+DE) - BP$$

$$\begin{aligned} \text{Now consider } \Delta \text{le BQD, } \quad \cos r &= \frac{DQ}{BD} = \frac{t}{BD} \Rightarrow BD = \frac{t}{\cos r} \\ &\Rightarrow BD = DE = \frac{t}{\cos r} \end{aligned}$$

$$\begin{aligned} \text{Now consider } \Delta \text{le BQD, } \quad \tan r &= \frac{BQ}{DQ} = \frac{BQ}{t} \Rightarrow BQ = t \cdot \tan r \\ &\Rightarrow BQ = QE = t \cdot \tan r \end{aligned}$$

$$\therefore \mu (BD+DE) = \mu \left( \frac{t}{\cos r} + \frac{t}{\cos r} \right) = \mu \left( \frac{2t}{\cos r} \right) = \frac{2\mu t}{\cos r}$$

Now consider  $\Delta$ le BEP

$$\begin{aligned} \sin i &= \frac{BP}{BE} \\ &\Rightarrow BP = BE \sin i \\ &= (BQ+QE) \sin i \\ BP &= (2t \tan r) \sin i = 2t \cdot \frac{\sin r}{\cos r} \cdot \sin i \times \frac{\sin r}{\sin r} \\ &= 2t \cdot \frac{\sin^2 r}{\cos r} \times \frac{\sin i}{\sin r} \quad \left( \text{from Snell's Law } \mu = \frac{\sin i}{\sin r} \right) \\ &= 2\mu t \frac{\sin^2 r}{\cos r} \end{aligned}$$

$$\begin{aligned} \therefore \text{Path difference} &= \mu (BD+DE) - BP \\ &= \frac{2\mu t}{\cos r} - 2\mu t \frac{\sin^2 r}{\cos r} \\ &= \frac{2\mu t}{\cos r} (1 - \sin^2 r) \\ &= \frac{2\mu t \cos^2 r}{\cos r} = 2\mu t \cos r \end{aligned}$$

$$\therefore \text{Path difference} = 2\mu t \cos r$$



### Case (i):- Bright Band

If the film will appear bright (or) constructive interference will form, if the path difference is

$$2\mu t \cos r = (2n+1) \lambda/2 \quad \text{- where } n= 0,1,2,3 \dots$$

### Case (ii):- Dark Band

If the film will appear dark (or) destructive interference will form, if the path difference is

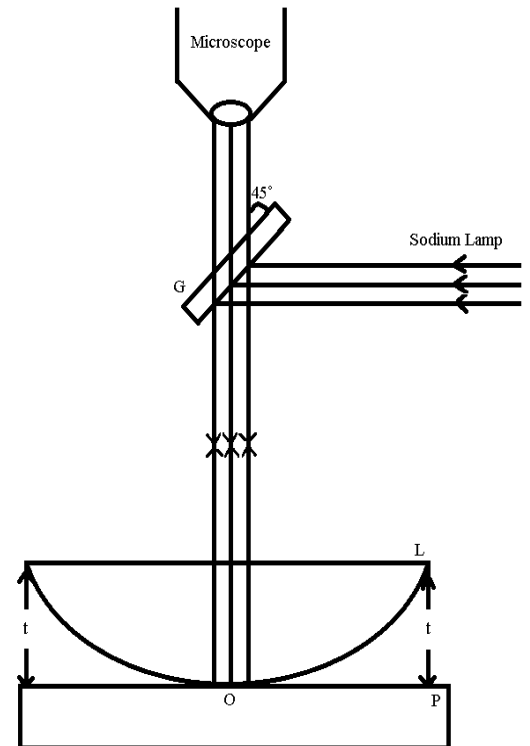
$$2\mu t \cos r = (2n) \lambda/2 = n\lambda \quad \text{- where } n=1, 2, 3 \dots$$

## Newton's Rings:-

### Experimental arrangement:-

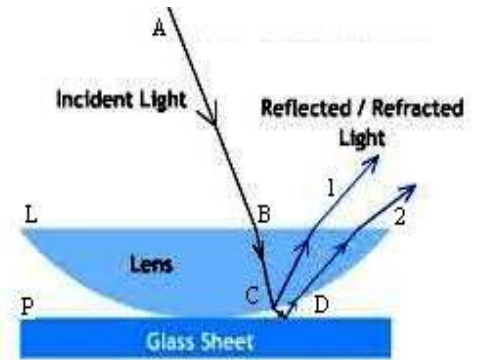
The experimental arrangement of obtaining Newton's rings is shown in below figure.

Let "L" is Plano convex lens of radius of large curvature. This lens with its convex surface is placed on a glass plate "P". The lens "L" contact with the plate P at point "O". A monochromatic source such as sodium lamp fall on a glass plate "G" held at an angle 45° with the vertical. The glass plate "G" reflects normally a part of incident light towards the lens "L" and glass plate "P". A part of incident light is reflected by the curved surface of the lens "L" and a part is transmitted which is reflected back from the plane surface of the plate "P". These two reflected rays interfere and give rise to an interference pattern in the form of circular rings. It can be seen with a microscope. This phenomenon was first described by Newton, that it why the rings are known as Newton's rings.



### Explanation:-

The formation of Newton's rings can be explained with the help of the figure. AB is a monochromatic ray of light which falls on the system. A part of light is reflected at "C", which goes out in the form of ray-1 without any phase. The other part of light is refracted along CD. At point "D" it is again reflected and goes out in the form of ray-2 with a phase of  $\pi$ . The reflected rays 1 and 2 are in a position to produce interference pattern and appears as a bright and dark rings.



As the rings are observed in the reflected light, the path difference between them is  $(2\mu t \cos r + \lambda/2)$ .

It  $\mu=1$ ,  $r=0$

Then, the path difference =  $(2t + \lambda/2)$

At the point of contact  $t=0$ , then the path difference is  $\lambda/2$ .

**Condition for Bright ring is:-**

$$2t + \lambda/2 = n\lambda$$

$$2t = n\lambda - \lambda/2$$

$$2t = \frac{2n\lambda - \lambda}{2}$$

$$2t = \frac{(2n-1)\lambda}{2}, \text{ where } n=1,2,3,\dots$$

**Condition for Dark ring is:-**

$$2t + \lambda/2 = (2n+1)\lambda/2$$

$$2t = \frac{2n\lambda}{2} + \frac{\lambda}{2} - \frac{\lambda}{2}$$

$$2t = n\lambda, \text{ where } n = 0,1,2,3,\dots$$

### Calculating the diameter of the ring:-

Let LOL' be the lens placed on the glass plate "P". The curved surface LOL' is part of the spherical surface with the centre at "C". Let 'R' be the radius of curvature and 'r' be the radius of Newton's rings corresponding to constant film thickness "t".

From the property of the circle

$$NQ \times NS = NO \times ND$$

$$r \times r = t \times (2R - t)$$

$$r^2 = 2tR - t^2 \approx 2tR$$

$$r^2 = 2tR$$

$$t = r^2/2R$$

Thus, for a bright ring

$$2t = \frac{(2n-1)\lambda}{2}$$

$$2 \times \frac{r^2}{2R} = \frac{(2n-1)\lambda}{2}$$

$$r^2 = \frac{(2n-1)\lambda R}{2}$$

Replacing "r" by D/2, where "D" is Diameter

$$\frac{D^2}{4} = \frac{(2n-1)\lambda R}{2}$$

$$D_n^2 = 2\lambda R(2n-1)$$

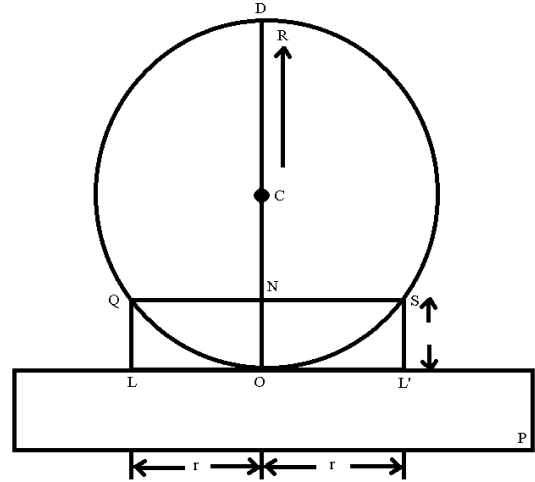
$$D_n = \sqrt{2\lambda R(2n-1)}$$

Similarly for the Dark ring

$$2t = n\lambda$$

$$2 \times \frac{r^2}{2R} = n\lambda$$

$$r^2 = nR\lambda$$





Replacing “r” by D/2,  $\frac{D^2}{4} = nR\lambda$   
 $D_n^2 = 4nR\lambda$   
 $D_n = 2\sqrt{nR\lambda}$

Thus, the diameter of the rings is proportional to the square root of the natural numbers.

**Calculating the wavelength of light:-**

We have for the diameter of the n<sup>th</sup> Dark ring  $D_n^2 = 4nR\lambda$

Similarly diameter for the (n+m)<sup>th</sup> Dark ring  $\therefore D_{n+m}^2 - D_n^2 = 4(n+m)R\lambda - 4nR\lambda$   
 $= 4nR\lambda + 4mR\lambda - 4nR\lambda$   
 $= 4mR\lambda$   
 $\therefore D_{n+m}^2 - D_n^2 = 4mR\lambda$

$$\lambda = \frac{D_{n+m}^2 - D_n^2}{4mR}$$

**How diffraction is different from interference:**

<b>Interference</b>	<b>Diffraction</b>
1. It is due to superposition of two different wave fronts originating from two coherent sources.	1. It is due to superposition of secondary wavelets originating from the different parts of the same wave front.
2. Interference bands are of equal width	2. Diffraction bands decrease in their widths as the order decreases
3. All the bright fringes are of the same intensity	3. The bright fringes are of varying intensity
4. All the dark fringes have zero intensity.	4. The intensity of dark fringes is not zero

## **DIFFRACTION**

### **Introduction:-**

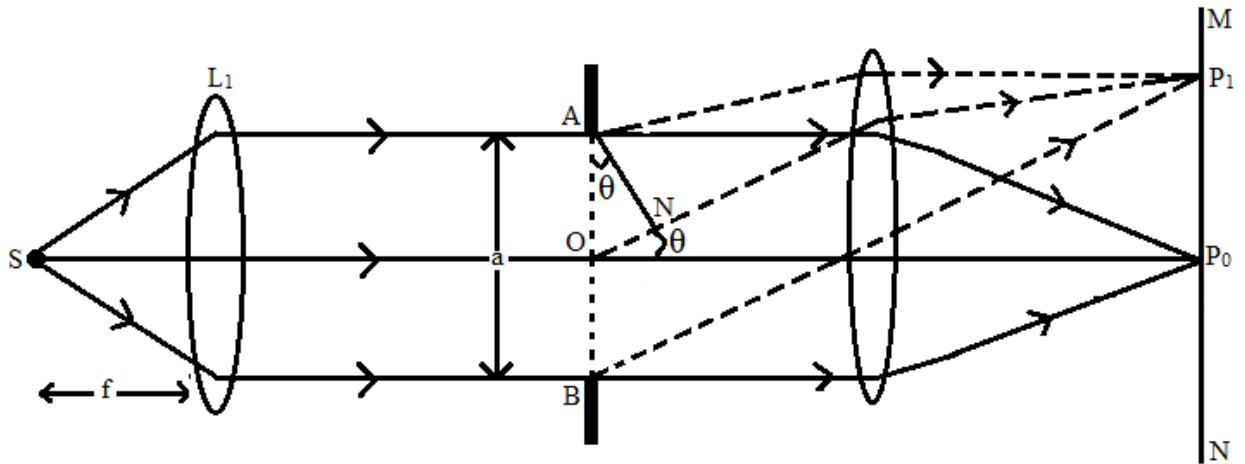
Sound waves propagate in the form of waves. Sound produced inside one room reaches another room after bending the edge of the doors (or) walls.

- The bending of waves around the obstacle is called Diffraction.
- If the wavelength of light is extremely small, there is no bending of light waves.
- If the size of the obstacle is comparable with the wavelength
- The bending of light ray around the corners of an obstacle and spreading of light ray into the geometrical shadow. This phenomenon is called Diffraction.
- The phenomenon of diffraction is divided into two types
  1. Fresnel's Diffraction
  2. Fraunhofer Diffraction

### **Difference between Fresnel's and Fraunhofer Diffraction:-**

<b>Fresnel's Diffraction</b>	<b>Fraunhofer Diffraction</b>
Either point source (or) illuminated narrow slit is used	Extended source at infinite distance is used
The wave front undergoes diffraction is either spherical (or) cylindrical	The wave front undergoes diffraction is plane wave front
The distance between the source and Screen is finite.	The distance between the source and Screen is infinite.
No lenses is used to focus the rays	Converging lenses is used to focus the rays

**Fraunhofer diffraction at single slit:**



Let 'S' be a point source of monochromatic light.  $L_1$  is the collimating lens of focal length ' $f$ ' at a distance from the source, so that the lens renders parallel rays as shown in figure. AB is the single slit of width  $a$ . the light passing through this slit is collected by lens  $L_2$  which forms the final image on the screen MN.

Let us consider a plane wave front incident normally on the slit AB. Each and every point on the wave front will act as secondary source of light. First of all, let us consider the wave travelling along  $OP_0$ . i.e. if we consider wave from point A, O and B, they travel straight and focus at  $P_0$  and produce constructive interference and hence point  $P_0$  has maximum intensity.

Now let us consider the secondary waves travelling with an angle  $\theta$  to reach point  $P_1$  on the screen. Now we have to find whether the point  $P_1$  will have maximum or minimum intensity. To calculate this let us first draw a line perpendicular to AN from the point A. this ray is diffracted from the point B. now the distance BN gives the path difference between the secondary waves from A and N reaching point  $P_1$ .

Let us consider the

$$\Delta le ANB \Rightarrow \sin \theta = \frac{BN}{AB}$$

$$BN = AB \sin \theta$$

$$AB = a$$



Hence the path difference =  $a \sin \theta$ .

Where  $a$  is the width of the slit  
 $\theta$  is the angle of diffraction

**Case (i):** let us assume that this path difference is  $\lambda$

Let us divide the slit AB into two halves AO and OB. The path difference between waves from point A and O reaching  $P_1$  will be  $\lambda/2$  and hence destructive interference takes place, then  $P_1$  will be of minimum intensity.

$$a \sin \theta = \lambda$$

$$\sin \theta = \frac{\lambda}{a}$$

if  $\theta$  is small, then  $\sin \theta = \theta$

$$\text{then } \theta = \frac{\lambda}{a}$$

**Case (ii):** Let us assume that this path difference is  $2\lambda$

Then the slit AB has to be divided into four equal parts of width  $a/4$  each. Then the path difference is  $\lambda/4$  and hence destructive interference takes place, then  $P_1$  will be of minimum intensity.

$$a \sin \theta = n\lambda$$

where  $n=1,2,3,\dots$

**Case (iii):** When the path difference is integral multiples of  $\lambda$ , minimum intensity is obtained. Then, what happens when the path difference is odd multiples of  $\lambda/2$ .

Let the path difference be  $3\lambda/2$ , i.e. between  $\lambda$  and  $2\lambda$ . Now the slit AB of width 'a' has to be divided into equal parts of width  $a/3$  each. Now the waves from corresponding points from first and second part will have a path difference  $\lambda/2$ . Hence it undergoes destructive interference. But the remaining second and third part will contribute towards intensity at point  $P_1$ . Hence  $P_1$  will be maximum intensity. Therefore the path difference

$$a \sin \theta = (2n+1)\lambda/2$$

where  $n=1,2,3,\dots$  etc is the condition for secondary maxima  
~ 10 ~

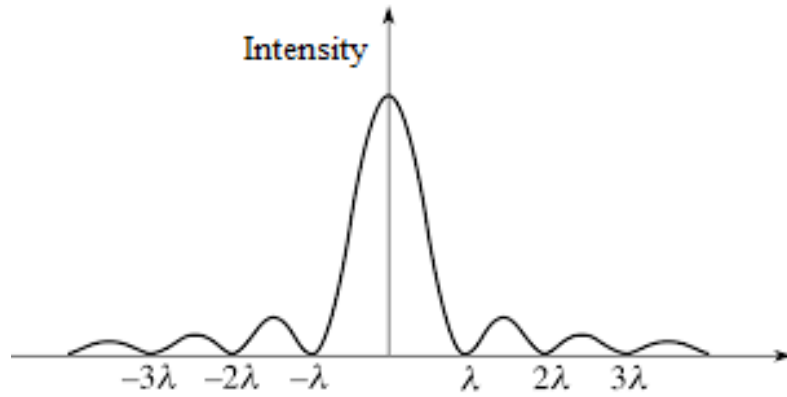
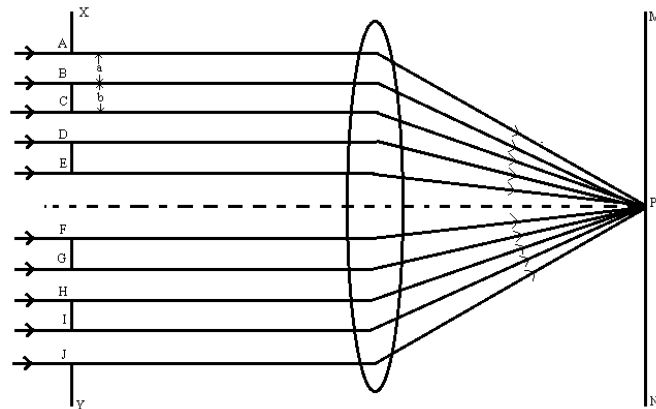


Fig: Intensity distribution due to diffraction at single slit

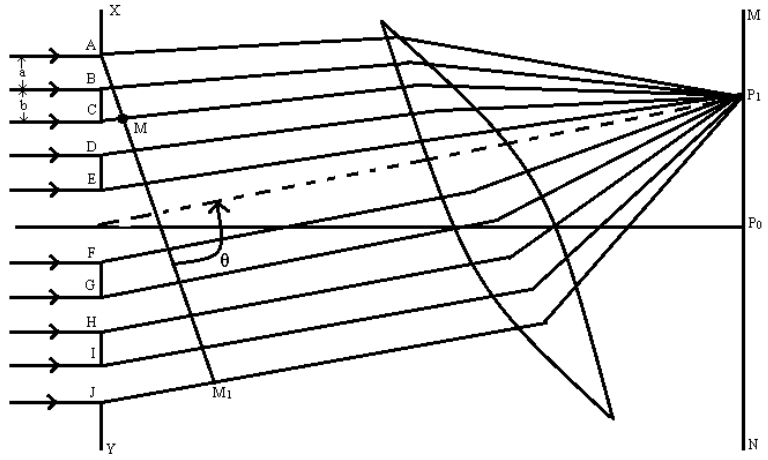
Thus the diffraction pattern due to a single slit consists of a central bright maximum with secondary maxima and minima on both sides as shown in above figure.

### Diffraction Grating:-

An arrangement which consists of large number of parallel slits of equal width and separating by equal space is called diffraction grating.



Let AB, CD, EF represents the slits of width “a”, while BC, DE, FG represents ruling width of “b” each. Now (a+b) is the combined width and it is also the distance between the two slits. When a light ray is incident normally on a grating, the plane wave front will be acts as a secondary source of light, and then the incident light rays will be moves in all directions, on the other side of the grating at point P<sub>0</sub>. P<sub>0</sub> is the central maxima.



The secondary diffracted waves moving in a direction which makes an angle “θ”.

A line is drawn normal to AMM’. So that image is seen at point P<sub>1</sub>. To calculate the central maxima,

Consider a Δle ACM  $\Rightarrow \sin \theta = \frac{CM}{AC}$   
 $\Rightarrow CM = AC \sin \theta$   
 $\Rightarrow CM = (a+b) \sin \theta$

Central maxima:-

$$(a+b) \sin \theta = m\lambda \dots\dots\dots (1)$$

$$\Rightarrow \sin \theta = \frac{m\lambda}{(a+b)}$$

$$\Rightarrow \sin \theta = mN\lambda \dots\dots\dots (2)$$

Where  $N = \frac{1}{a+b}$  is the number of grating elements

**Grating spectrum:-**

If m=1  $\rightarrow (a+b) \sin \theta = m \lambda$

$$(a+b) \sin \theta = \lambda$$

If m=2  $\rightarrow (a+b) \sin \theta = 2\lambda$

## OBJECTIVE QUESTIONS FOR ONLINE EXAMINATION

### INTERFERENCE

- Light has ----- nature
  - Particle
  - Wave
  - Both Particle and wave**
  - none
- The wave nature of light is evidences
  - Photo electric effect
  - Compton effect
  - Blackbody radiation
  - Interference**
- In interference, the intensity of light gets:
  - Modified**
  - Remains same
  - A and B
  - None
- Path difference between two coherent wave is
  - Zero
  - Constant
  - A and B**
  - None
- Two light becomes interference have their amplitudes 2:1 ratio, then the intensity ratio of bright and dark fringes is:
  - 1:2
  - 1:4
  - 9:1**
  - 4:1
- In Newton's rings experiment the diameter of bright ring is proportional to
  - $\sqrt{n-1}$
  - $\frac{(2n-1)\lambda}{2}$
  - $\sqrt{2n-1}$
  - $\sqrt{n}$
- In Newton's ring experiment, the wavelength  $\lambda =$ 
  - $\frac{D_m^2 - D_n^2}{4R(m-n)}$
  - $\frac{R(D_m^2 - D_n^2)}{4R(m-n)}$
  - $\frac{4(D_m^2 - D_n^2)}{R(m-n)}$
  - $\frac{4R(m-n)}{R(D_m^2 - D_n^2)}$
- Colors in soap bubbles are observed due to phenomenon of
  - Interference**
  - Diffraction
  - Polarization
  - Double refraction
- In young's double slit experiment, when d is the separation between the two coherent

sources,  $D$  is the distance between the coherent sources and the screen, and  $\lambda$  is the wave

- a.  $B = \lambda d/D$       b.  $\beta = \lambda D/d$       c.  $\beta = dD/\lambda$       d.  $\lambda dD$

10. In constructive interference the path difference is equal to

- a.  $n\lambda$       b.  $(2n+1)\lambda$       c.  $(2n-1)\lambda$       d.  $(2n+1)\lambda/2$

11. The basis for interference of light is principle of

- a. **Superposition**      b. Newton's corpuscular theory      c. Fermat      d. Reflection

12. To sustain the interference pattern the source must be

- a. Incoherent      b. Two real      c. Highly intense      d. **Coherent**

13. Division of amplitude is the principle use in the study

- a. Fresnel's Biprism      b. **Newton's rings**      c. Grating      d. Prism

14. Colors in thin films is explained using the study of

- a. Polarization      b. Diffraction      c. Dispersion      d. **Interference**

15. In the study of Newton's rings experiment the lens used is

- a. Double concave      b. Double convex      c. **Plano convex**      d. Plano concave

16. Using Newton's rings experiment one can determine

- a. **Refractive index**      b. Frequency      c. Speed of light      d. Density

17. In Newton's rings experiment, the plane glass plate is placed at this angle with the beam direction

- a.  $60^\circ$       b.  **$45^\circ$**       c.  $90^\circ$       d.  $30^\circ$

18. With white light, Newton's rings are

- a. Bright      b. Dark      c. Alternately dark and bright      d. **Colored**  
e.

19. In interference

- a. There is loss of energy in great amount    c. There is loss of energy in less amount  
**b. There is no loss energy**    d. Diffraction of light

20. The path difference for thin films is given as  $x=2\mu t \cos r \lambda/2$ . In this case interference is given by

- a. Reflected light**    b. Transmitted light    c. Refracted light    d. Light wave

21. In the study of interference due to thin films by reflected light the central part is

- a. Bright    **b. Dark**    c. Dull    d. Colored

22. Superposition of crest and trough results into ----- interference

- a. Constructive    **b. Destructive**    C. Damped    d. Undamped



## OBJECTIVE QUESTIONS FOR ONLINE EXAMINATION

### DIFFRACTION

23. The penetration of light waves into the regions of the geometrical shadow is

- a. interference    b. **Diffraction**    c. Polarization    d. Reflection

24. Diffraction phenomenon is usually divided into ----- classes

- a. 1    b. **2**    c. 3    d. 4

25. The wave front undergoes into Fraunhofer diffraction is

- a. Spherical    b. **Plane**    c. Elliptical    d. Cylindrical

26. In a single slit experiment if the slit width is reduced

- a. **The fringes become wider**    c. The fringes become narrower  
b. No change    d. The fringes become brighter

27. Diffraction grating is

- a. **Large number of equidistant slits**    c. Single slit    d. Double slit  
b. Large number of random distant slits

28. When white light is incident on a diffraction grating, the light diffracted more will be

- a. Blue    b. **Red**    c. Green    d. Orange

29. Resolving power of a grating is

- a. **Directly proportional to N**    c. Directly proportional to  $N^2$   
b. Inversely proportional to N    d. Inversely proportional to  $N^2$

30. If 'e' is the slit width, d is distance between the slits then the condition for principal maximum in diffraction grating is:

- a.  **$(e+d) \sin \theta = n\lambda$**     b.  $(e-d) \sin \theta = n\lambda$     c.  $e \sin \theta = d\lambda$     d.  $d \sin \theta = ne\lambda$

31. Instead of red color source, if blue color source is used in single slit experiment

- a. The diffraction bands become wider    c. The diffraction pattern disappears

- b. The diffraction bands becomes narrower and crowded together    a. The diffraction pattern does not change
32. The dispersive power of a grating is  
 a.  $D = nN\lambda/d\lambda$     b.  $n/(e+d) \sin\theta$     c.  $nN/\cos\theta$     d.  $nN/\sin\theta$
33. The first diffraction grating was constructed by  
 a. Newton    b. **Joseph von Fraunhofer**    c. Einstein    d. Augustin –Jean Freanel
34. Wavelength of spectral lines can be measured using  
 a. Interference    b. Diffraction    c. Polarization    d. **Diffraction Grating**
35. Diffraction grating consists of  
 a. Double slit    c. Large number of random distant slits  
 b. **Large number of equidistant slits**    d. Single slit
36. In diffraction pattern due to single slit, the width of the central maximum will be  
 a. Less for narrow slit    c. **Greater for narrow slit**  
 b. Less for wide slit    d. Greater for wide slit
37. The criterion of resolution of optical instruments was given by  
 a. Newton    b. Ransden    c. **Rayleigh**    d. Huygens
38. Maximum number of orders available with a grating is  
 a. Independent of the grating element    c. Inversely proportional to the grating element  
 b. **Directly proportional to the grating element**    d. Directly proportional to the wavelength
39. The resolving power of a grating having N slits in nth order will be  
 a.  $(n+N)$     b.  $(n-N)$     c.  $nN$     d.  $n/N$
40. A limit on the performance of a resolving instrument is set by  
 a. Interference    b. **Diffraction**    c. Polarization    d. Quantum nature of light

# POLARIZATION

## **Introduction:-**

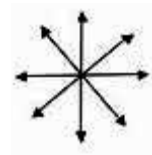
Interference and Diffraction phenomenon prove that the propagation of light is in the form of wave motion. But these phenomenon do not reveal the character of these wave motions i.e. whether it is longitudinal and transverse.

- In longitudinal waves, the particles of the medium moves to and fro in the direction of propagation of waves.  
Example: - Sound waves.
- In transverse waves, the particles of the medium vibrate up and down in the direction of propagation of waves  
Example: - Ripples on a water surface

The phenomenon of polarization was discovered, that it was established that waves are transverse waves.

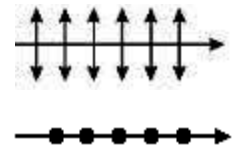
## **Unpolarized light:-**

The light rays which are moving in all direction is shown as Unpolarized light.



## **Polarized light:-**

The light which has moving only one direction is called polarized light.



## **Polarization:-**

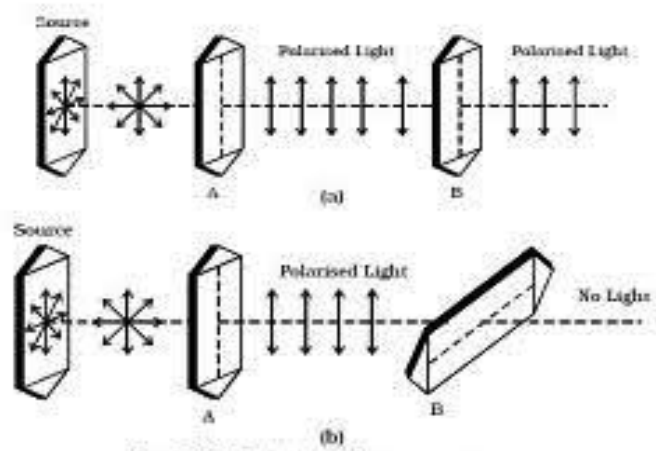
Polarization is the process of converting the unpolarized light into polarized light.

## Propagation of Light Waves:-

Consider an ordinary beam of light is passed through a tourmaline crystal as shown in figure. The tourmaline crystal has the property of allowing the components of light that are vibrate parallel to its axis to pass through it, and it allow all the other corresponding of light that are not parallel to the axis of the crystal.

The emergent beam from the tourmaline crystal vibrates in only one direction, i.e. it vibrates parallel to the axis of the crystal. The light rays that are vibrating in only one direction are known as polarized light.

- The first tourmaline crystal that produces the polarized light is called the polarizer.
- The second tourmaline crystal is used to analyze the polarized light. Therefore, the second crystal is said to be an analyzer.



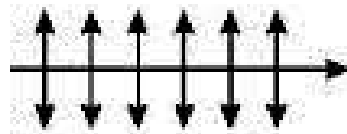
## Types of Polarized light:-

There are four types of polarized light.

1. Plane polarized light
2. Partially polarized light
3. Circularly polarized light
4. Elliptically polarized light

### 1). Plane polarized light:-

If the vibrations are confined to a single plane i.e. either in the direction along the parallel to the plane of the paper (or) in the direction perpendicular to the plane of the paper, then it is called Plane polarized light.



- The vibrations along parallel to the plane of the paper as shown in figure represented by arrows



- While the vibrations perpendicular to the plane of the paper are represented by dots as shown in figure.

## 2). partially polarized light:-

If the linearly polarized light contain small additional component of unpolarized light, it becomes partially plane polarized light. Then it is represented by either more arrows and less dots as shown in figure (a) (or) more dots as shown in figure (b).

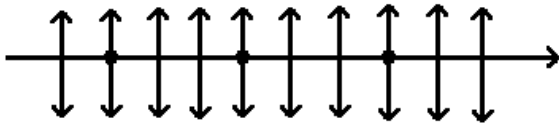


Figure (a):- More verticle and less horizontal vibrations

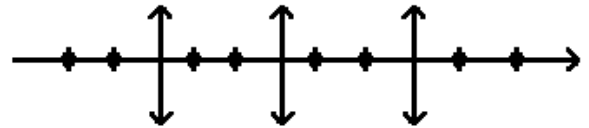
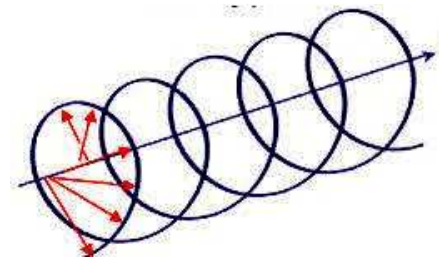


Figure (b):- More horizontal and less verticle vibrations

## 3). circularly polarized light:-

In circularly polarization, the electric vector of constant amplitude no longer oscillates, but rotates while proceeding in the form of a helix (or) circular.

- If the vector rotates in the clockwise direction with respect to the direction of preparation, it results in right – circularly polarized light.
- While the rotation in the anti-clockwise direction results in left circularly polarized light.



## 4). elliptically polarized light:-

If the amplitude of the electric vector is not a constant, but varies periodically, then it is elliptically polarized light.



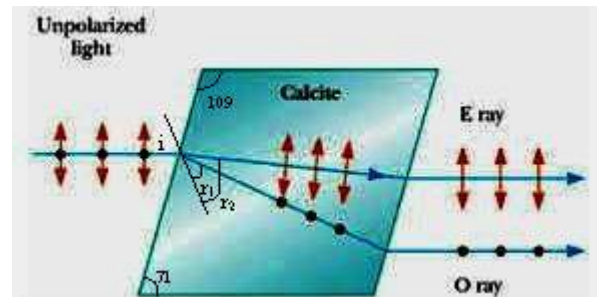
## Double Refraction:-

Double Refraction was discovered by Bartholinus.

- When ordinary light passed through a calcite crystal, it splits into two plane polarized refracted light.

1. Ordinary ray
2. Extra ordinary ray

- The one which obeys laws of refraction and has vibrations perpendicular to the principle section is known as “Ordinary ray”
- The other does not obey the law of refraction and has vibrations parallel to the principle section is called “Extra ordinary ray”.
- The phenomenon is known as Double refraction.



The refractive indices of O-ray and E-ray for calcite crystal are given by

$$\mu_o = \frac{\sin i}{\sin r_1}$$

$$\mu_e = \frac{\sin i}{\sin r_2}$$

It is clear that  $\mu_o$  is same for all the angles of incidence, while  $\mu_e$  varies with angle of incidence.

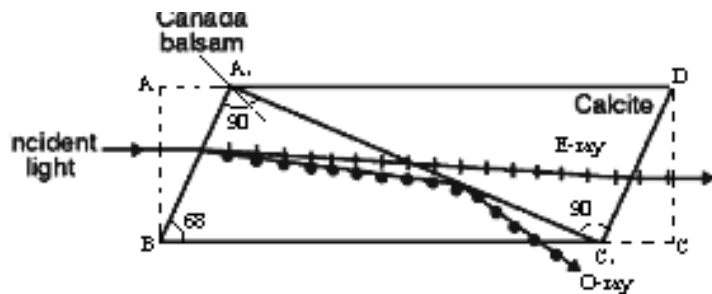
- Hence O-ray travel with same velocity and it is represented by a spherical wave front.
- Whereas E-ray travel with different velocity in different directions and is given by ellipsoidal wave front.



## Nicol's prism:-

Nicol's prism is one of the most important device used to produce plane polarized light. This was invented by Willum Nicol in 1828.

A calcite crystal whose length is three times breath is taken. The two ends AB and CD of the crystal are cut, so that the angles ABC reduces from  $71^\circ$  to



$68^\circ$  as shown in figure. Then the crystal is cut into two halves along the plane AC, which passes through the blunt corners and perpendicular to both the principle section and end faces.  $A_1C_1$  makes an angle of  $90^\circ$  with  $C_1D$  and  $A_1B$ . The two cut faces are well polished and cemented together using a thin layer of Canada balsam, a clear transparent material has refractive index 1.55 for  $\lambda=5893 \text{ \AA}$ .

When unpolarized light enters the Nicol prism, it splits into O-ray and E-ray.

- O-ray has the refractive index 1.6584, while the e-ray has the refractive index varies from 1.4864 to 1.6584.
- Inside the crystal when O-ray meets the thin layer of Canada balsam, it has to travel from denser medium to rarer medium.
- Because of shaping of the crystal face, the O-ray is refracted more so that the angle of incidence at the Canada balsam interface is greater than the critical angle.
- Hence it undergoes total internal reflection as shown in figure. Hence E-ray alone emerges out of the other face of the prism.
- Nicol prism is good polarizer and analyzer and can be used to produce and analyze the plane polarized light.

## Quarter Wave Plate:-

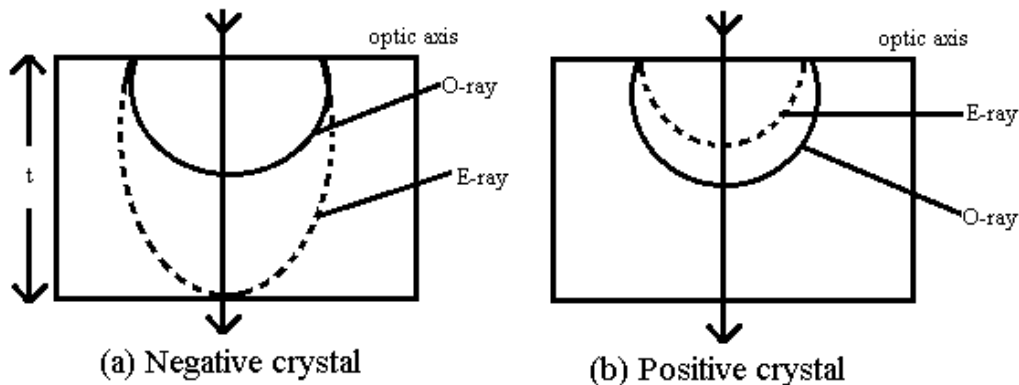
A quarter wave plate (QWP) is a device from a doubly refracting uniaxial crystal, whose refracting faces are cut parallel to the direction of the optic axis as shown in figure. When plane polarized light is incident on calcite crystal, the light splits up into O-ray and E-ray. As a result, a phase difference is introduced between them.

Let  $\mu_e$  and  $\mu_o$  be the refractive indices of the extraordinary and ordinary rays respectively. Let “t” be the thickness of the crystal.

$$\text{Optical path of O-ray} = \mu_o t$$

$$\text{Optical path of E-ray} = \mu_e t$$

$$\text{Hence Optical path difference} = \mu_o t - \mu_e t = (\mu_o - \mu_e)t$$



Then for a negative crystal ( $\mu_o > \mu_e$ ), then the path difference between these two rays is  $(\mu_o - \mu_e)t$

The QWP is cut in such a way that it can produce a path difference of  $\lambda/4$ .

$$\text{i.e. } (\mu_o - \mu_e) t = \frac{\lambda}{4}$$

$$t = \frac{\lambda}{4(\mu_o - \mu_e)}$$

Then for a positive crystal ( $\mu_e > \mu_o$ ), path difference between the ordinary and extra ordinary rays is  $(\mu_e - \mu_o) t$ .

The thickness of a QWP made up of positive crystal can be obtained by equating their path difference to  $\lambda/4$ .

$$\text{i.e. } (\mu_e - \mu_o) t = \frac{\lambda}{4}$$

$$t = \frac{\lambda}{4 (\mu_e - \mu_o)}$$

### Half Wave Plate:-

A half wave plate (HWP) is a doubly refracting uniaxial negative (or) positive crystal cut in such a way that it can produce a phase difference of “ $\pi$ ” (or) path difference of “ $\lambda/2$ ” between the ordinary and extra ordinary rays.

Let  $\mu_e$  and  $\mu_o$  be the refractive indices of the extraordinary and ordinary rays respectively. Let “ $t$ ” be the thickness of the crystal.

$$\text{Optical path of O-ray} = \mu_o t$$

$$\text{Optical path of E-ray} = \mu_e t$$

$$\text{Hence Optical path difference} = \mu_o t - \mu_e t = (\mu_o - \mu_e) t$$

Then for a negative crystal ( $\mu_o > \mu_e$ ), then the path difference between these two rays is  $(\mu_o - \mu_e) t$

The HWP is cut in such a way that it can produce a path difference of  $\lambda/2$ .

$$\text{i.e. } (\mu_o - \mu_e) t = \frac{\lambda}{2}$$

$$\Rightarrow t = \frac{\lambda}{2 (\mu_o - \mu_e)}$$

Then for a positive crystal ( $\mu_e > \mu_o$ ), path difference between the ordinary and extra ordinary rays is  $(\mu_e - \mu_o) t$ .

The thickness of a HWP made up of positive crystal can be obtained by equating their path difference to  $\lambda/2$ .

$$\text{i.e. } (\mu_e - \mu_o) t = \frac{\lambda}{2}$$

$$\Rightarrow t = \frac{\lambda}{2 (\mu_e - \mu_o)}$$

$$\sim 24 \sim 2 (\mu_e - \mu_o)$$

## OBJECTIVE QUESTIONS FOR ONLINE EXAMINATION

### POLARISATION

41. Polarization of light is evidence for

- a. **Transverse nature of light**
- b. Longitudinal nature of light
- c. Quantum nature of light
- d. Corpuscular nature of light

42. Nicol prism is based on the principle of

- a. Retraction
- b. **Double refraction**
- c. Reflection
- d. Scattering

43. The O-ray obeys

- a. **Snell's Law**
- b. Does not obey Snell's Law
- c. Brewster Law
- d. Does not obey Brewster Law

44. In double refraction

- a. **The velocity of ordinary ray is same in all direction**
- b. The velocity of extra ordinary ray is same in all directions
- c. Both a and b
- d. First o-ray have high velocity, finally e-ray have high velocity

45. In sound, following phenomenon does not takes place

- a. Interference
- b. **Polarization**
- c. Diffraction
- d. Scattering

46. The examples for negative crystal

- a. **calcite**
- b. Ice
- c. Quartz
- d. Water

47. On rotating the analyzer, the intensity of emergent light varies between a maximum and minimum which is zero, then the light incident in the analyzer is

- a. Unpolarized
- b. Circularly polarized
- c. Elliptically polarized
- d. **Plane polarized**

48. An axis of two nicol prisms is parallel to one another. One of two nicols is rotated through an angle of  $60^\circ$ . The rotation of the intensity of transmitted light before and after rotation is

- a. 1:2      b. 2:1      c. 1:4      d. **4:1**

49. In elliptically polarized light

- a. Amplitude of vibrations changes in direction only
- b. Amplitude of vibrations changes in magnitude only
- c. Amplitude of vibrations changes in magnitude and direction**
- d. Amplitude of vibrations does not changes in magnitude and direction

50. A glass slab has the angles of polarization  $62^{\circ}24'$ , then the refractive index of the glass slab is

- a. **1.91**      b. 19.1      c. 0.191      d. 191

51. The light waves are transverse in nature establish beyond doubt by

- a. Fresnel's diffraction    b. Fraunhofer diffraction    **c. Polarization**    d. Interference

52. The plane of polarization is that plane in which

- a. Few vibrations occur      **c. No vibrations occur**
- b. Number of vibrations occurs      d. Mixed vibrations occur

53. The plane of vibration is that plane in which

- a. **Vibrations occur at right angles to the plane of polarization**      c. No vibrations occur
- b. Few vibrations occur      d. Mixed vibrations occur

54. The angle of polarization is related to

- a. **Brewster's Law**    b. Snell's Law    c. Newton's Law    d. Bragg's Law

55. When a monochromatic light is passed through nicol prism, it splits in two rays, one ray through the nicol prism is

- a. Ordinary ray    b. **Extra ordinary ray**    c. Scattered ray    d. Reflected ray

56. The plane polarized light can be produced by using

- a. Rochon Prism    b. Wallaston prism    **c. Nicol Prism**    d. Some other prism

57. The refractive index of Canada Balsam is
- a. Greater than ordinary ray
  - b. Less than extra ordinary ray
  - c. Equal to the ordinary ray
  - d. Lies between ordinary and extraordinary ray**
58. The phenomenon by which splitting of light into e-ray and o-ray is known as
- a. Refraction
  - b. Selective absorption
  - c. Double refraction**
  - d. Scattering
59. A birefringent crystal has refractive indices  $\mu_o$  and  $\mu_e$  for the rays. If the crystal is to function as a quarter wave plate, the number of wavelengths, each component as the waves travel within the crystal thickness must differ by
- a. 4
  - b.  $\lambda/4$
  - c.  $\lambda$
  - d.  $\lambda/2$
60. A half wave plate introduces ----- phase difference between the ordinary and extraordinary rays
- a. 0
  - b.  $\pi/2$
  - c.  $\pi$
  - d.  $2\pi$



## PROBLEMS

1. Newton's rings are observed in the reflected light of wavelength  $5900\text{\AA}$ . The diameter of  $10^{\text{th}}$  dark ring is 0.5 cm. find the radius of curvature of lens used.

Sol: Given that

The wavelength of reflected light  $\lambda = 5900\text{\AA} = 5900 \times 10^{-10} \text{ m}$

The diameter of  $10^{\text{th}}$  dark ring  $D_{10} = 0.5 \text{ cm} = 0.0005 \text{ m} = 5 \times 10^{-3} \text{ m}$ .

The expression for the diameter of  $n^{\text{th}}$  dark ring

$$R = \frac{D_n^2}{4n\lambda} = \frac{(5 \times 10^{-3})^2}{4 \times 10 \times 5900 \times 10^{-10}} = 1.059 \text{ m}$$

The radius of curvature of the lens  $R = 1.059 \text{ m}$ .

2. In Newton's rings experiment the diameter of  $15^{\text{th}}$  ring was found to be 0.59 cm and that of  $5^{\text{th}}$  ring 0.336 cm. the radius of curvature of the lens is 100 cm. Find the wavelength of light.

Sol: Given that

The diameter of  $15^{\text{th}}$  ring,  $D_{15} = 0.59 \text{ cm} = 5.9 \times 10^{-3} \text{ m}$

The diameter of  $5^{\text{th}}$  ring,  $D_5 = 0.336 \text{ cm} = 3.36 \times 10^{-3} \text{ m}$

The radius of curvature of lens  $R = 100 \text{ cm} = 1 \text{ m}$

The expression for wavelength of light is

$$\lambda = \frac{D_{n+m}^2 - D_n^2}{4mR}$$

Here  $(n + p) = 15$  and  $n = 5$ , therefore

$$\lambda = \frac{D_{15}^2 - D_5^2}{4 \times 10 \times R} = \frac{(5.9 \times 10^{-3})^2 - (3.36 \times 10^{-3})^2}{4 \times 10 \times 1} = 0.588 \times 10^{-6} \text{ m} = 588 \text{ nm}$$

The wavelength of the light  $\lambda = 588 \text{ nm}$

3. In Newton's rings experiment the diameter of  $8^{\text{th}}$  ring was found to be 0.35 cm and that of  $18^{\text{th}}$  ring 0.65 cm. if the wavelength of the light  $6000\text{\AA}$  is used than find the radius of curvature of the plano-convex lens.

Sol: Given that

The diameter of the  $8^{\text{th}}$  ring  $D_8 = 0.35 \text{ cm}$

The diameter of the  $18^{\text{th}}$  ring,  $D_{18} = 0.65 \text{ cm}$

The wavelength of the light,  $\lambda = 6000\text{\AA} = 6000 \times 10^{-8} \text{ cm}$ .

The expression for wavelength of light

$$\lambda = \frac{D_{n+m}^2 - D_n^2}{4mR}$$

$$R = \frac{D_{n+m}^2 - D_n^2}{4m\lambda} = \frac{D_{18}^2 - D_8^2}{4 \times 10 \times \lambda}$$

$$R = \frac{(0.65)^2 - (0.352)^2}{4 \times 10 \times 6000 \times 10^{-8}} = 125 \text{ cm}$$

The radius of curvature of the lens  $R=125 \text{ cm}=1.25 \text{ m}$ .

4. Calculate the thickness of the air film at 10<sup>th</sup> dark ring in Newton's rings system viewed normally by a reflected light of wavelength 500 nm. The diameter of 10<sup>th</sup> dark ring is 2 mm.

Sol: Given that

The wavelength of reflected light  $\lambda=500 \text{ nm} = 500 \times 10^{-9} \text{ m}$

The diameter of 10<sup>th</sup> dark ring  $D_{10}=2 \text{ mm}=2 \times 10^{-3} \text{ m}$

The expression for diameter of dark ring

$$D_n^2 = 4n\lambda R$$

$$\therefore R = \frac{D_n^2}{4n\lambda} = \frac{(2 \times 10^{-3})^2}{4 \times 10 \times 500 \times 10^{-9}} = 0.2 \text{ m}$$

The radius of curvature of the convex lens  $R=0.2 \text{ m}$

The expression for the thickness of air film at nth dark ring is

$$2t = \frac{D_n^2}{4R}$$

$$t = \frac{D_n^2}{8R}$$

$$\text{If } n=10, \text{ then } t = \frac{(2 \times 10^{-3})^2}{8 \times 0.2} = 2.5 \times 10^{-6} \text{ m} = 2.5 \mu\text{m}$$

5. A plane transmission grating having 4250 lines per cm is illuminated with sodium light normally. In second order spectrum the spectral lines deviated by 30° are observed. Find the wavelength of the spectral line.

Sol: Given that

The number of lines per cm on grating  $N=4250$

The angle of diffraction  $\theta=30^\circ$

Order of the spectrum  $n=2$

We know that  $\lambda = \frac{\sin \theta}{Nn}$

$$\lambda = \frac{\sin 30}{4250 \times 2} = \frac{0.5}{8500} = 5.882 \times 10^{-5} \text{ cm} = 5882 \text{ \AA}$$

The wavelength of the spectral line  $\lambda=5882 \text{ \AA}$

6. Calculate the thickness of a mica sheet required for making a quarter wave plate for  $\lambda=5460\text{\AA}$ . The indices of refraction for the ordinary and extraordinary rays in mica are 1.586 and 1.592.

Sol: Given that,

The wavelength of light  $\lambda=5460\text{\AA}=5460\times 10^{-10}\text{ m}$

The refractive index of mica for ordinary ray  $\mu_o=1.586$

The refractive index of mica for extraordinary ray  $\mu_e=1.592$

We know that the thickness of a quarter wave plate is

$$t = \frac{\lambda}{4(\mu_e - \mu_o)}$$

$$t = \frac{5460 \times 10^{-10}}{4(1.592 - 1.586)} = 2.275 \times 10^{-5} \text{ m} = 22.75 \mu\text{m}$$

The thickness of quarter wave plate  $t= 22.75 \mu\text{m}$

7. Calculate the thickness of a quarter wave plate for light of wavelength  $6000\text{\AA}$ . ( $\mu_e=1.554, \mu_o=1.544$ )

Sol: Given that,

The wavelength of light  $\lambda=6000\text{\AA}=6000\times 10^{-10}\text{ m}$

The refractive index of mica for ordinary ray  $\mu_o=1.554$

The refractive index of mica for extraordinary ray  $\mu_e=1.544$

We know that the thickness of a quarter wave plate is

$$t = \frac{\lambda}{4(\mu_o - \mu_e)}$$

$$t = \frac{6000 \times 10^{-10}}{4(1.554 - 1.544)} = 1.5 \times 10^{-5} \text{ m}$$

The thickness of quarter wave plate  $t= 1.5 \times 10^{-5} \text{ m}$

8. Calculate the thickness of half wave plate of quartz for a wavelength  $500\text{ nm}$ . Here ( $\mu_e=1.553, \mu_o=1.544$ )

Sol: Given that

The wavelength of light  $\lambda=500\text{ nm}=500 \times 10^{-9}\text{ m}$

The refractive index of mica for ordinary ray  $\mu_o=1.553$

The refractive index of mica for extraordinary ray  $\mu_e=1.544$

We know that the thickness of a quarter wave plate is

$$t = \frac{\lambda}{2(\mu_o - \mu_e)}$$

$$t = \frac{500 \times 10^{-9}}{2(1.553 - 1.544)} = 2.777 \times 10^{-5} \text{ m} = 27.77 \mu\text{m}$$

# UNIT – II

Chapter – I: LASERS

Chapter – II: Fiber optics

Course Outcome:

**CO 2:** Analyze and understand various types of lasers & optical fibers. (Analyze) **K4**

## UNIT-II

### LASER

#### **Introduction:-**

The word “Laser” stands for “Light Amplification by Stimulated Emission of Radiation”

- Laser is outcome of “MASER”. It stands for “Microwave amplification and Stimulated Emission of Radiation”.
- Laser Principle is extended up to X-Rays and Gamma rays. Gamma ray lasers are called “GASERS”
- Laser are used many fields of Science and technology like radio astronomy, satellite communication, optical fiber communication, holography, data processing, testing and welding of materials, medicine etc.

#### **Characteristics of Lasers:-**

Laser differs from conventional light source in a number of ways such as sun, a flame or incandescent lamps. The most striking features of laser beam are

1. High Directional
2. High Intensity
3. High Monochromaticity
4. High Degree of Coherence

#### **High Directionality:-**

The conventional light sources emit light in all directions due to spontaneous emission. On the other hand, laser emits light only in one direction due to stimulated emission. The directionality of the laser beam is also expressed in terms of divergence.

$$\text{The divergence } \Delta\theta = \frac{r_2 - r_1}{d_2 - d_1},$$

Where  $r_1$  and  $r_2$  are the radii of laser beam spots at distance  $d_1$  and  $d_2$  respectively.

### **High Intensity:-**

The laser gives out light into a narrow beam and its energy is concentrated in a small region and has a great intensity.

Ex: 1 Watt of laser would appear many thousand times more intense than 100 watt of ordinary lamp.

So, laser light is intense light, because of all the emitted waves are in phase or coherent. In coherent beam, the resultant intensity is the sum of individual intensities and is proportional to  $na^2$

Where 'n' is the number of waves

'a' is the amplitude of each wave

### **High Monochromaticity:-**

Laser light is a monochromatic light, because all the photons have the same energy ( $E_2-E_1$ ) above the ground state and they have the same frequency.

$$\text{i.e } h\nu = E_2 - E_1$$

$$\nu = E_2 - E_1 / h$$

The laser source is the best monochromatic source.

### **High Degree of Coherence:-**

The laser has a high degree of coherence, because all the photons or waves are in phase. That is, purity of the spectral line is expressed in terms of coherence. Due to its coherence only, it is possible to create high power in space with laser beam.

### **Fundamental terms of lasers:-**

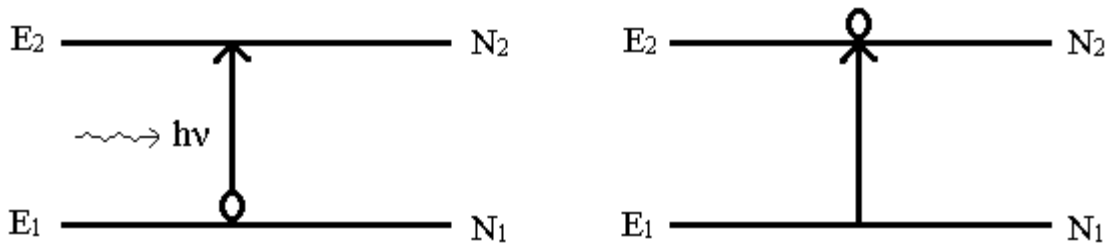
1. Absorption
2. Spontaneous emission
3. Stimulated emission
4. Population Inversion

### Absorption:-

The process of moving electron from ground state to excited state with the supply of energy is known as “Absorption”.

(Or)

An atom (or) molecules in the ground state  $E_1$  can absorb a photon of energy  $h\nu$  and go to the excited state  $E_2$ . This process is known as “Absorption”



$$\text{Rate of Absorption} = B_{12} \rho (\nu) N_1$$

Where  $B_{12}$  - Einstein coefficient of absorption

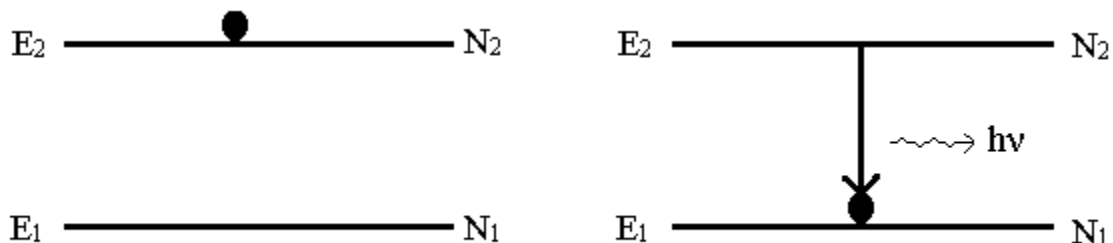
$N_1$  - Concentration of electron in ground state

$\rho (\nu)$  – Energy density

### Spontaneous emission:-

The emission of electron from excited state to the ground state on their own without any supply of external energy is known as “Spontaneous emission”.

The excited atom does not remain in that state for a long time. After a short interval of time i.e.  $10^{-8}$  sec, it falls to its ground state by emitting a photon as shown in figure.



Here the excited atoms jumps back to its ground state on its own and hence the process is called Spontaneous emission

i.e. Rate of Spontaneous emission =  $A_{21}N_2$

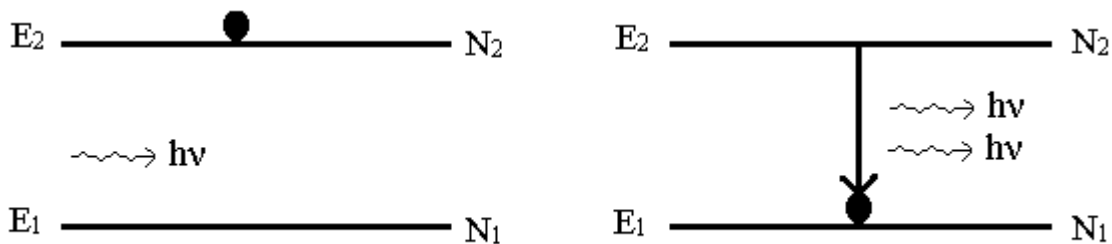
Where  $A_{21}$  - Einstein coefficient of absorption

$N_2$  - Concentration of electron in excited state

**Stimulated emission:-**

The emission of electron from the excited state to ground state with the supply of external energy is known as stimulated emission.

The process of speeding up the atomic transition from the excited state to ground state is called Stimulated emission.



Rate of Stimulated emission =  $B_{21} \rho (v) N_2$

Where  $B_{21}$  - Einstein coefficient of absorption

$N_2$  - Concentration of electron in excited state

$\rho (v)$  – Energy density

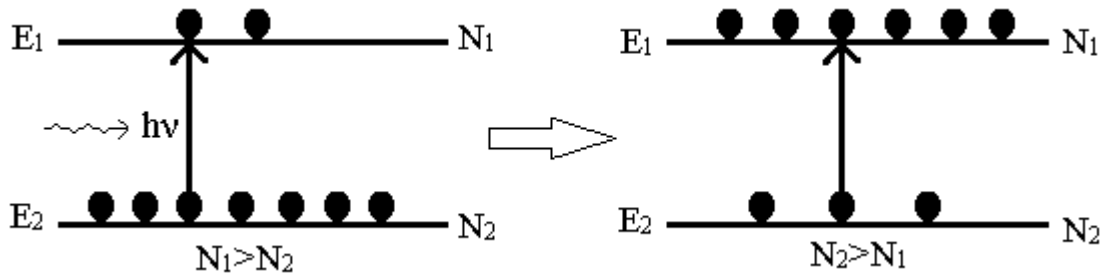


## Difference between Spontaneous emission Stimulated emissions:-

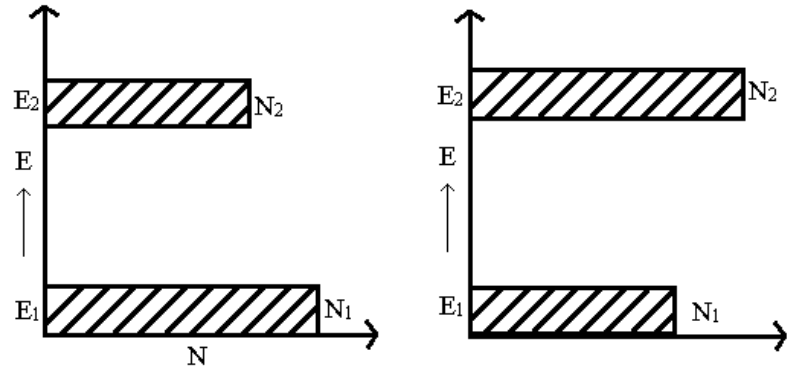
Spontaneous emission	Stimulated emissions
The emission of electron moving on their own from excited state to ground state is known as Spontaneous emission	The emission of electron from the excited state to ground state with the supply of external energy is known as stimulated emission.
It is non- directional	It is highly directional
Spontaneous emission is in-coherent	Stimulated emissions is coherent
Intensity of laser radiation is low.	Intensity of laser radiation is high.
Spontaneous emission cannot be controlled from outside	Stimulated emissions can be controlled from outside
sodium lamp and mercury lamp are the examples of Spontaneous emission	Ruby laser and He-Ne gas lasers are the examples of Stimulated emissions

## Population Inversion:-

Finding more number of electrons in higher energy level than lower energy level is known as population inversion.



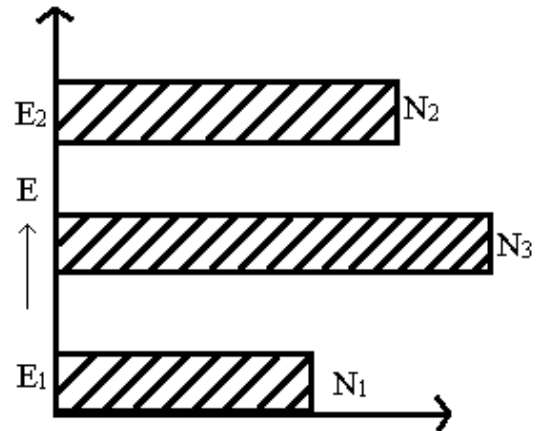
Population inversion is a pre required condition in order to have laser radiation. This population inversion can be understood in terms of metastable energy level.



**Metastable State:-**

The energy level which exists in between ground state energy level and excited energy level is known as metastable energy level.

Metastable state is a particular excited state of an atom that has longer life time than the ordinary excited states. A metastable state is an intermediate state. The life time of an atom in the metastable state is  $10^{-3}$  sec.



**Pumping:**

Pumping is the process of supplying energy. The pump is an external source that supplies energy needed to transfer atoms from lower energy level to upper energy level. There are a number of techniques for pumping. Optical pumping and electric discharge are some of the methods. In optical pumping a light source is used and in electric discharge method an electric field is used to transfer atoms from lower energy level to upper energy level.

**Life Time:**

The amount of time in which an atom stays in an energy state is called life time. The life time of an atom in the ground state is unlimited but it is  $10^{-8}$  sec in the excited state.

## Types of Lasers:-

Lasers are of different types basing on types basing on the type of active medium that we have with the Laser. Some of the important lasers are

1. Solid state laser – Ruby laser
2. Gas laser – He-Ne gas laser

## Ruby Laser:-

The Ruby Laser namely called as “solid state laser”. This is being constructed in the year 1961 by T.H. Maiman.

Ruby laser has the laser output to be in visible region. i.e. the output of ruby laser is red in color. Ruby laser finds good number of applications in so many fields like atmospheric sciences, engineering medicine and the technology.

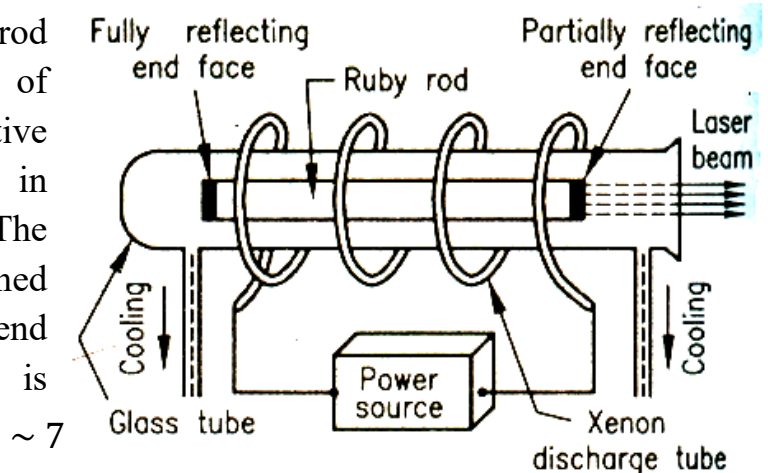
## Construction of Laser:-

Ruby laser can be identified by means of three important parts of it.

1. Energy source – Xenon flash tube
2. Active medium – Ruby rod
3. Active centers –  $\text{Cr}^{3+}$  ions.
4. Reflecting mirrors – Polished opposite ends

## Description:-

A ruby laser consists of ruby rod prepared out of  $\text{Al}_2\text{O}_3$  with 0.05% of  $\text{Cr}_2\text{O}_3$ .  $\text{Cr}^{3+}$  ions will act as active centers. The ruby rod appears pink in color due to presence of  $\text{Cr}^{3+}$  ions. The ends face of the rod is highly polished and then silvered while the other end partially reflecting. The ruby rod is



enclosed in a glass tube and surrounded by xenon flash lamp, which provides the pumping light to raise the ions to upper energy level. The flash tube not only emits light but also emits heat, so a cooling system is used.

**Working Principle:-**

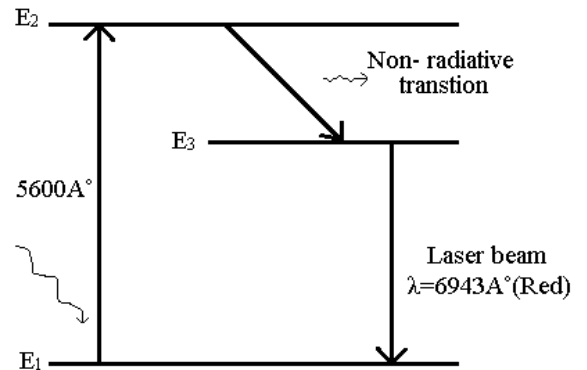
Almost all atoms are initially in ground state. When the energy is applied by a xenon flash lamp, which emits a radiation of  $5600\text{\AA}$ . The atoms move to an upper energy level by absorbing light photons of wavelength. It will absorb the energy at two specific wavelengths. i.e. blue, green. Upon absorption of energy the  $\text{Cr}^{3+}$  ions will go excited to the energy level  $E_2$  and  $E_3$  respectively. The ruby laser we came across two prominent transitions  $E_2 \rightarrow E_3$ ,  $E_3 \rightarrow E_1$ .

**Energy level Diagram:-**

Ruby laser is a three level energy laser. i.e. it consists of three energy levels.

i.e.  $E_1(\text{G.S}), E_2(\text{E.S}), E_3(\text{M.S})$

Now the excited atoms can return in to two steps. The first transition from  $E_2-E_3$  (Metastable state). Which is a shorter jump and energy emitted in this transition is not radiated in the form of photons and this transition is called “Non radiative transition” or “Radiation less transition”. The  $\text{Cr}^{3+}$  atoms return to  $E_3$  level can remain in this state atom long time. The accumulation of atoms in  $E_3$  level increases the population at  $E_3$  level. After a short interval of time population inversion is created between  $E_3$  and  $E_1$ .



From  $E_3$  to  $E_1$  by the process of stimulated emission a high intense light of wavelength  $6943\text{\AA}$  is emitted. i.e. a laser produced in visible region.

**Merits:-**

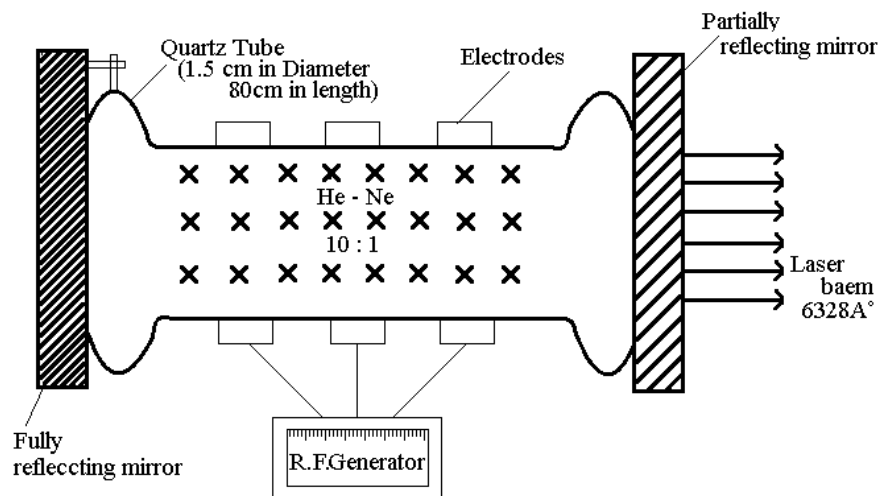
1. The output of ruby laser falls in visible region
2. It can cause output only in one color (red)
3. The laser radiation is not harmful.

### Demerits:-

1. Ruby laser cannot produce continuous beams of lasers
2. The output of this laser is discontinuous
3. Intensity of laser radiation is very less
4. Unnecessary heat will generated at active medium
5. Separate cooling arrangement is required

### Gas Lasers (Helium – Neon Laser):-

He – Ne gas laser was first gas laser constructed in the year 1961, by scientist “Alijavan”. This laser can cause continuous laser beam . He – Ne gas laser finds good number of application.

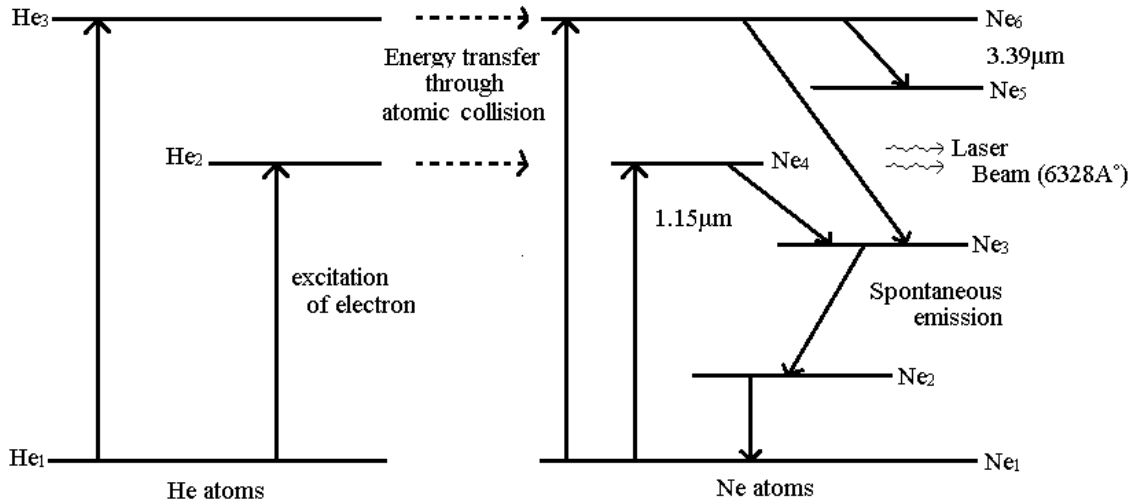


### Description:-

He – Ne gas laser consists of long narrow quartz tube of 80cm length and 1.5cm in diameter. This will get filled with He – Ne gas laser mixture taken in the ratio 10:1 with a pressure of about 1mm of mercury. Both ends of the tube are silvered. One of them is partially silvered, so that sufficiently built laser beam passes through it. A high frequency electric discharge, acts as a pumping source, to excite the He – Ne mixture. The reflecting mirrors are arranged at the opposite ends of the quartz tube.

## Working Principle:-

When electric discharge is passed through the He – Ne mixture, electrons from the discharge source collide with He – Ne atoms and thus transfer of their energy to these atoms. As a result He – Ne atoms are excited to higher state He<sub>2</sub> and He<sub>3</sub> in case of helium and Ne<sub>4</sub> and Ne<sub>6</sub> in case of neon. These are their metastable states.



The atoms remain in these levels for a sufficiently longer time. After their life time He atoms collide with Ne atoms, which are in the ground state, while the He atoms return to their metastable states. As the energy exchange continuous, the population of Ne atoms, in the excited states increases more and more and population inversion takes place.

Thus by the process of stimulated emission between Ne<sub>6</sub> and Ne<sub>3</sub> laser light is emitted with a wavelength of 6328 Å. The Ne atoms are returned to the ground state by non-radiative transition. After arriving ground state once again the Ne atoms are excited to higher state by the collisions of the atoms. This process continuous and continuous laser beam is produced.

## Merits:-

1. No exchange heat can develop
2. We require no separate cooling arrangement
3. The output result will last for long time.

## **Applications of Laser:-**

Laser finds wide and variety of applications in various fields basing on Directionality, Intensity, Monochromaticity and Coherence.

The applications of laser are wide spread over various scientific disciplines such as physics, chemistry, biology, medicine etc. They have exciting potential applications in industry, communication and warfare.

### **Lasers in Medicine:-**

1. Laser radiation is often used in controlling hemorrhages.
2. Lasers are used in destroying kidney stones.
3. Lasers are used in ophthalmology to reattach a detached Retina.
4. They are used in cancer diagnosis and therapy and also stomatology.

### **Lasers in Industry/Engineering:-**

1. They are used to blast holes in diamond and hard steel.
2. Lasers are used to test quality of a fabric.
3. It has been observed that finger prints on documents, current bills, cloths etc.
4. Laser cutting technology is widely used in the fabrication of Spacecrafts.

### **Lasers in Science and Research:-**

1. Lasers are best suited for optical communication.
2. Lasers are used to study the internal structure of micro organisms and cells.
3. Lasers are used for isotopic separation.
4. Lasers are used to produce certain chemical reactions.
5. Lasers have several applications in the field of computers.
6. To transmit memory blanks from one computer to another.
7. Laser is used in computer printers.
8. It is used to read the data from CD-ROM.

## OBJECTIVE QUESTIONS FOR ONLINE EXAMINATION

- Emission of photon when an electron jumps higher energy state to lower energy state due to interaction of external energy is called
  - Spontaneous emission
  - Stimulated emission**
  - Induced absorption
  - Amplified emission
- The population of the various energy level of a system in thermal equilibrium is given by
  - Boltzmann distribution law**
  - Einstein relation
  - Plank's law
  - Beer's law
- The color of the laser output from a ruby laser is
  - Green
  - Blue
  - Red**
  - Violet
- In Ruby lasing material the percentage of chromium ions in aluminum oxide is
  - 0.5
  - 0.05**
  - 5
  - 0.005
- In He-Ne laser, the ratio of He-Ne is in the order
  - 1 : 10
  - 1 : 1
  - 100 : 1
  - 10 : 1**
- The wavelength of emission from He-Ne laser is
  - 10.64 $\mu\text{m}$
  - 337.1nm
  - 794.3nm
  - 632.8 nm**
- Laser radiation is
  - Monochromatic
  - Highly directional
  - Coherent and stimulated
  - Highly directional, monochromatic, coherent and stimulated**
- He-Ne gas laser is
  - Pulsed laser
  - Semiconductor laser
  - Solid state laser
  - Continuous laser**



9. The lasing action is possible only if there is
- a. A set of reflecting mirrors                      **c. Population inversion**  
 b. A black body    d. Incandescent lamp
10. The pumping process used in Ruby laser is
- a. Optical pumping**    c. Atom – atom in elastic collision  
 b. Electric discharge    d. Chemical reaction
11. In He-Ne laser, atoms involved in laser emission are
- a. Ne-atoms**    b. He-atoms                      c. Both                      d. none
12. Rate of stimulated emission is proportional to
- a. Brightness    b. Monochromaticity    c. Directionality    **d. Coherence**
13. Emission of photon when an electron jumps higher energy state to lower energy state due to interaction with another photon is called
- a. Spontaneous emission    c. Induced absorption  
**b. Stimulated emission**    d. Amplified emission
14. Ruby laser is the best example for a ----- level system
- a. Two                      **b. Three**                      c. Four                      d. single
15. He- Ne laser is a ----- level laser system
- a. Two                      b. Three                      **c. Four**                      d. Single
16. Important characteristics of laser beam is
- a. Interference    b. Diffraction                      c. Dispersion    **d. coherence**
17. Supply of energy to atoms for excitation is called
- a. Glowing    b. bombarding                      c. incidenting    **d. Pumping**
18. The lifetime of an atom in a metastable state is of the order of
- a. Few seconds    b. Unlimited time    c. Electric discharge    **d. None**

19. The emission of photons without being aided by any external agency is called

- a. Light amplification
- b. Induced absorption
- c. Stimulated emission
- d. **Spontaneous emission**

20. The rate of stimulated emission is equal to

- a.  $B_{12} \rho(\nu) N_1$
- b.  **$B_{21} \rho(\nu) N_2$**
- c.  $B_{12} \rho(\nu) N_2$
- d.  $B_{21} \rho(\nu) N_1$

# FIBER OPTICS

## **Introduction:**

Fiber optics is the branch of physics, which deals with the transmission of light rays using optical fibers.

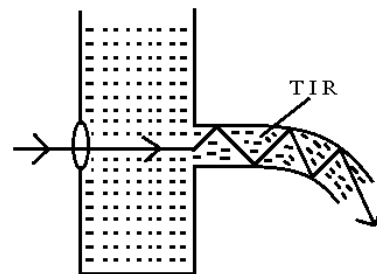
## **Optical Fiber:-**

An optical fiber is a dielectric wave guide, which having high tensile strength, which will be capable to transmitting the signals with no loss of energy.

(Or)

Optical Fibers are wave guide inside which light signals can travel.

- Optical Fiber causes the communication basing on the principal of “Total Internal Reflection (TIR)”
- The optical fiber communication is being realized in terms of simple observations of “Sir John Tyndoll” in the year 1857.
- According to these observations light rays can be made to pass through narrow streams of water coming out of small opening by “Total Internal Reflection (TIR)”
- The optical fiber are being first utilized in the year 1927 in U.S.A, for transmission of voice signals and it has began from 1957.
- Laser along with optical fibers have brought in so much of unprecedential development in the field of communication.

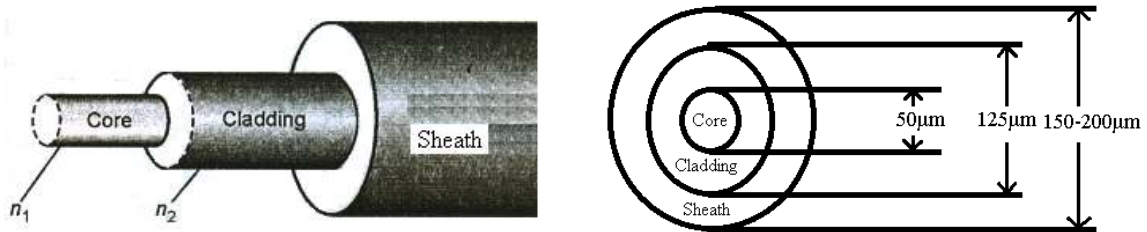


## **Structure of optical fiber:-**

Optical Fiber is a very thin, narrow and flexible wire having a cylinder shape consisting of three parts or sections.

1. Core
2. Cladding
3. Buffer (or) Sheath (or) Outer Jacket.

The structure of optical fiber is shown in the below figure



### Core and cladding:-

The core is the central part of an optical fiber to facilitate the “Total Internal Reflection (TIR)”. The core is made up of pure silica ( $\text{SiO}_2$ ) glass and its diameter is  $\sim 50\mu\text{m}$ , which is surrounded by a cladding whose diameter is  $\sim 125\mu\text{m}$ . The cladding is formed by the addition of small amounts of boron, germanium (or) phosphorus in  $\text{SiO}_2$  glass to increase the refractive index of the core than cladding to satisfy the condition of Total Internal Reflection.

### Sheath:-

To protect the fiber material and also give mechanical support, there is a protective cover called the Outer Jacket. It is made up of plastic material. In order to avoid damages it acts as a protective cover.

The optical fiber will get basically prepared in terms of either the glass (or) plastic material, depending on the way that we prepare the optical fiber. Optical fiber can broadly be divided into 3 types, they are

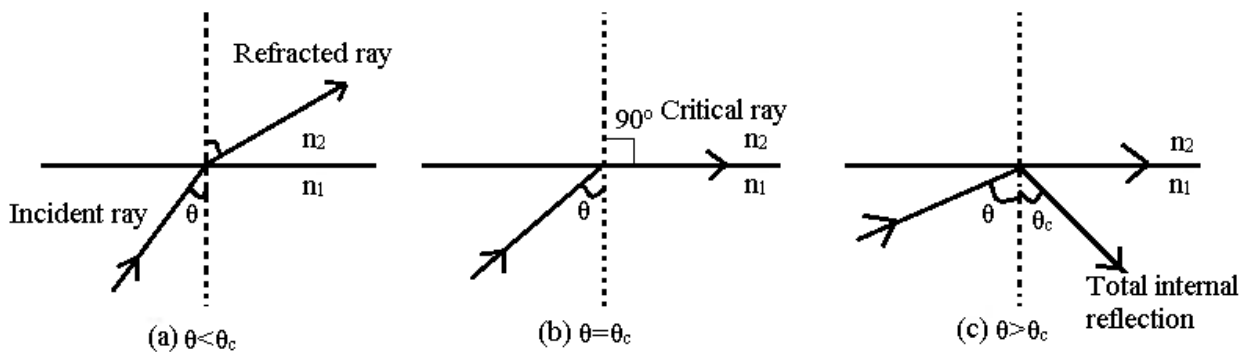
1. Glass – Glass Fiber
2. Glass – Plastic Fiber
3. Plastic – Plastic Fiber

## Principal of Optical Fiber: - (Total Internal Reflection)

The light travels through optical fiber on the principle of “Total Internal Reflection”

The Total Internal Reflection takes place only when the following two conditions are satisfied.

1. The refractive index of the core ( $n_1$ ) must be greater than that of the cladding ( $n_2$ ).
2. The angle of incidence ( $\theta$ ) must be greater than the critical angle ( $\theta_c$ ).



When light ray travels from core of refractive index  $n_1$  to the cladding of refractive index  $n_2$ , refraction occurs. Since light travels from denser to rarer medium, the angle of refraction is greater than the angle of incidence, which is shown in figure (a).

With the increase in angle of incidence, the angle of refraction also increases, and for a particular angle of incidence ( $\theta = \theta_c$ ) the refracted ray simply pass through the interface between the core and cladding. That angle we called it an incidence angle, which is shown in figure (b).

When the angle of incidence is further increased the ray is reflected back into the core. This phenomenon is called “Total Internal Reflection”

According to law of refraction  $n_1 \sin \theta_1 = n_2 \sin \theta_2 \dots\dots\dots (1)$

When  $\theta_1 = \theta_c$

$\theta_2 = 90^\circ$       Therefore       $n_1 \sin \theta_c = n_2 \sin 90$

$$\therefore \sin \theta_c = \frac{n_2}{n_1} \dots\dots\dots (2)$$

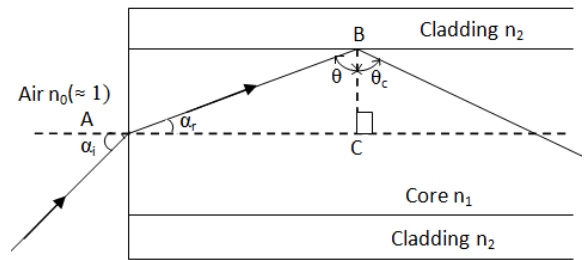
$$\theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right) \dots\dots\dots (3)$$

From equation (3) is the condition for “Total Internal Reflection”

“The reflection of light rays into the same medium, when the light rays are made to pass from denser medium to rarer medium with an angle of incidence greater than the critical angle is known as “Total Internal Reflection”.

**Acceptance angle:-**

When a light beam launched into a fiber at its one end, the entire light may not pass through the core, only the rays make the angle of incidence greater than the critical angle at the core, cladding interface undergo “Total Internal Reflection” and propagate through core. The other rays are refracted into the cladding material and are lost. Hence it is essential to know at what angle known as “Acceptance angle”. We have launched the beam at its end to enable the entire light to propagate through the core.



The light is launched form a medium (air) of refractive index  $n_0$  ( $\approx 1$ ) into core of refractive index  $n_1$ . The ray enters with an angle of incidence  $\alpha_i$  to the fiber end face at a point “A” and proceeds after refraction at an angle  $\alpha_r$ . Then it undergoes total internal reflection at “B” on core wall at an angle of incidence “ $\theta$ ”.

According to Snell’s law, we have  $\frac{\sin \alpha_i}{\sin \alpha_r} = \frac{n_1}{n_0} \dots\dots\dots (1)$

$$\sin \alpha_i = \frac{n_1}{n_0} \sin \alpha_r$$

From  $\Delta$ le ABC  $\Rightarrow \alpha_r = (90-\theta)$

$$\therefore \sin \alpha_i = \frac{n_1}{n_0} \cos \theta \dots\dots\dots (2)$$

$\alpha_i$  be the maximum possible angle of incidence at “A”, when  $\theta = \theta_c$

The equation (2) can be written as

$$\therefore \sin \alpha_i(\max) = \frac{n_1}{n_0} \cos \theta_c \dots\dots\dots (3)$$

$$\begin{aligned} \therefore \cos \theta_c &= \sqrt{1 - \sin^2 \theta_c} \\ &= \sqrt{1 - \frac{n_2^2}{n_1^2}} && \therefore \sin \theta_c = \frac{n_2}{n_1} \\ &= \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}} \dots\dots\dots (4) \end{aligned}$$

Substitute equation (4) in equation (3)

$$\begin{aligned} \sin \alpha_i(\max) &= \frac{n_1}{n_0} \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}} \\ &= \frac{n_1}{n_0} \frac{\sqrt{n_1^2 - n_2^2}}{n_1} \\ &= \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \end{aligned}$$

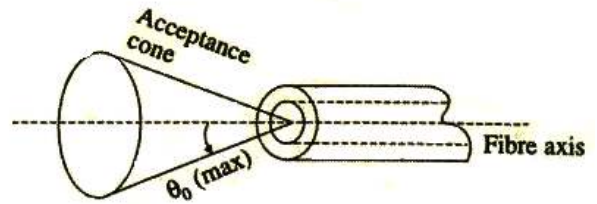
! Here  $n_0$  is refractive index of air ( $n_0 = 1$ ).

$$\begin{aligned} \sin \alpha_i(\max) &= \sqrt{n_1^2 - n_2^2} \\ \alpha_i(\max) &= \sin^{-1} \left( \sqrt{n_1^2 - n_2^2} \right) \end{aligned}$$

Thus “maximum angle  $\alpha_i$  is called “acceptance angle”.

## Acceptance Cone:-

Rotating the acceptance angle about the fiber axis describes the acceptance cone of the fiber. Half angle is called acceptance cone.



## Numerical Aperture (NA):-

Light collecting capacity of the fiber is expressed in terms of Numerical aperture (Or)

Light gathering ability of an optical fiber in order to propagate through it is known as “Numerical Aperture” (Or)

Sine of maximum acceptance angle is known as “Numerical Aperture”

$$\text{i.e. Numerical Aperture} = \sin \alpha_i(\text{max}) = \sqrt{n_1^2 - n_2^2} \quad \dots\dots\dots (1)$$

$$\text{Let } \Delta = \frac{n_1^2 - n_2^2}{2n_1^2} \quad \dots\dots\dots (2)$$

$$\therefore n_1^2 - n_2^2 = 2n_1^2 \Delta$$

$$\sqrt{n_1^2 - n_2^2} = n_1 \sqrt{2\Delta} \quad \dots\dots\dots (3)$$

Substituting equation (3) in eq (1)

$$\text{NA} = n_1 \sqrt{2\Delta}$$

$\Delta$  is the fractional difference between the refractive indices of the core and cladding known as fractional refractive index change.

The Numerical Aperture of the fiber is effectively depends on the refractive indices of core and cladding material, but not on the fiber dimensions. The value of numerical aperture ranges from 0.13 to 0.50.



### **Advantages of optical fiber in communication system:-**

1. Extremely wide band width, the rate at which information can be transmitted is directly related to signals frequency. Light has very high frequency in the range  $10^{14}$  to  $10^{15}$  Hz. So, the optical signal can transmit information at a higher rate.
2. Smaller diameter, light weight cables.
3. Lack of cross talk between parallel fibers.
4. Potential of delivering signals at low cost
5. Much safer than copper cables.
6. Longer life span
7. High temperature resistance.
8. Optical fibers are more reliable and easy to maintain than copper cables.

## OBJECTIVE QUESTIONS FOR ONLINE EXAMINATION

21. In an optical fiber, if  $n_1$  is the refractive index of core and  $n_2$  is the reflective index of cladding. Then

e.  $\frac{n_1 - n_2}{n_1} < 1$                       f.  $\frac{n_1 - n_2}{n_1} > 1$                       g.  $\frac{n_1 - n_2}{n_1} = 0$                       h.  $\frac{n_1 - n_2}{n_1} = 1$

22. Choose the correct statement

- a. Cladding has higher refractive index than core                      c. Cladding is for providing mechanical strength to the fiber
- b. Core has higher refractive index than cladding**                      d. The loss in wave guides without cladding is very low

23. In the case of an optical fiber the acceptance angle equal to

a.  $\sin^{-1}(n_1^2 - n_2^2)^{1/2}$                       b.  $\sin^{-1}(n_1^2 - n_2^2)$                       c.  $\sin^{-1}(n_1 - n_2)$                       d.  $\sin^{-1}(n_1 - n_2)^{1/2}$

24. Total internal reflection takes place when light travels from

- a. **Denser to rarer medium**                      c. Air to liquid or solid
- b. Any medium to other different medium                      d. A lighter to denser medium

25. The refractive index of core and cladding are 1.563 and 1.498 respectively then Numerical Aperture (NA) is

- a. **0.446**                      b. 0.246                      c. 0.1999                      d. 0.346

26. If the angle of incidence of ray is greater than the critical angle at the interface of core and cladding then the ray travels

- a. **In the core**                      b. In the cladding                      c. Along the interface                      d. In buffer

27. To enter an optical signals into the core the angle of incidence at the interface of air and core should be

- a. Equal to critical angle                      c.  $>$  acceptance angle
- b.  $<$  critical angle                      **d.  $<$  acceptance angle**

28. The numerical aperture of an optical fiber with core material of refractive index 1.55 and cladding material 1.30 is -----
- a. 0.142                      b. 0.254                      c. **0.844**                      d. 0.562
29. The NA of an optical fiber is 0.39 and the difference in refractive indices of the material of its core and cladding is 0.05, the refractive index of material of the core is
- a. 0.532                      b. **1.521**                      c. 2.431                      d. 0.254
30. The NA for an optical fiber with core and cladding refractive index being 1.48 and 1.45 respectively is
- a. 0.1468                      b. 0.5478                      c. 0.0468                      d. **0.2965**
31. The fundamental optical parameter of an optical fiber is -----
- a. **Refractive index**              b. Focal length              c. Optical power              d. Numerical aperture
32. The light ray should travel from ----- medium to ----- medium for total internal reflection to take place
- a. Denser to denser              b. **Denser to rarer**              c. Rarer to denser              d. Rarer to rarer
33. Propagation of light through fiber core is due to
- a. Diffraction                      b. Interference                      c. Refraction                      d. **TIR**
34. The numerical aperture of a fiber is
- a. A function of the fiber dimensions                      c. Independent on the refractive indices of the core and cladding
- b. **Dependent on the refractive indices of the core and cladding**                      d. A function of length of the optical fiber
35. The refractive index of core and cladding are 1.50 and 1.44 respectively and then acceptance angle is
- a. 23°.45'                      b. 23°                      c. 26°                      d. **24°.50'**
36. The condition for propagation of light in a fiber is given by ( $\theta$ =launch angle)
- a.  $\sin \theta = 1$                       b.  $\sin \theta > NA$                       c.  $\sin \theta \times NA = 1$                       d.  **$\sin \theta < NA$**
37. The refractive index of a cladding of a fiber with core refractive index 1.5 and NA

0.244 is

- a. 1.4                                      b. 1.325                                      c. **1.48**                                      d. 1.656

38. Optical fibers do not pick up electricity, because they are

- a. Transparent material                                      c. Magnetic materials  
b. **Non-metallic materials**                                      d. Electric materials

39. The refractive indices of core and cladding of an optical fiber with a numerical aperture of 0.33 and their fractional difference of refractive indices being 0.02 are

- a. **1.65, 1.617**                                      b. 1.325, 1.317                                      c. 1.48, 1.45                                      d. 1.656, 1.78

40. An optical fiber has a numerical aperture of 0.20 and a cladding refractive index of 1.59. the acceptance angle for the fiber in water whose refractive index is 1.33 is

- a. **8°.39'**                                      b. 10°                                      c. 26°                                      d. 24°.50'

## Problems on Fiber Optics

1. A glass fiber has core material of refractive index 1.466, cladding material of refractive index 1.46. If it is surrounded by air medium, compute the critical angle at the core-cladding interface.

Sol: Given that

The refractive index of core  $n_1 = 1.466$

The refractive index of cladding  $n_2 = 1.46$

We know that critical angle

$$\theta_c = \sin^{-1} \left[ \frac{n_2}{n_1} \right]$$
$$\theta_c = \sin^{-1} \left[ \frac{1.46}{1.466} \right] = 84.81$$

The critical angle at the core-cladding interface  $\theta_c = 84.81$ .

2. An optical fiber has a core material of refractive index of 1.55 and cladding material of refractive index 1.50. The light is launched into it in air. Calculate its numerical aperture.

Sol: Given that

The refractive index of core  $n_1 = 1.55$

The refractive index of cladding  $n_2 = 1.50$

$$NA = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

For air medium  $n_0 = 1$

$$\begin{aligned} NA &= \sqrt{n_1^2 - n_2^2} \\ &= \sqrt{1.55^2 - 1.50^2} \\ &= 0.39 \end{aligned}$$

The numerical aperture of an optical fiber,  $NA = 0.39$ .

3. The numerical aperture of an optical fiber is 0.39. if the fractional refractive index change of the material of its core and cladding is 0.05, calculate the refractive index of the material of the core.

Sol: Given that

The numerical aperture of an optical fiber,  $NA=0.39$

Fractional refractive index change  $\Delta=0.05$

$$NA = n_1 \sqrt{2\Delta}$$
$$n_1 = \frac{NA}{\sqrt{2\Delta}} = \frac{0.39}{\sqrt{2 \times 0.05}} = 1.233$$

The refractive index of the material of the core,  $n_1=1.233$ .

4. Calculate the acceptance angle and the numerical aperture of a given optical fiber. If the refractive indices of core and cladding are 1.563 and 1.498 respectively.

Sol: Given that

The refractive index of core  $n_1= 1.563$

The refractive index of cladding  $n_2= 1.498$

We know that the expression for Numerical aperture is

$$NA = \sqrt{n_1^2 - n_2^2}$$
$$= \sqrt{1.563^2 - 1.498^2} = 0.4461$$

The numerical aperture of an optical fiber,  $NA=0.4461$

We know that the expression for Numerical aperture is

$$\sin \theta_c = \left( \sqrt{n_1^2 - n_2^2} \right) = NA$$

$$\theta_c = \sin^{-1}(NA)$$

$$\theta_c = \sin^{-1}(0.4461) = 26.49$$

The acceptance angle of the optical fiber  $\theta_c=26.49$ .

5. The numerical aperture of optical fiber is 0.2 when surrounded by air. Given the refractive index of the cladding is 1.59. Find the acceptance angle when the fiber is in water. Assume the refractive index of water as 1.33.

Sol: Given that

The numerical aperture of an optical fiber  $NA=0.2$

The refractive index of cladding  $n_2= 1.59$

The refractive index of water  $n_0= 1.33$

We know that the numerical aperture of optical fiber is

$$NA = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

For air medium  $n_0=1$ , then

$$\begin{aligned} NA &= \sqrt{n_1^2 - n_2^2} \\ n_1^2 &= \sqrt{(NA)^2 + n_2^2} \\ &= \sqrt{(0.2)^2 + (1.59)^2} \\ &= 1.602 \end{aligned}$$

We know that the expression for acceptance angle

$$\begin{aligned} \theta_a &= \sin^{-1} \left( \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \right) \\ \theta_a &= \sin^{-1} \left( \frac{\sqrt{(1.602)^2 - (1.59)^2}}{1.33} \right) = 8.46 \end{aligned}$$

The acceptance angle of the fiber in water  $\theta_a=8.46$

# UNIT – III

Chapter – I: Crystallography

Chapter – II: X-Ray Diffraction

Course Outcome:

**CO 3:** Identifies the crystal structures and XRD techniques. **(Remember) K1**



## UNIT-III

### CRYSTALLOGRAPHY

#### **Introduction:**

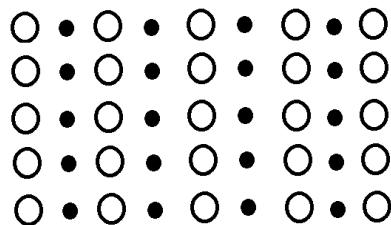
Matter is available in the universe in different forms; those are Solids, Liquids and Gases.

**Solids:** - A Solid consists of a large number of closely packed atoms or molecules. Solids are classified into two categories based on the arrangement of atoms. They are

1. Crystalline Solids
2. Amorphous solids

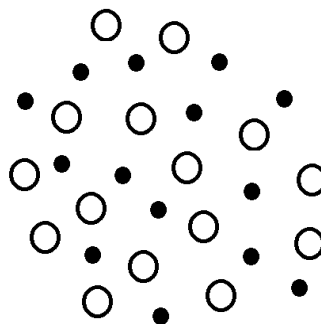
#### **Crystalline Solids:-**

In Crystalline Solids atoms or molecules are arranged in a regular manner. The Crystalline Solids have directional property; hence they are called as anisotropic substances.



#### **Amorphous solids:-**

In Amorphous solids atoms or molecules are arranged in a random manner. The Amorphous solids have no directional property; hence they are called as isotropic substances.

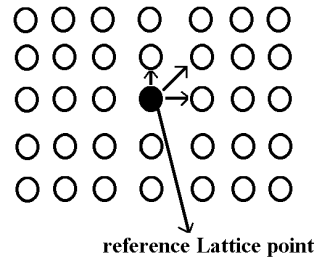


#### **Crystal:-**

Crystal is a transparent material, which has the regular periodic array of atoms.

## Fundamental terms of Crystal structure:-

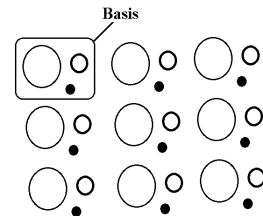
**Space Lattice:-** Space Lattice is nothing but a lattice, but it will have similar environment to see with reference lattice point is called **Space Lattice**.



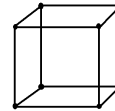
**Basis:** - Lattice point with one or more atoms.

(or)

Group of atoms is called **Basis**



**Unit cell:** - The fundamental block of a crystal structure is known as **Unit Cell**.



**Lattice parameters:** - Lattice parameters are six in numbers. They are Three are Primitives and Three are inter axial / inter facial angles.  $a$ ,  $b$ ,  $c$  and  $\alpha$ ,  $\beta$ ,  $\delta$  are known as **lattice Parameters**. From these lattice parameters we can find out the external structural of the crystal.

**Lattice:** - The regular periodic array of atoms is called **Lattice**

**Crystal Structure:** - Lattice + Basis = **Crystal Structure**.

**Crystallographic axis:** - A line is drawn to touch the Three faces of unit cell is called **Crystallographic axis**.

**Inter axial / Inter facial angles:** - The angular inclination varies between crystallographic axis is called **Inter axial / Inter facial angles**.

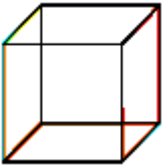
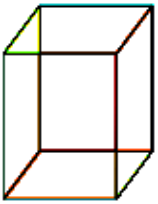
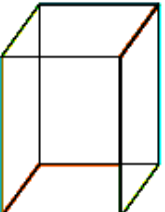
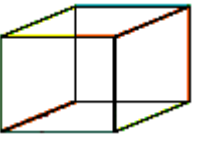
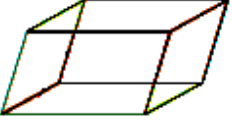
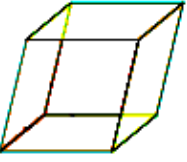
**Primitive cell or Simple cell:** - The primitive cell is defined as a unit cell which contains lattice points at corners only. Such cell contains effectively one lattice point per cell i.e. one lattice point is associated with each primitive cell.

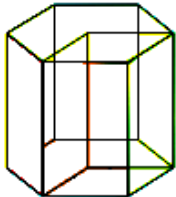
**Non-primitive cell:** - If a unit cell contains more than one lattice point then it is called Non-primitive cell (or) multiple cell.

## Crystal systems and Bravais Lattice:-

Crystals are classified into seven systems on the basis of the shape of the unit cell or lattice parameters, angles of the unit cell.

The seven crystal systems are classified into 14 different types of unit cell. Those 14 types of systems are called Bravais Lattice.

S.No	Crystal systems	Lattice parameters	Types of Bravais Lattice	No. Of Bravais Lattice	Figure
1.	Cubic	$a=b=c$ $\alpha=\beta=\delta=90^\circ$	P I F	3	
2.	Tetragonal	$a=b \neq c$ $\alpha=\beta=\delta=90^\circ$	P I	2	
3.	Orthorhombic	$a \neq b \neq c$ $\alpha=\beta=\delta=90^\circ$	P I F C	4	
4.	Mono Clinic	$a \neq b \neq c$ $\alpha=\beta=90^\circ \neq \delta$	P C	2	
5.	Tri Clinic	$a \neq b \neq c$ $\alpha \neq \beta \neq \delta \neq 90^\circ$	P	1	
6.	Rhombohedral 54,49,59, 64, 63	$a=b=c$ $\alpha=\beta=\delta \neq 90^\circ$	P	1	

7.	Hexagonal	$a=b \neq c$ $\alpha = \beta = 90^\circ$ $\delta = 120^\circ$	P	1	
Bravais Lattice				14	

The classification of Bravais lattice is based on the following crystal lattice

- 1. Primitive Cell (P):** - In this lattice, the unit cell consists of eight corner atoms and all these corner atoms contribute only one effective atom.
- 2. Body Centered Lattice (I):** - In addition to the eight corner atoms, it consists of an extra atom at the center of the body.
- 3. Face Centered Lattice (F):** - Along with corner atoms, each face will have one center atom.
- 4. Base Centered Lattice (C):** - Along with corner atoms, Base and opposite face will have center atom.

### Crystal Structures:-

To study the Crystal Structures, we require to know some important parameters. They are

- 1. Effective number of atoms per unit cell (or) Number of atoms present in a unit cell:-**

The effective number of atoms per unit cell is the total number of atoms considering the corner atoms, center atoms and face centered atoms.

- 2. Nearest neighboring distance(a):-**

The distance between two nearest neighboring atoms is known as nearest neighboring distance.

- 3. Atomic Radius (r):-**

Atomic Radius is the half of the distance between the atoms.

#### 4. Coordination Number(N):-

The number of equidistant neighboring atoms is called Coordination Number.

#### 5. Atomic Packing Factor(APF):-

Atomic Packing Factor is defined as the total volume occupied by the atoms in unit cell to the volume of the unit cell.

$$APF = \frac{\text{Total volume occupied by the atoms in unit cell}}{\text{volume of the unit cell}}$$

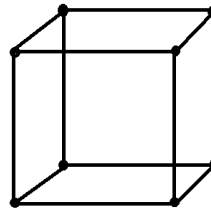
$$\frac{v}{V} = \frac{\text{No.of atoms present in a unit cell X volume of one atom}}{\text{volume of the unit cell}}$$

#### 6. Void Space:

The empty space available in a crystal lattice is called Void Space.

#### Simple Cubic Structure:

Simple cubic structure consists of 8 corners and 6 faces. All corners of the unit cell are touch with one another.



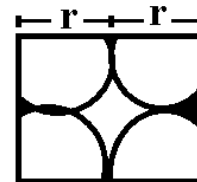
##### 1. Effective number of atoms:-

The total number of atoms considering the corner atoms is called Effective number of atoms.

$$\text{Total number of atoms and Corner atoms} = 8 \times \frac{1}{8} = 1$$

##### 2. Nearest neighboring distance(a):-

The distance between two nearest neighboring atoms is known as nearest neighboring distance.



The nearest neighboring distance for Simple Cubic is

$$a = 2r$$

~ 5 ~

### 3. Atomic Radius (r):-

Atomic Radius is the half of the distance between the atoms.

$$\text{Atomic Radius } r = a/2.$$

### 4. Coordination Number(N):-

The number of equidistant neighboring atoms is called Coordination Number.

Coordination Number for Simple cubic structure = 6

### 5. Atomic Packing Factor(APF):-

Atomic Packing Factor is defined as the total volume occupied by the unit cell to the volume of the unit cell.

$$\frac{v}{V} = \frac{\text{No.of atoms present in a unit cell} \times \text{volume of one atom}}{\text{volume of the unit cell}}$$

$$v = \text{volume of one atom} = \frac{4}{3} \pi r^3$$

$$V = \text{volume of unit cell} = a^3 = (2r)^3 = 8r^3$$

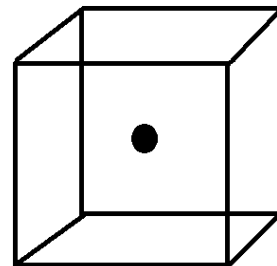
$$\text{Number of atoms present in unit cell} = \frac{1}{8} \times 8 = 1$$

$$\frac{v}{V} = \frac{\text{No.of atoms present in a unit cell} \times \text{volume of one atom}}{\text{volume of the unit cell}} = \frac{1 \times \frac{4}{3} \pi r^3}{8r^3} = \frac{\pi}{6} \approx 0.52 \text{ (or) } 52\%$$

Thus 52% of the volume of the simple cubic structure is occupied by the atoms. Hence it is loosely packed structure.

### Body Centered Cubic Structure:

Body centered cubic structure consists of 6 faces and 8 corners atoms along with one body centered atom. All corners atoms will be touch with one another via body centered atom.



#### 1. Effective number of atoms:-

The total number of atoms considering the corner atoms and center atoms is called Effective number of atoms.

$$\text{Total number of atoms and Corner atoms} = 8 \times \frac{1}{8} + 1 = 2$$

## 2. Nearest neighboring distance(a):-

The distance between two nearest neighboring atoms is known as nearest neighboring distance.

consider  $\Delta$  ABD

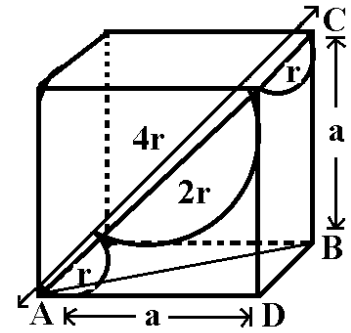
$$\Rightarrow AB^2 = AD^2 + DB^2 = a^2 + a^2 = 2a^2$$

consider  $\Delta$  ABC

$$\Rightarrow AC^2 = AB^2 + BC^2$$

$$(4r)^2 = 2a^2 + a^2$$

$$16r^2 = 3a^2 \Rightarrow a^2 = \frac{16}{3}r^2 \Rightarrow a = \frac{4}{\sqrt{3}}r$$



## 3. Atomic Radius (r):-

Atomic Radius is the half of the distance between the atoms.

$$\text{Atomic Radius } r = r = \frac{\sqrt{3}}{4}a$$

## 4. Coordination Number(N):-

The number of equidistant neighboring atoms is called Coordination Number.

Coordination Number for Simple cubic structure = 8

## 5. Atomic Packing Factor(APF):-

Atomic Packing Factor is defined as the total volume occupied by the unit cell to the volume of the unit cell.

$$\frac{v}{V} = \frac{\text{No. of atoms present in a unit cell} \times \text{volume of one atom}}{\text{volume of the unit cell}}$$

$$v = \text{volume of one atom} = \frac{4}{3}\pi r^3$$

$$V = \text{volume of unit cell} = a^3 = \left(\frac{4}{\sqrt{3}}r\right)^3$$

$$\text{Number of atoms present in unit cell} = \frac{1}{8} \times 8 + 1 = 2$$

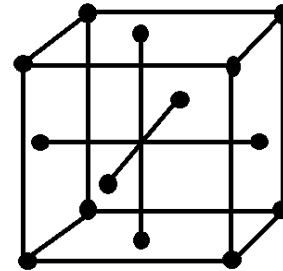
$$\frac{v}{V} = \frac{\text{No. of atoms present in a unit cell} \times \text{volume of one atom}}{\text{volume of the unit cell}}$$

$$= \frac{2X \frac{4}{3} \pi r^3}{\frac{64}{3\sqrt{3}} r^3} = \frac{\pi\sqrt{3}}{8} \approx 0.68(\text{or})68\%$$

Thus 68% of the volume of the Body centered cubic structure is occupied by the atoms. Hence it is closely packed structure.

### Face Centered Cubic Structure:

Face centered cubic structure consists of 8 corners and 6 faces along with six face centered atom.



#### 1. Effective number of atoms:-

The total number of atoms considering the corner atoms and center atoms is called Effective number of atoms.

$$\text{Total number of atoms and Corner atoms} = 8X \frac{1}{8} + 6X \frac{1}{2} = 4$$

#### 2. Nearest neighboring distance(a):-

The distance between two nearest neighboring atoms is known as nearest neighboring distance.

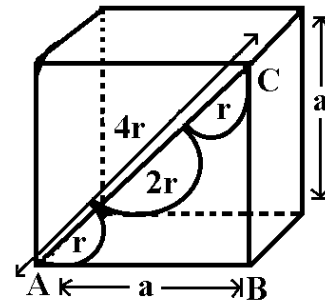
consider  $\Delta ABC$

$$\Rightarrow AC^2 = AB^2 + BC^2$$

$$(4r)^2 = a^2 + a^2$$

$$16r^2 = 2a^2 \Rightarrow a^2 = 8r^2$$

$$\Rightarrow a = 2\sqrt{2}r$$



#### 3. Atomic Radius (r):-

Atomic Radius is the half of the distance between the atoms.

$$\text{Atomic Radius } r = r = \frac{a}{2\sqrt{2}}$$



#### 4. Coordination Number(N):-

The number of equidistant neighboring atoms is called Coordination Number.

Coordination Number for Simple cubic structure = 12

#### 5. Atomic Packing Factor(APF):-

Atomic Packing Factor is defined as the total volume occupied by the unit cell to the volume of the unit cell.

$$\frac{v}{V} = \frac{\text{No.of atoms present in a unit cell X volume of one atom}}{\text{volume of the unit cell}}$$

$$v = \text{volume of one atom} = \frac{4}{3}\pi r^3$$

$$V = \text{volume of unit cell} = a^3 = (2\sqrt{2}r)^3 = 16\sqrt{2}r^3$$

$$\text{Number of atoms present in unit cell} = \frac{1}{8} \times 8 + 6 \times \frac{1}{2} = 4$$

$$\frac{v}{V} = \frac{\text{No.of atoms present in a unit cell X volume of one atom}}{\text{volume of the unit cell}}$$

$$= \frac{4 \times \frac{4}{3}\pi r^3}{16\sqrt{2}r^3} = \frac{\pi}{3\sqrt{2}} \approx 0.74(\text{or})74\%$$

Thus 74% of the volume of the Face centered cubic structure is occupied by the atoms. Hence it is very closely packed structure.

## OBJECTIVE QUESTIONS FOR ONLINE EXAMINATION

- The number of Lattice points in a primitive cell is
  - 2
  - 3
  - 1**
  - 4
- Highest packing factor occurs in
  - Body center cubic crystal
  - Face center cubic crystal**
  - Diamond
  - Simple cubic crystals
- The packing factor is defined as the ratio of
  - The number of all atoms in the unit cell to the volume of the unit cell
  - The volume of the unit cell to the number of atoms
  - The volume of the unit cell to the volume of all the atoms
  - The volume of all atoms in the unit cell to the volume of the unit cell**
- The crystal system, which exhibits all the four types of structure, is
  - Tetragonal
  - Cubic
  - Hexagonal
  - Orthogonal**
- The number of atoms per unit cell in the case of BCC, is
  - 2**
  - 1
  - 3
  - 4
- Coordination number is the
  - Number of atoms per unit cell
  - Number of unit cell per unit volume
  - Number of nearest neighbours in a unit cell
  - Number of equidistant atoms in a unit cell**
- According to Bravais, total number of lattices are
  - 11
  - 12
  - 13
  - 14**

8. The tiny fundamental block which, when repeated in space in indefinitely generates a crystal is called
- a. Primitive cell      b. Lattice cell      **c. Unit cell**      d. Simple cell
9. There are ----- basic crystal systems
- a. Four      b. Five      c. Six      **d. Seven**
10. Effective number of atoms belonging to the unit cell of BCC structure is
- a. 8      b. 1      **c. 2**      d. 9
11. The coordination number of FCC structure is
- a. 6      b. 8      **c. 12**      d. 4
12. The nearest neighboring distance in the case of Simple cubic structure is
- a.  $\frac{a}{\sqrt{2}}$       **b. a**      c.  $a\sqrt{\frac{3}{2}}$       d.  $a\sqrt{\frac{3}{4}}$
13. Atomic packing factor of FCC structure is
- a. 0.68      **b. 0.74**      c. 0.52      d. 1.00
14. There are ----- distinguishable ways of arranging point in three dimensional space called Bravais lattices
- a. 7      **b. 14**      c. 8      d. 5
15. In a crystal if the primitives  $a=b=c$ , and the interfacial angles  $\alpha=\beta=\gamma=90^\circ$ , then it belongs to the system
- a. Cubic**      b. Tetragonal      c. Mono clinic      d. Orthorhombic
16. In a crystal if the primitives  $a=b=c$ , and the interfacial angles  $\alpha=\beta=\gamma\neq 90^\circ$ , then it belongs to the system
- a. Cubic      b. Tetragonal      **c. Trigonal**      d. Orthorhombic
17. In a crystal if the primitives  $a=b\neq c$ , and the interfacial angles  $\alpha=\beta=\gamma=90^\circ$ , then it belongs to the system
- a. Cubic      **b. Tetragonal**      c. Mono clinic      d. Orthorhombic

18. In a crystal if the primitives  $a \neq b \neq c$ , and the interfacial angles  $\alpha \neq \beta \neq \gamma \neq 90^\circ$ , then it belongs to the system

- a. Cubic      **b. Triclinic**      c. Monoclinic      d. Orthorhombic

19. The element crystallizing in FCC structure

- a. Copper      b. Sodium      C. Zinc      **d. Diamond**

20. The coordination number of BCC structure is

- a. 6      **b. 8**      c. 12      d. 4

21. Atomic packing factor of BCC structure is

- a. 0.68**      b. 0.74      c. 0.52      d. 1.00

22. The cubic structure which has minimum value of packing factor is

- a. Simple cubic**      b. BCC      c. HCP      d. FCC

# X-RAY DIFFRACTION

## Introduction:-

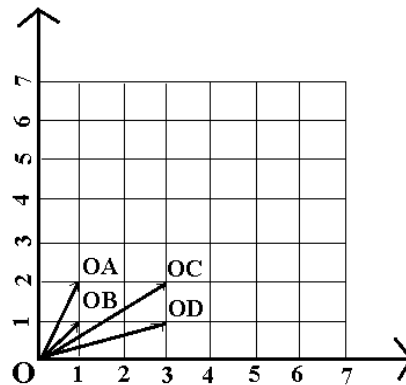
X- Rays are electromagnetic waves. The wavelength of X- Rays is of the order of  $1\text{\AA}$ .

- X- Rays are produced, when fast moving electron hit the target (or) material then it produces the radiation. These radiations are considered as X- Rays.
- X- Rays Diffraction is defined as bending of X- Rays around the obstacle.
- The X- Rays Diffraction occurs the size of the obstacle must be compared with the wavelength of radiation.
- X- Rays of wavelength is less than opaque obstacle it cannot produce X- Rays Diffraction.

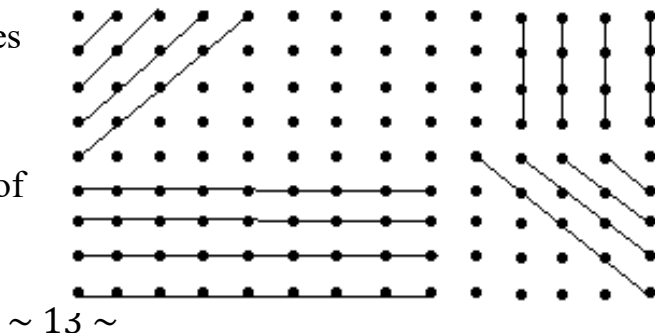
## Directions and Planes in Crystals:-

Crystals are the regular periodic array of atoms in 3-dimentional space. All these atoms fall by various directions in various planes of the crystal. These are known as Crystals Directions and Crystals Planes respectively.

In figure “O” is the origin and the directions are OA, OB, OC, and OD. The directions is described by giving the first inter point (XY) through which the line passes. The line OA is passing through the first point  $[X=1, Y=2]$ , so the direction of OA is given by  $[12]$ . Similarly the directions OB, OC, OD are represented by  $[11]$ ,  $[32]$ ,  $[31]$ .



The crystal planes are the planes which has highest lattice point on the planes. The lattice points forming space lattice may occupy various set of parallel planes.



## Miller Indices:-

The set of parallel planes are represented by certain numbers is called as Miller Indices.

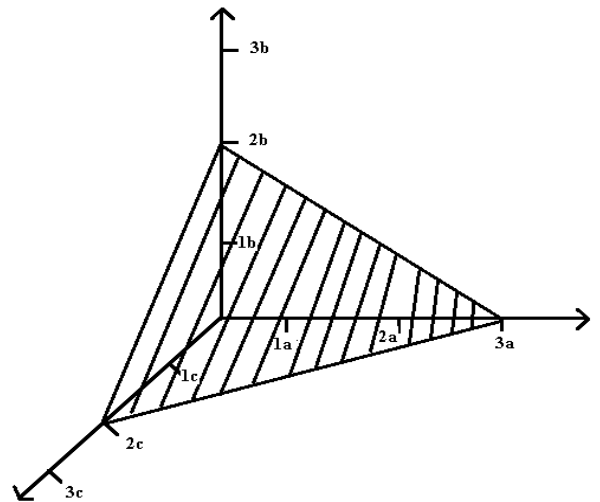
(Or)

There are smallest possible integers taken out of reciprocal of the intercepts made by crystal planes along three crystal axis.

### Rules to find Miller Indices:-

1. Note intercepts made by the crystal planes along three crystal axis.
2. Take a coordinator of the intercepts made by the planes.
3. Find the inverse of the coordinates.
4. Multiplying with the LCM in order to have proper integer value.
5. Keep the value thus obtained in the parent.

i.	3a	2b	2c
ii.	3	2	2
iii.	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$
iv.	$\frac{1}{3} \times 6$	$\frac{1}{2} \times 6$	$\frac{1}{2} \times 6$
v	(2	3	3)



- Miller Indices can be generally represented as (h k l)
- The crystal direction can be represented as [h k l]

### Important points of Miller Indices:-

1. Miller Indices always represented as parallel planes in crystals.
2. Miller Indices of the parallel crystal planes would be one and same.
3. Any plane passing parallel to any of the crystal axis will leave an at infinity. Then the miller indices value will be taken as zero.
4. If (h k l) are the Miller Indices of a crystal planes than the crystallographic intercepts made by the planes can be taken as a/h, b/k, c/l.

5. Miller indices can be used to find the inter planner distance of separation between the crystal planes.

$$\text{Inter planner distance of separation } d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

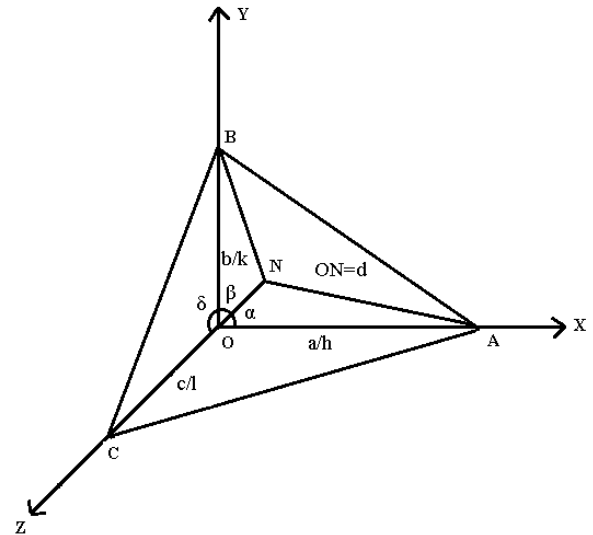
6. The angular inclination between the crystal direction of  $[U_1, V_1, W_1]$  and  $[U_2, V_2, W_2]$  can be given as

$$\cos \theta = \frac{U_1 U_2 + V_1 V_2 + W_1 W_2}{\sqrt{U_1^2 + V_1^2 + W_1^2} \sqrt{U_2^2 + V_2^2 + W_2^2}}$$

**Distance of separation between successive (h k l) Planes:-**

Here we shall derive an expression for the spacing between two parallel planes in a given crystal lattice. For Convenience, we shall take a simple unit cell in which coordinates axis are mutually perpendicular. Then we can use certain coordinates for calculating interplanner spacing.

In figure OX, OY and OZ are orthogonal axis. The origin is ‘O’. Consider any set of crystal planes represented by miller indices (h k l), the reference plane passing through the origin “o”, the next plane cutting the intercepts a/h, b/k, c/l on X, Y, Z axis respectively.



i.e  $OA = a/h$   
 $OB = b/k$   
 $OC = c/l$

A normal “ON” is drawn to the plane ABC from the origin. The length of this from the origin to the plane will be the interplanner separation.

Let  $\alpha$ ,  $\beta$  and  $\delta$  is the angles made by On with X, Y and Z directions respectively.

$$\left. \begin{aligned} \cos \alpha &= \frac{ON}{OA} = \frac{d}{a/h} \\ \cos \beta &= \frac{ON}{OB} = \frac{d}{b/k} \\ \cos \delta &= \frac{ON}{OC} = \frac{d}{c/l} \end{aligned} \right\} \dots\dots\dots (1)$$

The law of direction cosines is represented as

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \delta = 1 \quad \dots\dots\dots (2)$$

Substituting the values from equation (1) in equation (2)

$$\left[ \frac{d}{a/h} \right]^2 + \left[ \frac{d}{b/k} \right]^2 + \left[ \frac{d}{c/l} \right]^2 = 1$$

$$d^2 \left[ \frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2} \right] = 1$$

$$d^2 = \left[ \frac{1}{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}} \right]$$

The Interplanar distance of separation is given by  $d = \frac{1}{\sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}}}$

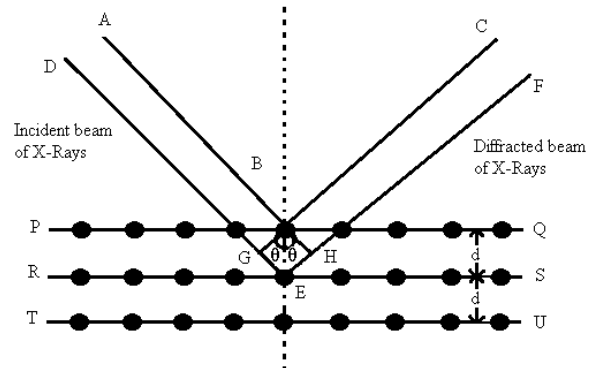
The above equation is known as general equation for interplanar separation for any set of planes. In Cubic system, as we know that  $a=b=c$ , then the expression becomes equal to

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

**Bragg's Law:-**

Bragg's law is known as one of the fundamental law of crystal physics. This is being invented by W.H. Bragg and his son W.L. Bragg in the year 1913. This law is being awarded with Nobel Prize in the year 1915. This Bragg's law enables us to have the complete structure of crystal.

Consider a set of parallel planes PQ, RS, TU separated by a distance "d". Let AB and DE be the incident beam of X-Rays, BC and EF are the diffracted beam of X-Rays.





Let  $\theta$  be the Bragg's angle(Or) Glancing angle. Let BG and BH be the normal drawn to BE and EF respectively.

From figure we have  $\angle GBE = \theta$ ,  $\angle EBH = \theta$

The path difference between the diffracted beams of X-Rays can be given as

$$\Delta = GE + EH \dots\dots\dots (1)$$

$$\text{From the } \Delta \text{le } GBE \neq \sin \theta = \frac{GE}{BE} = \frac{GE}{d}$$

$$\Rightarrow GE = d \sin \theta \dots\dots\dots (2)$$

$$\text{Similarly from the } \Delta \text{le } EBH \neq \sin \theta = \frac{EH}{BE} = \frac{EH}{d}$$

$$\Rightarrow EH = d \sin \theta \dots\dots\dots (3)$$

Substituting equation (2) and (3) in Equation (1)

$$\begin{aligned} \Delta &= d \sin \theta + d \sin \theta \\ &= 2d \sin \theta \dots\dots\dots (4) \end{aligned}$$

The condition for the constructive interference of diffracted beam of X-Rays is equal to “ $n\lambda$ ”

$$\Delta = n\lambda \dots\dots\dots (5)$$

From equation (4) and (5)

$$2d \sin \theta = n\lambda$$

The Bragg's Law states that “The Path difference between the diffracted light rays is an integral multiple of the Wavelength “ $\lambda$ ”.

**Limiting conditions for Bragg's law:-**

$$\text{From Bragg's law } 2d \sin \theta = n\lambda$$

We know that  $\sin \theta \leq 1$   $\sin \theta = \frac{n\lambda}{2d}$

$$\Rightarrow \frac{n\lambda}{2d} \leq 1$$

$$\Rightarrow n\lambda \leq 2d$$

For first order diffraction  $n=1$ ,  
 $\Rightarrow \lambda \leq 2d$

This is the limitation on wavelength of X-rays to diffraction

## Important Crystal Planes of Cubic Crystal Systems:-

There are three important crystal planes in the cubic crystal system. They are (100), (110), (111). This can be represented as,

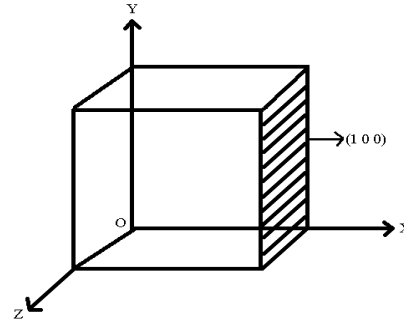
1. (100)

Indices - ( 100)

Intercepts - (a/h, b/k, c/l)

For Cubic Unit Cell a=b=c=1

$(1/1 \ 1/0 \ 1/0) = (1 \ \infty \ \infty) = (1 \ 0 \ 0)$



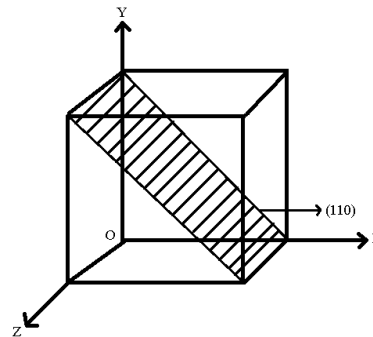
2. (110)

Indices - ( 110)

Intercepts - (a/h, b/k, c/l)

For Cubic Unit Cell a=b=c=1

$(1/1 \ 1/1 \ 1/0) = (1 \ 1 \ \infty) = (1 \ 1 \ 0)$



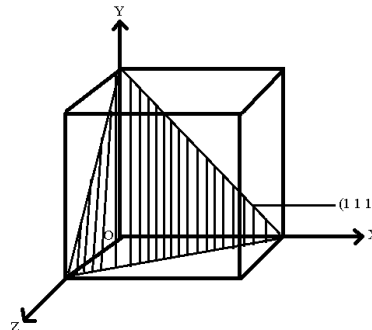
3. (111)

Indices - ( 111)

Intercepts - (a/h, b/k, c/l)

For Cubic Unit Cell a=b=c=1

$(1/1 \ 1/1 \ 1/1) = (1 \ 1 \ 1)$



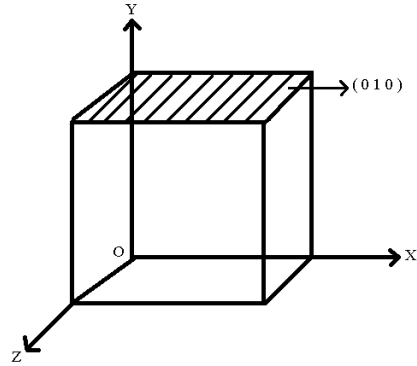
4. (010)

Indices - (010)

Intercepts - (a/h, b/k, c/l)

For Cubic Unit Cell a=b=c=1

(1/0 1/1 1/0) = (0 1 0)



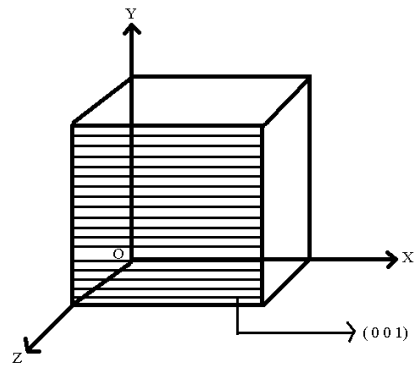
5. (001)

Indices - (001)

Intercepts - (a/h, b/k, c/l)

For Cubic Unit Cell a=b=c=1

(1/0 1/0 1/1) = (0 0 1)



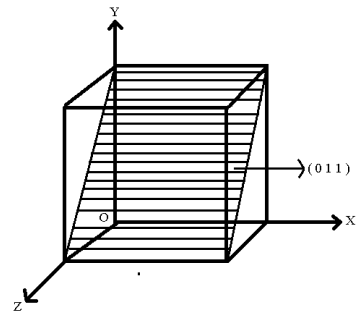
6. (011)

Indices - (001)

Intercepts - (a/h, b/k, c/l)

For Cubic Unit Cell a=b=c=1

(1/0 1/1 1/1) = (0 1 1)



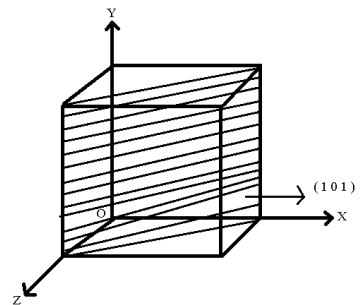
7. (101)

Indices - (101)

Intercepts - (a/h, b/k, c/l)

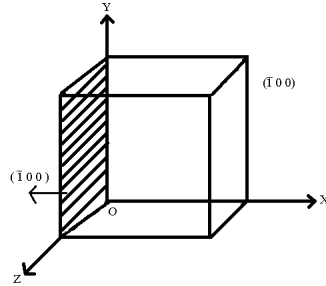
For Cubic Unit Cell a=b=c=1

(1/1 1/0 1/1) = (1 0 1)



8.  $(\bar{1}01)$

The Miller Indices can be positive as well as negative. The negative Miller Indices can be represented as  $(\bar{1}00)$



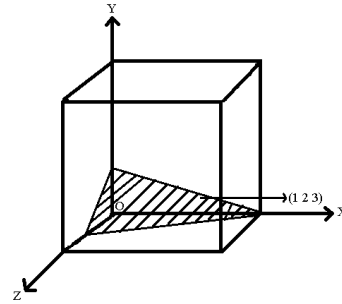
9. Indices -  $(123)$

Intercepts -

$$\left(\frac{1}{1} \frac{1}{2} \frac{1}{3}\right)$$

$$\left(\frac{1}{1} \times 6 \frac{1}{2} \times 6 \frac{1}{3} \times 6\right)$$

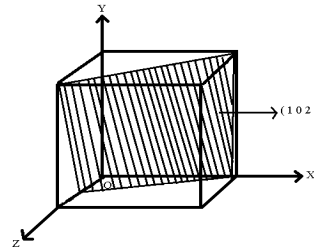
$$(632)$$



10. Indices -  $(102)$

Intercepts -  $\left(\frac{1}{1} \frac{1}{0} \frac{1}{2}\right)$

$$(102)$$



## OBJECTIVE QUESTIONS FOR ONLINE EXAMINATION

23. A direction line is drawn through the origin and it passes through the first integer point given by  $x=2$ ,  $y=3$ ,  $z=2$ . Then its direction is represented by

- e. (223)                      f. (322)                      **g. (232)**                      h. (222)

24. The Miller indices (h k l) represents

- a. A plane                      b. System of planes                      c. The direction                      **d. a set of parallel planes**

25. A plane parallel to xy plane is represented by the Miller Indices

- a. (111)                      b. (100)                      c. (010)                      **d. (001)**

26. A plane cuts the three axes at  $5a$ ,  $5b$  and  $5c$ , where  $a$ ,  $b$ ,  $c$  are lattice constants along  $x$ ,  $y$ ,  $z$  directions. Then the Miller indices of this plane

- a. **(111)**                      b. (110)                      c. (101)                      d. (011)

27. Let 'a' be the lattice constant of a cubic crystal then the interplanar separation for [111] planes is

- a. a                      b.  $a/\sqrt{2}$                       **c.  $a/\sqrt{3}$**                       d.  $a/2$

28. The interplanar separation for a set of planes [221] whose lattice parameter is  $6 \text{ \AA}$  of a cubic crystal is

- a.  $2 \text{ \AA}$**                       b.  $1 \text{ \AA}$                       c.  $3 \text{ \AA}$                       d.  $6 \text{ \AA}$

29. X-rays are

- a. Electric waves                      b. Magnetic waves                      **c. Electromagnetic waves**                      d. Mechanical waves

30. The wavelength of X-rays are nearly equal to

- a. 1 cm                      **b.  $1 \text{ \AA}$**                       c. 1 meter                      d.  $5000 \text{ \AA}$

31. X-ray diffraction is noticed with

- a. Plane grating space                      **b. Single crystal**                      c. rubber                      d. Glass

32. Bragg's Law is defined as

- a.  $d \sin\theta = n\lambda$       b.  $d \sin\theta = (n+1/2)\lambda$       c.  $2d \sin\theta = (n+1/2)\lambda$       **d.  $2d \sin\theta = n\lambda$**

33. In Bragg's relation  $2d \sin\theta = n\lambda$ , the factor n is called

- a. **Order of diffraction**      b. Refractive index      c. Bragg number      d. Density of x-rays

34. In Bragg's relation  $2d \sin\theta = n\lambda$ , the factor d is called

- a. Lattice parameters      **b. Interplaner spacing**      c. Density of the material      d. diameter

35. Crystal directions are represented by

- a. 123      b. (123)      **c. [123]**      d. <123>

36. Miller indices are used to specify

- a. Structure of the material      b. Unit cell      **c. Planes and directions in the crystal**      d. Intensity of Bragg lines

37. The Miller Indices of the plane parallel to x-axis is represented by

- a. (111)      b. (100)      **c. (011)**      d. (101)

38. The radiations that produces observable diffraction as the they pass through the crystal

- a.  $\gamma$ -rays      b. visible rays      **c. X-rays**      d.  $\alpha$ -rays

39. The Miller indices of a plane in a cubic crystal are (121). What are their intercepts on three crystallographic axes?

- a. a, b, c      **b. 2a, b, 2c**      c.  $\infty, 0, \infty$       d. a, 2b, c

40. A beam of X-rays of wavelength  $\lambda = 1.25 \text{ \AA}$  is made to fall on a material and its first Bragg's reflection is noticed at  $\theta = 45^\circ$ , what is the interplaner separation of this reflection

- a.  **$0.88 \text{ \AA}$**       b.  $8.8 \text{ \AA}$       c.  $0.5 \text{ \AA}$       d.  $0.6 \text{ \AA}$

41. A monochromatic X-ray beam of wave length  $1.25 \text{ \AA}$  falls on a set of planes

whose separation is  $2.5 \text{ \AA}$ . calculate the maximum Bragg's reflections that can be observed in this case

- a. 1                      b. 2                      c. 3                      **d. 4**

42. Different planes in a crystal are indicated by the same miller indices (h k l). this means the planes

- a. Are parallel and equidistant to each other**                      c. Inclined to each other  
b. Are perpendicular to each other                      d. Intersect each other

## Problems on XRD

1. Show that in a simple cubic lattice the separation between the successive lattice planes (100), (110) and (111) are in the ratio of 1:0.71:0.58.

Or

Calculate the ratio  $d_{(100)} : d_{(110)} : d_{(111)}$  for a simple cubic structure.

Sol: Given that

The three planes are (100), (110) and (111)

We know that the expression for interplanar spacing for a cubic crystal

$$d_{(hkl)} = \left( \frac{a}{\sqrt{h^2 + k^2 + l^2}} \right)$$
$$d_{(100)} = \left( \frac{a}{\sqrt{1^2 + 0^2 + 0^2}} \right) = a$$
$$d_{(110)} = \left( \frac{a}{\sqrt{1^2 + 1^2 + 0^2}} \right) = \frac{a}{\sqrt{2}}$$
$$d_{(111)} = \left( \frac{a}{\sqrt{1^2 + 1^2 + 1^2}} \right) = \frac{a}{\sqrt{3}}$$

$$\begin{aligned} \text{Hence } d_{(100)} : d_{(110)} : d_{(111)} &= a : \frac{a}{\sqrt{2}} : \frac{a}{\sqrt{3}} \\ &= 1 : \frac{1}{\sqrt{2}} : \frac{1}{\sqrt{3}} \\ &= \sqrt{6} : \sqrt{3} : \sqrt{2} = 1 : \frac{1}{1.414} : \frac{1}{1.732} \end{aligned}$$

2. In a tetragonal lattice  $a=b=2.5A$ ,  $c=1.8A$ . deduce the value of interplanar separation for planes given by (111).

Sol: Given that  $a=b=2.5A$ ,  $c=1.8A$

We know that the expression for interplanar spacing for any lattice

$$d_{(hkl)} = \frac{1}{\sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}}}$$

~ 24 ~



$$d_{(111)} = \frac{1}{\sqrt{\frac{1}{(2.5)^2} + \frac{1}{(2.5)^2} + \frac{1}{(1.8)^2}}} = 1.26 \text{ \AA}$$

The interplanar separation for (111) planes is  $d=1.26 \text{ \AA}$

3. Calculate the interplanar separation for a plane of simple cubic structure whose Miller indices are (321). Assume lattice constant as  $4.2 \times 10^{-10}$  meter.

Sol: Given that

$$\text{Lattice constant } a = 4.2 \times 10^{-10} \text{ meter}$$

We know that the expression for interplanar separation for a cubic structure

$$d_{(hkl)} = \left( \frac{a}{\sqrt{h^2 + k^2 + l^2}} \right)$$

$$d_{(321)} = \left( \frac{4.2 \times 10^{-10}}{\sqrt{9 + 4 + 1}} \right) = \frac{4.2 \times 10^{-10}}{\sqrt{14}} = 1.12 \times 10^{-10} \text{ m}$$

4. A beam of x-rays of wavelength 0.071 nm is diffracted by (110) plane of rock salt with lattice constant of 0.28 nm. Find the glancing angle for the second order diffraction.

Sol: Given that

$$\text{Wavelength } \lambda = 0.071 \text{ nm} = 0.071 \times 10^{-9} \text{ m}$$

$$\text{Miller indices of diffraction plane } (hkl) = (110)$$

$$\text{Lattice constant } a = 0.28 \text{ nm} = 0.28 \times 10^{-9} \text{ m}$$

$$\text{Diffraction order } n = 2$$

We know that the expression for interplanar spacing for a cubic crystal

$$d_{(hkl)} = \left( \frac{a}{\sqrt{h^2 + k^2 + l^2}} \right)$$

$$d_{(110)} = \left( \frac{0.28 \times 10^{-9}}{\sqrt{1^2 + 1^2 + 0^2}} \right) = \frac{0.28 \times 10^{-9}}{\sqrt{2}} = 0.197 \times 10^{-9} \text{ m}$$

The Bragg's law is  $2d\sin\theta=n\lambda$

$$\sin\theta = \left[ \frac{n\lambda}{2d} \right]$$

$$\theta = \sin^{-1} \left[ \frac{n\lambda}{2d} \right]$$

$$\theta = \sin^{-1} \left[ \frac{2 \times 0.071 \times 10^{-9}}{2 \times 0.197 \times 10^{-9}} \right] = \sin^{-1}(0.360) = 21.12$$

The glancing angle  $\theta=21.12$ .

5. A beam of x-rays is incident on NaCl crystal with lattice spacing 0.282nm. Calculate the wavelength of x-rays if the first order Bragg's reflection takes place at a glancing angle of 8.35. Also calculate the maximum order of diffraction possible.

Sol: Given that

Glancing angle  $\theta=8.35$

Lattice spacing  $d=0.282 \text{ nm}=0.282 \times 10^{-9} \text{ m}$

Diffraction order  $n=1$

We know that Bragg's law is  $2d\sin\theta=n\lambda$

$$2 \times 0.282 \times 10^{-9} \times \sin(8.35) = 1 \times \lambda$$

$$\lambda = 0.0841 \times 10^{-9} \text{ m} = 0.841 \times 10^{-10} \text{ m} = 0.0841 \text{ \AA}$$

Maximum order of Diffraction  $n = \frac{2d \sin\theta}{\lambda}$

For maximum order of diffraction the Bragg's angle,  $\theta=90^\circ$

$$n = \frac{2 \times (0.282 \times 10^{-9}) \sin 90}{0.841 \times 10^{-10}} = 6.7$$

Maximum order diffraction,  $n=6$

6. What is the glancing angle at which the third order reflection of X-rays of  $0.79 \text{ \AA}$  wavelength can occur in a calcite crystal of  $3.04 \times 10^{-8} \text{ cm}$  spacing?

Sol: Given that

$$\text{Wavelength } \lambda = 0.79 \text{ \AA} = 0.79 \times 10^{-9} \text{ m}$$

$$\text{Lattice spacing } d = 3.04 \times 10^{-8} \text{ cm} = 3.04 \times 10^{-10} \text{ m.}$$

$$\text{Diffraction order } n = 3.$$

According to Bragg's law,  $2d \sin \theta = n\lambda$

$$\sin \theta = \left[ \frac{n\lambda}{2d} \right]$$

$$\theta = \sin^{-1} \left[ \frac{n\lambda}{2d} \right]$$

$$\theta = \sin^{-1} \left[ \frac{3 \times 0.79 \times 10^{-10}}{2 \times 3.04 \times 10^{-10}} \right] = \sin^{-1}(0.3898) = 22.942$$

$$\text{Bragg's angle } \theta = 22.942.$$

7. Monochromatic X-rays of wavelength  $1.5 \text{ \AA}$  are incident on a crystal face having an interplanar spacing of  $1.6 \text{ \AA}$ . Find the highest order for which Bragg's reflection maximum can be seen.

Sol: Given that

$$\text{Wavelength of X-rays } \lambda = 1.5 \text{ \AA} = 1.5 \times 10^{-10} \text{ m}$$

$$\text{Lattice spacing } d = 1.6 \times 10^{-8} \text{ cm} = 1.6 \times 10^{-10} \text{ m.}$$

We know that Bragg's law is  $2d \sin \theta = n\lambda$

$$n = \frac{2d \sin \theta}{\lambda}$$

For maximum order of diffraction the Bragg's angle,  $\theta = 90^\circ$

$$n = \frac{2 \times (1.6 \times 10^{-10}) \sin 90}{1.5 \times 10^{-10}} = 2.133$$

$$\text{Maximum order diffraction, } n = 2.$$

# UNIT – IV

Chapter – I: Magnetic materials

Chapter – II: Superconductivity

Course Outcome:

**CO 4:** Knowing the applications of magnetic and superconducting materials in engineering field.  
**(Remember) K1**

## UNIT-IV

### MAGNETIC PROPERTIES

#### **Introduction:-**

The story of magnetic materials began with a mineral known as “Magnetite”. This magnetite is available abundantly at a place called “Magnesia”. The “magnet” comes from the word “Magnetite”

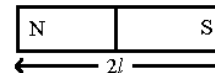
The materials which get magnetized in the presence of magnetic field are known as “Magnetic material”. These materials are known to mankind from 800 B.C. But the actual development of magnetic material has taken place from 15<sup>th</sup> century efforts are due to scientist “Gilbert”. The 19<sup>th</sup> century development efforts are due to “Orsted”. According to him electric current will always be accompanied by magnetic fields, starting from the invention of compass box (mariners compass). These materials find wide and variety of applications in the field of science, engineering and technology.

#### **Magnetism:-**

The attracting property exhibited by the magnet.

#### **Magnetic Dipole:-**

The two ends are known as Magnetic poles and are called North Pole and South Pole.



If a system consisting of two equal and opposite magnetic poles separated by a distance “2l”.

#### **Magnetic moment ( $\mu_m$ ):-**

If “m” is the magnetic pole strength and “2l” is the length of the magnet, then its magnetic dipole moment is given by the product of “m” and “2l” is called Magnetic moment.

## Magnetic field:-

The space surrounding the magnet up to which its attracting influence is felt is known as “Magnetic field”.

## Magnetic Induction (B) (or) Magnetic flux density:-

It is defined as the number of magnetic lines of force passing through a unit area of cross- section of magnetic materials.

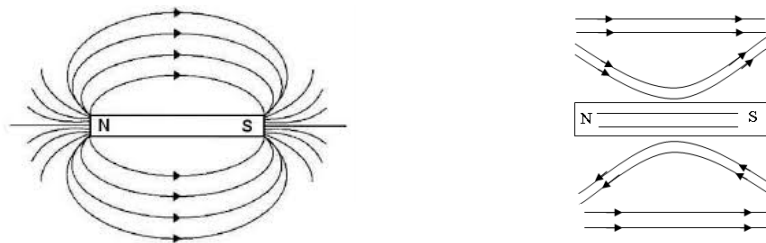
$$B = \Phi / A - \text{weber-m}^2 \text{ (or) Tesla}$$

## Magnetic field intensity (or) Magnetic field strength (H):-

It is the force experienced by a unit North Pole placed at a given point in a magnetic field.

## Magnetic permeability ( $\mu$ ):-

Consider an unmagnetised bar of a magnetic material in a uniform magnetic field as shown in figure. It has been observed that the bar gets magnetized by induction and a polarity. After magnetization, the magnetic lines in the bar possess N-Pole throughout region and then re-enters to the south S-Pole as shown in figure (a). These lines form a closed loop within the magnet. By passing from S-Pole to N-Pole, that the lines of the magnetized bar opposes the lines of the original field outside the magnet and favor inside the magnet, the result is shown in figure (b).



As a result of this, the magnetic field strength (H) is increased inside the bar and decreased outside the bar. Similarly the magnetic flux (B) becomes high inside the bar and low outside the bar. Thus we find that flux density (B) is directly proportional to the magnetic field strength (H).

$$B \propto H$$

$$\text{i.e. } B = \mu H \dots\dots\dots (1)$$

$$\mu = B / H$$

Where “ $\mu$ ” is a constant proportionality and is known as “Absolute Permeability”

If the flux density is established in air (or) vacuum (or) in a non- magnetic material, then the above equation can be written as

$$B_0 = \mu_0 H \dots\dots\dots (2)$$

$$\mu_0 = B_0 / H$$

Where  $B_0$  is the flux density in air (or) vacuum

$\mu_0$  is the absolute permeability of air (or) vacuum

**Magnetic permeability of free space ( $\mu_0$ ):-**

It is defined as the ratio of magnetic induction (or) magnetic flux ( $B_0$ ) of a free space to the applied magnetic field strength (H).

$$\mu_0 = B_0 / H$$

$$B_0 = \mu_0 H \dots\dots\dots (3) \quad \mu_0 = 4\pi \times 10^{-7} \text{ Hm}^{-1}$$

**Relative permeability ( $\mu_r$ ):-**

It is defined the ratio of permeability of a material to the permeability of free space.

$$\text{i.e. } \mu_r = \mu / \mu_0$$

$$\mu = \mu_r \mu_0 \dots\dots\dots (4)$$

**Magnetization (M):-**

Magnetization refers to the process of converting from unmagnetised bar into magnetized bar. (Or)

The magnetic moment per unit volume

**Magnetic susceptibility ( $\chi$ ):-**

It is the ratio of the intensity of magnetization produced in the sample to the magnetic field intensity which produced the magnetization.

$$\chi = M / H \dots\dots\dots (5)$$

**Relation between Magnetic Induction (B), Magnetization (M), Relative permeability ( $\mu_r$ ), and Magnetic susceptibility ( $\chi$ ):-**

We know that

$$B = \mu H \qquad \mu_r = \mu / \mu_0$$

$$= \mu_0 \mu_r H \qquad \mu = \mu_0 \mu_r$$

$$i.e. B = \mu_0 \mu_r H + \mu_0 H - \mu_0 H$$

$$= \mu_0 H + \mu_0 H (\mu_r - 1)$$

$$= \mu_0 H + \mu_0 M$$

$$B = \mu_0 (H + M) \quad \text{where } M = H(\mu_r - 1)$$

Where the magnetization M is equal to = H ( $\mu_r - 1$ )

$$i.e. B = \mu_0 (H+M).$$

The first term on the right side of equation (5), is due to external field. The second term is due to magnetization.

Thus the magnetic induction (B) in a solid is  $B = \mu_0 (H+M)$

$$\mu_0 = B / (H+M)$$

Relative Permeability  $\mu_r = \mu / \mu_0$

$$= \frac{\frac{B}{H}}{\frac{B}{H+M}} = \frac{H+M}{H} = 1 + \frac{M}{H} \qquad \left( \because \chi = \frac{M}{H} \right)$$

$$\mu_r = 1 + \chi$$



## Classification of Magnetic materials:-

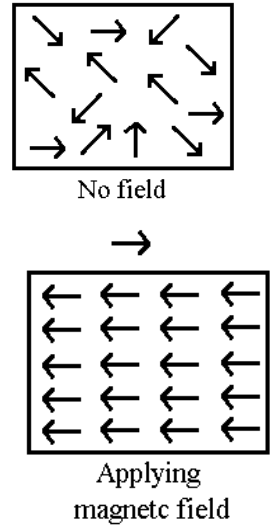
Magnetic materials are broadly classified into two categories. Those are

1. Diamagnetic materials
2. Paramagnetic materials
3. Ferromagnetic materials

### Diamagnetic materials:-

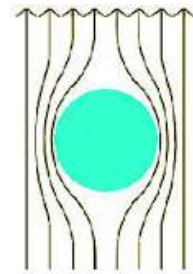
Materials composed of atoms or molecules, having zero magnetic moment are called diamagnetic materials

- In diamagnetic materials, the dipoles are oriented randomly.
- When external magnetic field is applied it causes the magnetic moment of its atoms will align in the opposite direction of the applied field.



### Properties:-

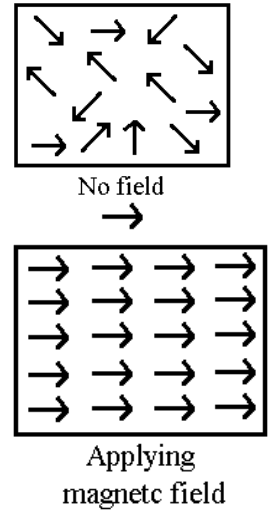
1. Permanent dipoles are absent.
2. The relative permeability ( $\mu_r$ ) of these materials is less than 1 ( $\mu_r < 1$ ), and susceptibility is ( $\chi$ ) is negative (-ve).
3. Diamagnetic materials are independent of temperature
4. When placed inside a magnetic field, magnetic lines of force are repelled.
5. The intensity of magnetization is negative.
6. No spin alignment is present.



### Paramagnetic materials:-

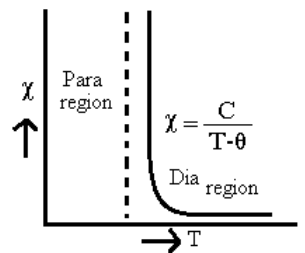
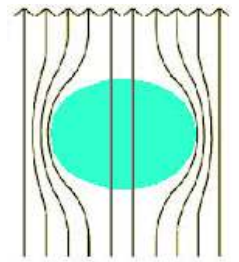
If the paramagnetic materials have net magnetic moment, these magnetic moments are weakly coupled to each other.

- If the permanent dipoles do not interact with each other is called paramagnetic materials.
- When these are placed in a magnetic field, acquired feeble magnetism, the moment start to align, but only a small fraction is deflected into the field direction.



### Properties:-

1. Paramagnetic materials permanent dipoles are present
2. The relative permeability ( $\mu_r$ ) of these materials is greater than 1 ( $\mu_r > 1$ ), and susceptibility is ( $\chi$ ) is positive (+ve).
3. Paramagnetic materials are greatly depends on temperature
4. When placed inside a magnetic field, it attracts the magnetic lines of force.
5. The intensity of magnetization is positive (+ve) and moderate.
6. The temperature “T” is less than Curie temperature, and then the material converted into diamagnetic material.



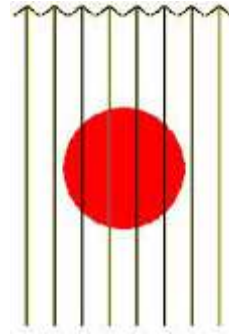
### Ferromagnetic material:-

In ferromagnetic materials, the magnetic interaction between any two dipoles aligns themselves parallel to each other.

- Thus, ferromagnetic substances possess magnetic moment even in the absence of the applied magnetic field. This magnetization is known as “Spontaneous magnetization”.

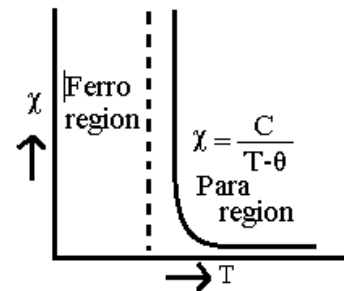
## Properties:-

1. The ferromagnetic materials possess spontaneous magnetization.
2. The Relative permeability ( $\mu_r$ ) of these materials is very greater than 1 ( $\mu_r \gg 1$ ), value of susceptibility is ( $\chi$ ) is positive (+ve) and very large.
3. Ferro magnetic materials are greatly depends on temperature
4. When placed inside a magnetic field, it attracts the magnetic lines of force very strongly.
5. The intensity of magnetization is positive (+ve) and high
6. The temperature below Curie temperature, the material would have ferromagnetic behavior. Temperature above Curie temperature the material is converted into paramagnetic material.



Depending upon the spin orientation the electrons, ferromagnetic materials are classified into two types. They are

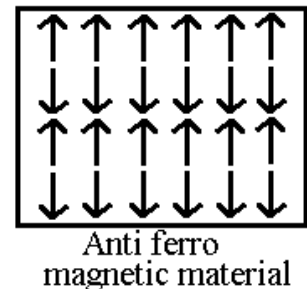
1. Anti Ferro magnetic material
2. Ferri magnetic material



## Anti Ferro magnetic material:-

The materials with anti- parallel magnetic moment (spin magnetic moment) are known as Anti Ferro magnetic materials.

In Anti Ferro magnetic materials, the magnetic interaction between any two adjacent dipoles align themselves anti parallel to each other. If the material is composed of atoms. One of the atoms having their moment oriented in one direction and the other of atoms having their spin moments in the opposite direction as shown in figure.

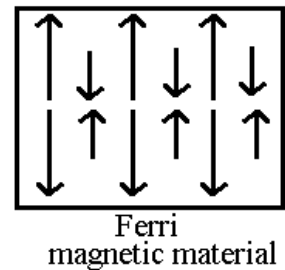


**Properties:-**

1. Anti-Ferro magnetic materials possess permanent dipoles.
2. The relative permeability ( $\mu_r$ ) of these materials is greater than 1 ( $\mu_r > 1$ ), and susceptibility is ( $\chi$ ) is positive (+ve).
3. Anti-Ferro magnetic materials are greatly depends on temperature
4. When placed inside a magnetic field, it attracts the magnetic lines of force.
5. The temperature “T” is less than Curie temperature, and then the material converted into diamagnetic material.
6. The intensity of magnetization is positive (+ve) and moderate.

**Ferri magnetic materials:-**

The substances which consist of anti-parallel magnetic moment of different magnitudes are known as Ferri magnetic substances.



(Or)

In Ferri magnetic materials, the magnetic interaction between any two adjacent dipoles, align themselves anti parallel to each other.

**Properties:-**

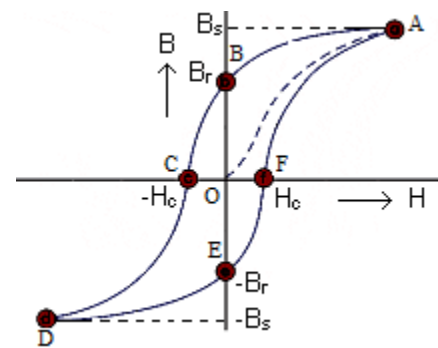
1. The Ferri magnetic materials possess spontaneous magnetization.
2. The Relative permeability ( $\mu_r$ ) of these materials is very greater than 1 ( $\mu_r \gg 1$ ), value of susceptibility is ( $\chi$ ) is positive (+ve) and very large.
3. The temperature below Curie temperature, the material would have ferromagnetic behavior. Temperature above Curie temperature the material is converted into paramagnetic material.
4. When placed inside a magnetic field, it attracts the magnetic lines of force very strongly.

5. In the presence of magnetic field, the magnetic lines of force are highly attracted towards the center of the material and hence, the magnetic field induction inside the material is very greater than the outside.

### Hysteresis Loop:-

The hysteresis of Ferro magnetic materials refers to the lag of magnetization behind the magnetic field. It gives the relationship between the magnetic flux density (B) and magnetic field (H) referred as B-H curve.

Consider an unmagnetised ferromagnetic material is placed in a magnetic field. When the material is slowly magnetized and the magnetic flux density (B) increases with increase of magnetic field (H) initially through OA and reaches saturation at point 'A' known as "saturation magnetization ( $B_s$ )". When external magnetic field (H) is removed, the magnetization slowly decreases but not comes to zero (at  $H=0$ ), which is referred to as residual magnetization ( $B_r$ ) at the point OB this phenomenon is called "retentivity". When reverse magnetic field is applied, the residual magnetism decreases and comes to zero for a particular value of reverse magnetic field known as coercive field ( $H_c$ ) at the point OC and these phenomenon is called as "coercivity". After reaching the saturation level D, when the magnetizing field is reversed, the curve closes to the point A, completing the cycle. The loop OABCDEFA is called hysteresis loop as shown in figure.



The retentivity and coercivity are important characteristics of different ferromagnetic materials. Based on the degree of their values, the ferromagnetic materials may be classified as soft and hard magnetic materials.

## Hard and soft magnetic materials:-

The magnetic materials are classified into two types.

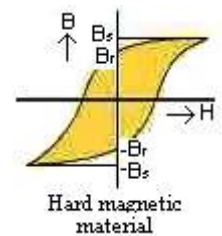
1. Hard magnetic materials
2. Soft magnetic materials

### Hard magnetic materials:-

The materials which are very difficult to magnetize and demagnetized are said to be hard magnetic materials.

#### Properties:-

1. Hard magnetic materials have large hysteresis loss due to large hysteresis loop area.
2. These materials have low value of susceptibility and permeability.
3. The coercivity and retentivity are large
4. These materials have large amount of impurities



#### Applications of Hard magnetic materials:

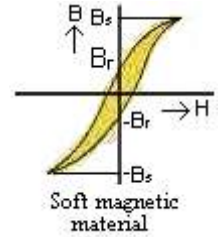
1. They are used in digital computers
2. They are used for making permanent magnets
3. They are used in transducers
4. They are used in magnetic tapes.

### Soft magnetic materials:-

The materials which are easy to magnetize and demagnetized are said to be soft magnetic materials.

#### Properties:-

1. Soft magnetic materials have low hysteresis loss due to small hysteresis loop area.
2. These materials have large value of susceptibility and permeability.
3. The coercivity and retentivity are small
4. These materials are free from irregularities like strain and impurities.



### Applications of Soft magnetic materials:

1. They are used in electromagnets.
2. They are used in AC current machinery.
3. They are used in communication equipment's.
4. They are used in audio and video transformers.

## SUPER CONDUCTIVITY

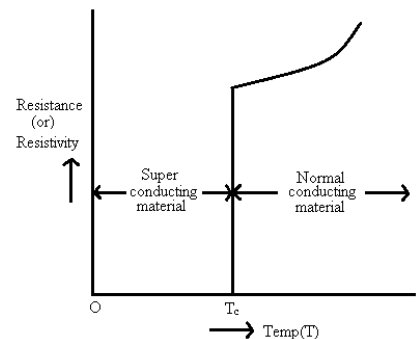
### Introduction:-

Resistance (or) resistivity is the property exhibits by thee materials. This is mainly due to the scattering of electrons, while interacting with positive ions present in the materials. When temperature of material is decreased to a low value, then due to lower energy, scattering of electrons decreases and as a result resistance (or) resistivity decreases. Then the conductivity increases.

The phenomenon of attains zero resistivity (or) infinite conductivity at low temperature is known as super conductivity.

The materials which exhibit superconductivity are known as super conductors.

Super conductivity was first observed by Kammerlingh onne's in the year 1911 in the case of mercury. When temperature of mercury is decreases



then the resistance also decreases and it is zero at 4.2 K temperature as shown in figure.

The temperature at which the material undergoes a transition from normal state to superconducting state is known as critical temperature (or) transition temperature.

Different materials have different  $T_c$  values

1. Aluminium  $T_c - 1.19$  K
2. Lead  $T_c - 7.2$  K
3. Tungsten  $T_c - 0.01$  K
4. Tin  $T_c - 1.19$  K
5. Cadmium  $T_c - 1.19$  K

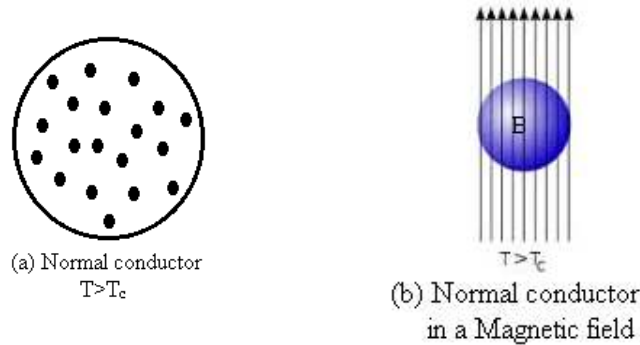
### **General properties:-**

1. Super conductivity is a low temperature phenomenon
2. The transition from normal state to superconducting state occurs below the Critical Temperature( $T_c$ ).
3. Different materials will have different critical temperatures.
4. The current ones setup in a super conductor persists (stay) for a long time due to zero resistivity.
5. Super conductors do not allow magnetic field through them and behaves as a diamagnetic material. This property of expulsion (repulsion) of magnetic field is known as “Meissner effect”
6. The magnetic field at which a super conductor loses its superconductivity and becomes a normal conductor is known as Critical Magnetic field ( $H_c$ ).
7. Super conductivity occurs in metallic elements in which the number of valence electrons lies between 2 to 8.
8. Super conducting materials are not good conductors at room temperature



**Meissner effect:-**

Consider a normal conductor at room temperature as shown in Figure (a). When a magnetic field (H) is applied to it then it allows the magnetic lines to pass through it. Thus we have a magnetic induction field (B) in a conductor as shown in figure (b).



When the entire system is cooled so that  $T < T_c$ , then the normal conductor becomes a super conductor and it will not allow the magnetic lines to pass through it. It expels the magnetic lines. This effect, observed by Meissner, is known as “Meissner effect”. Thus the superconductor does not allow the magnetic lines through it (or) expels the magnetic lines as shown in figure (c).

For a normal conductor, magnetic induction field “B” is given by

$$B = \mu_0 (H+M)$$

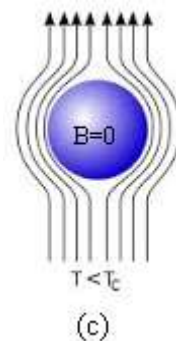
Where  $\mu_0$  is the permeability of free space (or) air

M is the magnetization of the normal conductor

For a super conductor,  $B=0$

$$\mu_0 (H+M) = 0$$

$$H = -M$$



i.e. applied magnetic field induces magnetization in opposite direction

$$\chi = M / H = -1$$

$$\chi = -1$$

## Types of super conductors:-

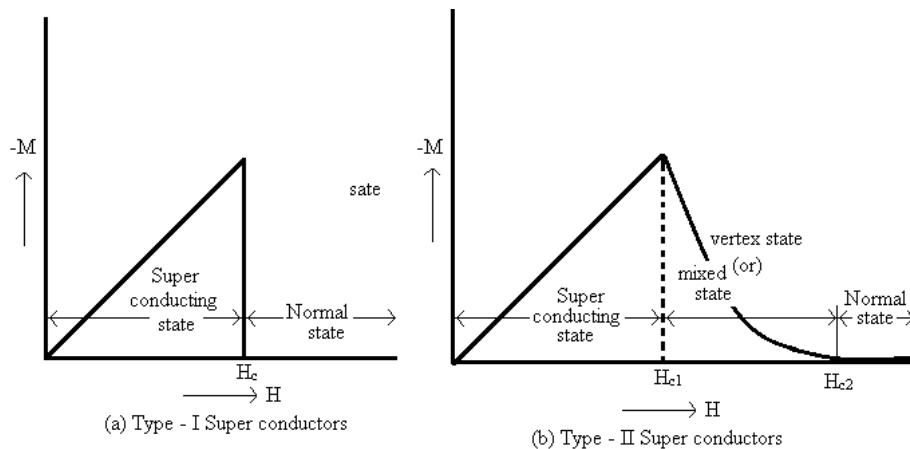
In the presence of critical magnetic field, a super conductor converts into a normal conductor. Based on the conversion process, super conductors are classified into two types.

1. Type – I super conductors
2. Type – II super conductors

Based on diamagnetic response, super conductors can be classified as Type - I and Type – II super conductors.

The super conductors exhibit a complete Meissner effect (perfect diamagnetism) are called Type-I super conductors are also known as soft super conductors.

When the magnetic field strength is gradually increased from its initial value  $H < H_c$  at  $H_c$ , the diamagnetism abruptly disappears and the transition from super conducting state to normal state is sharp as shown in figure(a).



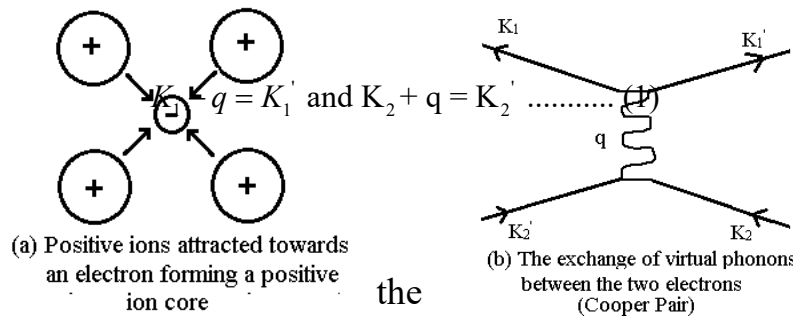
Type –II superconductors are shown in figure (b), up to the field, the specimen is in a pure superconducting state. The magnetic flux lines are rejected. When the field is increased beyond  $H_{c1}$  (the lower critical field), the magnetic flux lines starts penetrating. The specimen is in a normal state. This means that the Meissner effect is incomplete in the region between  $H_{c1}$  and  $H_{c2}$ . This region is known as vertex state (or) mixed state. Type –II super conductors are known as hard super conductors.

## BCS Theory:-

BCS Theory of super conductivity was put forward by Bardeen, Cooper and Schrieffer in 1957 and hence named as BCS theory. This theory explains the super conducting state of super conductor. This theory involves the electron interaction through phonon as mediators.

### (i). Electron – Electron interaction via lattice deformation:-

Let us consider an electron passing through the lattice of positive ions. The electron is attracted by the neighboring positive ions, forms a positive ion core as shown in figure (a) and gets screened by them. The screening greatly reduces the effective charge of this electron. Due to the attraction between the electron and the ion core, the lattice gets deformed on local state.



another  
by the side  
the

the

Now if  
electron passes  
assembly of  
electron and

the ion core, it gets attracted with the first electron via lattice deformation. This interaction is said to be due to the exchange of virtual phonon  $q$ , between the two electrons. This interaction process can be written in terms of the wave vector  $K$  as i.e  $K_1 + K_2 = K_1' + K_2'$ . This gives the net wave vector of the pair is conserved. The momentum is transferred between the electrons. These two electrons together form a cooper pair and are known as Cooper electron.

## (ii). Cooper Pair:-

To understand the mechanism of Cooper pair formation, let us consider the distribution of electrons in metals as given by Fermi – Dirac distribution function

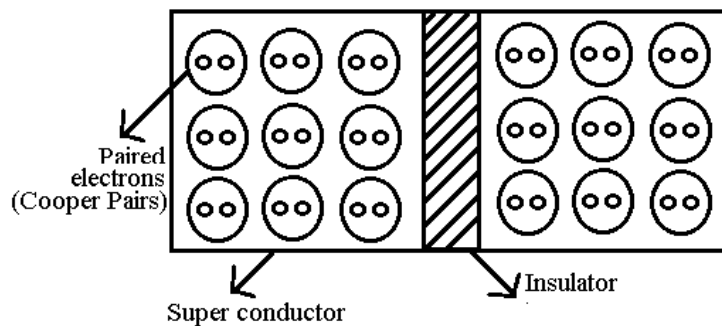
$$F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{K_T}\right)}$$

At  $T=0K$ , all the energy state below Fermi level  $E_F$  are completely filled.  
all the energy state above Fermi level  $E_F$  are completely empty.

Let us see, what happens when two electrons are added to a metal at absolute zero. Since all the quantum states with energies  $E \leq E_F$  are filled. They are forced to occupy states having energies  $E \geq E_F$ . Cooper showed that if there is an attraction between the two electrons, they are able to form a bound state so that their total energy is less than  $2E_F$ . These electrons are paired to form a single system. These two electrons together form a Cooper pair and are known as Cooper electrons.

## Josephson Effect:-

Consider two super conductors which are joined together with the help of a thin insulating layer as shown in figure. These super conductors consist of paired electrons known as Cooper pair in the super conducting state. These Cooper pairs will try to penetrate or tunnel through the thin insulator and constitute a small super current. The insulator which forms the junctions between super conductors is known as Josephson junction and this effect is known as “Josephson Effect”.



This effect is of two types, they are

1. DC – Josephson Effect
2. AC – Josephson Effect

### DC – Josephson Effect:-

Without any applied voltage across the junction due to tunneling of cooper pairs, a small direct super current (DC) flows across the junction. This effect is known as DC – Josephson Effect.

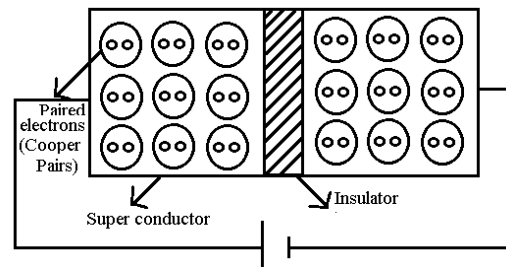
Let the propagation of cooper pair be in the form of waves. The phase difference between the two parts of the waves on either side of the junctions, in terms of wave function is  $\phi_0 = \phi_2 - \phi_1$

The tunneling current is given by  $I = I_0 \sin \phi_0$

Where  $I_0$  is the maximum current that flows through the junction without any voltage across the junction. The above equation represents a direct current (DC) that flows across the junction.

### AC – Josephson Effect:-

When a static potential  $V_0$  is applied across the junction, then the cooper pairs starts oscillating through the insulating layer. As a result, an alternating current (AC) flows through the junction. This effect is known as AC – Josephson Effect.



Due to  $V_0$  an additional phase difference of  $\Delta\phi = \frac{Et}{\hbar}$  is introduced for the cooper pairs, where E is the total energy of the cooper pair at any time 't'.

$$E = (2e) V_0$$

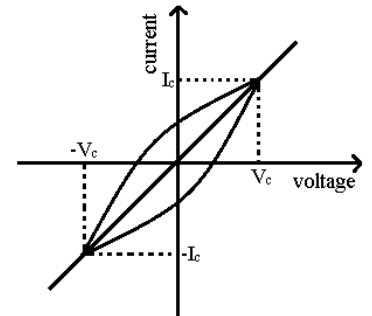
$$\Delta\phi = \frac{2eV_0t}{\hbar}$$

$$\sim 17 \sim$$

The tunneling current can be written as

$$\begin{aligned}
 I &= I_0 \sin(\phi_0 + \Delta\phi) \\
 &= I_0 \sin\left(\phi_0 + \frac{2eV_0 t}{\hbar}\right) \\
 &= I_0 \sin(\phi_0 + \omega t) \quad \omega = \frac{2eV_0}{\hbar} = \text{angular frequency}
 \end{aligned}$$

This represents an AC with angular frequency  $\omega$ , current-voltage characteristics of a Josephson junction as shown in below graph.



1. When  $V_0=0$ , there is a constant flow of DC current  $I_c$  through the junction. This current is called super conducting current and the effect is DC-Josephson effect.
2. When  $V_0 < V_c$ , a constant DC-current  $I_c$  flows.
3. When  $V_0 > V_c$ , the junction has a finite resistance and the current oscillates with frequency  $\omega = \frac{2eV_0}{\hbar}$ . This effect is AC-Josephson effect.

## Applications of Super conductors:-

### 1. Electric generators:-

Super conducting generators are smaller in size with less weight and consume very low energy. The low loss super conducting coil rotated in a strong magnetic field. This is the basis of new generation of energy-saving power system.

### 2. Low loss transmission and transformers:-

When super conducting wires are used as electric cables then the transmission losses are minimized. If super conductors are used for winding of a transformer, the power losses will be very small.

### 3. Magnetic levitation:-

Dia magnetic property of a super conductor is the basis of magnetic levitation. This effect can be used for high-speed transportation.

#### **4. Generation of high magnetic field:-**

The application of magnetic field generator than the changes the superconducting state to normal state and removal of the field reverse the process. This principle is used in switching element.

#### **5. Fast electrical switching:-**

The application of magnetic field greater than the changes the superconducting state to normal state and removal of the field reverse the process. This principle is used in switching element.

#### **6. Logic and storage functions in computers:-**

The I-V characteristic of Josephson Effect is used for memory elements in computers. Thus super conductors are used to perform logic and storage function in computers.

#### **7. Super Conducting Quantum Interference Devices (SQUIDS):-**

Super Conducting Quantum Interference Devices is basically in the form of a tiny Super conducting ring consists of two super conductors separated by thin insulating layer so as to form two Josephson junctions. SQUIDS can measure magnetic field of the order of  $10^{-14}$  tesla. SQUIDS are used in the construction of sensitive magnetometers to measure accurately the magnetic field produced in case of heart and brain also the earth magnetic field at different place.

## OBJECTIVE QUESTIONS FOR ONLINE EXAMINATION

1. The susceptibility of diamagnetic material is
  - a. Positive
  - b. **Negative**
  - c. Zero
  - d. None
2. In ferro magnetic material, the spin alignment is
  - a. **Regular**
  - b. Irregular
  - c. No alignment
  - d. None
3. The relation between B and H is
  - a.  **$B = \mu H$**
  - b.  $H = \mu B$
  - c.  $\mu = BH$
  - d.  $H = B\mu$
4. The units of magnetic susceptibility are
  - a.  $\text{Amp}\cdot\text{m}^2$
  - b. Henry/m
  - c. **No units**
  - d. Ampere m
5. The units of magnetic dipole moment are
  - a. **Amp-m**
  - b.  $\text{Amp}\cdot\text{m}^2$
  - c. Amp/m
  - d.  $\text{Amp}/\text{m}^2$
6. The magnetism exhibited by all materials is
  - a. **Dia**
  - b. Para
  - c. Ferro
  - d. Ferri
7. Spontaneous magnetization is present in
  - a. Diamagnetic
  - b. Paramagnetic
  - c. **Ferromagnetic**
  - d. Antiferro
8. The value of one Bohr magneton is
  - a.  **$9.273 \times 10^{-24} \text{ A}\cdot\text{m}^2$**
  - b.  $9.273 \times 10^{24} \text{ A}\cdot\text{m}^2$
  - c.  $9.273 \times 10^{-27} \text{ A}\cdot\text{m}^2$
  - d.  $9.273 \times 10^{27} \text{ A}\cdot\text{m}^2$
9. If the magnetic moment of atoms of a material is zero, the material is called
  - a. **Diamagnetic**
  - b. Paramagnetic
  - c. Ferromagnetic
  - d. Antiferro
10. The units of relative permeability is
  - a. **No units**
  - b. Weber
  - c. Tesla
  - d. Amp/meter
11. Above curie temperature, a magnetic material becomes
  - a. Diamagnetic
  - b. **Paramagnetic**
  - c. Ferromagnetic
  - d. Antiferro



1. The condition required for superconductivity phenomenon in a material is
  - a. **H=0, R=0, below  $T_c$**
  - b. H=0, R=0, above  $T_c$
  - c. H $\neq$ 0, R=0, above  $T_c$
  - d. H $\neq$ 0, R=0, below  $T_c$
2. Superconductors are ----- materials
  - a. Perfectly Ferro magnetic
  - b. Dielectric
  - c. Perfectly Paramagnetic
  - d. **Perfectly Diamagnetic**
3. The magnetic susceptibility of a superconductor is
  - a. **-1**
  - b. +1
  - c. 0
  - d.  $\infty$
4. Persistent current is the characteristic property of
  - a. **Superconductor**
  - b. Meissner Effect
  - c. Josephson Effect
  - d. Hall Effect
5. Magnetic induction is zero inside a superconductor is
  - a. **Meissner Effect**
  - b. Hall Effect
  - c. Josephson Effect
  - d. Isotope Effect
6. For Type –I superconductor
  - a. Two critical magnetic fields exist
  - b. Two critical electric fields exist
  - c. **Only one critical magnetic field exist**
  - d. Only one critical electric field exist
7. For Type –II superconductor
  - a. **Two critical magnetic fields exist**
  - b. Two critical electric fields exist
  - c. Only one critical magnetic field exist
  - d. Only one critical electric field exist
8. Examples of Type-I superconductors are
  - a. Al, Nb and Ta
  - b. **Al, Zn and Hg**
  - c. Ta, V and Nb
  - d. Hg, Na and Zn

9. Below transition temperature, a superconducting material exhibit
- Only zero resistance
  - Only diamagnetic property
  - Zero resistance and diamagnetism**
  - Zero resistance and ferromagnetism
10. The magnetic of a superconductor is
- 0
  - H
  - 1
  - H**
11. In a superconducting state
- Entropy alone changes
  - Electric specific heat alone changes
  - Both entropy and electronic specific heat change**
  - Both remains constant
12. In Type-I superconductors the coherence length is of the order of
- $10^6$  m
  - $10^{-3}$  m
  - $10^3$  m
  - $10^{-6}$  m**
13. When a material becomes superconductors
- The properties of lattice structure do not change**
  - The properties of lattice structure do not change
  - It becomes ferromagnetic in nature
  - Magnetic property does not change
14. In superconducting state, the energy gap
- Is large compared to semiconductors and insulators
  - Is zero
  - Is very small as compared to semiconductors and insulators**
  - Does not change

## Problems on Magnetic Properties

1. Find the relative permeability of a ferromagnetic material if a field of strength 220 amp/m produces a magnetization of 3300 amp/m in it.

Sol: Given that

The field strength of ferromagnetic material if a field  $H = 220$  amp/m

Magnetization  $I = 3300$  amp/m

We know that the expression for relative permeability

$$\begin{aligned}\mu_r &= 1 + \frac{I}{H} \\ &= 1 + \frac{3300}{220} = 16\end{aligned}$$

The relative permeability of a ferromagnetic material  $\mu_r = 16$ .

2. A magnetic material has a magnetization of 3300 amp/m and flux of 0.0044 weber/m<sup>2</sup>. Calculate the magnetizing field and the relative permeability of the material. The permeability of vacuum is  $4\pi \times 10^{-7}$  H/m.

Sol: Given that

Magnetization of the material  $I = 3300$  amp/m

The flux density  $B = 0.0044$  weber/m<sup>2</sup>

The permeability of vacuum  $\mu_0 = 4\pi \times 10^{-7}$  H/m.

We know that the expression for magnetic flux density is

$$\begin{aligned}B &= \mu_0 (H + M) \\ H &= \frac{B}{\mu_0} - I \\ &= \frac{0.0044}{4 \times 3.142 \times 10^{-7}} - 3300 = 200 \text{ amp/m}\end{aligned}$$

And the expression for relative permeability is

$$\mu_r = 1 + \frac{I}{H} = 1 + \frac{3300}{200} = 17.5$$

The expression for magnetic flux density  $H = 200$  amp/m

The relative permeability of the magnetic material  $\mu_r = 17.5$

3. The magnetic field in the interior of a certain solenoid has the value of  $6.5 \times 10^{-4}$  T. when the solenoid is empty. When it is filled with iron, the field becomes 1.4T. Find the relative permeability of iron.

Sol: Given that

The magnetic field of solenoid without iron  $B_0 = 6.5 \times 10^{-4}$  T

The magnetic field of solenoid with iron  $B = 1.4$  T

We know that the expression for magnetic flux density,  $B = \mu H$

$$B = \mu_0 \mu_r H \text{ and } B_0 = \mu_0 H$$

$$\mu_r = \frac{B}{B_0} = \frac{1.4}{6.5 \times 10^{-4}} = 2154.$$

# UNIT – V

Chapter – I: Acoustics

Chapter – II: Ultrasonics

Course Outcome:

**CO 5:** Identifies the use of Acoustics and Ultrasonics in engineering field. **(Analyze) K4**

# ACOUSTICS

## **Introduction:-**

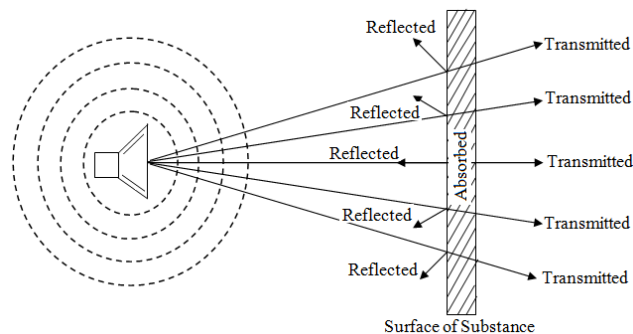
Acoustics is the branch of physics which deals with the process of generation, propagation and hearing of sound is called Acoustics. This covers many fields and is closely related to various branches of engineering such as design of acoustical instruments, electro acoustics (i.e., the branch relating to the method of sound production and recording) and architectural acoustics dealing with the design and construction of buildings, musical instruments.

In 1911, Sabine, professor of physics at Harvard university laid the foundations of acoustics engineering and architectural acoustics, it deals with the design and construction of acoustically good buildings, music halls, sound recording rooms and movie theatres where the audience the best sound quality.

## **Sound Absorption:-**

Sound Absorption measures the amount of sound energy absorbed by a material. The property of surface by which sound energy is converted into other form of energy is known as absorption. In the process of absorption sound energy is converted into heat due to frictional resistance inside the process of material. The fibrous and porous material absorbs sound energy more than other solid material.

When a sound energy strikes on any surface of substance, a part of it is absorbed or transmitted by the surface and rest is reflected as shown in figure.



Generally, the sound energy reflected from a surface is less than that incident on it. i.e. some of the sound is absorbed by the surface of substance. The absorption of sound by a substance depends on its size, shape position in the room and its physical properties. Every element in a room has a sound absorption factor. Sound absorption coefficient  $\alpha$  gives the sound absorbing properties of material.

Absorption of the material is given by  $a = \alpha s$

Where  $a$  = absorption of material

$\alpha$  = absorption coefficient of material

$s$  = Surface area

If there are different materials in a Hall, then total sound absorption is given by

$$A = a_1 + a_2 + a_3 + \dots$$

$$A = \alpha_1 s_1 + \alpha_2 s_2 + \alpha_3 s_3 + \dots$$

$$A = \sum_{i=1}^n \alpha_i s_i$$

Where  $A$  is the total absorption of sound

$\alpha_1 + \alpha_2 + \alpha_3 + \dots =$  absorption coefficient of materials

$s_1 + s_2 + s_3 + \dots =$  surface area of materials

## Reverberation:-

“The persistence of sound in an enclosure as a result of continuous reflections of sound at the walls even after the source of sound has been turned off is called reverberation”

If the intensity of sound falls below a minimum value, it is inaudible. This minimum intensity of sound is called threshold of audibility. When the source of sound is turned off, the listener hears it for some time until it reaches the threshold of audibility.

## Reverberation time:-

It is the time taken by a sound wave to fall from an average value to inaudibility. According to Sabine, time taken by a sound to reduce its intensity to one-millionth of its intensity when the source is cut off is known as standard reverberation time.

According to Sabine, reverberation time for a hall of volume “V” is

$$T = \frac{0.161 V}{A}$$

Where A = total absorption

## Absorption coefficient and its measurements:-

The absorption coefficient of a material gives the ability of a material to absorb sound. It is denoted by “ $\alpha$ ”. The values of ‘a’ are expressed as a numbers between 0 and 1. If  $\alpha=0$ , total reflection and  $\alpha=1$ , total absorption. The coefficient of absorption “ $\alpha$ ” of a material is defined as

$$\alpha = \frac{\text{sound energy absorbed by the surface}}{\text{total sound energy incident on the surface}}$$

## Measurement of absorption coefficient:-

Consider that  $T_1$  be the reverberation time measured in absence of absorbing material and  $T_2$  be the reverberation time when absorbing material present in the room by using Sabine’s formula.

$$\begin{aligned} T_1 &= \frac{0.161 V}{A} = \frac{0.161 V}{\sum \alpha s} \\ \therefore \frac{1}{T_1} &= \frac{\sum \alpha s}{0.161 V} \quad \text{and} \quad \frac{1}{T_2} = \frac{\sum \alpha s + \alpha_1 s_1}{0.161 V} \\ \therefore \frac{1}{T_2} - \frac{1}{T_1} &= \frac{\sum \alpha s}{0.161 V} + \frac{\alpha_1 s_1}{0.161 V} - \frac{\sum \alpha s}{0.161 V} \\ \therefore \frac{1}{T_2} - \frac{1}{T_1} &= \frac{\alpha_1 s_1}{0.161 V} \end{aligned}$$

$$\alpha_1 = \frac{0.161 V}{s_1} \left( \frac{1}{T_2} - \frac{1}{T_1} \right)$$

~ 3 ~

Here  $\alpha_1$  = absorption coefficient of material  
of area of the surface  $S_1$   
V = Total volume of Hall



## Sabine Formula:-

If “T” is the reverberation time, “V” is the volume of the room and A is the surface area of absorbing material, then Sabine’s formula can be expressed as

$$T = \frac{0.161 V}{A}$$

### Proof:-

Let’s assume that the walls in our room are completely absorbing. i.e. the sound that hits the wall is absorbed and there is no reflection sound. Assume that the energy density (E) is constant throughout the room.

We know that  $P = AI \dots\dots\dots (1)$

Where  $P =$  dissipated power in the walls

$A =$  area of the walls

$I =$  intensity of sound hitting walls.

We know that Energy density  $E = \frac{\text{Energy}}{\text{volume}(V)}$   
 $\text{Energy} = EV$

The power lost to the walls must equal to the time rate of change of energy

$$\text{i.e. } \frac{d(EV)}{dt} = -P = -AI \dots\dots\dots (2)$$

We know that, intensity (I) equal to the energy density (E) times the speed of sound (v) divided by 4. Here, we divide by 4 because sound is going out in all directions (left, right, forward, backward). Here V is constant.

! From equation (2)

$$\begin{aligned} V \frac{dE}{dt} &= -A \cdot \frac{E}{4} v \\ \frac{dE}{dt} &= -\frac{Av}{4V} E \\ \frac{dE}{E} &= -\frac{Av}{4V} dt \dots\dots\dots (3) \\ &\sim 4 \sim \end{aligned}$$

$$\left( \begin{aligned} \therefore I &= E \times v \\ &= \frac{E}{V} \times \frac{d}{T} \\ &= \frac{E}{Ad} \times \frac{d}{T} \\ &= \frac{E}{AT} \end{aligned} \right)$$

Integrating equation (3), we get

$$\int_{E_0}^E \frac{dE}{E} = - \int_0^T \frac{A v}{4V} dt$$

$$[\log E]_{E_0}^E = - \frac{A v}{4V} [T]_0^T$$

$$\log \left( \frac{E}{E_0} \right) = - \frac{A v}{4V} [T]$$

We know that

$$\log_e 10^{-6} = - \frac{A v}{4V} T$$

$$T = - \frac{4V}{A v} \times \log 10^{-6}$$

$$= \frac{-4V \times (-13.8)}{A \times 344}$$

since V= velocity of sound = 344 m/s

$$T = \frac{0.161 V}{A}$$

### Eyring Equation:-

Sabine formula is used to determine the standard reverberation time for the hall. When the absorption coefficient of the sound absorbing material  $\alpha = 1$ , then all the sound energy incident on the surface of the material is totally absorbed. Therefore, the reverberation time for that hall is zero. i.e.  $T=0$ .

When the reverberation time for the hall is zero ( $T=0$ ), then the hall is known as dead room. According to Sabine formula, the reverberation time is not equal to zero. i.e.  $T \neq 0$ . On the other hand, the experimental observations shows that Sabine formula is valid only when  $\alpha \leq 0.2$ .

In order to overcome these limitations, based on the method of images Eyring derived an equation for the reverberation time.

When  $\alpha = 1$ , from equation (1), T will become zero. But according to Sabine formula, when  $\alpha = 1$ , the reverberation time  $T \neq 0$ .

Eyring equation (1) is equal to Sabine formula, when the limitation is applied.

$$i.e. \alpha^0 [-\log(1-\alpha)] = \alpha$$

$$i.e. T = \frac{0.161 V}{s[-\log(1-\alpha)]} \dots\dots\dots (1)$$

Then equation (1) is called the Eyring equation.

**Basic requirements of acoustically good hall:**

1. There should be sufficient loudness and no echoes should be present.
2. The total quality of speech and music must be unchanged i.e. the relative intensities of the several components of a complex sound must be maintained.
3. The shape of the hall and ceiling should be so as to provide uniform distribution of sound through the hall.
4. The reverberation time should be optimum, i.e. neither too large nor too short. The reverberation time should be 1-2sec for music and 0.5-1 sec for speech.
5. The shape of the hall and ceiling should be so as to provide uniform distribution of sound through the hall.
6. The volume of the auditorium is decided by the type of program to be conducted there and also the seats to accommodate. A music hall requires a large volume where as a lecture hall requires a small volume.

# ULTRASONICS

## INTRODUCTION

- The word *ultrasonic* combines the Latin roots *ultra*, meaning 'beyond' and *sonic*, or *sound*.
- The sound waves having frequencies above the audible range (20Hz-20,000Hz) i.e., above 20,000Hz are called *Ultrasonics* or *Supersonics*. The term *supersonics* is generally used nowadays for sound waves having velocities greater than that of sound.
- Generally, these waves are called as *high frequency waves*. (Very small wavelengths.)
- The broad sectors of society that regularly apply ultrasonic technology are the medical community, industry and the military.
- Sound waves of frequencies less than 20Hz are called *Infrasonic*.

### ***Properties:***

- They have high energy content.
- These waves also travel with the speed of sound.
- Just like ordinary sound waves, ultrasonic waves get reflected, refracted and absorbed.
- They can be transmitted over large distances with no appreciable loss of energy.
- If an arrangement is made to form stationary waves of ultrasonics in a liquid, it serves as a diffraction grating. It is called an *acoustic grating*.
- They produce intense heating effect when passed through a substance.

### **Production of Ultrasonic Waves:**

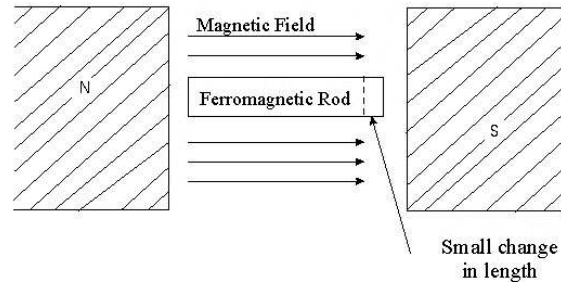
There are different methods for the production of Ultrasonic's. However, the most commonly used methods are,

- (1) Magnetostriction generator or oscillator
- (2) Piezo-electric generator or oscillator

## Magneto-striction Method (Magneto-striction generator or oscillator):

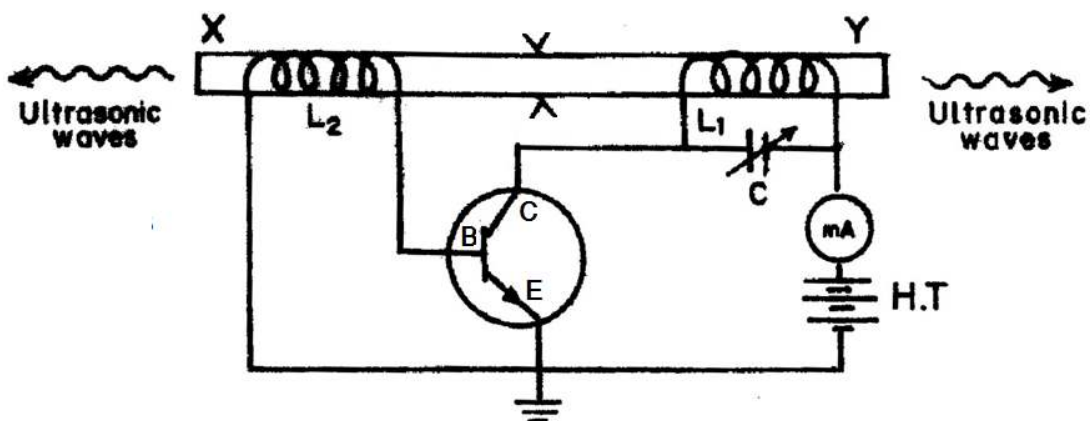
### Principle:

When a ferromagnetic rod like iron or nickel is placed in a magnetic field parallel to its length, the rod experiences a small change in its length. This is called Magnetostriction effect.



### Construction:

The circuit diagram of Magnetostriction ultrasonic generator is as shown in the figure. A short permanently magnetized nickel rod (normally the rod is invar-36% Ni and 64% Fe) is placed in the middle between two knife-edges. A coil  $L_1$  is wound on the right-hand portion of the rod.  $C$  is a variable capacitor.  $L_1$  and  $C_1$  form the resonant circuit of the collector-tuned oscillator. Coil  $L_2$  wound on the left-hand side of the rod is connected in the base circuit. The coil  $L_2$  is used as a feedback loop.



### Working:

When the battery is switched on, the resonant circuit  $L_1C_1$  set up an alternating current of frequency

$$F = \frac{I}{2\pi\sqrt{L_1C_1}}$$

This circuit flowing round the coil  $L_1$  produces an alternating magnetic field of frequency  $F$  along the length of the nickel rod. The rod starts vibrating due to magneto strictive effect. The vibrations of the rod create ultrasonic waves.

The longitudinal expansion and contraction of the rod produce an e.m.f in the coil  $L_2$ . This e.m.f is applied to the base of the transistor. Hence the amplitude of high frequency of high oscillation in the coil  $L_1$  is increased due to positive feedback.

The developed alternating current frequency can be tuned with the natural frequency of the rod by adjusting the capacitor.

Condition for resonance is

***Frequency of the oscillator circuit = Frequency of the vibrating rod***

$$F = \frac{1}{2\pi\sqrt{L_1C_1}} = \frac{1}{2l}\sqrt{\frac{E}{\rho}}$$

Where,  $l$  is the length of the rod

$E$  is the young's modulus of the rod

$\rho$  is the density of the material of the rod.

The resonance condition is indicated by the rise in the collector current shown in milliammeter

### Advantages

- Magnetostriction oscillators are mechanically rugged.
- The design of this oscillator is very simple and its production cost is low

- At low ultrasonic frequencies, the large power output can be produced without the risk of damage of the oscillatory circuit.

**Disadvantages:**

- It has low upper frequency limit and cannot generate ultrasonic frequency above 3000 kHz (ie.3MHz).
- The frequency of oscillations depends on temperature.
- Breadth of the resonance curve is large; it is due to the vibrations of elastic constants of ferromagnetic material with the degree of magnetization. So there will be losses of energy due to hysteresis and eddy current and we cannot get a constant single frequency.

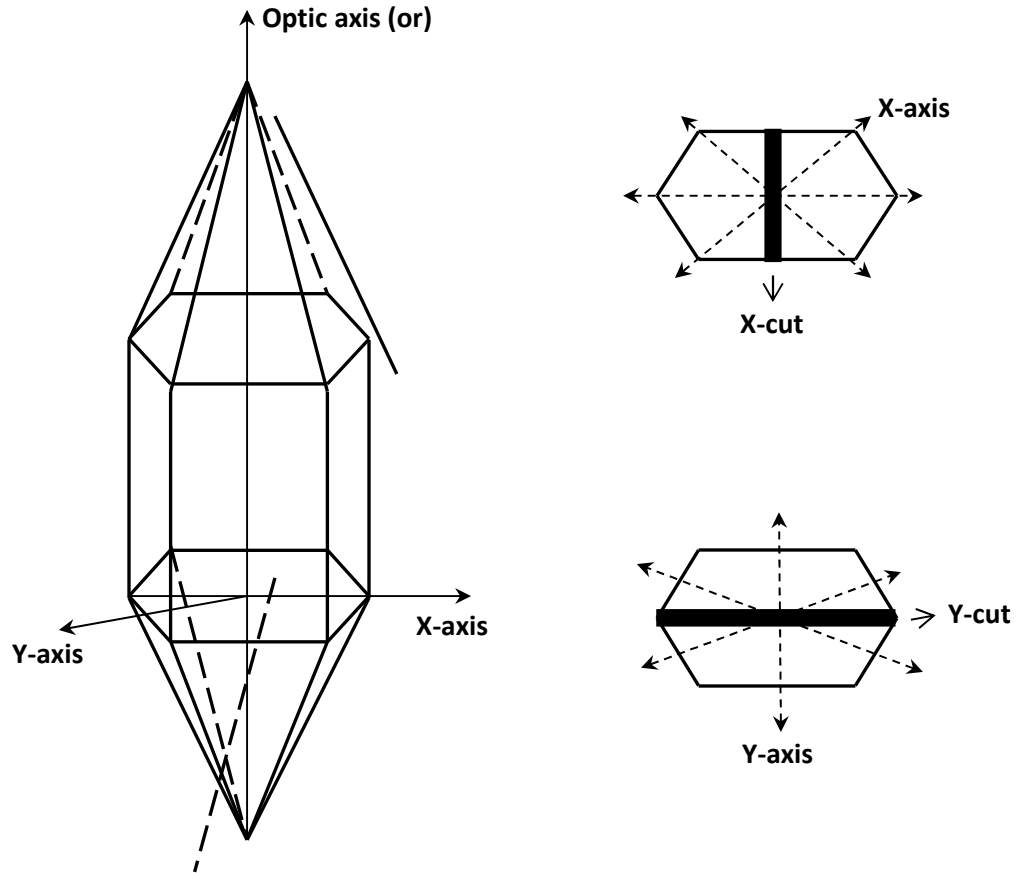
**Piezo Electric crystals:**

The crystals which produce piezo-electric effect and inverse piezo-electric effect are termed as Piezo-electric crystal.

Examples: Quartz, Tourmaline, Rochelle salts etc.

A typical piezo-electric crystal(Quartz) is as shown in the figure. It has hexagonal shape with pyramids attached at both ends. It consists of three axes,

- I. Electric axis or X-axis ; which joins the corners of hexagon.
- II. Mechanical axis or Y-axis ; which joins the center or sides of the hexagon.
- III. Optic axis or Z-axis ; which join the edges of the pyramid. (Blunt corners) As shown



**X-cut and Y-cut crystals:**

**X-Cut crystal:**

when a crystal is cut perpendicular to the X-axis, as shown, then it is called X-crystal. Generally, these are used to produce longitudinal ultrasonic waves.

**Y-Cut crystal:**

when a crystal is cut perpendicular to the Y-axis, as shown, then it is called Y-cut crystal. Generally, these are used to produce transverse ultrasonic waves.

**Piezo electric effect:**

It was discovered by brothers J. and P. Curie in the year 1880.

“When a mechanical stress is applied to the mechanical axis with respect to optical axis, a potential difference is developed across the electrical axis with respect to optical axis”



### Inverse piezo-electric effect:

“When an alternating electric field is applied to electrical axis with respect to optical axis, expansion or contraction takes place in the mechanical axis with respect to optical axis.”

### Piezo electric Effect (Piezo-electric generator or oscillator):

Piezo electric generator was developed by Langevin in 1917

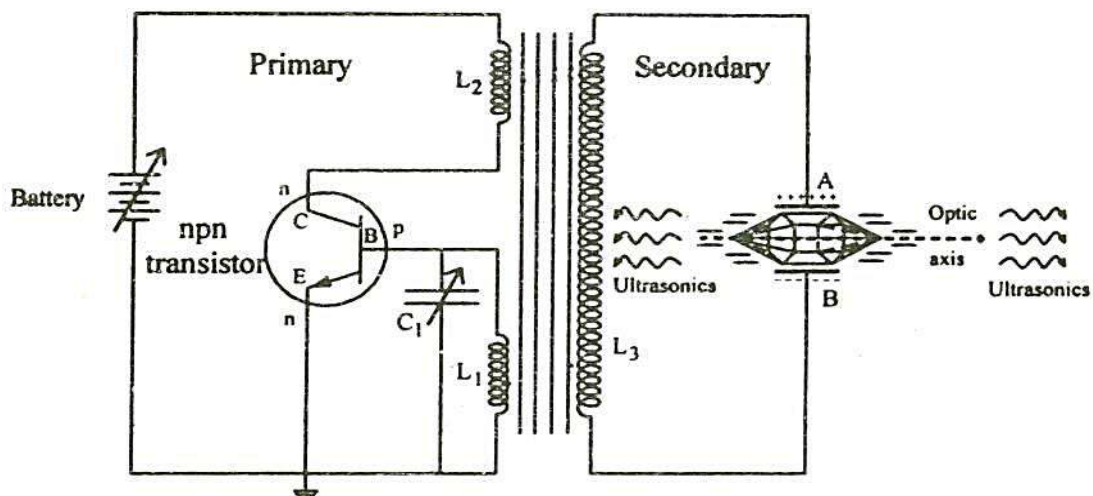
### Principle:

This is based on the inverse piezo-electric effect. When a quartz crystal is subjected to an alternating potential difference along the electric axis, the crystal is set into elastic vibrations along its mechanical axis. If the frequency of electric oscillations coincides with the natural frequency of the crystal, the vibrations will be of large amplitude. If the frequency of the electric field is in the ultrasonic frequency range, the crystal produces ultrasonic waves.

### Construction:

The circuit diagram is as shown in the figure. It is base tuned oscillator circuit. A slice of quartz crystal is placed between the metal plates A and B so as to form a parallel plate capacitor with the crystal as a dielectric. This is coupled to the electronic oscillator through the primary coil  $L_3$  of the transformer

Coils  $L_2$  and  $L_1$  of oscillator circuit are taken for the primary of the transformer. The collector coil  $L_2$  is individually coupled to base coil  $L_1$  and variable capacitor  $C$  from the tank circuit of the oscillator.



### Working:

When the battery is switched on, the oscillator produces high frequency oscillations. As oscillatory e.m.f is induced in the coil  $L_3$  due to transformer action. So the crystal is now under high frequency alternating voltage.

The capacitance of  $C_1$  is varied so that the frequency of oscillations produced is in resonance with the natural frequency of the crystal. Now the crystal vibrates with larger amplitude due to resonance. Thus, high power ultrasonic waves are produced.

Condition for resonance is

*Frequency of the oscillator circuit = Frequency of the vibrating rod*

$$F = \frac{1}{2\pi\sqrt{L_1 C_1}} = \frac{1}{2l} \sqrt{\frac{E}{\rho}}$$

Where,  $l$  is the length of the rod

$E$  is the young's modulus of the rod

$\rho$  is the density of the material of the rod.

### Advantages

- Ultrasonic frequencies as high as 500 MHz can be obtained with this arrangement.
- The output of this oscillator is very high.
- It is not affected by temperature and humidity.
- It is more efficient than the magnetostriction oscillator
- The breadth of the resonance curve is very small. So we can get a stable and constant frequency of ultrasonic waves.

### Disadvantages

- The cost of piezo electric material ( quartz crystal) is very high
- The cutting and shaping of quartz crystal are very complex

## APPLICATIONS:

1. Ultrasonics are used to find the velocity sound waves in liquid and gases.
2. Ultrasonics are used to study the elastic symmetry of crystals.
3. Ultrasonics are used to find the depth of sea, to detect the position of submerged rocks, submarines etc.
4. Ultrasonic waves are used for directional signaling.
5. Ultrasonic waves of high intensity are used to form certain alloys (Ex: iron-lead, zinc-lead, copper lead and aluminum- cadmium)
6. Metals can be soldered by putting them in ultrasonic vibrations.
7. Ultrasonic waves are used to form emulsions of immiscible liquids such as water and mercury or water and oil.
8. Ultrasonic waves are used for washing clothes mainly silken fabrics, in cleaning of watches and metals.
9. Ultrasonic waves are used to detect the defects in an object without harming the object by **NDT** (Nondestructive testing).
10. Ultrasonic waves are used to create a black-and-white picture of something within the human body by **Sonogram**.

## What are the basic requirements of acoustically good hall?

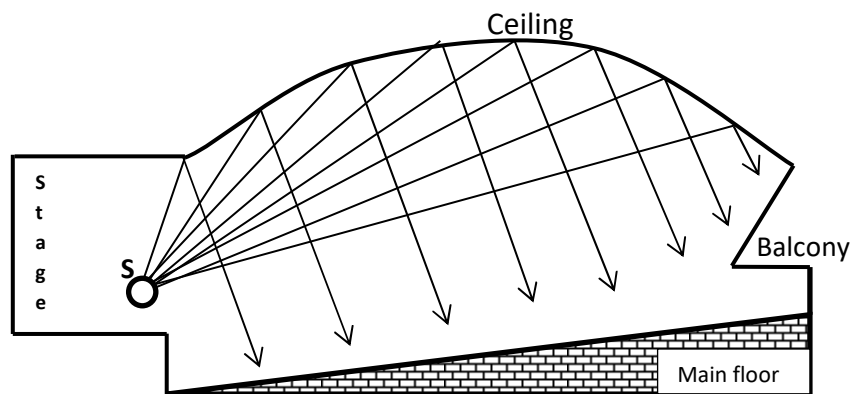
- The reverberation time should be optimum, i.e. neither too large nor too short. To control the reverberation time the sound absorbing materials are to be chosen carefully.

The reverberation time should be 1-2sec for music and 0.5-1 sec for speech

- The volume of the auditorium is decided by the type of program to be conducted there and also the seats to accommodate. A music hall requires a large volume where as a lecture hall requires a small volume.

In deciding the volume of the hall its height plays important role than its length and breadth. The ratio between the ceiling height and breadth should be 2:3

- The shape of the hall and ceiling should be so as to provide uniform distribution of sound through the hall.
- There should be sufficient loudness and no echoes should be present.
- The total quality of speech and music must be unchanged i.e. the relative intensities of the several components of a complex sound must be maintained.



- For the sake of the clarity, the successive syllables spoken must be clear and distinct, i.e. there must be no confusion due to overlapping of syllables.
- There should be no concentration of sound in any part of the hall.
- The boundaries should be sufficiently sound proof to exclude extraneous noise.
- There should be no *Echelon effect* i.e. reflection of sound from stair case.
- There should be no resonance with in the building.
- The hall must be full of audience.

**OBJECTIVE QUESTIONS FOR ONLINE EXAMINATION**

1. Frequency range for ultrasonic sound is  
a. **20,000 Hz and above**      b. Less than 20,000 Hz      c. 20-20,000 Hz      d. Less than 20 Hz
2. Velocity of ultrasound wave is  
a. More than audible      b. Less than audible sound      **c. Equal to audible sound**      d. Greater than light wave
3. The wavelength of ultrasonic wave of frequency 1 MHz travelling with a velocity 1000 m/s is  
a. 1 cm      **b. 1 mm**      c. 1 m      d. 10 m
4. Ultrasonic waves are  
a. Transverse in nature      **b. Longitudinal in nature**      c. Either transverse or longitudinal      d. Neither transverse or longitudinal
5. Ultrasonic waves are produced by  
a. Piezo electric effect      b. Magnetostriction effect      **c. Both a & b**      d. None
6. The expression for frequency of ultrasonic waves is  
a.  $f = \sqrt{\frac{Y}{\rho}}$       b.  $f = \frac{1}{l} \sqrt{\frac{Y}{\rho}}$       c.  $f = \frac{1}{2l} \frac{Y}{P}$       d.  $f = \frac{1}{2l} \sqrt{\frac{Y}{\rho}}$
7. The piezoelectric crystal is  
a. Quartz      b. Tourmaline      c. Rochelle Salt      **d. All the above**
8. The X-cut crystal is  
**a. Normal to X-axis**      b. Normal to Y-axis      c. Parallel to Y-Axis      d. None
9. The Y-cut crystal is  
a. Normal to X-axis      **b. Normal to Y-axis**      c. Parallel to Y-Axis      d. None
10. The high frequency ultrasonic waves are produced by  
a. Galton's Whistle      b. Magnetostriction method      **c. Piezo electric method**      d. None

1. A hall of volume of 1500 m<sup>3</sup>. Its total absorption is equivalent to 100 m<sup>2</sup> of open window. What will be the effect on the reservation time, if the absorption is increased by 100 m<sup>2</sup> of open window, by filling the hall with audience.

Sol:

Given data, volume  $V=1500 \text{ m}^3$

Absorption of sound by hall  $\sum a_1 S = A_1 = 100 \text{ m}^2$

Sound y audience  $\sum a_2 S = A_2 = 100 \text{ m}^2$

Total Absorption of sound= Absorption of sound by the hall + Absorption of audience

$$= 100 + 100 = 200$$

Reverberation time when room is empty

$$= \frac{0.161V}{\sum a_1 S} = \frac{0.161 \times 1500}{100} = \frac{241.5}{100} = 2.415$$

When room filled

$$= \frac{0.161V}{\sum a_1 S + \sum a_2 S} = \frac{0.161 \times 1500}{(100+100)} = \frac{241.5}{200} = 1.2075$$

Change of reverberation time=2.4-1.2=1.2.

When hall is filled, the reverberation time reduced to 1.2

2. A hall of volume 1000 m<sup>3</sup> has a sound absorbing surface of area 400 m<sup>2</sup>. If the average absorption coefficient of the hall is 0.2. Find the reverberation time of the hall.

Sol.

Given data Volume = 1000 m<sup>3</sup>,

S=12000 m<sup>2</sup>

A=0.2 Sabine, T=?

$$T = \frac{0.161V}{aS} = \frac{0.161 \times 1000}{0.2 \times 400} = \frac{161}{80} = 2.012 \text{ sec}$$

3. The volume of an auditorium is 12000m<sup>3</sup>. Its reverberation time is 1.5 sec. if the average absorption coefficient of interior surface is 0.4 sabine. Determine the area of interior surface.

Sol:

Given Data V=12000m<sup>3</sup>, T=1.5 Sec a=0.4 Sabine S=?

We know

$$T = \frac{0.161V}{aS} \Rightarrow S = \frac{0.161V}{aT}$$
$$S = \frac{0.161 \times 12000}{0.4 \times 1.5} = \frac{1980}{0.6} = 3340 \text{ m}^2$$